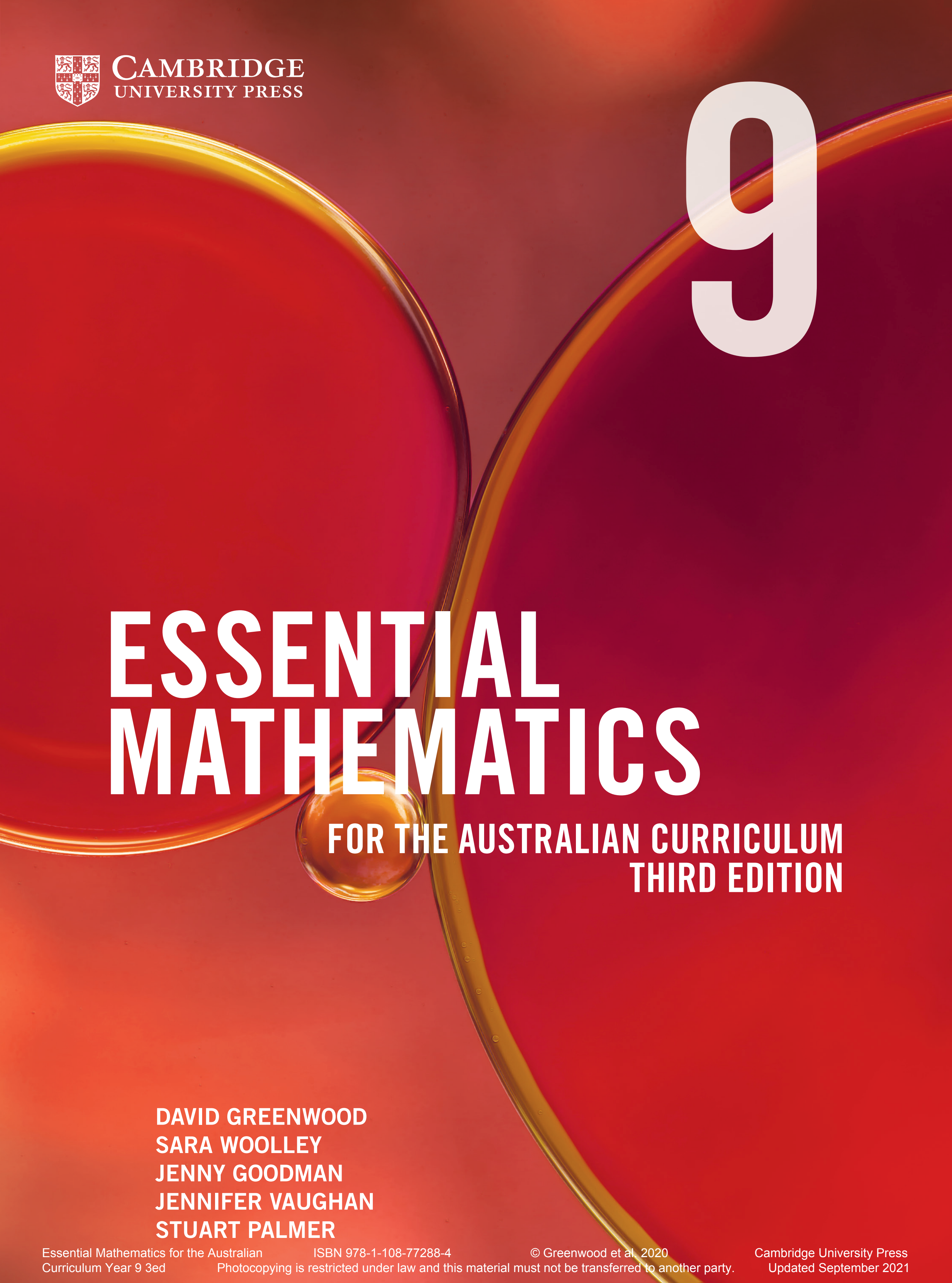

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ESSENTIAL MATHEMATICS

FOR THE AUSTRALIAN CURRICULUM
THIRD EDITION

DAVID GREENWOOD
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CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of the University of Cambridge.

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First Edition © David Greenwood, Jenny Goodman, Jennifer Vaughan, Sara Woolley, GT Installations, Georgia Sotiriou and Voula Sotiriou 2011
Second and Third Edition © David Greenwood, Sara Woolley, Jenny Goodman and Jennifer Vaughan 2015, 2020

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First published 2011
Second Edition 2015
Third Edition 2020
20 19 18 17 16 15 14 13 12 11 10 9 8 7 6

Cover and text designed by Loupe Studio
Typeset by diacriTech
Printed in China by C & C Offset Printing Co., Ltd.

A catalogue record for this book is available from the National Library of Australia at www.nla.gov.au

ISBN 978-1-108-77288-4 Paperback

Additional resources for this publication at www.cambridge.edu.au/GO

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About the authors

David Greenwood is the Head of Mathematics at Trinity Grammar School in Melbourne and has 25+ years' experience teaching mathematics from Years 7 to 12. He has run numerous workshops within Australia and overseas regarding the implementation of the Australian Curriculum and the use of technology for the teaching of mathematics. He has written more than 30 mathematics titles and has a particular interest in the sequencing of curriculum content and working with the Australian Curriculum proficiency strands.

Sara Woolley was born and educated in Tasmania. She completed an Honours degree in Mathematics at the University of Tasmania before completing her education training at the University of Melbourne. She has taught mathematics in Victoria from Years 7 to 12 since 2006 and is currently a Head of Mathematics. She has written more than 15 mathematics titles and specialises in lesson design and differentiation.

Jenny Goodman Jenny Goodman has taught in schools for over 25 years and is currently teaching at a selective high school in Sydney. Jenny has an interest in the importance of literacy in mathematics education, and in teaching students of differing ability levels. She was awarded the Jones Medal for education at Sydney University and the Bourke Prize for Mathematics. She has written for *CambridgeMATHS NSW* and was involved in the *Spectrum* and *Spectrum Gold* series.

Jennifer Vaughan has taught secondary mathematics for over 30 years in New South Wales, Western Australia, Queensland and New Zealand, and has tutored and lectured in mathematics at Queensland University of Technology. She is passionate about providing students of all ability levels with opportunities to understand and to have success in using mathematics. She has had extensive experience in developing resources that make mathematical concepts more accessible; hence, facilitating student confidence, achievement and an enjoyment of maths.

Stuart Palmer has been a head of department in two schools and is now an educational consultant who conducts professional development workshops for teachers all over New South Wales and beyond. He is a Project Officer with the Mathematical Association of New South Wales, and also works with pre-service teachers at The University of Sydney and Western Sydney University.

Introduction

This third edition of *Essential Mathematics for the Australian Curriculum* includes some substantial new features in the print and digital versions of the textbook, as well as in the Online Teaching Suite. The main new features are listed below.

Now you try

Every worked example now contains additional questions, without solutions, called ‘Now you try’. Rather than expect students to absorb the worked example by passively reading through it, these questions give students immediate practice at the same type of question. We also anticipate these questions will be useful for the teacher to do in front of the class, given that students will not have seen the solution beforehand.

Building understanding and changes to the exercise structure

To improve the flow of ideas from the beginning of each lesson through to the end of the exercise, a few structural changes have been made in each lesson. First, the Understanding questions have been taken out of the exercise, simplified into discussion-style questions, and placed immediately after the Key ideas. These questions are now called ‘Building understanding’ and are intended to consolidate the skills and concepts covered by the Key ideas, which students will then encounter in the worked examples. Each exercise now starts at Fluency, and the first question in each exercise has been revised to ensure that it links directly to the first worked example in the lesson. The exercise then continues as before through Problem-solving, Reasoning and Enrichment.

Learning intentions and Success criteria checklist

At the beginning of every lesson is a set of Learning intentions that describe what the student can expect to learn in the lesson. At the end of the chapter, these appear again in the form of a Success criteria checklist; students can use this to check their progress through the chapter. Every criterion is listed with an example question to remind students of what the mathematics mentioned looks like. These checklists can also be downloaded and printed off so that students can physically check them off as they accomplish their goals.

Modelling and more extended-response

A modelling activity now accompanies the Investigation in each chapter, with the goal of familiarising students with using the modelling process to define, solve, verify and then communicate their solutions to real-life problems. Also included in each chapter is a set of three applications and problem-solving questions. These extended-response style problems apply the mathematics of the chapter to realistic contexts and provide important practice at this type of extended-response work before any final test is completed.

Workspaces and self-assessment

In the Interactive Textbook, students can complete almost any question from the textbook inside the platform via **workspaces**. Questions can be answered with full worked solutions using three input tools: ‘handwriting’ using a stylus, inputting text via a keyboard and in-built symbol palette, or uploading an image of work completed elsewhere. Then students can critically engage with their own work using the **self-assessment** tools, which allow them to rate their confidence with their work and also red-flag to the teacher any questions they have not understood. All work is saved, and teachers will be able to see both students’ working-out and how they’ve assessed their own work via the Online Teaching Suite.

Note that the workspaces and self-assessment feature is intended to be used as much or as little as the teacher wishes, including not at all. However, the ease with which useful data can be collected will make this feature a powerful teaching and learning tool when used creatively and strategically.

Guide to the working programs

As with the second edition, *Essential Mathematics for the Australian Curriculum Third Edition* contains working programs that are subtly embedded in every exercise. The suggested working programs provide three pathways through each book to allow differentiation for Foundation, Standard and Advanced students.

Each exercise is structured in subsections that match the Australian Curriculum proficiency strands of Fluency, Problem-solving and Reasoning, as well as Enrichment (Challenge). (Note that Understanding is now covered by 'Building understanding' in each lesson.) In the exercises, the questions suggested for each pathway are listed in three columns at the top of each subsection:

- The left column (lightest shaded colour) is the Foundation pathway
- The middle column (medium shaded colour) is the Standard pathway
- The right column (darkest shaded colour) is the Advanced pathway.

Foundation	Standard	Advanced
FLUENCY		
1, 2–4(½)	2–5(½)	2–5(½)
PROBLEM-SOLVING		
6, 7	6–8	7–9
REASONING		
10	10–12	12–14
ENRICHMENT		
–	–	15

The working program for Exercise 3A in Year 7

Gradients within exercises and proficiency strands

The working programs make use of the gradients that have been seamlessly integrated into the exercises. A gradient runs through the overall structure of each exercise – where there is an increasing level of mathematical sophistication required from Fluency through to Reasoning and Enrichment – but also within each proficiency strand; the first few questions in Fluency, for example, are easier than the last few, and the last Problem-solving question is more challenging than the first Problem-solving question.

The right mix of questions

Questions in the working programs are selected to give the most appropriate mix of *types* of questions for each learning pathway. Students going through the Foundation pathway should use the left tab, which includes all but the hardest Fluency questions as well as the easiest Problem-solving and Reasoning questions. An Advanced student can use the right tab, proceed through the Fluency questions (often half of each question), and have their main focus be on the Problem-solving and Reasoning questions, as well as the Enrichment questions. A Standard student would do a mix of everything using the middle tab.

Choosing a pathway

There are a variety of ways to determine the appropriate pathway for students through the course. Schools and individual teachers should follow the method that works for them. If required, there are two types of chapter pre-tests (now found online) that can be used as a tool for helping students select a pathway. For the prior-knowledge pre-test, the following are recommended guidelines:

- A student who gets 40% or lower should complete the Foundation questions
- A student who gets between 40% and 85% should complete the Standard questions
- A student who gets 85% or higher should complete the Advanced questions.

For schools that have classes grouped according to ability, teachers may wish to set one of the Foundation, Standard or Advanced pathways as their default setting for their entire class and then make individual alterations depending on student need. For schools that have mixed-ability classes, teachers may wish to set a number of pathways within the one class, depending on previous performance and other factors.

* The nomenclature used to list questions is as follows:

- 3, 4: complete all parts of questions 3 and 4
- 1-4: complete all parts of questions 1, 2, 3 and 4
- 10(½): complete half of the parts from question 10 (a, c, e, ... or b, d, f, ...)
- 2-4(½): complete half of the parts of questions 2, 3 and 4
- 4(½), 5: complete half of the parts of question 4 and all parts of question 5
- — : do not complete any of the questions in this section.

Guide to this resource

PRINT TEXTBOOK FEATURES

1 **Australian Curriculum:** content strands, sub-strands and content descriptions are listed at the beginning of the chapter (see the teaching program for more detailed curriculum documents)

2 **Working with unfamiliar problems:** a set of problem-solving questions not tied to a specific topic

3 NEW **Learning intentions:** sets out what a student will be expected to learn in the lesson

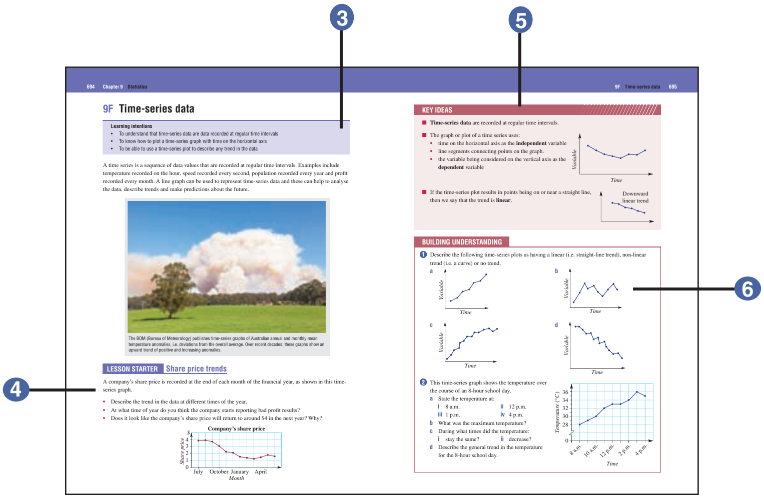
4 **Lesson starter:** an activity, which can often be done in groups, to start the lesson

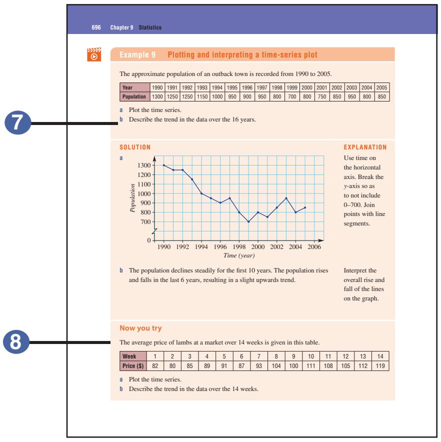
5 **Key ideas:** summarises the knowledge and skills for the lesson

6 NEW **Building understanding:** a small set of discussion questions to consolidate understanding of the Key ideas (replaces Understanding questions formerly inside the exercises)

7 **Worked examples:** solutions and explanations of each line of working, along with a description that clearly describes the mathematics covered by the example

8 NEW **Now you try:** try-it-yourself questions provided after every worked example in exactly the same style as the worked example to give immediate practice





9 **Revised exercise structure:** the exercise now begins at Fluency, with the first question always linked to the first worked example in the lesson

10 **Working programs:** differentiated question sets for three ability levels in exercises

11 **Example references:** show where a question links to a relevant worked example – the first question is always linked to the first worked example in a lesson

12 NEW **Non-CAS TI and Casio calculator activities** added for Years 9 (online) and 10&10A (print)

13 NEW **Modelling activities** in every chapter allow students to practise solving problems using a systematic modelling process

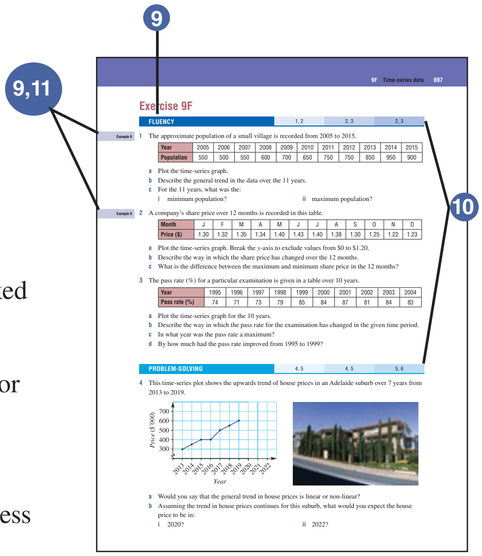
14 NEW **Applications and problem-solving:** a set of three extended-response questions across two pages that give practice at applying the mathematics of the chapter to real-life contexts

15 **Problems and challenges:** in each chapter provide practice with solving problems connected with the topic

16 NEW **Success criteria checklist:** a checklist of the learning intentions for the chapter, with example questions

17 **Chapter reviews:** with short-answer, multiple-choice and extended-response questions; questions that are extension are clearly signposted

18 **Solving unfamiliar problems poster:** at the back of the book outlines a strategy for solving any unfamiliar problem



9F Time-series data 697

Exercise 9F

FLUENCY 1, 2 | 2, 3 | 2, 3

Example 9 1 The approximate population of a small village is recorded from 2005 to 2015.

Year	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Population	550	500	550	600	700	650	750	750	850	950	900

a Plot the time-series graph.
b Describe the general trend in the data over the 11 years.
c For the 11 years, what was the:
i minimum population? ii maximum population?

Example 9 2 A company's share price over 12 months is recorded in this table.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Price ($)	1.30	1.32	1.35	1.34	1.40	1.43	1.40	1.38	1.30	1.25	1.22	1.23

a Plot the time-series graph. Break the y-axis to exclude values from \$0 to \$1.20.
b Describe the way in which the share price has changed over the 12 months.
c What is the difference between the maximum and minimum share price in the 12 months?

3 The pass rate (%) for a particular examination is given in a table over 10 years.

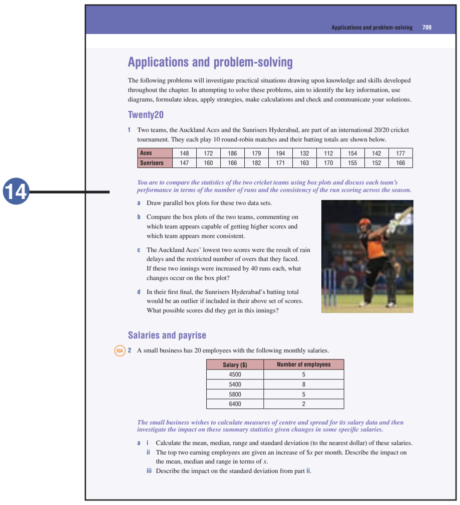
Year	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004
Pass rate (%)	74	71	73	79	85	84	87	81	84	83

a Plot the time-series graph for the 10 years.
b Describe the way in which the pass rate for the examination has changed in the given time period.
c In what year was the pass rate a maximum?
d By how much had the pass rate improved from 1995 to 1999?

PROBLEM-SOLVING 4, 5 | 4, 5 | 5, 6

4 This time-series plot shows the upwards trend of house prices in an Adelaide suburb over 7 years from 2013 to 2019.

a Would you say that the general trend in house prices is linear or non-linear?
b Assuming the trend in house prices continues for this suburb, what would you expect the house price to be in:
i 2020? ii 2022?



Applications and problem-solving 709

Applications and problem-solving

The following problems will investigate practical situations drawing upon knowledge and skills developed throughout the chapter. In attempting to solve these problems, aim to identify the key information, use diagrams, formulate ideas, apply strategies, make calculations and check and communicate your solutions.

Twenty20

1 Two teams, the Auckland Aces and the Sunrisers Hyderabad, are part of an international 20/20 cricket tournament. They each play 10 round-robin matches and their batting totals are shown below.

Aces	148	172	186	179	194	132	112	154	142	177
Sunrisers	147	160	166	182	171	163	170	155	152	166

You are to compare the statistics of the two cricket teams using box plots and discuss each team's performance in terms of the number of runs and the consistency of the run scoring across the season.

a Draw parallel box plots for these two data sets.
b Compare the box plots of the two teams, commenting on which team appears capable of getting higher scores and which team appears more consistent.
c The Auckland Aces' lowest two scores were the result of rain delays and the restricted number of overs that they faced. If these two innings were increased by 40 runs each, what changes occur on the box plot?
d In their first final, the Sunrisers Hyderabad's batting total would be an outlier if included in their above set of scores. What possible scores did they get in this innings?

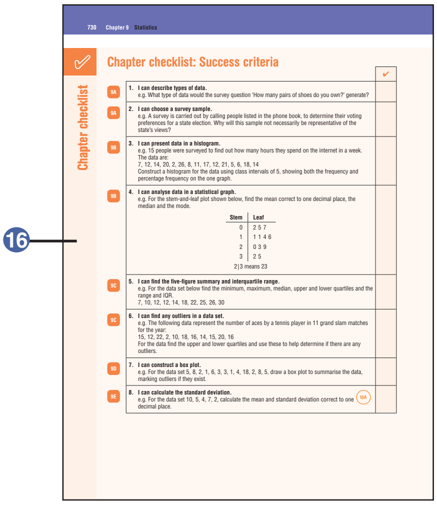
Salaries and payrise

10A 2 A small business has 20 employees with the following monthly salaries.

Salary ($)	Number of employees
4500	5
5400	8
5800	5
6400	2

The small business wishes to calculate measures of centre and spread for its salary data and then investigate the impact on these summary statistics given changes in some specific salaries.

a i Calculate the mean, median, range and standard deviation (to the nearest dollar) of these salaries.
ii The top two earning employees are given an increase of \$$x$ per month. Describe the impact on the mean, median and range in terms of x.
iii Describe the impact on the standard deviation from part ii.



730 Chapter 9 Statistics

Chapter checklist

Chapter checklist: Success criteria ✓

9A 1. **I can describe types of data.**
e.g. What type of data would the survey question 'How many pairs of shoes do you own?' generate? ☐

9A 2. **I can choose a survey sample.**
e.g. A survey is carried out by calling people listed in the phone book, to determine their voting preferences for a state election. Why will this sample not necessarily be representative of the state's views? ☐

9B 3. **I can present data in a histogram.**
e.g. 15 people were surveyed to find out how many hours they spend on the internet in a week. The data are:
7, 12, 14, 20, 2, 26, 8, 11, 17, 12, 21, 5, 6, 18, 14
Construct a histogram for the data using class intervals of 5, showing both the frequency and percentage frequency on the one graph. ☐

9B 4. **I can analyse data in a statistical graph.**
e.g. For the stem-and-leaf plot shown below, find the mean correct to one decimal place, the median and the mode. ☐

Stem	Leaf
0	2 5 7
1	1 1 4 6
2	0 3 9
3	2 5

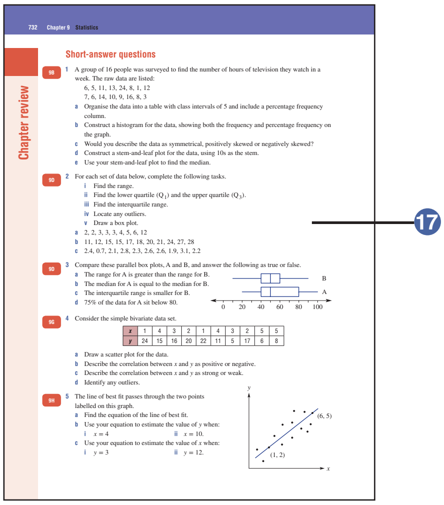
2|3 means 23

9C 5. **I can find the five-figure summary and interquartile range.**
e.g. For the data set below find the minimum, maximum, median, upper and lower quartiles and the range and IQR.
7, 10, 12, 12, 14, 18, 22, 25, 26, 30 ☐

9C 6. **I can find any outliers in a data set.**
e.g. The following data represent the number of aces by a tennis player in 11 grand slam matches for the year:
15, 12, 22, 2, 10, 18, 16, 14, 15, 20, 16
For the data find the upper and lower quartiles and use these to help determine if there are any outliers. ☐

9D 7. **I can construct a box plot.**
e.g. For the data set 5, 8, 2, 1, 6, 3, 3, 1, 4, 18, 2, 8, 5, draw a box plot to summarise the data, marking outliers if they exist. ☐

9E 8. **I can calculate the standard deviation.**
e.g. For the data set 10, 5, 4, 7, 2, calculate the mean and standard deviation correct to one decimal place. 10A ☐



732 Chapter 9 Statistics

Chapter review

Short-answer questions

9B 1 A group of 16 people was surveyed to find the number of hours of television they watch in a week. The raw data are listed:
6, 5, 11, 13, 24, 8, 1, 12
7, 6, 14, 10, 9, 16, 8, 3
a Organise the data into a table with class intervals of 5 and include a percentage frequency column.
b Construct a histogram for the data, showing both the frequency and percentage frequency on the graph.
c Would you describe the data as symmetrical, positively skewed or negatively skewed?
d Construct a stem-and-leaf plot for the data, using 10s as the stem.
e Use your stem-and-leaf plot to find the median.

9D 2 For each set of data below, complete the following tasks.
i Find the range.
ii Find the lower quartile (Q_1) and the upper quartile (Q_3).
iii Find the interquartile range.
iv Locate any outliers.
v Draw a box plot.
a 2, 2, 3, 3, 3, 4, 5, 6, 12
b 11, 12, 15, 15, 17, 18, 20, 21, 24, 27, 28
c 2.4, 0.7, 2.1, 2.8, 2.3, 2.6, 2.6, 1.9, 3.1, 2.2

9D 3 Compare these parallel box plots, A and B, and answer the following as true or false.
a The range for A is greater than the range for B.
b The median for A is equal to the median for B.
c The interquartile range is smaller for B.
d 75% of the data for A sit below 80.

9G 4 Consider the simple bivariate data set.

x	1	4	3	2	1	4	3	2	5	5
y	24	15	16	20	22	11	5	17	6	8

a Draw a scatter plot for the data.
b Describe the correlation between x and y as positive or negative.
c Describe the correlation between x and y as strong or weak.
d Identify any outliers.

9H 5 The line of best fit passes through the two points labelled on this graph.
a Find the equation of the line of best fit.
b Use your equation to estimate the value of y when:
i $x = 4$ ii $x = 10$.
c Use your equation to estimate the value of x when:
i $y = 3$ ii $y = 12$.

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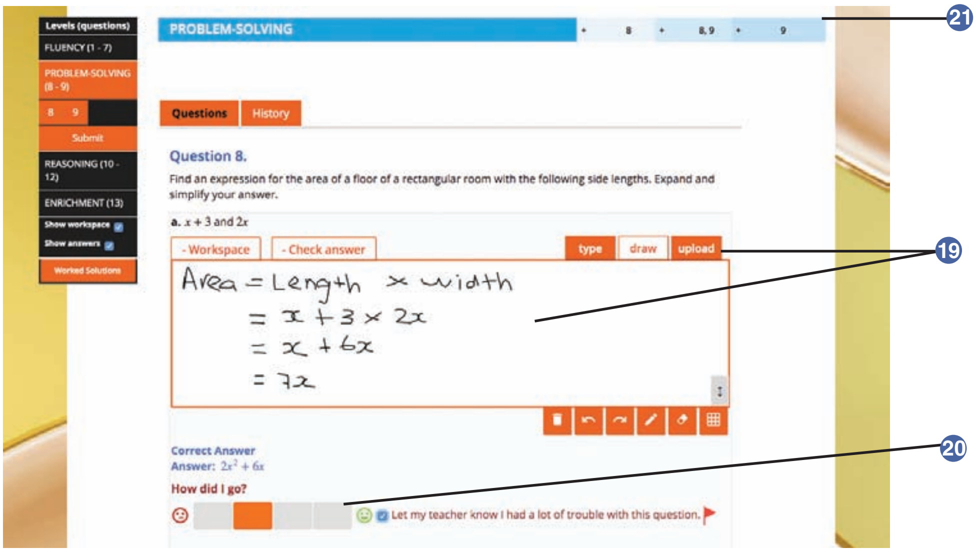

Question 8.

Find an expression for the area of a floor of a rectangular room with the following side lengths. Expand and simplify your answer.

a. $x + 3$ and $2x$

Area = Length × width

$= x + 3 \times 2x$

$= x + 6x$

$= 7x$

Correct Answer

Answer: $2x^2 + 6x$

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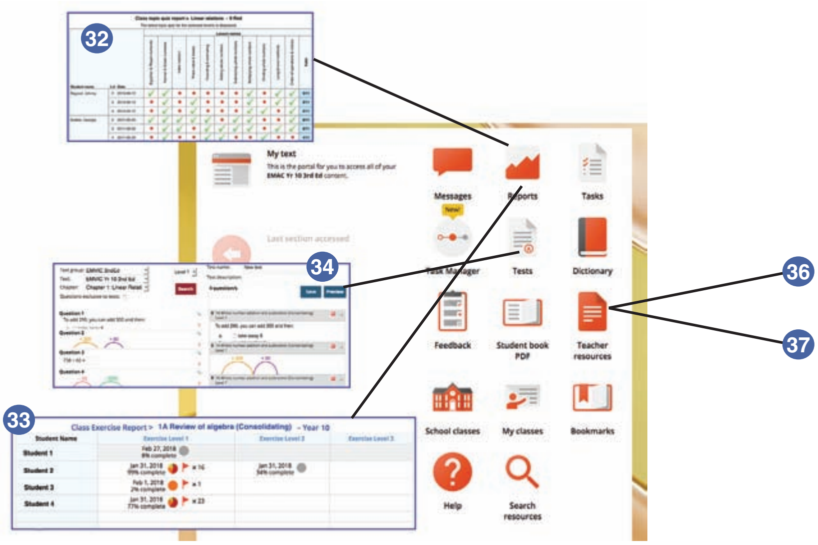

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Working with unfamiliar problems: Part 1

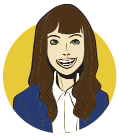
The questions on the next four pages are designed to provide practice in solving unfamiliar problems. Use the 'Working with unfamiliar problems' poster at the back of this book to help you if you get stuck.

In Part 1, apply the suggested strategy to solve these problems, which are in no particular order. Clearly communicate your solution and final answer.

1 Determine the exact answer for the following calculations:

   a $999\,999\,999^2$ **b** $10^{2x} - 10^x + 1$ when $x = 10$

2 In a school of 999 students all the lockers are closed but unlocked. The first student opens all the lockers, then each next student changes the 'state' (i.e. open to shut or shut to open) of some lockers: student 2 changes every second locker; student 3 changes every third locker, etc. Imagine that this continues until all the 999 students have had a turn with the 999 lockers. At the end, how many lockers are open and which locker numbers are they?

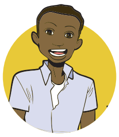


For Questions 1 and 2, start by working with smaller numbers and look for a pattern.

3 What is the smallest number that, when divided by 11, has a remainder of 6 and when divided by 6 has a remainder of 1?

4 When 9 is added to certain two-digit numbers the digits are reversed; that is, $AB + 9 = BA$. Find all the two-digit numbers for which this is true.

For Questions 3 and 4, try making a list or table.

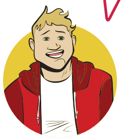


For Questions 5 and 6, try drawing diagrams to help you visualise the problem.

5 A new east–west highway is 12 km north of Adina and 39 km north of Birubi. Town residents want the shortest possible road that will connect both towns to the same highway entry point. The east–west distance between the towns is 68 km. Determine the total length of the new road and the length and true bearing from each town along the new road to the highway. Round all answers to two decimal places.

6 A rectangle has coordinates $(0, 0)$, $(0, 4)$, $(6, 4)$ and $(6, 0)$. If a point (x, y) is randomly chosen in this rectangle, determine the probability that $y \geqslant x$.

7 Future travel to Mars could be with rockets powered by nuclear fusion. Estimate how many days a one way Earth–Mars trip would take if a fusion-powered rocket travels at 321 800 km/h and the journey distance is 563 800 000 km.



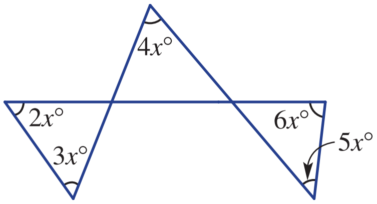
For Question 7, use rounding to estimate the answer.

8 Estimate the average time that a Year 9 student takes to walk between classes at your school.

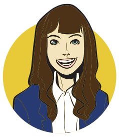
For Question 8, try estimating by taking a sample.

9 A plane flight between two cities takes t hours. If the plane increases its speed by 25% by what percentage will the flight time decrease?

10 Find the value of x in this diagram. The diagram is not necessarily to scale.



For Questions 9–13, try using algebra as a tool to work out the unknowns.



11 Three numbers have a median of 14 and a mean that is 9 more than the smallest number and 11 less than the largest number. Find the sum of the three numbers.

12 An even integer is tripled and added to double the next consecutive even integer. If the result is 364, determine the value of the first even integer.

13 A school has two rooms that are square. The larger room has sides 5 m longer than the sides of the smaller room. If the total floor area of these rooms is 157 m^2, find the side length of each room.

14 **a** A diagonal line is ruled on a rectangular sheet of paper. In your own words, describe the three-dimensional object that this line forms when the sheet of paper is rolled up to make a cylinder.

b Calculate the length of handrail required for a spiral staircase with steps of length 80 cm attached to a central pole of radius 10 cm and height 2.5 m. Round to two decimal places.

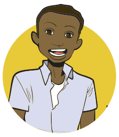
c An amusement park has a spiral track for a section of the rollercoaster ride. The track has 2 revolutions and is designed around a virtual cylinder with diameter 12 m and height 8 m. Find the length of this spiral track correct to two decimal places.

For Questions 14 and 15, try using concrete, everyday materials to help you understand the problem.

15 If 27 dots are used to form a cube, with 9 dots on each of its faces and one dot in the middle of the cube, how many lines containing exactly three dots can be drawn?

16 Given that $a^{2q} = 5$, find the value of $2a^{6q} + 4$.

17 What is the value of $\dfrac{x - \dfrac{1}{y}}{y - \dfrac{1}{x}}$?



For Questions 16 and 17, try using a mathematical procedure to find a shortcut to the answer.

Working with unfamiliar problems: Part 2

For the questions in Part 2, again use the 'Working with unfamiliar problems' poster at the back of this book, but this time choose your own strategy (or strategies) to solve each problem. Clearly communicate your solution and final answer.

1 The number $\frac{79}{21}$ can be written in the form $3 + \cfrac{1}{x + \cfrac{1}{y + \cfrac{1}{5}}}$. Find the values of x and y.

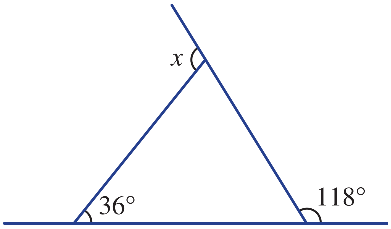
2 How many solids can you name that have eight vertices?

3 Increasing a number by 25% then decreasing the result by $x\%$ gives the original number again. What is the value of x in this case?

4 Prove that the sum of two odd numbers is an even number.

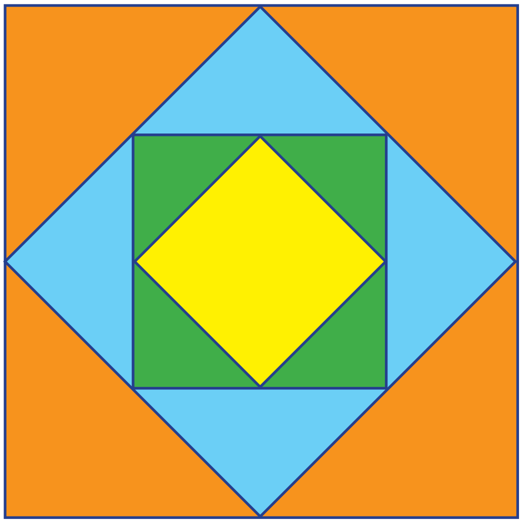
5 $x + y + xy = 34$ and x and y are both positive integers. What is the value of xy?

6 Find the value of the angle marked x in this diagram.



7 Given that n is a perfect square, write an expression, in terms of n, for the next largest perfect square.

8 A square of side length 8 cm has the midpoints of its sides joined to form a smaller square inside it. The midpoints of that square are joined to form an even smaller square. This method is repeated to create more squares.



- **a** Find the area of the first five squares. Write these area values as a sequence of powers of a prime number.
- **b** Continue your sequence from part **a** to find the areas of the 7th square and the 10th square.
- **c** Write a rule for the area of the nth square. Use this rule to find the area of the 15th square.

ISBN 978-1-108-77288-4

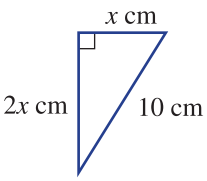
Cambridge University Press
Updated September 2021

9 If the exterior angles of a triangle are in the ratio $4:5:6$, then what is the ratio of the interior angles of the same triangle?

10 An operation exists where $\text{A} \# \text{B} = \text{A}^{\text{B}} + \text{B}^{\text{A}}$. If $\text{A} \# 6 = 100$, then A has what value?

11 Six years ago Sam was five times Noah's age. In ten years' time Sam will be two more than twice Noah's age. How old are they now?

12 Find the area of this triangle.

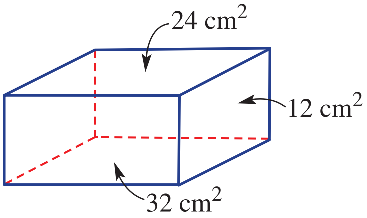


13 Simplify these expressions.

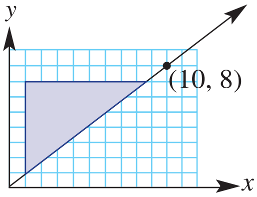
a $3^{x-2} + 3^{x-2} + 3^{x-2}$

b $(-1)^1 + (-1)^2 + (-1)^3 + \ldots + (-1)^{345} + (-1)^{346}$

14 **a** What is the volume of the rectangular prism shown?



b Find the exact area of the shaded triangle, given each grid square is 1 unit^2.



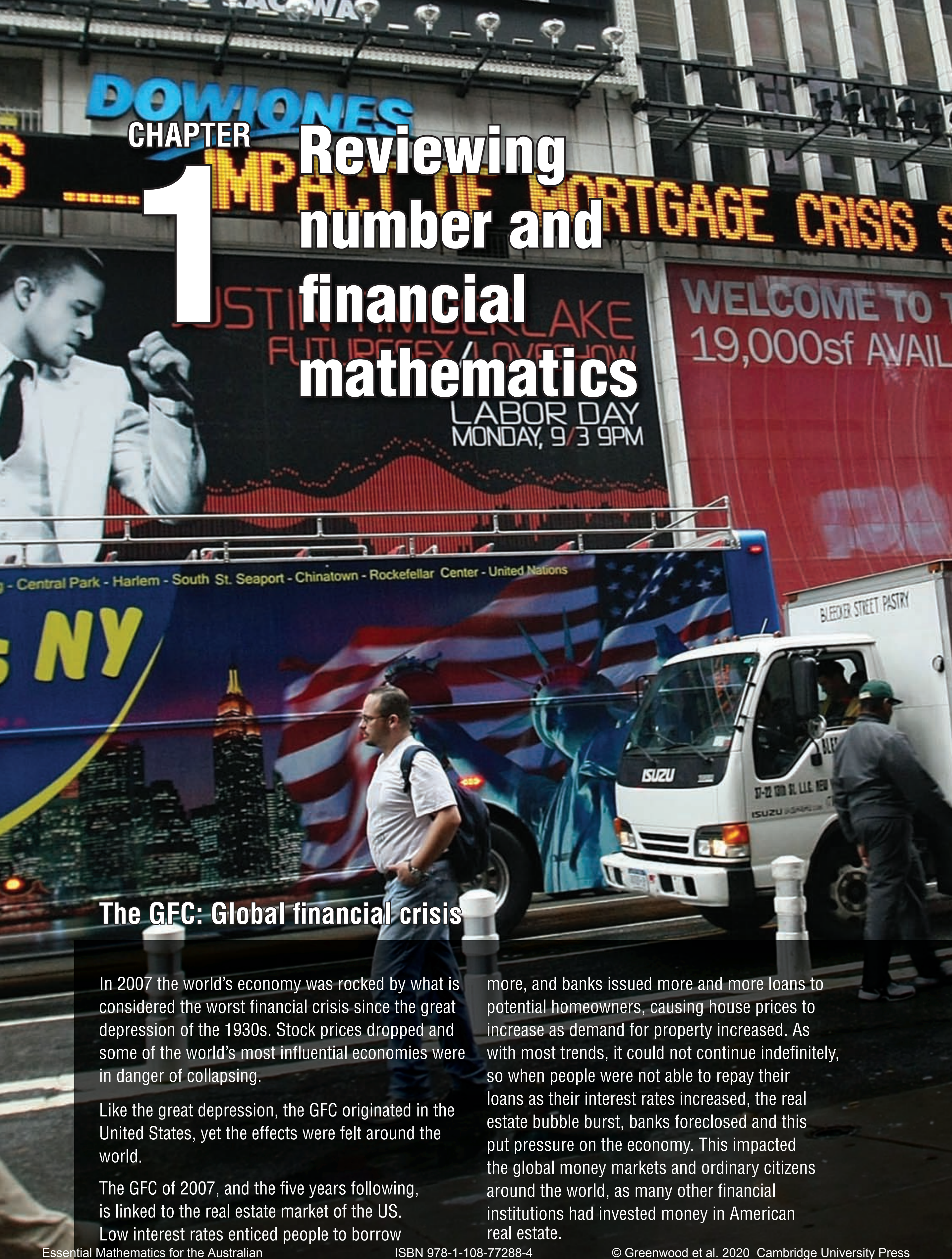
15 When a certain number is added to 18 and the same number is subtracted from 21 the product is 350. Find two possible values of this number.

16 On a training ride Holly cycled at an average speed of 20 km/h, stopped for a 15 minute break and then completed her trip at an average speed of 24 km/h. If the total distance was 68 km and the total time 3.5 hours (including the stop), determine the time taken for each stage of the ride.

17 A triangle has vertices $A(-7, -3)$, $B(8, 7)$ and $C(6, -3)$. A point D is on line AB so that CD is perpendicular to AB. Determine the exact ratio of the area of triangle ABC to the area of triangle DBC.

CHAPTER

1 Reviewing number and financial mathematics



The GFC: Global financial crisis

In 2007 the world's economy was rocked by what is considered the worst financial crisis since the great depression of the 1930s. Stock prices dropped and some of the world's most influential economies were in danger of collapsing.

Like the great depression, the GFC originated in the United States, yet the effects were felt around the world.

The GFC of 2007, and the five years following, is linked to the real estate market of the US. Low interest rates enticed people to borrow more, and banks issued more and more loans to potential homeowners, causing house prices to increase as demand for property increased. As with most trends, it could not continue indefinitely, so when people were not able to repay their loans as their interest rates increased, the real estate bubble burst, banks foreclosed and this put pressure on the economy. This impacted the global money markets and ordinary citizens around the world, as many other financial institutions had invested money in American real estate.

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Australia survived this period due to our strong mining industry and trade with Asia. Interest rates were lowered to stimulate growth in our economy and the housing market. It remains to be seen what the future brings as interest rates fluctuate over time.

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

In this chapter

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1J Simple interest
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Australian Curriculum

NUMBER AND ALGEBRA
Money and financial mathematics

Solve problems involving simple interest (ACMNA211)

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1A Integer operations CONSOLIDATING

Learning intentions

- To know the numbers in the set of integers
- To know the rules for the mathematical operations applied to negative numbers
- To understand the notation for powers and roots
- To be able to evaluate operations on integers including negative numbers, powers and roots and apply order of operations

Throughout history, mathematicians have developed number systems to investigate and explain the world in which they live. The Egyptians used hieroglyphics to record whole numbers as well as fractions, the Babylonians used a place-value system based on the number 60, and the ancient Chinese and Indians developed systems using negative numbers. Our current base 10 decimal system (the Hindu–Arabic system) has expanded to include positive and negative numbers, fractions (rational numbers), as well as numbers that cannot be written as a fraction (irrational numbers), for example, π and $\sqrt{2}$. All the numbers in our number system, not including imaginary numbers, are called real numbers.

The abacus is an efficient calculating tool and was widely used for thousands of years across China and Europe.

LESSON STARTER Special sets of numbers

Here are some special groups of numbers. Can you describe what special property each group has? Try to use the correct vocabulary, for example, factors of 12.

- 7, 14, 21, 28, …
- 1, 4, 9, 16, 25, …
- 1, 2, 3, 4, 6, 9, 12, 18, 36.
- 1, 8, 27, 64, 125, …
- 0, 1, 1, 2, 3, 5, 8, 13, …
- 2, 3, 5, 7, 11, 13, 17, 19, …

KEY IDEAS

- The **integers** include …, −3, −2, −1, 0, 1, 2, 3, …

- **The rules of negative numbers**
  - $a + (-b) = a - b$ For example: $5 + (-2) = 5 - 2 = 3$
  - $a - (-b) = a + b$ For example: $5 - (-2) = 5 + 2 = 7$
  - $a \times (-b) = -ab$ For example: $3 \times (-2) = -6$
  - $-a \times (-b) = ab$ For example: $-4 \times (-3) = 12$
  - $a \div (-b) = -\frac{a}{b}$ For example: $8 \div (-4) = -2$
  - $-a \div (-b) = \frac{a}{b}$ For example: $-8 \div (-4) = 2$

- **Squares and cubes**
  - $a^2 = a \times a$ and $\sqrt{a^2} = a$ (if $a \geq 0$) For example: $6^2 = 36$ and $\sqrt{36} = 6$
  - $a^3 = a \times a \times a$ and $\sqrt[3]{a^3} = a$ For example: $4^3 = 64$ and $\sqrt[3]{64} = 4$

- **LCM, HCF and primes**
  - The **lowest common multiple** (LCM) of two numbers is the smallest multiple shared by both numbers. For example: the LCM of 6 and 9 is 18.
  - The **highest common factor** (HCF) of two numbers is the largest factor shared by both numbers. For example: the HCF of 24 and 30 is 6.
  - **Prime numbers** have only two factors: 1 and the number itself. The number 1 is not considered a prime number.
  - **Composite numbers** have more than two factors.

- **Order of operations**
  - Deal with brackets first.
  - Deal with indices next.
  - Do multiplication and division next from left to right.
  - Do addition and subtraction last from left to right.

$$\begin{aligned}&5 \times (3 - (-2)^2) + 3^3 \div (-9)\\&= 5 \times (3 - 4) + 27 \div (-9)\\&= 5 \times (-1) + (-3)\\&= -5 + (-3)\\&= -8\end{aligned}$$

BUILDING UNDERSTANDING

1 State these sets of numbers.

a The factors of 56
b The HCF (highest common factor) of 16 and 56
c The first 7 multiples of 3
d The LCM (lowest common multiple) of 3 and 5
e The first ten prime numbers starting from 2

2 Evaluate the following.

a 11^2 **b** $\sqrt{144}$ **c** 3^3 **d** $\sqrt[3]{8}$

3 Evaluate the following.

a $-3 + 2$ **b** $2 + (-3)$ **c** $11 - (-4)$
d $2 \times (-3)$ **e** $-11 \times (-2)$ **f** $18 \div (-2)$

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Example 1 Operating with integers

Evaluate the following.

a $-2-(-3\times 13)+(-10)$ **b** $(-20\div(-4)+(-3))\times 2$ **c** $\sqrt[3]{8}-(-1)^2+3^3$

SOLUTION	EXPLANATION
a $-2-(-3\times 13)+(-10)=-2-(-39)+(-10)$ $=-2+39+(-10)$ $=37-10$ $=27$	Deal with the operations in brackets first. $-a-(-b)=-a+b$ $a+(-b)=a-b$
b $(-20\div(-4)+(-3))\times 2=(5+(-3))\times 2$ $=2\times 2$ $=4$	$-a\div(-b)=\frac{a}{b}$ Deal with the operations inside brackets before doing the multiplication. $5+(-3)=5-3$.
c $\sqrt[3]{8}-(-1)^2+3^3=2-1+27$ $=28$	Evaluate powers first. $\sqrt[3]{8}=2$ since $2^3=8$ $(-1)^2=-1\times(-1)=1$ $3^3=3\times 3\times 3=27$

Now you try

Evaluate the following.

a $-4-(-1\times 12)+(-3)$ **b** $(-24\div(-6)+(-2))\times 2$ **c** $\sqrt[3]{27}-(-2)^2-4^3$

Exercise 1A

FLUENCY	1, 2–5(½)	2–6(½)	2(¼), 3–6(½)

1 Evaluate the following.

Example 1a **a** i $-1-(-2\times 4)+(-2)$ ii $2+(3\times(-2))-(-1)$

Example 1b **b** i $(-10\div 2+(-1))\times 2$ ii $(4\div(-1)-3)\times(-3)$

Example 1c **c** i $\sqrt[3]{1}-(-2)^2+2^3$ ii $\sqrt[3]{125}-(-3)^3-2^2$

Example 1a,b

2 Evaluate the following showing your steps.

a $-4-3\times(-2)$ **b** $-3\times(-2)+(-4)$

c $-2\times(3-8)$ **d** $2-7\times(-2)$

e $2-3\times 2+(-5)$ **f** $4+8\div(-2)-3$

g $(-24\div(-8)+(-5))\times 2$ **h** $-7-(-4\times 8)-15$

i $-3-12\div(-6)\times(-4)$ **j** $4\times(-3)\div(-2\times 3)$

k $(-6-9\times(-2))\div(-4)$ **l** $10\times(-2)\div(-7-(-2))$

m $6\times(-5)-14\div(-2)$ **n** $(-3+7)-2\times(-3)$

o $-2+(-4)\div(-3+1)$ **p** $-18\div((-2-(-4))\times(-3))$

q $-2\times 6\div(-4)\times(-3)$ **r** $(7-14\div(-2))\div 2$

s $2-(1-2\times(-1))$ **t** $20\div(6\times(-4\times 2)\div(-12)-(-1))$

ISBN 978-1-108-77288-4

3 Find the LCM of these pairs of numbers.

a 4, 7 **b** 8, 12 **c** 11, 17 **d** 15, 10

4 Find the HCF of these pairs of numbers.

a 20, 8 **b** 100, 65 **c** 37, 17 **d** 23, 46

Example 1c

5 Evaluate the following.

a $2^3 - \sqrt{16}$ **b** $5^2 - \sqrt[3]{8}$ **c** $(-1)^2 \times (-3)$

d $(-2)^3 \div (-4)$ **e** $\sqrt{9} - \sqrt[3]{125}$ **f** $1^3 + 2^3 - 3^3$

g $\sqrt[3]{27} - \sqrt{81}$ **h** $\sqrt[3]{27} - \sqrt{9} - \sqrt[3]{1}$ **i** $(-1)^{101} \times (-1)^{1000} \times \sqrt[3]{-1}$

6 Evaluate these expressions by substituting $a = -2$, $b = 6$ and $c = -3$.

a $a^2 - b$ **b** $a - b^2$ **c** $2c + a$

d $b^2 - c^2$ **e** $a^3 + c^2$ **f** $3b + ac$

g $c - 2ab$ **h** $abc - (ac)^2$ **i** $\sqrt{abc} - ac^2$

PROBLEM-SOLVING	7, 9	7–9	8–11

7 Insert brackets into these statements to make them true.

a $-2 \times 11 + (-2) = -18$ **b** $-6 + (-4) \div 2 = -5$

c $2 - 5 \times (-2) = 6$ **d** $-10 \div 3 + (-5) = 5$

e $3 - (-2) + 4 \times 3 = -3$ **f** $(-2)^2 + 4 \div (-2) = -2^2$

8 How many different answers are possible if any number of pairs of brackets is allowed to be inserted into this expression?

$-6 \times 4 - (-7) + (-1)$

9 Margaret and Mildred meet on a Eurostar train travelling from London to Paris. Margaret visits her daughter in Paris every 28 days. Mildred visits her son in Paris every 36 days. When will Margaret and Mildred have a chance to meet again on the train?

10 a The sum of two numbers is 5 and their difference is 9 . What are the two numbers?

b The sum of two numbers is -3 and their product is -10. What are the two numbers?

11 Two opposing football teams have squad sizes of 24 and 32. For a training exercise, each squad is to divide into smaller groups of equal size. What is the largest number of players in a group if the group size for both squads is the same?

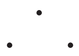
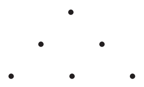
REASONING	12	12	12, 13

12 **a** Evaluate:
  i 4^2
  ii $(-4)^2$
b If $a^2 = 16$, write down the possible values of a.
c If $a^3 = 27$, write down the value of a.
d Explain why there are two values of a for which $a^2 = 16$ but only one value of a for which $a^3 = 27$.
e Find $\sqrt[3]{-27}$.
f Explain why $\sqrt{-16}$ cannot exist (using real numbers).
g -2^2 is the same as -1×2^2. Now evaluate:
  i -2^2
  ii -5^3
  iii $-(-3)^2$
  iv $-(-4)^2$
h Decide if $(-2)^2$ and -2^2 are equal.
i Decide if $(-2)^3$ and -2^3 are equal.
j Explain why the HCF of two distinct prime numbers is 1.
k Explain why the LCM of two distinct prime numbers a and b is $a \times b$.

13 If a and b are both positive numbers and $a > b$, decide if the following are true or false.
a $a - b < 0$
b $-a \times b > 0$
c $-a \div (-b) > 0$
d $(-a)^2 - a^2 = 0$
e $-b + a < 0$
f $2a - 2b > 0$

ENRICHMENT: Special numbers	–	–	14

14 **a** Perfect numbers are positive integers that are equal to the sum of all their factors, excluding the number itself.
  i Show that 6 is a perfect number.
  ii There is one perfect number between 20 and 30 . Find the number.
  iii The next perfect number is 496. Show that 496 is a perfect number.
b Triangular numbers are the number of dots required to form triangles as shown in this table.
  i Complete this table.

Number of rows	1	2	3	4	5	6
Diagram						
Number of dots (triangular number)	1	3				

  ii Find the 7th and 8th triangular numbers.
c Fibonacci numbers are a sequence of numbers in which each number is the sum of the two preceding numbers. The first two numbers in the sequence are 0 and 1.
  i Write down the first ten Fibonacci numbers.
  ii If the Fibonacci numbers were to be extended in the negative direction, what would the first four negative Fibonacci numbers be?

1B Decimals and significant figures

Learning intentions

- To know the rules for rounding to a required number of decimal places
- To understand what is meant by significant figures and how to count them
- To be able to round to a required number of significant figures
- To be able to estimate calculations using rounding

Numbers with and without decimal places can be rounded depending on the level of accuracy required. When using numbers with decimal places, it is common to round off the number to leave only a certain number of decimal places. The time for a 100 m sprint race, for example, might be 9.94 seconds.

Due to the experimental nature of science and engineering, not all the digits in all numbers are considered important or 'significant'. In such cases we are able to round numbers to within a certain number of significant figures (sometimes abbreviated to sig. fig. or simply s.f.). The number of cubic metres of gravel required for a road, for example, might be calculated as 3485 but rounded to 3500. This number is written using two significant figures.

When civil engineers design a new road cutting through a hill, the volume of earth to be removed only needs to be calculated to two or three significant figures.

LESSON STARTER Plausible but incorrect

Johny says that the number 2.748 when rounded to one decimal place is 2.8 because:

- the 8 rounds the 4 to a 5
- then the new 5 rounds the 7 to an 8.

What is wrong with Johny's theory?

KEY IDEAS

- To round a number to a required number of **decimal places**:
  - Locate the digit in the required decimal place.
  - Round down (leave as is) if the next digit (**critical digit**) is 4 or less.
  - Round up (increase by 1) if the next digit is 5 or more.

  For example:
  - To two decimal places, 1.543 rounds to 1.54 and 32.9283 rounds to 32.93.
  - To one decimal place, 0.248 rounds to 0.2 and 0.253 rounds to 0.3.

ISBN 978-1-108-77288-4

- To round a number to a required number of **significant figures**:
  - Locate the first non-zero digit counting from left to right.
  - From this first significant digit, count the number of significant digits including zeros.
  - Stop at the required number of significant digits and round this last digit.
  - Replace any non-significant digit to the left of a decimal point with a zero.

  For example, these numbers are all rounded to 3 significant figures:
  $2.5391 \approx 2.54$, $0.002713 \approx 0.00271$, $568\,810 \approx 569\,000$.

BUILDING UNDERSTANDING

1 Choose the number to answer each question.

a Is 44 closer to 40 or 50?
b Is 266 closer to 260 or 270?
c Is 7.89 closer to 7.8 or 7.9?
d Is 0.043 closer to 0.04 or 0.05?

2 Choose the correct answer if the first given number is rounded to three significant figures.

a 32 124 is rounded to 321, 3210 or 32 100
b 431.92 is rounded to 431, 432 or 430
c 5.8871 is rounded to 5.887, 5.88 or 5.89
d 0.44322 is rounded to 0.44, 0.443 or 0.44302

3 Using one significant figure rounding, 324 rounds to 300, 1.7 rounds to 2 and 9.6 rounds to 10.

a Calculate $300 \times 2 \div 10$.
b Use a calculator to calculate $324 \times 1.7 \div 9.6$.
c What is the difference between the answer in part **a** and the exact answer in part **b**?

Example 2 Rounding to a number of decimal places

Round each of these numbers to two decimal places.

a 256.1793 **b** 0.04459 **c** 4.8972

SOLUTION	EXPLANATION
a $256.1793 \approx 256.18$	The number after the second decimal place is 9, so round up (increase the 7 by 1).
b $0.04459 \approx 0.04$	The number after the second decimal place is 4, so round down. 4459 is closer to 4000 than 5000.
c $4.8972 \approx 4.90$	The number after the second decimal place is 7, so round up. Increasing by 1 means 0.89 becomes 0.90.

Now you try

Round each of these numbers to two decimal places.

a 138.4681 **b** 0.03268 **c** 2.7961

ISBN 978-1-108-77288-4

Updated September 2021

Example 3 Rounding to a number of significant figures

Round each of these numbers to two significant figures.

a 2567 **b** 23 067.453 **c** 0.04059

SOLUTION	EXPLANATION
a $2567 \approx 2600$	The first two digits are the first two significant figures. The third digit is 6, so round up. Replace the last two non-significant digits with zeros.
b $23\,067.453 \approx 23\,000$	The first two digits are the first two significant figures. The third digit is 0, so round down.
c $0.04059 \approx 0.041$	Locate the first non-zero digit, i.e. 4. So 4 and 0 are the first two significant figures. The next digit is 5, so round up.

Now you try

Round each of these numbers to two significant figures.

a 3582 **b** 49 018.47 **c** 0.02864

Example 4 Estimating using significant figures

Estimate the answer to the following by rounding each number in the problem to one significant figure. Use your calculator to check how reasonable your answer is.

$27 + 1329.5 \times 0.0064$

SOLUTION	EXPLANATION
$27 + 1329.5 \times 0.0064$ $\approx 30 + 1000 \times 0.006$ $= 30 + 6$ $= 36$	Round each number to one significant figure and evaluate. Recall that multiplication occurs before the addition.
The estimated answer is reasonable.	By calculator (to one decimal place): $27 + 1329.5 \times 0.0064 = 35.5$

Now you try

Estimate the answer to the following by rounding each number in the problem to one significant figure. Use your calculator to check how reasonable your answer is.

$19 + 2143.1 \times 0.0032$

ISBN 978-1-108-77288-4

Exercise 1B

FLUENCY	1, 2–7($\frac{1}{2}$)	2–7($\frac{1}{2}$)	2($\frac{1}{4}$), 4–6($\frac{1}{2}$), 7($\frac{1}{3}$)

1 Round each of these to two decimal places.

Example 2a **a** **i** 124.2694 **ii** 830.4428

Example 2b **b** **i** 0.02163 **ii** 0.06714

Example 2c **c** **i** 7.6941 **ii** 13.4953

Example 2 **2** Round each of the following numbers to two decimal places.

a 17.962 **b** 11.082 **c** 72.986 **d** 47.859
e 63.925 **f** 23.807 **g** 804.5272 **h** 500.5749
i 821.2749 **j** 5810.2539 **k** 1004.9981 **l** 2649.9974

3 Round these numbers to the nearest integer.

a 6.814 **b** 73.148 **c** 129.94 **d** 36 200.49

4 Use division to write these fractions as decimals rounded to three decimal places.

a $\frac{1}{3}$ **b** $\frac{2}{7}$ **c** $\frac{13}{11}$ **d** $\frac{400}{29}$

Example 3 **5** Round each of these numbers to two significant figures.

a 2436 **b** 35 057.4 **c** 0.06049 **d** 34.024
e 107 892 **f** 0.00245 **g** 2.0745 **h** 0.7070

6 Round these numbers to one significant figure.

a 32 000 **b** 194.2 **c** 0.0492 **d** 0.0006413

Example 4 **7** Estimate the answers to the following by rounding each number in the problem to one significant figure. Check how reasonable your answer is with a calculator.

a $567 + 3126$ **b** $795 - 35.6$ **c** 97.8×42.2
d $965.98 + 5321 - 2763.2$ **e** $4.23 - 1.92 \times 1.827$ **f** $17.43 - 2.047 \times 8.165$
g $0.0704 + 0.0482$ **h** 0.023×0.98 **i** $0.027 \div 0.0032$
j 41.034^2 **k** 0.078×0.9803^2 **l** $1.8494^2 + 0.972 \times 7.032$

PROBLEM-SOLVING	8, 9	8–10	9–11

8 An electronic timer records the time for a running relay. Team A's time is 54.283 seconds and team B's time is 53.791 seconds. What would be the difference in the times for teams A and B if the times were written down using:

a 1 decimal place?
b 4 significant figures?
c 2 significant figures?
d 1 significant figure?

9 $28.4 \times 2.94 \times 11.31$ is calculated by first rounding each of the three numbers. Describe the type of rounding that has taken place if the answer is:

a 900 **b** 893.2 **c** 924

10 150 m of fencing and 18 posts are used to create an area in the shape of an equilateral triangle. Posts are used in the corners and are evenly spaced along the sides. Find the distance between each post. Write your answer in metres rounded to the nearest centimetre.

11 A tonne (1000 kg) of soil is to be equally divided between 7 garden beds. How much soil is placed in each garden bed? Write your answer in tonnes rounded to the nearest kilogram.

REASONING	12	12, 13	13, 14

12 Should 2.14999 be rounded down or up if it is to be rounded to one decimal place? Give reasons.

13 A scientific experiment uses very small amounts of magnesium (0.0025 g) and potassium (0.0062 g). Why does it make sense to use two significant figures instead of two decimal places when recording numbers in a situation like this?

14 Consider the two numbers 24 and 26.

a Calculate:

i $24 + 26$ **ii** 24×26

b Find the sum (+) of the numbers after rounding each number to one significant figure.

c Find the product (×) of the numbers after rounding each number to one significant figure.

d What do you notice about the answers for parts **b** and **c** as compared to part **a**? Give an explanation.

ENRICHMENT: *n*th decimal place	–	–	15

15 **a** Use division to express $\frac{2}{11}$ as a decimal correct to 8 decimal places.

b Using the decimal pattern from part **a**, find the digit in the:

i 20th decimal place **ii** 45th decimal place **iii** 1000th decimal place.

c Express $\frac{1}{7}$ as a decimal correct to 13 decimal places.

d Using the decimal pattern from part **c** find the digit in the:

i 20th decimal place **ii** 45th decimal place **iii** 1000th decimal place.

e Can you find any fraction whose decimal representation is non-terminating and has no pattern? Use a calculator to help.

1C Rational numbers CONSOLIDATING

Learning intentions

- To understand that the real number system is made up of different types of numbers
- To be able to determine if a number is rational or irrational
- To review how to simplify fractions and convert between fractions and decimals
- To know how to compare fractions using a common denominator

Under the guidance of Pythagoras in about 500 BCE, it was discovered that some numbers could not be expressed as a fraction. These special numbers, called irrational numbers, when written as a decimal continue forever and do not show any pattern. So to write these numbers exactly, you need to use special symbols such as $\sqrt{\ }$ and π. If, however, the decimal places in a number terminate or if a pattern exists, the number can be expressed as a fraction. These numbers are called rational numbers.

This is $\sqrt{2}$ to 100 decimal places:

1.41421356237309504880168872420969807856967187537694
80731766797379907324784621070388503875343276415727

Numerous people, including scientists, engineers and trade workers, regularly use formulas that contain rational and irrational numbers. This steel sphere is used for chemical analysis and has volume: $V = \frac{4}{3}\pi r^3$.

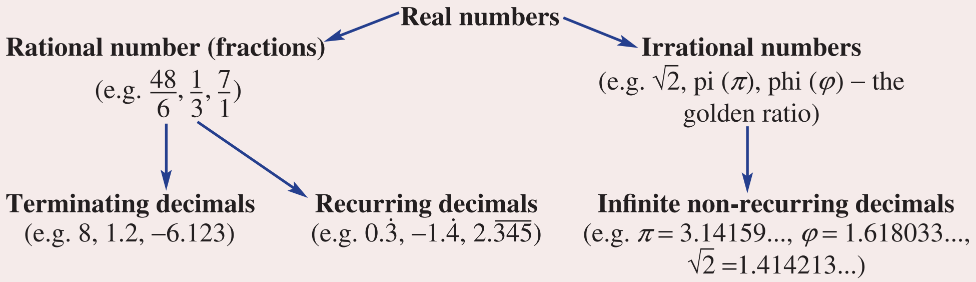
LESSON STARTER Approximating π

To simplify calculations, the ancient and modern civilisations have used fractions to approximate π. To ten decimal places, $\pi = 3.1415926536$.

- Using single-digit numbers, what fraction best approximates π? $\left(\text{For example: } \frac{5}{2}\right)$.
- Using single and/or double-digit numbers, find a fraction that is a good approximation of π. Compare with other students to see who has the best approximation. $\left(\text{For example: } \frac{35}{11}\right)$.

ISBN 978-1-108-77288-4

KEY IDEAS



Real numbers

Rational number (fractions)
(e.g. $\frac{48}{6}, \frac{1}{3}, \frac{7}{1}$)

Irrational numbers
(e.g. $\sqrt{2}$, pi (π), phi (φ) – the golden ratio)

Terminating decimals
(e.g. 8, 1.2, –6.123)

Recurring decimals
(e.g. $0.\dot{3}$, $-1.\dot{4}$, $2.\overline{345}$)

Infinite non-recurring decimals
(e.g. $\pi = 3.14159...$, $\varphi = 1.618033...$, $\sqrt{2} = 1.414213...$)

- An **infinite** decimal is one in which the decimal places continue indefinitely.
- **Equivalent fractions** have the same value.
  For example: $\frac{2}{3} = \frac{6}{9}$
- Recall that $\frac{a}{b}$ means $a \div b$.
- A fraction can be simplified by dividing the **numerator** and the **denominator** by their highest common factor.
  For example: $\frac{\cancel{8}^{2}}{\cancel{20}_{5}} = \frac{2}{5}$
- If $\frac{a}{b}$ is a **proper fraction** then $a < b$.
  For example: $\frac{2}{7}$
- If $\frac{a}{b}$ is an **improper fraction** then $a > b$.
  For example: $\frac{10}{3}$
- A **mixed number** is written as a whole number plus a proper fraction.
  For example: $2\frac{3}{5}$
- Fractions can be compared using a **common denominator**. This should be the lowest common multiple of both denominators.
  For example: $\frac{2}{3} < \frac{4}{5}$ because $\frac{10}{15} < \frac{12}{15}$
- A dot or bar is used to show a pattern in a recurring decimal number.
  For example: $\frac{1}{6} = 0.16666\ldots = 0.1\dot{6}$ or $\frac{3}{11} = 0.272727\ldots = 0.\overline{27}$

BUILDING UNDERSTANDING

1 Give these numbers as mixed numbers.

a $\frac{7}{5}$ **b** $\frac{13}{3}$ **c** $\frac{48}{11}$

2 Give these numbers as improper fractions.

a $1\frac{4}{7}$ **b** $5\frac{1}{3}$ **c** $9\frac{1}{2}$

3 Simplify these fractions by cancelling.

a $\frac{4}{10}$ **b** $\frac{8}{58}$ **c** $4\frac{20}{120}$

4 Give the missing number.

a $\frac{3}{5} = \frac{\square}{15}$ **b** $\frac{5}{6} = \frac{20}{\square}$ **c** $\frac{\square}{4} = \frac{21}{28}$ **d** $\frac{3}{\square} = \frac{15}{50}$

ISBN 978-1-108-77288-4

Updated September 2021

Example 5 Writing fractions as decimals

Write these fractions as decimals.

a $3\frac{3}{8}$

b $\frac{5}{13}$

SOLUTION

a
$$\begin{array}{r} 0.\ 3\ 7\ 5 \\ 8\overline{)3.{}^{3}0{}^{6}0{}^{4}0} \end{array}$$

$$3\frac{3}{8} = 3.375$$

b
$$\begin{array}{r} 0.\ 3\ 8\ 4\ 6\ 1\ 5\ 3 \\ 13\overline{)5.{}^{5}0{}^{11}0{}^{6}0{}^{8}0{}^{2}0{}^{7}0{}^{5}0} \end{array}$$

$$\frac{5}{13} = 0.\overline{384615}$$

EXPLANATION

Find a decimal for $\frac{3}{8}$ by dividing 8 into 3 using the short-division algorithm.

Divide 13 into 5 and continue until the pattern repeats.
Add a bar over the repeating pattern.
Writing $0.\dot{3}8461\dot{5}$ is an alternative.

Now you try

Write these fractions as decimals.

a $4\frac{5}{8}$

b $\frac{2}{11}$

Example 6 Writing decimals as fractions

Write these decimals as fractions.

a 0.24

b 2.385

SOLUTION

a
$$0.24 = \frac{24}{100} = \frac{6}{25}$$

b
$$2.385 = 2\frac{385}{1000} = 2\frac{77}{200}$$

OR

$$\frac{2385}{1000} = \frac{477}{200} = 2\frac{77}{200}$$

EXPLANATION

Write as a fraction using the smallest place value (hundredths) and then simplify using the HCF of 4.

The smallest place value is thousandths. Simplify to an improper fraction or a mixed number.

Now you try

Write these decimals as fractions.

a 0.45

b 1.425

Example 7 Comparing fractions

Decide which is the larger fraction.

$\frac{7}{12}$ or $\frac{8}{15}$

SOLUTION	EXPLANATION
LCM of 12 and 15 is 60.	Find the lowest common multiple of the two denominators (lowest common denominator).
$\frac{7}{12} = \frac{35}{60}$ and $\frac{8}{15} = \frac{32}{60}$ $\therefore \frac{7}{12} > \frac{8}{15}$	Write each fraction as an equivalent fraction using the common denominator. Then compare numerators (i.e. $35 > 32$) to determine the larger fraction.

Now you try

Decide which is the larger fraction.

$\frac{3}{5}$ or $\frac{7}{11}$

Exercise 1C

FLUENCY	1, 2–5(½), 7	2–5(⅓), 6, 7	2–5(¼), 6, 7(½)

1 Write these fractions as decimals.

Example 5a **a** **i** $2\frac{3}{8}$ **ii** $6\frac{7}{8}$

Example 5b **b** **i** $\frac{3}{13}$ **ii** $\frac{7}{15}$

Example 5a **2** Write these fractions as decimals.

a $\frac{11}{4}$ **b** $\frac{7}{20}$ **c** $3\frac{2}{5}$ **d** $\frac{15}{8}$

e $2\frac{5}{8}$ **f** $3\frac{4}{5}$ **g** $\frac{37}{16}$ **h** $\frac{7}{32}$

Example 5b **3** Write these fractions as recurring decimals.

a $\frac{3}{11}$ **b** $\frac{7}{9}$ **c** $\frac{9}{7}$ **d** $\frac{5}{12}$

e $\frac{10}{9}$ **f** $3\frac{5}{6}$ **g** $7\frac{4}{15}$ **h** $\frac{29}{11}$

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Example 6

4 Write these decimals as fractions.

a 0.35	**b** 0.06	**c** 3.7	**d** 0.56
e 1.07	**f** 0.075	**g** 3.32	**h** 7.375
i 2.005	**j** 10.044	**k** 6.45	**l** 2.101

Example 7

5 Decide which is the larger fraction in the following pairs.

a $\frac{3}{4}$ or $\frac{5}{6}$	**b** $\frac{13}{20}$ or $\frac{3}{5}$	**c** $\frac{7}{10}$ or $\frac{8}{15}$	**d** $\frac{5}{12}$ or $\frac{7}{18}$
e $\frac{7}{16}$ or $\frac{5}{12}$	**f** $\frac{26}{35}$ or $\frac{11}{14}$	**g** $\frac{7}{12}$ or $\frac{19}{30}$	**h** $\frac{7}{18}$ or $\frac{11}{27}$

6 Place these fractions in descending order.

a $\frac{3}{8}, \frac{5}{12}, \frac{7}{18}$ **b** $\frac{1}{6}, \frac{5}{24}, \frac{3}{16}$ **c** $\frac{8}{15}, \frac{23}{40}, \frac{7}{12}$

7 Express the following quantities as simplified fractions.

a \$45 out of \$100

b 12 kg out of 80 kg

c 64 baskets out of 90 shots in basketball

d 115 mL out of 375 mL

PROBLEM-SOLVING

8, 9	8(½), 9, 10	9–11

8 These sets of fractions form a pattern. Find the next two fractions in the pattern.

a $\frac{1}{3}, \frac{5}{6}, \frac{4}{3}, \frac{\square}{\square}, \frac{\square}{\square}$ **b** $\frac{6}{5}, \frac{14}{15}, \frac{2}{3}, \frac{\square}{\square}, \frac{\square}{\square}$

c $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{\square}{\square}, \frac{\square}{\square}$ **d** $\frac{1}{2}, \frac{4}{7}, \frac{9}{14}, \frac{\square}{\square}, \frac{\square}{\square}$

9 The 'Weather forecast' website says there is a 0.45 chance that it will rain tomorrow. The 'Climate control' website says that the chance of rain is $\frac{14}{30}$. Which website gives the smaller chance that it will rain?

10 A jug has 400 mL of half-strength orange juice. The following amounts of full-strength juice are added to the mix. Find a fraction to describe the strength of the orange drink after the full-strength juice is added.

a 100 mL **b** 50 mL **c** 120 mL **d** 375 mL

ISBN 978-1-108-77288-4

Updated September 2021

11 If x is an integer, determine the values that x can take in the following.

a The fraction $\frac{x}{3}$ is a number between (and not including) 10 and 11.

b The fraction $\frac{x}{7}$ is a number between (and not including) 5 and 8.

c The fraction $\frac{34}{x}$ is a number between 6 and 10.

d The fraction $\frac{23}{x}$ is a number between 7 and 12.

e The fraction $\frac{x}{14}$ is a number between (and not including) 3 and 4.

f The fraction $\frac{58}{x}$ is a number between 9 and 15.

REASONING	12	12, 13	12, 13

12 $a\frac{b}{c}$ is a mixed number with unknown digits a, b and c. Write it as an improper fraction.

13 Given that $\frac{a}{b}$ is a fraction, answer the following questions with reasons.

a Is it possible to find a fraction that can be simplified by cancelling if one of a or b is prime?

b Is it possible to find a fraction that can be simplified by cancelling if both a and b are prime? Assume $a \neq b$.

c If $\frac{a}{b}$ is a fraction in simplest form, can a and b both be even?

d If $\frac{a}{b}$ is a fraction in simplest form, can a and b both be odd?

ENRICHMENT: Converting recurring decimals to fractions	–	–	14

14 Here are two examples of how to convert recurring decimals to fractions.

$$\begin{aligned} 0.\dot{6} &= 0.6666\ldots \\ \text{Let } x &= 0.6666\ldots \quad (1) \\ 10x &= 6.6666\ldots \quad (2) \\ (2)-(1)\ 9x &= 6 \\ x &= \frac{6}{9} = \frac{2}{3} \\ \therefore 0.\dot{6} &= \frac{2}{3} \end{aligned}$$

$$\begin{aligned} 1.\overline{27} &= 1.272727\ldots \\ \text{Let } x &= 1.272727\ldots \quad (1) \\ 100x &= 127.2727\ldots \quad (2) \\ (2)-(1)\ 99x &= 126 \\ x &= \frac{126}{99} \\ \therefore 1.\overline{27} &= \frac{126}{99} = 1\frac{27}{99} = 1\frac{3}{11} \end{aligned}$$

Convert these recurring decimals to fractions using the method above.

a $0.\dot{8}$
b $1.\dot{2}$
c $0.\overline{81}$
d $3.\overline{43}$
e $9.\overline{75}$
f $0.\overline{132}$
g $2.\overline{917}$
h $13.\overline{8125}$

1D Operations with fractions CONSOLIDATING

Learning intentions

- To understand that to add or subtract fractions a common denominator is required
- To be able to add and subtract fractions, including those in mixed number form, using the lowest common denominator
- To understand that to multiply fractions it is simpler to first cancel any common factors between numerators and denominators
- To know that to divide a number by a fraction we multiply by the reciprocal of the fraction
- To know to express mixed numbers as improper fractions before multiplying or dividing
- To be able to multiply and divide fractions

Operations with integers can be extended to include rational numbers, which are numbers that can be expressed as fractions. The operations include addition, subtraction, multiplication and division. Addition and subtraction of fractions is generally more complex than multiplication and division because there is the added step of finding common denominators.

Can you name some everyday applications of fractions illustrated by this photo?

LESSON STARTER The common errors

Here are incorrect solutions to four problems involving fractions.

- $\frac{2}{3} \times \frac{5}{3} = \frac{2 \times 5}{3} = \frac{10}{3}$
- $\frac{2}{3} + \frac{1}{2} = \frac{2+1}{3+2} = \frac{3}{5}$
- $\frac{7}{6} \div \frac{7}{3} = \frac{7}{6} \div \frac{14}{6} = \frac{\frac{1}{2}}{6} = \frac{1}{12}$
- $1\frac{1}{2} - \frac{2}{3} = 1\frac{3}{6} - \frac{4}{6} = -1\frac{1}{6}$

In each case describe what is wrong and give the correct solution.

KEY IDEAS

- To add or subtract fractions, first convert each fraction to **equivalent fractions** that have the same **denominator**.
  - Choose the lowest common denominator (LCD).
  - Add or subtract the numerators and retain the denominator.
- To multiply fractions, multiply the numerators and multiply the denominators after the following steps.
  - Convert mixed numbers to improper fractions before multiplying.
  - Cancel the highest common factor between any numerator and any denominator before multiplying.
  - Note: The word ‘of’ usually means ‘multiply’.

    For example: $\frac{1}{3}$ of $24 = \frac{1}{3} \times 24$
- The **reciprocal** of a number multiplied by the number itself is equal to 1.
  - For example: the reciprocal of 2 is $\frac{1}{2}$ since $2 \times \frac{1}{2} = 1$

    the reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$ since $\frac{3}{5} \times \frac{5}{3} = 1$
- To divide a number by a fraction, multiply by its reciprocal.

  For example: $\frac{2}{3} \div \frac{5}{6}$ becomes $\frac{2}{3} \times \frac{6}{5}$
  - Whole numbers can be written using a denominator of 1.

    For example: $3 = \frac{3}{1}$

BUILDING UNDERSTANDING

1 State the lowest common denominator for these pairs of fractions.

a $\frac{1}{2}, \frac{1}{3}$

b $\frac{3}{7}, \frac{5}{9}$

c $\frac{1}{2}, \frac{1}{4}$

d $\frac{5}{12}, \frac{7}{30}$

2 Convert these mixed numbers to improper fractions.

a $2\frac{1}{3}$

b $7\frac{4}{5}$

c $10\frac{1}{4}$

3 State the missing numbers to complete the given working.

a $\frac{3}{2} + \frac{4}{3} = \frac{\square}{6} + \frac{\square}{6}$
$= \frac{\square}{6}$

b $1\frac{1}{3} - \frac{2}{5} = \frac{\square}{3} - \frac{2}{5}$
$= \frac{\square}{15} - \frac{\square}{15}$
$= \frac{14}{\square}$

c $\frac{5}{3} \div \frac{2}{7} = \frac{5}{3} \times \frac{\square}{2}$
$= \frac{\square}{6}$

Example 8 Adding and subtracting fractions

Evaluate the following.

a $\frac{1}{2} + \frac{3}{5}$ **b** $1\frac{2}{3} + 4\frac{5}{6}$ **c** $3\frac{2}{5} - 2\frac{3}{4}$

SOLUTION

a
$$\begin{aligned}\frac{1}{2} + \frac{3}{5} &= \frac{5}{10} + \frac{6}{10}\\ &= \frac{11}{10} \quad \text{or} \quad 1\frac{1}{10}\end{aligned}$$

b
$$\begin{aligned}1\frac{2}{3} + 4\frac{5}{6} &= \frac{5}{3} + \frac{29}{6}\\ &= \frac{10}{6} + \frac{29}{6}\\ &= \frac{39}{6}\\ &= \frac{13}{2} \quad \text{or} \quad 6\frac{1}{2}\end{aligned}$$

Alternative method:

$$\begin{aligned}1\frac{2}{3} + 4\frac{5}{6} &= 1\frac{4}{6} + 4\frac{5}{6}\\ &= 5\frac{9}{6}\\ &= 6\frac{3}{6}\\ &= 6\frac{1}{2}\end{aligned}$$

c
$$\begin{aligned}3\frac{2}{5} - 2\frac{3}{4} &= \frac{17}{5} - \frac{11}{4}\\ &= \frac{68}{20} - \frac{55}{20}\\ &= \frac{13}{20}\end{aligned}$$

EXPLANATION

The lowest common denominator of 2 and 5 is 10. Rewrite as equivalent fractions using a denominator of 10 and add the numerators.

Change each mixed number to an improper fraction.

Remember the lowest common denominator of 3 and 6 is 6. Change $\frac{5}{3}$ to an equivalent fraction with denominator 6, then add the numerators and simplify by cancelling HCF of 3.

Alternatively, add whole numbers and fractions separately, after finding a common denominator for the fractions $\left(\frac{2}{3} = \frac{4}{6}\right)$.

$\frac{9}{6} = 1\frac{3}{6}$

Convert to improper fractions and then rewrite as equivalent fractions with the same denominator. Subtract the numerators.

Now you try

Evaluate the following.

a $\frac{1}{3} + \frac{2}{5}$ **b** $1\frac{1}{2} + 3\frac{5}{8}$ **c** $4\frac{3}{5} - 2\frac{3}{4}$

Example 9 Multiplying fractions

Evaluate the following.

a $\frac{2}{3} \times \frac{5}{7}$

b $1\frac{2}{3} \times 2\frac{1}{10}$

SOLUTION

a $\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7}$

$= \frac{10}{21}$

EXPLANATION

No cancelling is possible as there are no common factors between numerators and denominators. Multiply the numerators and the denominators.

b $1\frac{2}{3} \times 2\frac{1}{10} = \frac{{}^{1}\cancel{5}}{{}_{1}\cancel{3}} \times \frac{\cancel{21}^{7}}{\cancel{10}_{2}}$

$= \frac{7}{2} \text{ or } 3\frac{1}{2}$

Rewrite as improper fractions.

Cancel common factors between numerators and denominators and then multiply remaining numerators and denominators.

Now you try

Evaluate the following.

a $\frac{4}{5} \times \frac{3}{7}$

b $2\frac{1}{3} \times 3\frac{2}{7}$

Example 10 Dividing fractions

Evaluate the following.

a $\frac{4}{15} \div \frac{12}{25}$

b $1\frac{17}{18} \div 1\frac{1}{27}$

SOLUTION

a $\frac{4}{15} \div \frac{12}{25} = \frac{4}{15} \times \frac{25}{12}$

$= \frac{{}^{1}\cancel{4}}{{}_{3}\cancel{15}} \times \frac{\cancel{25}^{5}}{\cancel{12}_{3}}$

$= \frac{5}{9}$

EXPLANATION

To divide by $\frac{12}{25}$ we multiply by its reciprocal $\frac{25}{12}$.

Cancel common factors between numerators and denominators and then multiply fractions.

b $1\frac{17}{18} \div 1\frac{1}{27} = \frac{35}{18} \div \frac{28}{27}$

$= \frac{{}^{5}\cancel{35}}{{}_{2}\cancel{18}} \times \frac{\cancel{27}^{3}}{\cancel{28}_{4}}$

$= \frac{15}{8} \text{ or } 1\frac{7}{8}$

Rewrite mixed numbers as improper fractions.

Multiply by the reciprocal of the second fraction.

Now you try

Evaluate the following.

a $\frac{2}{9} \div \frac{4}{15}$

b $1\frac{11}{12} \div 1\frac{3}{8}$

Exercise 1D

FLUENCY	1, 2–8(1/2)	2–5(1/2), 6(1/3), 7, 8(1/3)	3(1/2), 5(1/2), 6(1/4), 7, 8(1/4)

1 Evaluate the following.

Example 8a

a **i** $\frac{2}{3}+\frac{1}{2}$ **ii** $\frac{2}{5}+\frac{1}{6}$

Example 8b

b **i** $1\frac{1}{3}+2\frac{5}{6}$ **ii** $3\frac{2}{3}+1\frac{1}{4}$

Example 8c

c **i** $3\frac{1}{5}-2\frac{1}{2}$ **ii** $5\frac{2}{7}-4\frac{3}{5}$

Example 8a

2 Evaluate the following.

a $\frac{2}{5}+\frac{1}{5}$ **b** $\frac{3}{9}+\frac{1}{9}$ **c** $\frac{5}{7}+\frac{4}{7}$ **d** $\frac{3}{4}+\frac{1}{5}$

e $\frac{1}{3}+\frac{4}{7}$ **f** $\frac{3}{8}+\frac{4}{5}$ **g** $\frac{2}{5}+\frac{3}{10}$ **h** $\frac{4}{9}+\frac{5}{27}$

Example 8b

3 Evaluate the following.

a $3\frac{1}{4}+1\frac{3}{4}$ **b** $2\frac{3}{5}+\frac{4}{5}$ **c** $1\frac{3}{7}+3\frac{5}{7}$

d $2\frac{1}{3}+4\frac{2}{5}$ **e** $2\frac{5}{7}+4\frac{5}{9}$ **f** $10\frac{5}{8}+7\frac{3}{16}$

4 Evaluate the following.

a $\frac{4}{5}-\frac{2}{5}$ **b** $\frac{4}{5}-\frac{7}{9}$ **c** $\frac{3}{4}-\frac{1}{5}$ **d** $\frac{2}{5}-\frac{3}{10}$

e $\frac{8}{9}-\frac{5}{6}$ **f** $\frac{3}{8}-\frac{1}{4}$ **g** $\frac{5}{9}-\frac{3}{8}$ **h** $\frac{5}{12}-\frac{5}{16}$

Example 8c

5 Evaluate the following.

a $2\frac{3}{4}-1\frac{1}{4}$ **b** $3\frac{5}{8}-2\frac{7}{8}$ **c** $3\frac{1}{4}-2\frac{3}{5}$

d $3\frac{5}{8}-2\frac{9}{10}$ **e** $2\frac{2}{3}-1\frac{5}{6}$ **f** $3\frac{7}{11}-2\frac{3}{7}$

Example 9

6 Evaluate the following (recall that $6=\frac{6}{1}$).

a $\frac{2}{5}\times\frac{3}{7}$ **b** $\frac{3}{5}\times\frac{5}{6}$ **c** $\frac{2}{15}\times\frac{5}{8}$ **d** $\frac{6}{21}\times1\frac{5}{9}$

e $6\times\frac{3}{4}$ **f** $8\times\frac{2}{3}$ **g** $\frac{5}{6}\times9$ **h** $1\frac{1}{4}\times4$

i $2\frac{1}{2}\times6$ **j** $1\frac{5}{8}\times16$ **k** $\frac{10}{21}\times1\frac{2}{5}$ **l** $\frac{25}{44}\times1\frac{7}{15}$

m $1\frac{1}{2}\times1\frac{1}{2}$ **n** $1\frac{1}{2}\times2\frac{1}{3}$ **o** $2\frac{2}{3}\times2\frac{1}{4}$ **p** $1\frac{1}{5}\times1\frac{1}{9}$

7 Write down the reciprocal of these numbers.

a 3 **b** $\frac{5}{7}$ **c** $\frac{1}{8}$ **d** $\frac{13}{9}$

ISBN 978-1-108-77288-4

Example 10

8 Evaluate the following (recall that the reciprocal of 8 is $\frac{1}{8}$).

a $\frac{4}{7} \div \frac{3}{5}$
b $\frac{3}{4} \div \frac{2}{3}$
c $\frac{5}{8} \div \frac{7}{9}$
d $\frac{3}{7} \div \frac{4}{9}$
e $\frac{3}{4} \div \frac{9}{16}$
f $\frac{4}{5} \div \frac{8}{15}$
g $\frac{8}{9} \div \frac{4}{27}$
h $\frac{15}{42} \div \frac{20}{49}$
i $15 \div \frac{5}{6}$
j $6 \div \frac{2}{3}$
k $12 \div \frac{3}{4}$
l $24 \div \frac{3}{8}$
m $\frac{4}{5} \div 8$
n $\frac{3}{4} \div 9$
o $\frac{8}{9} \div 6$
p $14 \div 4\frac{1}{5}$
q $6 \div 1\frac{1}{2}$
r $1\frac{1}{3} \div 8$
s $2\frac{1}{4} \div 1\frac{1}{2}$
t $4\frac{2}{3} \div 5\frac{1}{3}$

PROBLEM-SOLVING	10–12	9(1/2), 10, 12, 13	9(1/2), 12–14

9 Evaluate these mixed-operation problems.

a $\frac{2}{3} \times \frac{1}{3} \div \frac{7}{9}$
b $\frac{4}{5} \times \frac{3}{5} \div \frac{9}{10}$
c $\frac{4}{9} \times \frac{6}{25} \div \frac{1}{150}$
d $2\frac{1}{5} \times \frac{3}{7} \div 1\frac{3}{14}$
e $5\frac{1}{3} \times \frac{13}{24} \div 1\frac{1}{6}$
f $2\frac{4}{13} \times \frac{3}{8} \div 3\frac{3}{4}$

10 To remove impurities, a mining company filters $3\frac{1}{2}$ tonnes of raw material. If $2\frac{5}{8}$ tonnes of impurities are removed, what quantity of material remains?

11 When a certain raw material is processed it produces $3\frac{1}{7}$ tonnes of mineral and $2\frac{3}{8}$ tonnes of waste. How many tonnes of raw material were processed?

12 In a $2\frac{1}{2}$ hour maths exam, $\frac{1}{6}$ of that time is allocated as reading time. How long is the reading time?

13 A road is to be constructed with $15\frac{1}{2}$ m^3 of crushed rock. If a small truck can carry $2\frac{1}{3}$ m^3 of crushed rock, how many truckloads will be needed?

14 Regan worked for $7\frac{1}{2}$ hours in a sandwich shop. Three-fifths of her time was spent cleaning up and the rest serving customers. How much time did she spend serving customers?

REASONING	15	15, 16	16, 17

15 Here is an example involving the subtraction of fractions in which improper fractions are not used.

$$2\frac{1}{2} - 1\frac{1}{3}$$
$$= 2\frac{3}{6} - 1\frac{2}{6}$$
$$= 1\frac{1}{6}$$

Try this technique on the following problem and explain the difficulty that you encounter.

$$2\frac{1}{3} - 1\frac{1}{2}$$

16 **a** A fraction is given by $\frac{a}{b}$. Write down its reciprocal.

b A mixed number is given by $a\frac{b}{c}$. Write an expression for its reciprocal.

17 If a, b and c are integers, simplify the following.

a $\frac{b}{a} \times \frac{a}{b}$

b $\frac{a}{b} \div \frac{b}{a}$

c $\frac{a}{b} \div \frac{a}{b}$

d $\frac{a}{b} \times \frac{c}{a} \div \frac{a}{b}$

e $\frac{abc}{a} \div \frac{bc}{a}$

f $\frac{a}{b} \div \frac{b}{c} \times \frac{b}{a}$

ENRICHMENT: Fraction operation challenge	–	–	18(½)

18 Evaluate the following. Express your answers using improper fractions where necessary.

a $2\frac{1}{3} - 1\frac{2}{5} \times 2\frac{1}{7}$

b $1\frac{1}{4} \times 1\frac{1}{5} - 2\frac{1}{2} \div 10$

c $1\frac{4}{5} \times 4\frac{1}{6} + \frac{2}{3} \times 1\frac{1}{5}$

d $\left(1\frac{2}{3} + 1\frac{3}{4}\right) \div 3\frac{5}{12}$

e $4\frac{1}{6} \div \left(1\frac{1}{3} + 1\frac{1}{4}\right)$

f $\left(1\frac{1}{5} - \frac{3}{4}\right) \times \left(1\frac{1}{5} - \frac{3}{4}\right)$

g $\left(2\frac{1}{4} - 1\frac{2}{3}\right) \times \left(2\frac{1}{4} + 1\frac{2}{3}\right)$

h $\left(3\frac{1}{2} + 1\frac{3}{5}\right) \times \left(3\frac{1}{2} - 1\frac{3}{5}\right)$

i $\left(2\frac{2}{3} - 1\frac{3}{4}\right) \times \left(2\frac{2}{3} + 1\frac{3}{4}\right)$

j $\left(4\frac{1}{2} - 3\frac{2}{3}\right) \div \left(1\frac{1}{3} + \frac{1}{2}\right)$

1E Ratios, rates and best buys CONSOLIDATING

Learning intentions
- To understand how ratios and rates are used to compare quantities
- To be able to write ratios and rates in simplest form using correct notation
- To know how to apply the unitary method to divide a quantity in a ratio
- To be able to use ratios and rates to determine best buys when purchasing products

Fractions, ratios and rates are used to compare quantities. A leaf blower, for example, might require $\frac{1}{6}$ of a litre of oil to make a petrol mix of 2 parts oil to 25 parts petrol, which is an oil to petrol ratio of 2 to 25 or 2 : 25. The leaf blower's fan might then spin at a rate of 1000 revolutions per minute (1000 revs/min).

A vehicle's fuel efficiency is a rate, usually given in litres per 100 km. Fuel consumption rates can vary from 5 L/100 km for a small car up to 25 L/100 km for a large car towing a caravan.

LESSON STARTER The lottery win

\$100 000 is to be divided among three lucky people in a ratio of 2 to 3 to 5 (2 : 3 : 5). How should the money be divided?

- Clearly write down your method and answer. There may be many different ways to solve this problem.
- Write down and discuss the alternative methods suggested by other students in the class.

KEY IDEAS

- **Ratios** are used to compare quantities with the same units.
  - The ratio of a to b is written $a : b$.
  - Ratios in simplest form use whole numbers that have no common factor.
- The **unitary method** involves finding the value of one part of a total.
  - Once the value of one part is found, then the value of several parts can easily be determined.
- A **rate** compares related quantities with different units.
  - The rate is usually written with one quantity compared to a single unit of the other quantity. For example: 50 km per 1 hour or 50 km/h.
- Ratios and rates can be used to determine **best buys** when purchasing products.

ISBN 978-1-108-77288-4

BUILDING UNDERSTANDING

1 State the missing number.

a $2:5=\boxed{}:10$ **b** $3:7=\boxed{}:28$ **c** $8:\boxed{}=640:880$ **d** $\boxed{}:4=7.5:10$

2 Consider the ratio of boys to girls of 4 : 5.

- **a** What is the total number of parts?
- **b** What fraction of the total are boys?
- **c** What fraction of the total are girls?
- **d** If there were 18 students in total, how many of them are boys?
- **e** If there were 18 students in total, how many of them are girls?

3 A car is travelling at a rate (speed) of 80 km/h.

- **a** How far would it travel in:
  - **i** 3 hours? **ii** $\frac{1}{2}$ hour? **iii** $6\frac{1}{2}$ hours?
- **b** How long would it take to travel:
  - **i** 400 km? **ii** 360 km? **iii** 20 km?

4 How much would 1 kg cost if:

a 2 kg costs \$8? **b** 5 kg costs \$15? **c** 4 kg costs \$10?

Example 11 Simplifying ratios

Simplify these ratios.

a 38 : 24 **b** $2\frac{1}{2}:1\frac{1}{3}$ **c** 0.2 : 0.14

SOLUTION	EXPLANATION
a $38:24=19:12$	The HCF of 38 and 24 is 2, so divide both sides by 2.
b $2\frac{1}{2}:1\frac{1}{3}=\frac{5}{2}:\frac{4}{3}$ $=\frac{15}{6}:\frac{8}{6}$ $=15:8$	Write as improper fractions using the same denominator. Then multiply both sides by 6 to write as whole numbers.
c $0.2:0.14=20:14$ $=10:7$	Multiply by 100 to remove all the decimal places and simplify.

Now you try

Simplify these ratios.

a 12 : 8 **b** $2\frac{2}{3}:1\frac{1}{2}$ **c** 0.4 : 0.24

Example 12 Dividing into a given ratio

\$300 is to be divided in the ratio 2 : 3.
Find the value of the larger portion using the unitary method.

SOLUTION	EXPLANATION
Total number of parts is $2 + 3 = 5$. 5 parts = \$300	Use the ratio 2 : 3 to find the total number of parts.
1 part $= \frac{1}{5}$ of \$300 = \$60	Calculate the value of each part, $300 \div 5$.
Larger portion $= 3 \times \$60$ = \$180	Calculate the value of 3 parts.

Now you try

\$250 is to be divided in the ratio 1 : 4.
Find the value of the larger portion using the unitary method.

Example 13 Simplifying rates

Write these rates in simplest form.

a 120 km every 3 hours

b 5000 revolutions in $2\frac{1}{2}$ minutes

SOLUTION	EXPLANATION
a 120 km per 3 hours $= \frac{120}{3}$ km/h = 40 km/h	Divide by 3 to write the rate compared to 1 hour.
b 5000 revolutions per $2\frac{1}{2}$ minutes = 10 000 revolutions per 5 minutes	First multiply by 2 to remove the fraction.
$= \frac{10\,000}{5}$ revs/min = 2000 revs/min	Then divide by 5 to write the rate using 1 minute.

Now you try

Write these rates in simplest form.

a 70 km every 2 hours

b 600 revolutions in $1\frac{1}{2}$ minutes

ISBN 978-1-108-77288-4

Example 14 Finding best buys

Which is better value:

a 5 kg of potatoes for \$3.80 or 3 kg for \$2.20?

b 400 mL of shampoo A at \$3.20 or 320 mL of shampoo B at \$2.85? Compare the cost of 100 mL of each product and then decide which is the better buy. Assume the products are of similar quality.

SOLUTION	EXPLANATION
a **Method A.** Price per kg	
5 kg bag:	
1 kg costs \$3.80 ÷ 5 = \$0.76	Divide each price by the number of kilograms to find the price per kilogram.
3 kg bag:	
1 kg costs \$2.20 ÷ 3 = \$0.73	
∴ the 3 kg bag is better value.	Then compare.
Method B. Amount per \$1	
5 kg bag:	
\$1 buys 5 ÷ 3.8 = 1.32 kg	Divide each amount in kilograms by the cost to find the weight per \$1 spent.
3 kg bag:	
\$1 buys 3 ÷ 2.2 = 1.36 kg	
∴ the 3 kg bag is better value.	Then compare.
b Shampoo A:	
100 mL costs \$3.20 ÷ 4 = \$0.80	Alternatively, divide by 400 to find the cost of 1 mL and then multiply by 100.
Shampoo B:	
100 mL costs \$2.85 ÷ 3.2 = \$0.89	Alternatively, divide by 320 to find the cost of 1 mL and then multiply by 100.
∴ shampoo A is the better buy.	

Now you try

Which is better value:

7 kg of onions for \$5.60 or 5 kg of onions for \$4.25?

Exercise 1E

FLUENCY	1, 2–4(1/2), 5, 7(1/2), 9(1/2)	2–4(1/2), 6, 7(1/2), 8, 9(1/2), 10	2–4(1/3), 6–10(1/3)

Example 11

1 Simplify these ratios.

a 44 : 8
b $3\frac{1}{5} : 1\frac{1}{2}$
c 0.3 : 0.12

Example 11

2 Simplify these ratios.

a 6 : 30
b 8 : 20
c 42 : 28
d 52 : 39
e $1\frac{1}{2} : 3\frac{1}{3}$
f $2\frac{1}{4} : 1\frac{2}{5}$
g $\frac{3}{8} : 1\frac{3}{4}$
h $1\frac{5}{6} : 3\frac{1}{4}$
i 0.3 : 0.9
j 0.7 : 3.5
k 1.6 : 0.56
l 0.4 : 0.12

3 Write each of the following as a ratio in simplest form. (*Hint*: Convert to the same unit first.)

a 80c : \$8
b 90c : \$4.50
c 80 cm : 1.2 m
d 0.7 kg : 800 g
e 2.5 kg : 400 g
f 30 min : 2 hours
g 45 min : 3 hours
h 4 hours : 50 min
i 40 cm : 2 m : 50 cm
j 80 cm : 600 mm : 2 m
k 2.5 hours : 1.5 days
l 0.09 km : 300 m : 1.2 km

Example 12

4 Divide \$500 in these ratios using the unitary method.

a 2 : 3
b 3 : 7
c 1 : 1
d 7 : 13

5 420 g of flour is to be divided in a ratio of 7 : 3 for two different recipes. Find the smaller amount.

6 Divide \$70 in these ratios.

a 1 : 2 : 4
b 2 : 7 : 1
c 8 : 5 : 1

Example 13

7 Write these rates in simplest form.

a 150 km in 10 hours
b 3000 revolutions in $1\frac{1}{2}$ minutes
c 15 swimming strokes in $\frac{1}{3}$ of a minute
d 56 metres in 4 seconds
e 180 mL in 22.5 hours
f 207 heart beats in $2\frac{1}{4}$ minutes

8 Hamish rides his bike at an average speed of 22 km/h. How far does he ride in:

a $2\frac{1}{2}$ hours?
b $\frac{3}{4}$ hours?
c 15 minutes?

Example 14a

9 Determine the best buy (cheaper deal) in each of the following.

a 2 kg of washing powder for \$11.70 or 3 kg for \$16.20
b 1.5 kg of Red Delicious apples for \$4.80 or 2.2 kg of Royal Gala apples for \$7.92
c 2.4 litres of orange juice for \$4.20 or 3 litres of orange juice for \$5.40
d 0.7 GB of internet usage for \$14 or 1.5 GB for \$30.90 with different service providers

Example 14b

10 Find the cost of 100 g of each product below and then decide which is the best buy (cheaper deal).

a 300 g of coffee A at \$10.80 or 220 g of coffee B at \$8.58
b 600 g of pasta A for \$7.50 or 250 g of pasta B for \$2.35
c 1.2 kg of cereal A for \$4.44 or 825 g of cereal B for \$3.30

PROBLEM-SOLVING	11, 12	12–15	14–18

11 Kirsty manages a fresh fruit and vegetable market. Each day she buys watermelons and mangoes in the ratio of 3 : 2. How many watermelons did she buy if, on one day, the total number of watermelons and mangoes was 200?

12 If a prize of \$6000 was divided among Georgia, Leanne and Maya in the ratio of 5 : 2 : 3, how much did each girl get?

13 When a crate of twenty 375 mL soft drink cans is purchased it works out to be \$1.68 per litre. If a crate of 30 of the same cans is advertised as being a saving of 10 cents per can compared with the 20-can crate, calculate how much the 30-can crate costs.

14 The dilution ratio for a particular chemical with water is 2 : 3 (chemical to water). If you have 72 litres of chemical, how much water is needed to dilute the chemical?

15 Amy, Belinda, Candice and Diane invested money in the ratio of 2 : 3 : 1 : 4 in a publishing company. If the profit was shared according to their investment, and Amy's profit was \$2400, find the profit each investor made.

16 Julie is looking through the supermarket catalogue for her favourite cookies-and-cream ice-cream. The catalogue is advertising 2 L of triple-chocolate ice-cream for \$6.30. The cookies-and-cream ice-cream is usually \$5.40 for 1.2 L. What saving does there need to be on the price of the 1.2 L container of cookies-and-cream ice-cream for it to be of equal value to the 2 L triple-chocolate container?

17 The ratio of the side lengths of one square to another is 1 : 2. Find the ratio of the areas of the two squares.

18 A quadrilateral (with angle sum 360°) has interior angles in the ratio 1 : 2 : 3 : 4. Find the size of each angle.

REASONING	19	19, 20	20, 21

19 2.5 kg of cereal A costs \$4.80 and 1.5 kg of cereal B costs \$2.95. Write down at least two different methods for finding which cereal is the better buy (cheaper deal).

20 If $a : b$ is in simplest form, state whether the following are true or false.
- **a** a and b must both be odd.
- **b** a and b must both be prime.
- **c** At least one of a or b is odd.
- **d** The HCF of a and b is 1.

21 A ratio is $a : b$ with $a < b$ and a and b are positive integers. Write an expression for:
- **a** the total number of parts
- **b** the fraction of the smaller quantity out of the total
- **c** the fraction of the larger quantity out of the total.

ENRICHMENT: Mixing drinks	–	–	22

22 Four jugs of cordial have a cordial to water ratio as shown in the table. The total volume is also shown.

Jug	Cordial to water ratio	Total volume
1	1 : 5	600 mL
2	2 : 7	900 mL
3	3 : 5	400 mL
4	2 : 9	330 mL

- **a** How much cordial is in:
  - **i** jug 1?
  - **ii** jug 2?
- **b** How much water is in:
  - **i** jug 3?
  - **ii** jug 4?
- **c** Jugs 1 and 2 were mixed together to give 1500 mL of drink.
  - **i** How much cordial is in the drink?
  - **ii** Find the ratio of cordial to water in the drink.
- **d** Find the ratio of cordial to water when the following jugs are mixed.
  - **i** jugs 1 and 3
  - **ii** jugs 2 and 3
  - **iii** jugs 2 and 4
  - **iv** jugs 3 and 4
- **e** Which combination of two jugs gives the strongest cordial to water ratio?

ISBN 978-1-108-77288-4

1F Percentages and money CONSOLIDATING

Learning intentions

- To know how to convert between fractions or decimals and percentages
- To be able to express one quantity as a percentage of another quantity
- To be able to find a percentage of a quantity
- To be able to use division or the unitary method to find an original amount from a percentage of the amount

We use percentages for many different things in our daily lives. Some examples are loan rates, the interest rate on a term deposit and the percentage discount on purchases.

We know from our previous studies that a percentage is a number expressed out of 100. ‘Per cent’ comes from the Latin term *per centum* and means ‘out of 100’.

Businesses convert fractions to percentages to make comparison easier, such as comparing the fractions of total profit made by each department or branch of a business.

LESSON STARTER Which is the largest piece?

Four people receive the following portions of a cake:

Milly	25.5%
Tom	$\frac{1}{4}$
Adam	0.26
Mai	what’s left

a Which person gets the most cake and why?

b How much cake does Mai get? What is her portion written as a percentage, a decimal and a fraction?

KEY IDEAS

- To express a number as a **percentage**, *multiply* by 100.
- To express a percentage as a **fraction** or **decimal**, *divide* by 100.
- A percentage of a number can be found using multiplication.
  For example: $\begin{aligned}25\%\text{ of }\$26 &= 0.25 \times \$26\\ &= \$6.50\end{aligned}$
- One quantity can be expressed as a percentage of another quantity by writing as a fraction using the same units and converting to a percentage.
- To find an original amount, use the **unitary method** or use division.
  For example: 3% of an amount is \$36.
  - Using the unitary method: 1% of the amount is $\$36 \div 3 = \$12$
    $\therefore$ 100% of the amount is $\$12 \times 100 = \$1200$
  - Using division: 3% of the amount is \$36
    $$\begin{aligned}0.03 \times \text{amount} &= \$36\\ \text{amount} &= \$36 \div 0.03\\ &= \$1200\end{aligned}$$

BUILDING UNDERSTANDING

1 Divide these percentages by 100 to express them as fractions. For example, $9\% = \frac{9}{100}$. Simplify where possible.

a 3% **b** 11% **c** 35% **d** 8%

2 Divide these percentages by 100 to express them as decimals. For example, $9\% = 0.09$.

a 4% **b** 23% **c** 86% **d** 46.3%

3 Express these simple decimals and fractions as percentages.

a 0.5 **b** 0.25 **c** $\frac{3}{4}$ **d** $\frac{1}{5}$

Example 15 Converting between percentages, decimals and fractions

a Express 0.45 as a percentage.
b Express 25% as a decimal.
c Express $3\frac{1}{4}\%$ as a fraction.

SOLUTION	EXPLANATION
a $0.45 \times 100 = 45$ $\therefore\ 0.45 = 45\%$	Multiply by 100. This moves the decimal point two places to the right.
b $25\% = 25 \div 100 = 0.25$	Divide by 100. This moves the decimal point two places to the left.

c $3\frac{1}{4}\% = 3\frac{1}{4} \div 100$
$= \frac{13}{4} \times \frac{1}{100}$
$= \frac{13}{400}$

Divide by 100.
Write the mixed number as an improper fraction and multiply by the reciprocal of 100 (i.e. $\frac{1}{100}$).

Now you try

a Express 0.75 as a percentage.
b Express 60% as a decimal.
c Express $2\frac{3}{4}\%$ as a fraction.

Example 16 Expressing one quantity as a percentage of another quantity

Express 50c as a percentage of $2.50.

SOLUTION

$$\text{50c out of \$2.50} = \frac{\overset{1}{\cancel{50}}}{\underset{5}{\cancel{250}}} \times 100\% = 20\%$$

EXPLANATION

Convert to the same units ($2.50 = 250c) and write as a fraction. Multiply by 100, cancelling first.

Now you try

Express 20c as a percentage of $1.60.

Example 17 Finding a percentage of a quantity

Find 15% of $35.

SOLUTION

$$15\% \text{ of } \$35 = \frac{\overset{3}{\cancel{15}}}{\underset{20}{\cancel{100}}} \times \$35 = \$5.25$$

EXPLANATION

Write the percentage as a fraction out of 100 and multiply by $35.
(Note: ‘of’ means to ‘multiply’.)

Now you try

Find 35% of $24.

ISBN 978-1-108-77288-4

Example 18 Finding the original amount

Determine the original amount if 5% of the amount is \$45.

SOLUTION	EXPLANATION
Method 1: Unitary method 5% of the amount = \$45 1% of the amount = \$9 100% of the amount = \$900 So the original amount is \$900.	To use the unitary method, find the value of 1 part or 1% and then multiply by 100 to find 100%.
Method 2: Division 5% of the amount = \$45 0.05 × amount = \$45 amount = \$45 ÷ 0.05 = \$900 So the original amount is \$900.	Write 5% as a decimal and set up an equation using multiplication for 'of'. Then divide both sides by 0.05 to find the original amount.

Now you try

Determine the original amount if 4% of the amount is \$12.

Exercise 1F

FLUENCY	1, 2–4($\frac{1}{2}$), 5, 6, 8($\frac{1}{2}$)	2–4($\frac{1}{2}$), 5, 6–8($\frac{1}{2}$)	2–4($\frac{1}{4}$), 6–9($\frac{1}{3}$)

Example 15a **1** **a** Express each decimal as a percentage.

i 0.55 **ii** 0.91

Example 15b **b** Express each percentage as a decimal.

i 75% **ii** 61%

Example 15c **c** Express each percentage as a fraction.

i $2\frac{1}{2}\%$ **ii** $5\frac{1}{3}\%$

Example 15a **2** Express each decimal as a percentage.

a 0.34 **b** 0.4 **c** 0.06 **d** 0.7
e 1 **f** 1.32 **g** 1.09 **h** 3.1

Example 15b **3** Express each percentage as a decimal.

a 67% **b** 30% **c** 250% **d** 8%
e $4\frac{3}{4}\%$ **f** $10\frac{5}{8}\%$ **g** $30\frac{2}{5}\%$ **h** $44\frac{1}{4}\%$

Example 15c **4** Express each part of Question **3** as a simplified fraction.

5 Copy and complete these tables. Use the simplest form for fractions.

Percentage	Fraction	Decimal
10%		
	$\frac{1}{2}$	
5%		
		0.25
		0.2
	$\frac{1}{8}$	
1%		
	$\frac{1}{9}$	
		$0.\dot{2}$

Percentage	Fraction	Decimal
	$\frac{3}{4}$	
15%		
		0.9
37.5%		
$33\frac{1}{3}\%$	$\frac{1}{3}$	
$66\frac{2}{3}\%$		
		0.625
	$\frac{1}{6}$	

Example 16

6 Convert each of the following to a percentage.

a \$3 out of \$12 **b** \$6 out of \$18 **c** \$0.40 out of \$2.50

d \$44 out of \$22 **e** \$140 out of \$5 **f** 45c out of \$1.80

Example 17

7 Find:

a 10% of \$360 **b** 50% of \$420 **c** 75% of 64 kg

d 12.5% of 240 km **e** 37.5% of 40 apples **f** 87.5% of 400 m

g $33\frac{1}{3}\%$ of 750 people **h** $66\frac{2}{3}\%$ of 300 cars **i** $8\frac{3}{4}\%$ of \$560

Example 18

8 Determine the original amount when:

a 10% of the amount is \$12 **b** 6% of the amount is \$42

c 3% of the amount is \$9 **d** 40% of the amount is \$2.80

e 90% of the amount is \$0.18 **f** 35% of the amount is \$140

9 Determine the value of x in the following.

a 10% of x is \$54 **b** 15% of x is \$90 **c** 25% of x is \$127

d 18% of x is \$225 **e** 105% of x is \$126 **f** 110% of x is \$44

PROBLEM-SOLVING 10, 11 11–13 13–15

10 Bad weather stopped a cricket game for 35 minutes of a scheduled $3\frac{1}{2}$ hour match. What percentage of the scheduled time was lost?

11 Joe lost 4 kg and now weighs 60 kg. What percentage of his original weight did he lose?

12 About 80% of the mass of the human body is water. If Clare weighs 60 kg, how many kilograms of water make up her body weight?

13 In a class of 25 students, 40% have been to England. How many students have not been to England?

14 20% of the cross country runners in a school team weigh between 60 kg and 70 kg. If 4% of the school's 1125 students are in the cross country team, how many students in the team weigh between 60 and 70 kg?

15 One week Grace spent 16% of her weekly wage on a new bookshelf that cost \$184. What is her weekly wage?

REASONING	16	16	16, 17

16 Consider the equation $P\%$ of $a = b$ (like 20% of $40 = 8$ or 150% of $22 = 33$).

a For what value of P is $P\%$ of $a = a$?

b For what values of P is $P\%$ of $a < a$?

c For what values of P is $P\%$ of $a > a$?

17 What can be said about the numbers x and y if:

a 10% of $x =$ 20% of y?

b 10% of $x =$ 50% of y?

c 5% of $x =$ 3% of y?

d 14% of $x =$ 5% of y?

ENRICHMENT: More than 100%	–	–	18

18 **a** Find 120% of 60.

b Determine the value of x if 165% of x is 1.5.

c Write 2.80 as a percentage.

d Write 325% as a fraction.

e An investment of \$2000 in a bank account increases to \$5000 over a period of time. By how much has the amount increased as a percentage?

1G Percentage increase and decrease CONSOLIDATING

Learning intentions
- To know how to increase or decrease an amount by a percentage using multiplication
- To understand that increasing and then decreasing by the same percentage does not return the original amount
- To be able to find the percentage change when a value increases or decreases
- To be able to find the percentage error in measurements
- To be able to find the original amount from the new amount after a percentage increase or decrease

Percentages are often used to describe by how much a quantity has increased or decreased. The price of a car in the new year might be increased by 5%. On a \$70 000 car, this equates to a \$3500 increase. The price of a shirt might be marked down by 30% and if the shirt originally cost \$60, this provides an \$18 discount. It is important to note that the increase or decrease is calculated on the original amount.

Tourists often change money into another currency. A foreign exchange trader buys and sells money on the International Money Market. Up-to-date currency exchange rates and percentage increases and decreases are available online.

LESSON STARTER The quicker method

Nicky and Mila consider this question: When \$250 is increased by 15%, what is the final amount?
Nicky puts his solution on the board with two steps.

Step 1. $15\% \text{ of } \$250 = 0.15 \times \$250$
$= \$37.50$

Step 2. $\text{Final amount} = \$250 + \$37.50$
$= \$287.50$

Mila says that the same problem can be solved with only one step using the number 1.15.

- Can you explain Mila's method? Write it down.
- What if the question was altered so that \$250 is decreased by 15%. How would Nicky and Mila's methods work in this case?
- Which of the two methods do you prefer and why?

KEY IDEAS

- To increase an amount by a given percentage, multiply by 100% plus the given percentage.
  For example: to increase by 30%, multiply by $100\% + 30\% = 130\% = 1.3$
- To decrease an amount by a given percentage, multiply by 100% minus the given percentage.
  For example: to decrease by 25%, multiply by $100\% - 25\% = 75\% = 0.75$
- To find a **percentage change** or **absolute percentage difference** use

$$\text{Percentage change} = \frac{\text{change}}{\text{original amount}} \times 100\%$$

  - **Percentage error** is calculated in the same way.

$$\%\ \text{Error} = \frac{\text{difference between measured value and theoretical value}}{\text{theoretical value}} \times 100\%$$

BUILDING UNDERSTANDING

1 State the missing number.
  - **a** To increase a number by 40% multiply by _____.
  - **b** Multiplying by 1.21 increases a number by _____ .
  - **c** To decrease a number by 73% multiply by _____.
  - **d** Multiplying by 0.94 decreases a number by _____ .

2 The price of a watch increases from \$120 to \$150.
  - **a** What is the price increase?
  - **b** Express this increase as a percentage of the original price.

3 A person's weight decreases from 108 kg to 96 kg.
  - **a** What is the weight decrease?
  - **b** Express this decrease as a percentage of the original weight. Round to one decimal place.

Example 19 Increasing by a percentage

Increase \$70 by 15%.

SOLUTION	EXPLANATION
$100\% + 15\% = 115\%$ $= 1.15$	First add 15% to 100%.
$\$70 \times 1.15 = \$80.50$	Multiply by 1.15 to give \$70 plus the increase in one step.

Now you try

Increase \$40 by 25%.

Example 20 Decreasing by a percentage

Decrease \$5.20 by 40%.

SOLUTION	EXPLANATION
$100\% - 40\% = 60\%$ $= 0.6$	First subtract the 40% from 100% to find the percentage remaining.
$\$5.20 \times 0.6 = \$3.12$	Multiply by $60\% = 0.6$ to get the result.

Now you try

Decrease \$6.80 by 30%.

Example 21 Finding a percentage change

a The price of a mobile phone increased from \$250 to \$280. Find the percentage increase.

b The population of a town decreases from 3220 to 2985. Find the percentage decrease and round to one decimal place.

SOLUTION	EXPLANATION
a Increase $= \$280 - \$250$ $= \$30$	First find the actual increase.
Percentage increase $= \dfrac{30}{250} \times \dfrac{100}{1}\%$ $= 12\%$	Divide the increase by the original amount and multiply by 100%.
b Decrease $= 3220 - 2985$ $= 235$	First find the actual decrease.
Percentage decrease $= \dfrac{235}{3220} \times \dfrac{100}{1}\%$	Divide the decrease by the original population.
$= 7.3\%$ (to 1 d.p.)	Round as indicated.

Now you try

a The price of a TV increased from \$300 to \$375. Find the percentage increase.

b The population of a town decreases from 6250 to 5500. Find the percentage decrease.

Example 22 Finding the original amount

After rain, the volume of water in a tank increased by 24% to 2200 L. How much water was in the tank before it rained? Round to the nearest litre.

SOLUTION	EXPLANATION
$100\% + 24\% = 124\%$ $= 1.24$	Write the total percentage as a decimal.
Original volume $\times 1.24 = 2200$ $\therefore$ original volume $= 2200 \div 1.24$ $= 1774$ litres	The original volume is increased by 24% to give 2200 litres. Divide to find the original volume. Round as indicated.

Now you try

The area of a rectangular deck is increased by 32% to 300 m^2. What was the area of the original deck? Round to the nearest square metre.

ISBN 978-1-108-77288-4

Exercise 1G

FLUENCY	1, 2–3($\frac{1}{2}$), 4–6	2–3($\frac{1}{2}$), 4, 6, 7	2–3($\frac{1}{4}$), 5, 7, 8

Example 19

1 Increase:
- **a** $50 by 20%
- **b** $124 by 24%

Example 19

2 Increase the given amounts by the percentage given in the brackets.
- **a** $50 (5%)
- **b** 35 min (8%)
- **c** 250 mL (50%)
- **d** 1.6 m (15%)
- **e** 24.5 kg (12%)
- **f** 25 watts (44%)
- **g** $13 000 (4.5%)
- **h** $1200 (10.2%)

Example 20

3 Decrease the given amounts by the percentage given in the brackets.
- **a** 24 cm (20%)
- **b** 35 cm (30%)
- **c** 42 kg (7%)
- **d** 55 min (12%)
- **e** $90 (12.8%)
- **f** 220 mL (8%)
- **g** 25°C (28%)
- **h** $420 (4.2%)

Example 21a

4 The length of a bicycle sprint race is increased from 800 m to 1200 m. Find the percentage increase.

Example 21a

5 From the age of 10 to 17, Nick's height increased from 125 cm to 180 cm. Find the percentage increase.

Example 21b

6 The temperature at night decreased from 25°C to 18°C. Find the percentage decrease.

Example 21b

7 Brett, a rising sprint star, lowered his 100 m time from 13 seconds flat to 12.48 seconds. Find the percentage decrease.

8 Find the percentage change in these situations, rounding to one decimal place in each case.
- **a** 22 g increases to 27 g
- **b** 86°C increases to 109°C
- **c** 136 km decreases to 94 km
- **d** $85.90 decreases to $52.90

PROBLEM-SOLVING	9–11	9–12	11–14

Example 22

9 After a price increase of 20% the cost of entry to a museum rose to $25.80. Find the original entry fee.

Example 22

10 The average attendance at a sporting match rose by 8% in the past year to 32 473. Find the average attendance in the previous year to the nearest integer.

11 A car when resold had decreased in value by 38% to $9235. What was the original price of the car to the nearest dollar?

12 Calculate the % error of these experimental measures compared to the theoretical measures.

	Experimental	Theoretical
a	22 cm	20 cm
b	4.5 L	4 L
c	1.05 sec	1.25 sec
d	58 m^2	64 m^2

13 The total price of an item including GST (at 10%) is \$120. How much GST is paid to the nearest cent?

14 A consultant charges a school a fee of \$300 per hour including GST (at 10%). The school hires the consultant for 2 hours but can claim back the GST from the tax office. Find the net cost of the consultant for the school to the nearest cent.

REASONING	15	15, 16	16–18

15 An investor starts with \$1000.

- **a** After a bad day the initial investment is reduced by 10%. Find the balance at the end of the day.
- **b** The next day is better and the balance is increased by 10%. Find the balance at the end of the second day.
- **c** The initial amount decreased by 10% on the first day and increased by 10% on the second day. Explain why the balance on the second day didn't return to \$1000.

16 During a sale, all travel guides in a bookstore are reduced from \$30 by 20%. What percentage increase is required to take the price back to \$30?

17 The cost of an item is reduced by 50%. What percentage increase is required to return to its original price?

18 The cost of an item is increased by 75%. What percentage decrease is required to return to its original price? Round to two decimal places.

ENRICHMENT: Repeated increase and decrease	–	–	19, 20

19 If the cost of a pair of shoes was increased three times by 10%, 15% and 8% from an original price of \$80, then the final price would be

$\$80 \times 1.10 \times 1.15 \times 1.08 = \$109.30$

Use a similar technique to find the final price of these items. Round the answer to the nearest cent.

- **a** Skis starting at \$450 and increasing by 20%, 10% and 7%
- **b** A computer starting at \$2750 and increasing by 6%, 11% and 4%
- **c** A DVD player starting at \$280 and decreasing by 10%, 25% and 20%
- **d** A circular saw starting at \$119 and increasing by 18%, 37% and 11%

20 If an amount is increased by the same percentage each time, powers can be used.
For example, 50 kg increased by 12% three times would increase to

$$\begin{aligned} &50\text{ kg} \times 1.12 \times 1.12 \times 1.12 \\ &= 50\text{ kg} \times (1.12)^3 \\ &= 70.25\text{ kg (to 2 d.p.)} \end{aligned}$$

Use a similar technique to find the final value in these situations. Round the answer to two decimal places.

- **a** A rat with an initial mass of 60 grams grows at a rate of 10% every month for 3 months.
- **b** The cost of a new lawnmower with an initial cost of \$80 000 increases by 5% every year for 4 years.
- **c** The value of a house initially at \$380 000 decreases by 4% per year for 3 years.
- **d** As a result of being sharpened, the length of a pencil initially 16 cm long decreases by 15% each week for 5 weeks.

Progress quiz

1A **1** Evaluate the following.

a $-45 + (-3 \times 6 + 9)$ **b** $-4 \times 8 \div (-2) - 2^3$

1B **2** Round each of these numbers as indicated in the brackets.

a 3.45678 (2 decimal places) **b** 45.89985 (1 decimal place)

c 0.007856473 (2 significant figures) **d** 46 789 000 (3 significant figures)

1C **3** Write these fractions as decimals.

a $\frac{3}{4}$ **b** $\frac{4}{5}$ **c** $\frac{7}{20}$ **d** $\frac{1}{3}$

1C **4** Write these decimals as fractions.

a 0.9 **b** 0.85 **c** 0.125

1D **5** Evaluate the following.

a $\frac{1}{2} + \frac{5}{6}$ **b** $\frac{6}{7} - \frac{2}{3}$ **c** $1\frac{1}{2} \times \frac{4}{5}$ **d** $\frac{5}{8} \div \frac{3}{10}$

1E **6** Simplify these ratios.

a \$24 to 80 cents **b** 45 : 81 **c** 2.4 : 0.36 **d** $\frac{3}{4} : 8$

1E **7** Divide:

a \$400 in the ratio of 5 : 3

b 6 kg in the ratio of 3 : 7

c 1000 cm in the ratio of 4 : 5 : 6

1E **8** Write these rates in simplest form.

a \$350 in 5 hours

b 200 km in 2.5 hours

1E **9** Which is the best buy (cheaper deal)?

Product A, which costs \$3.45 for 500 grams, or product B, which costs \$4.38 for 680 grams.

1F **10** Write the following as percentages.

a $\frac{4}{5}$ **b** 0.96 **c** $3\frac{3}{4}$ **d** 40 cents of \$5

1F **11** Find 34% of 6000 cm.

1F **12** Determine the original amount if 8% of the amount is \$32.

1G **13** **a** Increase \$450 by 12%.

b Decrease 500 kg by 5%.

1G **14** Find the percentage change if 40 g is reduced to 24 g.

1G **15** The volume of a tank is increased by 12% to 952 L. Find the original volume.

ISBN 978-1-108-77288-4

1H Profits and discounts

Learning intentions

- To know the meaning of the key terms profit, discount, mark-up and selling price
- To be able to apply percentages to calculate the selling price based on discounts and mark-ups
- To be able to determine the percentage profit or loss made on a sale

Percentages are widely used in the world of finance. Profits, losses, commissions, discounts and taxation are often expressed and calculated using percentages.

Percentages are used by businesses to describe profits and losses.

LESSON STARTER The best discount

Two adjacent shops are selling the same jacket at a discounted price. The recommended retail price for the jacket is the same for both shops. Each shop has a sign near the jacket with the given details:

- Shop A: Discounted by 25%
- Shop B: Reduced by 20% and then take a further 10% off that discounted price.

Which shop offers the bigger discount?
Is the difference between the discounts equal to 5% of the retail price?

KEY IDEAS

- **Profit** is the amount of money made on a sale. If the profit is negative we say that a loss has been made.
  - Profit = selling price − cost price
- **Mark-up** is the amount added to the cost price to produce the selling price.
  - Selling price = cost price + mark-up
- The **percentage profit** or loss can be found by dividing the profit or loss by the cost price and multiplying by 100.
  - $\%\,\text{Profit or Loss} = \dfrac{\text{profit or loss}}{\text{cost price}} \times 100\%$
- **Discount** is the amount by which an item is marked down.
  - New price = original price − discount
  - Discount = % discount × original price

BUILDING UNDERSTANDING

① State the missing numbers in the table.

Cost price ($)	7	18		3250
Selling price ($)	10	15.50	11.80	
Profit/Loss ($)			4.50 profit	1180 loss

② The following percentage discounts are given on the price of various products. State the sale price as a percentage of the original price.

a 10%

b 15%

Example 23 Determining profit

A manufacturer produces an item for $400 and sells it for $540.

a Determine the profit made.

b Express this profit as a percentage of the cost price.

SOLUTION

a Profit = $540 − $400
= $140

b $\%\,\text{profit} = \dfrac{140}{400} \times 100\%$
$= 35\%$

EXPLANATION

Profit = selling price − cost price

$\%\,\text{profit} = \dfrac{\text{profit}}{\text{cost price}} \times 100\%$

Now you try

A manufacturer produces an item for $320 and sells it for $464.

a Determine the profit made.

b Express this profit as a percentage of the cost price.

Example 24 Calculating selling price from mark-up

An electrical store marks up all entertainment systems by 30%.
If the cost price of an entertainment system is $8000, what will be its selling price?

SOLUTION

$$\begin{aligned}\text{Selling price} &= 130\% \text{ of cost price}\\ &= 1.3 \times \$8000\\ &= \$10\,400\end{aligned}$$

EXPLANATION

As there is a 30% mark-up added to the cost price (100%), it follows that the selling price is 130% of the cost price.

Alternative method:

$$\begin{aligned}\text{Mark-up} &= 30\% \text{ of } \$8000\\ &= 0.3 \times \$8000\\ &= \$2400\end{aligned}$$

Change percentage to a decimal and evaluate.

$$\begin{aligned}\therefore\ \text{selling price} &= \$8000 + \$2400\\ &= \$10\,400\end{aligned}$$

Selling price = cost price + mark-up.

Now you try

A nursery marks up all plants by 40%.
If the cost price of a plant is \$12, what will be its selling price?

Example 25 Finding the discount amount

Harvey Norman advertises a 15% discount on all audio-visual equipment as a Christmas special. Find the sale price on a projection system that has a marked price of \$18 000.

SOLUTION

$$\begin{aligned}\text{New price} &= 85\% \text{ of } \$18\,000\\ &= 0.85 \times \$18\,000\\ &= \$15\,300\end{aligned}$$

EXPLANATION

Discounting by 15% means the new price is 85%, i.e. (100 − 15)%, of the original price.

Alternative method:

$$\begin{aligned}\text{Discount} &= 15\% \text{ of } \$18\,000\\ &= 0.15 \times \$18\,000\\ &= \$2700\end{aligned}$$

Change the percentage to a decimal and evaluate.

$$\begin{aligned}\therefore\ \text{new price} &= \$18\,000 - \$2700\\ &= \$15\,300\end{aligned}$$

New price is original price minus discount.

Now you try

A shop has items discounted by 16% for its Easter sale. Find the sale price if an item was marked at \$204.

ISBN 978-1-108-77288-4

Example 26 Calculating sale saving

Big W discounts a toy by 10% in a sale. If the sale price was \$10.80, what was the original price?

SOLUTION	EXPLANATION
Let \$$x$ be the original price. $0.9 \times x = 10.8$ $x = 10.8 \div 0.9$ $x = 12$	The percentage discount = 100% − 10% = 90% = 0.9. Thus \$10.80 is 90% of the original price. Write an equation representing this and solve.
The original price was \$12.	Write the answer in words.

Now you try

A car is discounted by 20%. If the sale price was \$32 000, what was the original price?

Exercise 1H

FLUENCY	1–6, 9, 11	2(½), 3, 4, 6, 7, 9–11	3, 5, 6, 8, 9, 11

Example 23

1 A shop buys an item for \$500 and sells it for \$575.
  a Determine the profit made.
  b Express this profit as a percentage of the cost price.

Example 23

2 A manufacturer produces and sells items for the prices shown.
  i Determine the profit made.
  ii Express this profit as a percentage of the cost price.
  a Cost price \$10, selling price \$12
  b Cost price \$20, selling price \$25
  c Cost price \$120, selling price \$136.80
  d Cost price \$1400, selling price \$3850

3 Dom runs a pizza business. Last year he took in \$88 000 and it cost him \$33 000 to run the business. What is his percentage profit for the year? Round to two decimal places.

4 It used to take 20 hours to fly to Los Angeles. It now takes 12 hours. What is the percentage decrease in travel time?

5 Rob goes to the races with \$600 in his pocket. He leaves at the end of the day with \$45. What is his percentage loss?

Example 24

6 Helen owns a handicrafts store that has a policy of marking up all of its items by 25%. If the cost price of one article is \$30, what will be its selling price?

Example 24

7 Lenny marks up all computers in his store by 12.5%. If a computer cost him \$890, what will be the selling price of the computer, to the nearest cent?

8 A used-car dealer purchases a vehicle for \$13 000 and sells it for \$18 500. Determine the percentage mark-up on the vehicle to one decimal place.

Example 25

9 A store is offering a 15% discount for customers who pay with cash. Rada wants a microwave oven marked at \$175. How much will she pay if she is paying with cash?

Example 25

10 A camera store displays a camera marked at \$595 and a lens marked at \$380. Sam is offered a discount of 22% if he buys both items. How much will he pay for the camera and lens?

Example 26

11 A refrigerator is discounted by 25%. If Paula pays \$460 for it, what was the original price? Round to the nearest cent.

PROBLEM-SOLVING	12	12, 13	13, 14

12 A store marks up a \$550 widescreen television by 30%. During a sale it is discounted by 20%. What is the percentage change in the original price of the television?

13 An armchair was purchased for a cost price of \$380 and marked up to a retail price. It was then discounted by 10% to a sale price of \$427.50. What is the percentage mark-up from the cost price to the sale price?

14 Pairs of shoes are manufactured for \$24. They are sold to a warehouse with a mark-up of 15%. The warehouse sells the shoes to a distributor after charging a holding fee of \$10 per pair. The distributor sells them to 'Fine Shoes' for a percentage profit of 12%. The store then marks them up by 30%.

a Determine the price of a pair of shoes if you buy it from one of the 'Fine Shoes' stores (round to the nearest 5 cents).

b What is the overall percentage mark-up of a pair of shoes to the nearest whole per cent?

REASONING	15	15, 16	16, 17

15 The price of an item to be sold includes a percentage mark-up as well as a sale discount. Does it make a difference in which order the mark-up and discount occur? Explain your answer.

16 Depreciation relates to a reduction in value. A computer depreciates in value by 30% in its first year. If its original value is \$3000, find its value after one year.

17 John buys a car for \$75 000. The value of the car depreciates at 15% per year. After 1 year the car is worth 85% of its original value, i.e. 85% of $\$75\,000 = 0.85 \times 75\,000 = \$63\,750$.

a What is the value of the car, to the nearest cent, after:

i 2 years? **ii** 5 years?

b After how many years will the car first be worth less than \$15 000?

ENRICHMENT: Deposits and discounts	–	–	18

18 A car company offers a special discount deal. After the cash deposit is paid, the amount that remains to be paid is discounted by a percentage that is one tenth of the deposit percentage.
For example, a deposit of \$8000 on a \$40 000 car represents 20% of the cost. The remaining \$32 000 will be discounted by 2%.

Find the amount paid for each car given the following car price and deposit. Round to two decimal places where necessary.

a Price = \$35 000, deposit = \$7000 **b** Price = \$45 000, deposit = \$15 000

c Price = \$28 000, deposit = \$3500 **d** Price = \$33 400, deposit = \$5344

e Price = \$62 500, deposit = \$5000 **f** Price = \$72 500, deposit = \$10 150

1I Income and taxation

Learning intentions

- To understand the different ways in which income can be earned
- To be able to calculate income based on wages, overtime, salaries and commission
- To know the difference between gross and net income
- To understand that employees pay tax and to be able to calculate tax based on a person's income

A laboratory technician might receive a 1% pay rise giving her a $600 p.a. increase in gross pay. This amount is then decreased by 32.5% tax, giving the technician a net pay increase of $405 p.a.

Most people's income is made up largely of the money they receive from their paid work – their job. Depending on the job, this payment can be made using a number of different methods. Many professional workers will receive an annual fixed *salary* which may be paid monthly or fortnightly. Casual workers, including those working in the retail area or restaurants, may receive a *wage* where they are paid a rate per hour worked. This rate may be higher out of regular working hours such as weekends or public holidays. Many sales people, including some real estate agents, may receive a weekly fee (a *retainer*) but may also receive a set percentage of the amount of sales they make (a *commission*). From their income, people have to pay living costs such as electricity, rent, groceries and other items. In addition, they have to pay tax to the government, which funds many of the nation's infrastructure projects and welfare. The method in which this tax is paid from their income may also vary.

LESSON STARTER Which job pays better?

Ben and Nick are both looking for part-time work and they spot the following advertisements.

Kitchen hand

$9.40 per hour, $14.10 per hour on weekends.

Office assistant

Receive $516 per month for 12 hours work per week.

- Nick gets the job as the kitchen hand. In his first week he works 4 hours during the week and 8 hours at the weekend. How much did he earn?
- Ben gets the job as the office assistant. How much does he earn per week if he works 4 weeks in a month? What does his hourly rate turn out to be?
- If Nick continues to work 12 hours in a week, does he earn more than Ben if he only works on week days? How many weekday hours must Nick work to match Ben's pay?
- Out of the 12 hours, what is the minimum number of hours Nick must work at the weekend to earn at least as much as Ben?

KEY IDEAS

- Workers who earn a **wage** (for example, a casual waiter) are paid a fixed rate per hour. Hours outside the normal working hours (public holidays etc.) are paid at a higher rate called **overtime**. This can occur in a couple of common ways:
  - **Time and a half**: pay is 1.5 times the usual hourly rate
  - **Double time**: pay is twice the usual hourly rate
- Workers who earn a **salary** (for example, an engineer) are paid a fixed amount per year, say \$95 000. This is often paid monthly or fortnightly.
  - 12 months in a year and approximately 52 weeks in a year $= 26$ fortnights
- **Commission** is a proportion of the overall sales amount. Salespeople may receive a commission of their sales as well as a set weekly or monthly fee called a **retainer**.
  - $\text{Commission} = \%\,\text{commission} \times \text{total sales}$
- A person's **gross income** is the total income that they earn. The **net income** is what is left after deductions, such as tax, are taken out.
  - $\text{Net income} = \text{gross income} - \text{deductions}$
- **Taxation** is paid to the government once a person's taxable income passes a set amount. The amount paid depends on the person's taxable income.

BUILDING UNDERSTANDING

① A young employee has an annual salary of \$47 424 and works 38-hour weeks. Find the average earnings:

a per month
b per week
c per hour.

② A shop assistant is paid \$11.40 per hour. Calculate their earnings for the following hours worked.

a 7 hours
b 5.5 hours
c 4 hours at twice the hourly rate (double time)
d 6 hours at 1.5 times the hourly rate (time and a half)

③ Find the commission earned on the given sales if the percentage commission is 20%.

a \$1000
b \$4500

④ Find the net annual income given that gross annual income is \$56 300, and tax paid is \$10 134.

Example 27 Comparing wages and salaries

Ken earns an annual salary of $88 450 and works a 38-hour week. His wife Brooke works part time in retail and earns $53.20 per hour.

a Calculate how much Ken earns per week.
b Determine who has the higher hourly rate of pay.
c If Brooke works on average 18 hours per week, what is her yearly income?

SOLUTION	EXPLANATION
a Weekly rate = \$88 450 ÷ 52 = \$1700.96 ∴ Ken earns \$1700.96 per week	\$88 450 pay in a year. There are approximately 52 weeks in a year.
b Brooke: \$53.20 per hour Ken: \$1700.96 ÷ 38 = \$44.76 per hour ∴ Brooke is paid more per hour.	Ken works 38 hours in week. Hourly rate = weekly rate ÷ number of hours. Round to the nearest cent. Compare hourly rates.
c Weekly income: \$53.20 × 18 = \$957.60 Yearly income = \$957.60 × 52 = \$49 795.20	Weekly income = hourly rate × number of hours. Multiply by 52 weeks to get yearly income.

Now you try

Maria earns an annual salary of $124 500 and works a 40-hour week. Her husband Gary works part time in catering and earns $64.50 per hour.

a Calculate how much Maria earns per week.
b Determine who has the higher hourly rate of pay.
c If Gary works an average of 20 hours per week, what is his yearly income?

Example 28 Calculating overtime

Georgie works some weekends and late nights in addition to normal working hours and has overtime pay arrangements with her employer.

a Calculate how much Georgie earns in one week if she works 16 hours during the week at the normal hourly rate of $18.50 and 6 hours on the weekend at time and a half.
b Georgie's normal hourly rate is changed. In a week she works 9 hours at the normal rate, 4 hours at time and a half and 5 hours at double time. If she earns $507.50, what is her new normal hourly rate?

SOLUTION	EXPLANATION
a Earnings at normal rate $= 16 \times \$18.50$ $= \$296$	16 hours at standard rate
Earnings at time and a half $= 6 \times 1.5 \times \$18.50$ $= \$166.50$	Time and half is 1.5 times the normal rate.
Total earnings $= \$296 + \$166.50$ $= \$462.50$	Combine earnings.
b Equivalent hours worked in the week $= 9 + (4 \times 1.5) + (5 \times 2)$ $= 9 + 6 + 10$ $= 25$ hours	Calculate the number of equivalent hours worked. 4 hours at time and a half is the same pay as for working 6 hours (4×1.5). 5 hours at double time is the same as working 10 hours (5×2).
Normal hourly rate $= \$507.50 \div 25$ $= \$20.30$	Divide weekly earnings by the 25 equivalent hours worked.

Now you try

Matt works some weekends and nights in addition to normal working hours and has overtime pay arrangements with his employee.

a Calculate how much Matt earns in one week if he works 20 hours at the normal hourly rate of \$15.20, and 5 hours at time-and-a-half.

b Matt's normal hourly rate is changed. In a week he works 10 hours at the normal rate, 4 hours at time-and-a-half and 2 hours at double time. If he earns \$330 in a week, what is his new normal hourly rate?

Example 29 Calculating commission

A saleswoman is paid a retainer of \$1500 per month. She also receives a commission of 1.25% on the value of goods she sells. If she sells goods worth \$5600 during the month, calculate her earnings for that month.

SOLUTION	EXPLANATION
Commission $= 1.25\%$ of $\$5600$ $= 0.0125 \times \$5600$ $= \$70$	Calculate the commission on sales. Change the percentage to a decimal and evaluate.
Earnings $= \$1500 + \$70$ $= \$1570$	Earnings = retainer + commission

Continued on next page

Now you try

A salesman is paid a retainer of $1250 per month. He also receives a commission of 1.1% on the value of the goods he sells. If he sells goods worth $4500 during the month, calculate his earnings for that month.

Example 30 Calculating tax to find net income

Liam has an annual salary of $85 000. His payslip each month shows deductions for taxation of $1500.

a Calculate Liam's net income each month.

b What percentage of Liam's monthly pay is being paid to the government by his employer for taxation? Round to one decimal place.

c The taxation for Liam's salary changes to $3572 plus 32.5 c for each $1 over $37 000. Calculate Liam's net income for the year.

SOLUTION	EXPLANATION
a $\text{Monthly pay} = \$85\,000 \div 12$ $= \$7083.33$	Calculate gross income per month.
$\text{Net monthly income} = \$7083.33 - \$1500$ $= \$5583.33$	Net income = gross income − taxation
b $\%\,\text{tax} = \dfrac{1500}{7083.33} \times 100\%$ $= 21.2\%$ (to 1 d.p.)	Calculate what fraction $1500 is of the monthly income $7083.33. Multiply by 100 to convert to a percentage.
c $\text{Salary for tax purposes} = \$85\,000 - \$37\,000$ $= \$48\,000$	First calculate how much is to be taxed at the rate of 32.5%.
$\text{Tax amount} = 32.5\% \text{ of } \$48\,000 + \$3572$ $= 0.325 \times \$48\,000 + \$3572$ $= \$19\,172$	Add the tax amount at the 32.5% rate to the amount payable for the amount under $37 000. Convert percentage to a decimal and evaluate.
$\text{Net income} = \$85\,000 - \$19\,172$ $= \$65\,828$	Net income = gross income − tax amount

Now you try

Shirley has an annual salary of $105 500. Her payslip each month shows deductions for taxation of $2000.

a Calculate Shirley's net income each month.

b What percentage of Shirley's monthly pay is being paid to the government by her employer for taxation? Round to one decimal place.

c If the annual taxation for Shirley's salary changes to $4200 plus 32.5 c for each $1 over $37 000. Calculate Shirley's net income for the year.

Exercise 1I

FLUENCY	1–3, 5, 7, 9	2, 3, 4($^1/_2$), 5, 7, 9, 10($^1/_2$)	4, 6, 8, 9–10($^1/_2$)

Example 27

1 Georgia earns an annual salary of $95 000 and works a 40-hour week. Her husband Mike works part time and earns $45.20 per hour.

- **a** Calculate how much Georgia earns per week.
- **b** Determine who has the higher hourly rate of pay.
- **c** If Mike works on average 16 hours per week, what is his yearly income?

Example 27

2 **a** Calculate the hourly rate of pay for a 38-hour working week with an annual salary of:

- **i** $38 532 **ii** $53 352 **iii** $83 980

b Calculate the yearly income for someone who earns $24.20 per hour and in a week works, on average:

- **i** 24 hours **ii** 35 hours **iii** 16 hours

Example 28a

3 A job has a normal working hours pay rate of $18.40 per hour. Calculate the pay, including overtime, for the following hours worked.

- **a** 3 hours and 4 hours at time and a half
- **b** 4 hours and 6 hours at time and a half
- **c** 14 hours and 3 hours at double time
- **d** 20 hours and 5 hours at double time
- **e** 10 hours and 8 hours at time and a half and 3 hours at double time
- **f** 34 hours and 4 hours at time and a half and 2 hours at double time

4 Calculate how many hours at the standard hourly rate the following working hours are equivalent to:

- **a** 3 hours and 2 hours at double time
- **b** 6 hours and 8 hours at time and a half
- **c** 15 hours and 12 hours at time and a half
- **d** 10 hours time and a half and 5 hours at double time
- **e** 20 hours and 6 hours at time and a half and 4 hours at double time
- **f** 32 hours and 4 hours at time and a half and 1 hour at double time

Example 28b

5 Jim, a part-time gardener, earned $261 in a week in which he worked 12 hours during normal working hours and 4 hours overtime at time and a half. What was his hourly rate of pay?

Example 28b

6 Sally earned $329.40 in a week in which she worked 10 hours during the week and 6 hours on Saturday at time and a half and 4 hours on Sunday at double time. What was her hourly rate of pay?

Example 29

7 Amy works at Best Bookshop. During one week she sells books valued at $800. If she earns $450 per week plus 5% commission, how much does she earn in this week?

Example 29

8 Jason works for an alarm system company and earns $650 per month plus 4% commission on sales. How much does he earn in a month in which he sells $84 000 worth of alarm systems?

Example 30a,b

9 For each of the following, find:

  i the annual net income

  ii the percentage of gross income paid as tax. Round to one decimal place where necessary.

a Gross annual income = \$48 241, tax withheld = \$8206

b Gross annual income = \$67 487, tax withheld = \$13 581.20

c Gross monthly income = \$4041, tax withheld = \$606.15

d Gross monthly income = \$3219, tax withheld = \$714.62

Example 30c

10 Calculate the amount of tax to be paid on the following annual salaries if the tax payable is \$3572 plus 32.5 c for each \$1 over \$37 000.

a salary = \$40 000

b salary = \$56 000

c salary = \$62 000

d salary = \$86 500

PROBLEM-SOLVING	11, 12	12–14	13–15

11 Arrange the following workers in order of most to least earnt in a week of work.

- Adam has an annual salary of \$33 384.
- Bill works for 26 hours at a rate of \$22.70 per hour.
- Cate earns \$2964 per month. (Assume 52 weeks in a year.)
- Diana does shift work 4 days of the week between 10 p.m. and 4 a.m. She earns \$19.90 per hour before midnight and \$27.50 per hour after midnight.
- Ed works 18 hours at the normal rate of \$18.20 per hour, 6 hours at time and a half and 4 hours at double time in a week.

12 Stephen earns an hourly rate of \$17.30 for the first 38 hours he works, time and a half for the next 3 hours and double time for each extra hour above that. Calculate his earnings if he works 44 hours in a week.

13 Jessica works for Woods Real Estate and earns \$800 per week plus 0.05% commission. If this week she sold three houses valued at \$334 000, \$210 000 and \$335 500 respectively, how much will she have earned?

14 A door-to-door salesman sells 10 security systems in one week at \$1200 each. For the week, he earns a total of \$850 including a retainer of \$300 and a commission. Find his percentage commission correct to two decimal places.

15 Mel has a net annual income of \$53 246 after 21% of her income is withheld for tax purposes. What was her gross income?

REASONING	16	16, 17	17, 18

16 Karl is saving and wants to earn \$90 from a casual job paying \$7.50 per hour.

a How many hours must he work to earn the \$90?

b Karl can also work some hours where he is paid time and a half. He decides to work x hours at the normal rate and y hours at time and a half to earn the \$90. If x and y are both positive integers, find the possible combinations for x and y.

17 A car salesman earns 2% commission on sales up to $60 000 and 2.5% on sales above $60 000.

a Determine the amount earned on sales worth:

i $46 000 **ii** $72 000

b Write a rule for the amount, A dollars, earned on sales of $\$x$ if:

i $x \leqslant 60000$ **ii** $x > 60000$

18 Kim has a job selling jewellery. She has to choose one of two new payment plans below.

Plan A: $220 per week plus 5% of sales

Plan B: 9% of sales and no set weekly retainer

a What value of sales gives the same return from each plan?

b Explain how you would choose between Plan A and Plan B.

ENRICHMENT: Taxation systems	–	–	19

19 Many countries use a progressive taxation system in which the percentage of tax paid is increased for higher incomes. An example is shown in this table.

Income	Tax rate	Tax payable
$0–$10 000	0%	$0
$10 001–$30 000	20%	$0 + 20% of each dollar over $10 000
$30 001–$100 000	30%	$4000 + 30% of (income – $30 000)
$100 001–	40%	$25 000 + 40% of (income – $100 000)

a Using the table above, find the tax payable on the following incomes.

i $20 000 **ii** $55 000 **iii** $125 000

b Copy and complete the details in the following progressive tax system.

Income	Tax rate	Tax payable
$0–$15 000	0%	$0
$15 001–$40 000	15%	$0 + 15% of each dollar over $15 000
$40 001–$90 000	25%	- -
$90 001–	33%	- -

c A different system on 'Taxation Island' looks like this.

Income	Tax rate	Tax payable
$0–$20 000	10%	10% of total income
$20 001–$80 000	30%	30% of total income
$80 001–	50%	50% of total income

Find the tax payable on an income of:

i $20 000 **ii** $21 000

iii $80 000 **iv** $80 001

d By referring to your answers in part **c**, describe the problems associated with the taxation system on Taxation Island.

ISBN 978-1-108-77288-4

Applications and problem-solving

The following problems will investigate practical situations drawing upon knowledge and skills developed throughout the chapter. In attempting to solve these problems, aim to identify the key information, use diagrams, formulate ideas, apply strategies, make calculations and check and communicate your solutions.

Bedroom makeover

1 Phil and Deb are mixing their own paint to form a special colour for their bedroom makeover. They have tested the following ratios and feel that these two options, A and B, suit their tastes.

Colour	White	Yellow	Red	Blue
Ratio A	20	3	1	3
Ratio B	24	4	2	3

The white paint costs \$8 per litre and the colours all cost \$25 per litre. The bedroom is 6 m long, 5 m wide and 2.5 m high, and the paint covers 10 m^2 per litre.

Phil and Deb are interested in the amount and cost of paint required to cover the walls and ceiling of the bedroom using one of these colour combinations.

a What fraction of:
  i red paint is in ratio A? **ii** blue paint in ratio B? **iii** white paint in ratio B?

b If 45 L of paint is required in total, find the amount of the following correct to the nearest litre.
  i White paint required using ratio A **ii** Yellow paint required using ratio B

c If the walls and ceiling require two coats of paint, find:
  i the total area of the walls and ceiling **ii** the amount of paint required in total
  iii the amount of red paint needed using ratio A, correct to one decimal place
  iv the amount of blue paint needed using ratio B, correct to one decimal place.

d All paint can only be purchased in a whole number of litres. Find the minimum cost of painting the room with two coats using:
  i ratio A **ii** ratio B.

Deals on wheels

2 During the summer, Casey makes two types of gift hampers and delivers them to customers' homes.

- 5 kg hamper, retail price: \$24.90
- 8 kg hamper, retail price: \$34.90

The cost to Casey to put the hampers together is:

- 5 kg hamper: \$15
- 8 kg hamper: \$23.

Casey also charges a \$5 delivery fee, but her cost on average per km of travel is \$0.50.

Casey is interested in the possible profits that can be made in her business and ways in which she can boost sales by offering special combination deals.

Where necessary round any decimal answers to the nearest cent, e.g. \$3.57.

a For the given hamper weights and retail prices:
  i which hamper is better value for money?
  ii if the 5 kg hamper was re-priced based on the cost per kg of the 8 kg hamper, what would be its new retail price?

ISBN 978-1-108-77288-4

Scenario A

Hamper	Sales
5 kg	45
8 kg	25

Scenario B

Hamper	Sales
5 kg	80
8 kg	50

b Casey looks at two sales scenarios.

- Scenario A: 60 deliveries covering 500 km
- Scenario B: 100 deliveries covering 1000 km

Find the total profit from:

i Scenario A **ii** Scenario B.

c To encourage sales, Casey offers a combination deal of one 5 kg hamper and one 8 kg hamper for $50.

i What is the overall profit for a combination deal, assuming a single delivery charge of $5 and a 10 km journey?

ii Compared to selling one of each of the 5 kg and 8 kg hampers, how much of a discount does the combination deal represent? Express as a percentage correct to one decimal place. Assume that the delivery charges are the same.

d In scenario B in part **b** above, 30 people buy two hampers, including one of each type of hamper, and all the other sales are single hamper purchases. Casey predicts that 50% of the customers who buy just one single hamper can be convinced to buy the combination deal. Will this mean that Casey is better off and by how much? (Note: The 30 people who bought both types of hampers now only pay $50.)

Taxing commission

3 Dihan works in sales for a large surfboard company and receives a $600 per week retainer, plus $50 for each surfboard he sells. Tax is taken out of his yearly income by the company according to the table opposite.

Income bracket	Tax rate	Tax payable
$0–$10 000	0%	$0
$10 001–$20 000	10%	$0 + 10% of each dollar over $10 000
$20 000–$50 000	20%	$$a$ + 20% of each dollar over $20 000
$50 001–	30%	$$b$ + 30% of each dollar over $50 000

Dihan is interested in the number of sales required for a particular income goal and the amount of tax payable on that income.

a Assume that Dihan works and is paid for 48 weeks in a year.

i Calculate the total income for the year if Dihan sells on average 6 surfboards each week.

ii Write a rule for Dihan's income per year, $$A$, if he averages n sales per week.

iii Find the minimum average number of sales per week if Dihan wants to earn $50 000 for the year.

b The values of a and b in the tax table can be calculated as the maximum amount of tax payable from the previous bracket. Find the values of a and b.

c Find Dihan's tax payable if he earns the following amounts in a year.

i $36 000 **ii** $85 000

d Find Dihan's yearly income if he is required to pay the following amounts of tax.

i $6000 **ii** $12 000

e Dihan's total living expenses are $$E$ per year and he wants to save close to $20 000 in a year after living expenses and taxes are paid. If $E = 25\,000$, find the average number of sales required each week to achieve his goal. Assume he works for 48 weeks and give a whole number answer.

1J Simple interest

Learning intentions

- To understand how simple interest is calculated
- To know the simple interest formula and the meaning of the variables
- To be able to apply the simple interest formula to calculate interest or an investment time period or rate
- To be able to calculate a final balance from a simple interest account

When paying back the amount borrowed from a bank or other financial institution, the borrower pays interest to the lender. It is like rent paid on the money borrowed. A financial institution might be the lender, giving you a loan, or the borrower, when you invest your savings with them (effectively when you lend them your money). In either case, interest is calculated as a percentage of the amount borrowed. For simple interest, the percentage is calculated on the amount originally borrowed or invested and is paid at agreed times, such as once a year.

Buying an item with a loan greatly increases its price. A car bought with a $750 deposit and a $10 000 loan at 8% p.a. simple interest repaid over 5 years, plus fees, has had its price increased to around $15 000.

LESSON STARTER Developing the rule

$5000 is invested in a bank and 5% simple interest is paid every year. In the table at right, the amount of interest paid is shown for Year 1, the amount of accumulated total interest is shown for Years 1 and 2.

Year	Interest paid that year	Accumulated total interest
1	$\frac{5}{100} \times \$5000 = \$250$	$1 \times \$250 = \$250$
2		$2 \times \$250 = \$500$
3		
4		
t		

- Complete the table, writing an expression in the last cell for the accumulated total interest after t years.
- Now write a rule using $\$P$ for the initial amount, t for the number of years and r for the interest rate to find the total interest earned, $\$I$.

KEY IDEAS

- To calculate **simple interest**, we apply the formula:

$$I = P \times \frac{r}{100} \times t \quad \text{or} \quad I = \frac{Prt}{100}$$

where

- I is the amount of **simple interest** (in $)
- P is the **principal** amount; the money borrowed or loaned (in $)
- $r\%$ is the rate per unit time; usually **per annum** (p.a.), which means per year
- t is the period of **time**, expressed in the stated units, usually years.

- When using simple interest, the principal amount is constant and remains unchanged from one period to the next.
- The total amount ($\$A$) equals the principal plus interest:
  $A = P + I$

BUILDING UNDERSTANDING

① \$12 000 is invested at 6% p.a. for 42 months.
- **a** What is the principal amount?
- **b** What is the interest rate?
- **c** What is the time period in years?
- **d** How much interest is earned each year?
- **e** How much interest is earned after 2 years?
- **f** How much interest is earned after 42 months?

② **a** How many years is the following number of months?
- **i** 36
- **ii** 18
- **iii** 66

b How many months is the following number of years?
- **i** 4
- **ii** 2.5
- **iii** 7.25

Example 31 Using the simple interest formula

Calculate the simple interest earned if the principal is \$1000, the rate is 5% p.a. and the time is 3 years.

SOLUTION	EXPLANATION
$P = 1000, r = 5, t = 3$	List the information given.
$I = \dfrac{Prt}{100}$	Write the formula and substitute the given values.
$= \dfrac{100\cancel{0} \times 5 \times 3}{10\cancel{0}}$	This is the same as using $I = P \times \dfrac{r}{100} \times t$.
$= 150$	
Interest = \$150	Answer the question.

Now you try

Calculate the simple interest earned if the principal is \$1500, the rate is 7% p.a. and the time is 4 years.

Example 32 Calculating the final balance

Allan and Rachel plan to invest some money for their child Kaylan. They invest \$4000 for 30 months in an account that pays 4.5% p.a. Calculate the simple interest and the amount available at the end of the 30 months.

SOLUTION	EXPLANATION
$P = 4000, r = 4.5, t = \frac{30}{12} = 2.5$	t is written in years since interest rate is per annum.
$I = \frac{Prt}{100}$ $= \frac{400\cancel{0} \times 4.5 \times 2.5}{10\cancel{0}}$ $= 450$ Interest $= \$450$	Write the formula, substitute the known information and evaluate. Alternatively, use $I = P \times \frac{r}{100} \times t$
Total amount $= \$4000 + \$450$ $= \$4450$	Total amount = principal + interest

Now you try

Rohan plans to invest some money to save for a new tractor. He invests \$20 000 for 42 months at the rate of 6.5%. Calculate the simple interest and the amount available at the end of the 42 months.

Example 33 Determining the investment period

Remy invests \$2500 at 8% p.a. simple interest for a period of time and receives \$50 interest. For how long (in months) did she invest the money?

SOLUTION	EXPLANATION
$I = 50, P = 2500, r = 8$	List the information given.
$I = \frac{Prt}{100}$ $50 = \frac{250\cancel{0} \times 8 \times t}{10\cancel{0}}$	Write the formula, substitute the known information and simplify.
$50 = 200t$ $t = \frac{50}{200}$ $= 0.25$	Solve the remaining equation for t.
Time $= 0.25$ years $= 0.25 \times 12$ months $= 3$ months	Convert decimal time to months where appropriate.

Now you try

Mia invests $4200 at 6% p.a. simple interest for a period of time and receives $189 interest. For how long (in months) did she invest the money?

Exercise 1J

FLUENCY	1–6	2, 3, 5–7	3, 5, 6, 8

Example 31

1 Use the rule $I = \frac{Prt}{100}$ to find the simple interest earned in these financial situations.

- **a** Principal $6000, rate 12% p.a., time 5 years
- **b** Principal $5200, rate 4% p.a., time 24 months

Example 31

2 Calculate the simple interest earned if the principal is $2000, the value is 6% p.a. and the time is 5 years.

Example 32

3 Wally invests $15 000 at a rate of 6% p.a. for 3 years. Calculate the simple interest and the amount available at the end of 3 years.

Example 32

4 Annie invests $22 000 at a rate of 4% p.a. for 27 months. Calculate the simple interest and the amount available at the end of 27 months.

5 A finance company charges 14% p.a. simple interest. If Lyn borrows $2000 to be repaid over 2 years, calculate her total repayment.

Example 33

6 Zac invests $3500 at 8% p.a. simple interest, for a period of time, to produce $210 interest. For how long did he invest the money?

Example 33

7 If $4500 earns $120 simple interest at a flat rate of 2% p.a., calculate the duration of the investment.

8 Calculate the principal amount that earns $500 simple interest over 3 years at a rate of 8% p.a. Round to the nearest cent.

PROBLEM-SOLVING	9–11	10–12	11, 12

9 Wendy wins $5000 during a chess tournament. She wishes to invest her winnings, and has the two choices given below. Which one gives her the greater total at the end of the time?
Choice 1: 8.5% p.a. simple interest for 4 years
Choice 2: 8% p.a. simple interest for 54 months

10 Charlotte borrows \$9000 to buy a secondhand car. The loan must be repaid over 5 years at 12% p.a. simple interest. Calculate:

- **a** the total amount to be repaid
- **b** the monthly repayment amount if the repayments are spread equally over the 5 years.

11 An investment of \$5000 earns \$6000 simple interest in 12 years. Calculate the interest rate.

12 Chris invests \$$P$ and wants to double this amount of money.

- **a** How much interest must be earned to double this initial amount?
- **b** What simple interest rate is required to double the initial amount in 8 years?
- **c** The simple interest rate is 5% p.a.
  - **i** How many years will it take to double the investment?
  - **ii** How many years will it take to triple the investment amount?
  - **iii** How do the investment periods in parts **i** and **ii** compare?

REASONING	13	13, 14	13, 14

13 To find the total amount \$$T$ including simple interest, the rule is $T = P\left(1 + \frac{rt}{100}\right)$.

- **a** Use this rule to find the total amount after 10 years when \$30 000 is invested at 7% p.a.
- **b** Use the rule to find the time that it takes for an investment to grow from \$18 000 to \$22 320 when invested at 6% p.a. simple interest.

14 Rearrange the rule $I = \frac{Prt}{100}$ to find a rule for:

- **a** P in terms of I, r and t
- **b** t in terms of I, P and r
- **c** r in terms of I, P and t.

ENRICHMENT: Property investing	–	–	15

15 Many investors use interest-only loans to buy shares or property. For such loans the principal stays constant and only the interest is paid back each month.

Sasha buys an investment property for \$300 000 and borrows the full amount at 7% p.a. simple interest. She rents out the property at \$1500 per month and pays \$3000 per year in rates and other costs to keep the property.

- **a** Find the amount of interest Sasha needs to pay back every month.
- **b** Find Sasha's yearly income from rent.
- **c** By considering the other costs in keeping the property, calculate Sasha's overall loss in a year.
- **d** Sasha hopes that the property's value will increase enough to cover any loss she is making. By what percentage of the original price will the property need to increase in value per year?

Using CAS and non-CAS calculators 1J: Number and interest problems

The activity is in the Interactive Textbook in the form of a printable PDF.

1K Compound interest EXTENDING

Learning intentions

- To understand how compound interest works
- To be able to calculate an account balance using compound interest
- To be able to calculate interest or an initial amount using the compound interest formula

When interest is added to an investment total before the next amount of interest is calculated, we say that the interest is compounded. Interest on a \$1000 investment at 8% p.a. gives \$80 in the first year. If the interest is compounded, the interest calculated in the second year is 8% of \$1080. This is repeated until the end of the investment period. Other forms of growth and decay work in a similar manner.

Assets, such as investment accounts, land and buildings, that grow by a percentage each year increase in total value at a faster rate over time. Their possible future value can be predicted using the compound interest formula.

LESSON STARTER Power play

\$10 000 is invested at 5% interest compounded annually. Complete this table showing the interest paid and the balance (original investment plus interest) at the end of each year.

Year	Interest paid that year	Balance
1	$0.05 \times \$10\,000 = \$\_\_\_\_$	$\$10\,000 \times 1.05 = \$\_\_\_\_$
2	$0.05 \times \$\_\_\_\_ = \$\_\_\_\_$	$\$10\,000 \times 1.05 \times 1.05$ $= \$10\,000 \times 1.05^2$ $= \$\_\_\_\_$
3	$0.05 \times \$\_\_\_\_ = \$\_\_\_\_$	$\$10\,000 \times \_\_\_\_$ $= \$10\,000 \times 1.05^{\square}$ $= \$\_\_\_\_$

- What patterns can you see developing in the table?
- How can you use the *power* button on your calculator to help find the balance at the end of each year?
- How would you find the balance at the end of 10 years without creating a large table of values?

KEY IDEAS

- A repeated product can be written and calculated using a power.
  - For example: $1.06 \times 1.06 \times 1.06 \times 1.06 = (1.06)^4$
    $0.85 \times 0.85 \times 0.85 = (0.85)^3$
- **Compound interest** is interest which is added to the investment amount before the next amount of interest is calculated.
  - For example: \$5000 invested at 6% compounded annually for 3 years gives $\$5000 \times 1.06 \times 1.06 \times 1.06 = 5000 \times (1.06)^3$.
  - 6% compounded annually can be written as 6% p.a.
  - p.a. means ‘per annum’ or ‘per year’.
  - The initial investment or loan is called the **principal**.
  - The total interest earned = final amount − principal.

ISBN 978-1-108-77288-4

BUILDING UNDERSTANDING

1 \$2000 is invested at 10% compounded annually for 3 years.
- **a** Find the interest earned in the first year.
- **b** Find the total balance at the end of the first year.
- **c** Find the interest earned in the second year.
- **d** Find the total balance at the end of the second year.
- **e** Find the interest earned in the third year.
- **f** Find the total balance at the end of the third year.

2 Find the value of the following correct to two decimal places. What do you notice?
- **a** $2500 \times 1.03 \times 1.03 \times 1.03$
- **b** $420 \times 1.22 \times 1.22 \times 1.22 \times 1.22$
- **c** $2500 \times (1.03)^3$
- **d** $420 \times (1.22)^4$

3 State the missing numbers to help describe each situation.
- **a** \$4000 is invested at 20% compounded annually for 3 years.
  $\$4000 \times (\_\_\_\_)^3$
- **b** \$15 000 is invested at 7% compounded annually for 6 years.
  $\$\_\_\_\_ \times (1.07)^{\square}$
- **c** \$825 is invested at 11% compounded annually for 4 years.
  $\$825 \times (\_\_\_\_)^{\square}$

Example 34 Calculating a balance using compound interest

Find the total value of the investment when \$8000 is invested at 5% compounded annually for 4 years.

SOLUTION	EXPLANATION
$100\% + 5\% = 105\%$ $= 1.05$	Add 5% to 100% to find the multiplying factor.
Investment total $= \$8000 \times (1.05)^4$ $= \$9724.05$	Multiplying by $(1.05)^4$ is the same as multiplying by $1.05 \times 1.05 \times 1.05 \times 1.05$.

Now you try

Find the total value of the investment when \$6000 is invested at 4% compounded annually for 3 years.

ISBN 978-1-108-77288-4

Example 35 Finding the initial amount

After 6 years a loan grows to \$62 150. If the interest was compounded annually at a rate of 9%, what is the amount of the initial loan to the nearest dollar?

SOLUTION	EXPLANATION
$100\% + 9\% = 109\%$ $= 1.09$	Add 9% to 100% to find the multiplying factor.
$\text{Initial amount} \times (1.09)^6 = \$62\,150$ $\text{Initial amount} = \$62\,150 \div (1.09)^6$ $= \$37\,058$	Write the equation including the final total. Divide by $(1.09)^6$ to find the initial amount and round as required.

Now you try

After 7 years a loan grows to \$43 200. If the interest was compounded annually at a rate of 8%, what is the amount of the initial loan to the nearest dollar?

Exercise 1K

FLUENCY	1–5	2(½), 3, 4(½), 5, 6(½)	2(½), 3, 4(½), 6(½)

Example 34

1 Find the total value of the investment when \$3200 is invested at 6% compounded annually for 5 years.

Example 34

2 Find the total balance of these investments given the interest rates and time period. Assume interest is compounded annually in each case. Round to the nearest cent.

a \$4000, 5%, 10 years

b \$6500, 8%, 6 years

c \$25 000, 11%, 36 months

d \$4000, 7%, 60 months

3 Barry borrows \$200 000 from a bank for 5 years and does not pay any money back until the end of the period. The compound interest rate is 8% p.a. How much does he need to pay back at the end of the 5 years? Round to the nearest cent.

4 Find the total percentage increase in the value of these amounts, compounded annually at the given rates. Round to one decimal place.

a \$1000, 4% p.a., 5 years

b \$20 000, 6% p.a., 3 years

c \$125 000, 9% p.a., 10 years

d \$500 000, 7.5% p.a., 4 years

Example 35

5 After 5 years a loan grows to \$45 200. If the interest was compounded annually at a rate of 6% p.a., find the size of the initial loan to the nearest dollar.

ISBN 978-1-108-77288-4

Example 35

6 Find the initial investment amount to the nearest dollar given these final balances, annual interest rates and time periods. Assume interest is compounded annually in each case.

a $26 500, 4%, 3 years
b $42 000, 6%, 4 years
c $35 500, 3.5%, 6 years
d $28 200, 4.7%, 2 years

PROBLEM-SOLVING	7, 8	8–10	9–11

7 Average house prices in Hobart are expected to grow by 8% per year for the next 5 years. What is the expected average value of a house in Hobart in 5 years time, to the nearest dollar, if it is currently valued at $370 000?

8 The population of a country town is expected to fall by 15% per year for the next 8 years due to the downsizing of the iron ore mine. If the population is currently 22 540 people, what is the expected population in 8 years' time? Round to the nearest whole number.

9 It is proposed that the mass of a piece of limestone lying out in the weather has decreased by 4.5% per year for the last 15 years. Its current mass is 3.28 kg. Find its approximate mass 15 years ago. Round to two decimal places.

10 Charlene wants to invest $10 000 long enough for it to grow to at least $20 000. The compound interest rate is 6% p.a. How many whole number of years does she need to invest the money for so that it grows to her $20 000 target?

11 A forgetful person lets a personal loan balance grow from $800 to $1440 at a compound interest rate of 12.5% p.a. For approximately how many years did the person forget about the loan?

REASONING	12	12	12, 13

12 $400 is invested for 5 years under the following conditions.

  i Simple interest at 7% p.a.
  ii Compound interest at 7% p.a.

a Find the percentage increase in the total value of the investment using condition **i**.
b Find the percentage increase in the total value of the investment using condition **ii**. Round to two decimal places.
c Explain why the total for condition **i** is less than the total for condition **ii**.

13 Find the total percentage increase in these compound interest situations to two decimal places.

a 5% p.a. for 3 years
b 12% p.a. for 2 years
c 4.4% p.a. for 5 years
d 7.2% p.a. for 9 years
e r% for t years

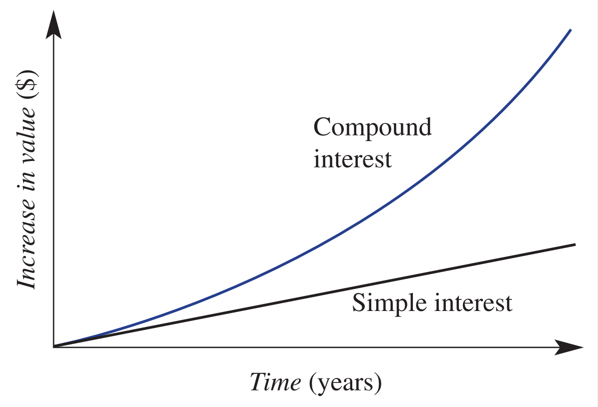
With compound interest, the balance grows at a faster rate as time passes.

ENRICHMENT: Comparing simple and compound interest	–	–	14–16

14 \$16 000 is invested for 5 years at 8% compounded annually.
- **a** Find the total interest earned over the 5 years to the nearest cent.
- **b** Find the simple interest rate that would deliver the same overall interest at the end of the 5 years. Round to two decimal places.

15 \$100 000 is invested for 10 years at 5.5% compounded annually.
- **a** Find the total percentage increase in the investment to two decimal places.
- **b** Find the simple interest rate that would deliver the same overall interest at the end of the 10 years. Round to two decimal places.

16 Find the simple interest rate that is equivalent to these annual compound interest rates for the given periods. Round to two decimal places.
- **a** 5% p.a. for 4 years
- **b** 10.5% p.a. for 12 years



This graph shows the effect of the same rate of compound and simple interest on the increase in value of an investment or loan, compared over time. Simple interest is calculated only on the principal, so yields a balance that increases by the same amount every year, forming a straight line graph. After the first year, compound interest is calculated on the principal and what has been added by the previous years' interest, so the balance increases by a greater amount each year. This forms an exponential graph.

ISBN 978-1-108-77288-4

Updated September 2021

Modelling

Investing in art

Matilda is a keen art investor and has the opportunity to purchase a new work from an auction house. The auctioneer is saying that the estimated value of the painting is $10 000.

Matilda's main investment goal is for each of her investments to at least double in value every 10 years.

Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate. Round all monetary values to the nearest cent.

Preliminary task

a If Matilda purchases the painting for $10 000 and assumes a growth rate of 5% p.a., calculate the value of the investment after: **i** 1 year **ii** 2 years **iii** 3 years.

b The rule connecting the value of the $10 000 investment ($$A$) growing at 5% p.a. after t years is given by $A = 10000\left(1 + \frac{5}{100}\right)^t$.

- **i** Check your answers to part **a** by substituting $t = 1$, $t = 2$ and $t = 3$ into the given rule and evaluating the value of A using technology.
- **ii** Construct a similar rule for an investment value of $12 000 and a growth rate of 3%.
- **iii** Construct a similar rule for an investment value of $8000 and a growth rate of 8%.

c Using $A = 10000\left(1 + \frac{5}{100}\right)^t$ find the value of a $10 000 investment at 5% p.a. after 10 years. (i.e. Calculate the value of A if $t = 10$.)

Modelling task

Formulate

a The problem is to determine an investment growth rate that delivers at least a doubling of Matilda's initial investment amount after 10 years. Write down all the relevant information that will help solve this problem.

b Explain what the numbers 10 000 and 5 mean in the rule $A = 10000\left(1 + \frac{5}{100}\right)^t$ in relation to Matilda's investment.

Solve

c Use the rule $A = 10000\left(1 + \frac{r}{100}\right)^t$ to determine the value of Matilda's $10 000 investment after 10 years for the following growth rates (r%).

i $r = 4$ **ii** $r = 7$ **iii** $r = 10$

d Choose your own values of r using one decimal place accuracy, and determine the growth rate for which the investment doubles in value after 10 years.

Evaluate and verify

e By considering values of r either side of your chosen value found in part **d**, demonstrate that your answer is correct to one decimal place.

f Refine your answer so that it is correct to two decimal places.

Communicate

g Summarise your results and describe any key findings.

Extension questions

a Decide if changing the initial investment value changes the total percentage increase in value after the same number of years. Justify your answer.

b If Matilda only paid $8000 for the artwork but still wanted it to be valued at $20 000 after 10 years, determine the required growth rate of the artwork. Round to two decimal places.

Compounding investments

Banks offer many types of investments that pay compound interest. Recall that for compound interest you gain interest on the money you have invested over a given time period. This increases your investment amount and therefore the amount of interest you gain in the next period.

Calculating yearly interest

Mary invests \$1000 at 6% per annum. This means Mary earns 6% of the principal every year in interest.

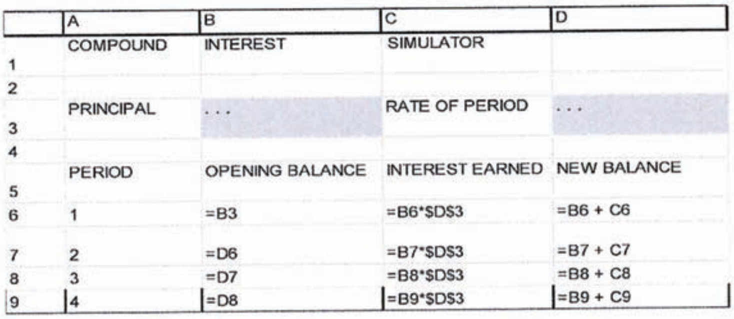
That is, after 1 year the interest earned is 6% of \$1000 $= \frac{6}{100} \times \$1000 = \$60$

- **a** The interest earned is added to the principal at the end of the year, and the total becomes the principal for the second year.
  - **i** How much interest will she earn at the end of the second and third year?
  - **ii** What total amount will Mary have at the end of the third year?
- **b** Write down a rule that calculates the total value of Mary's investment after t years. Use an initial investment amount of \$1000 and an annual interest rate of 6% p.a.
- **c** Use your rule from part **b** to calculate:
  - **i** the value of Mary's investment after 5 years
  - **ii** the time it takes for Mary's investment to grow to \$2000.

Using a spreadsheet

This spreadsheet will calculate the compound interest for you if you place the principal in cell B3 and the rate in cell D3.

In Mary's case put 1000 in B3 and $\frac{6}{100}$ in D3.



	A	B	C	D
1	COMPOUND	INTEREST	SIMULATOR	
2				
3	PRINCIPAL	. . .	RATE OF PERIOD	. . .
4				
5	PERIOD	OPENING BALANCE	INTEREST EARNED	NEW BALANCE
6	1	=B3	=B6*\$D\$3	=B6 + C6
7	2	=D6	=B7*\$D\$3	=B7 + C7
8	3	=D7	=B8*\$D\$3	=B8 + C8
9	4	=D8	=B9*\$D\$3	=B9 + C9

- **a** Copy the spreadsheet shown using 'fill down' at cells B7, C6 and D6.
- **b** What will be Mary's balance after 10 years? Extend your spreadsheet to find out.
- **c** Draw a graph of investment value versus time. Plot points using the results from your spreadsheet and join them with a smooth curve. Discuss the shape of the curve.
- **d** Now try altering the interest rate. What would Mary's investment grow to in 10 years if the interest rate was 10%?
- **e** What interest rate makes Mary's investment grow to \$2000 in 8 years? Use trial and error to get an answer correct to two decimal places. Record your investigation results using a table.
- **f** Investigate how changing the principal changes the overall investment amount. Record your investigations in a table, showing the principal amounts and investment balance for a given interest rate and period.

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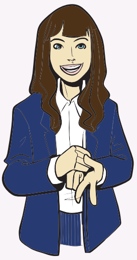
Problems and challenges

Up for a challenge? If you get stuck on a question, check out the 'Working with unfamiliar problems' poster at the end of the book to help you.



1 By only using the four operations +, −, × and ÷ as well as brackets and square root ($\sqrt{\ }$), the number 4 can be used exactly 4 times to give the answer 9 in the following way: $4 \times \sqrt{4} + 4 \div 4$. Use the number 4 exactly 4 times (and no other numbers) and any of the operations (as many times as you like) to give the answer 0 or 1 or 2 or 3 or … or 10.

2 What is the value of n if n is the largest of 5 consecutive numbers that multiply to give 95 040?

3 Evaluate.

a $\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{3}}}$

b $\dfrac{2}{1+\dfrac{2}{1+\dfrac{2}{5}}}$

4 A jug has 1 litre of 10% strength cordial. How much pure cordial needs to be added to make the strength 20%? (The answer is not 100 mL.)

5 An old table in a furniture store is marked down by 10% from its previous price on five separate occasions. What is the total percentage reduction in price correct to two decimal places?

6 What simple interest rate is equivalent to a compound interest rate of 6% p.a. over 10 years correct to two decimal places?

7 Brendon has a rectangular paved area in his yard.

a If he increases both the length and width by 20%, what is the percentage increase in area?

b If the length and width were increased by the same percentage and the area increases by 21%, what is the percentage increase of the length and width?

8 A rectangular sandpit is shown on a map that has a scale of 1 : 100. On the map the sandpit has an area of 20 cm^2. What is its actual area?

9 Arrange the numbers 1 to 15 in a row so that each adjacent pair of numbers sums to a square number.

Reviewing number and financial mathematics

Review of numbers

$2 + (-3) = -1$
$-4 - (-2) = -2$
$15 \div (-5) = -3$
$-7 \times (-3) = 21$
$(-2)^2 = 4$
$4^3 = 64$

For 8 and 12
HCF is 4
LCM is 24

$\sqrt{36} = 6$
$\sqrt[3]{27} = 3$

Operations with fractions

$$1\frac{5}{6} - \frac{7}{12} = \frac{11}{6} - \frac{7}{12} = \frac{22}{12} - \frac{7}{12} = \frac{15}{12} = 1\frac{1}{4}$$

$$1\frac{1}{2} \div \frac{7}{2} = \frac{3}{\cancel{2}_1} \times \frac{\cancel{2}^1}{7} = \frac{3}{7}$$

Rounding

1.284 to 1 d.p. is 1.3
2 sig. fig. is 1.3
472.543 to 1 d.p. is 472.5
1 sig. fig. is 500
0.0624 to 2 d.p. is 0.06
2 sig. fig. is 0.062

Rates, ratios and best buy

Rate: 180 km per 3 h = 60 km/h
Ratio: $\frac{3}{7} : \frac{1}{2} = \frac{6}{14} : \frac{7}{14} = 6 : 7$
$200 divided into 7 : 3
10 parts is $200
1 part is $20
3 parts is $60
7 parts is $140
Best buy: 3 kg of carrots for $6.45
5 kg for $10.20
3 kg bag: 6.45 ÷ 3 = $2.15/kg
5 kg bag: 10.20 ÷ 5 = $2.04/kg
∴ 5 kg is best buy

Rational numbers (fractions)

$1\frac{3}{8} = \frac{11}{8} = 1.375$
mixed number, improper fraction, terminating decimal

$\frac{1}{6} = 0.166... = 0.1\dot{6}$
proper fraction, recurring decimal

$\frac{2}{3}$ and $\frac{4}{6}$ are equivalent fractions
Irrational numbers cannot be expressed as a fraction

Compound interest (Ext)

$3000 at 5% for 3 years
Amount $= \$3000 \times (1.05)^3$
$= \$3472.88$

Simple interest

$2000 at 5% for 4 years
$I = \frac{Prt}{100} = \frac{\$2000 \times 5 \times 4}{100}$
$= \$400$

Percentages

$\frac{3}{4} = \frac{3}{4} \times 100\% = 75\%$
$25\frac{1}{2}\% = 25.5 \div 100\% = 0.255$
20% of 60 = 0.2 × 60 = 12
5% of amount = 32
∴ amount = 32 ÷ 0.05
= 640

Applications of percentages

Percentage profit or loss
$= \frac{\text{profit or loss}}{\text{cost price}} \times 100\%$
Mark-up and discount
Commission/tax

Percentage increase and decrease

Increase	Decrease
20 by 6%	20 by 5%
20 × 1.06 = 21.2	20 × 0.95 = 19

$\text{Percentage change} = \frac{\text{change}}{\text{original}} \times 100\%$

Income and tax

Employees can be paid:
wage: hourly rate with overtime at time and a half = 1.5 or double time
salary: annual amount
commission: % of sales
net income
= gross income − tax

ISBN 978-1-108-77288-4

Chapter checklist

Chapter checklist: Success criteria

		✔
1A	**1. I can work with positive and negative integers.** e.g. Evaluate $3 \times (-5) + -10 - (-2))$.	
1A	**2. I can work with roots and powers.** e.g. Evaluate $(-2)^2 - 3^2 \div \sqrt{9}$.	
1B	**3. I can round to a required number of decimal places.** e.g. Round 3.4572 to two decimal places.	
1B	**4. I can round to a required number of significant figures.** e.g. Round 24 532 and 0.0423 to two significant figures.	
1B	**5. I can estimate using significant figures.** e.g. Estimate the answer to $1986.2 + 37 \div 0.0234$ by rounding each number in the problem to one significant figure.	
1C	**6. I can express a fraction as a decimal.** e.g. Express $\frac{7}{12}$ as a decimal.	
1C	**7. I can express a decimal as a fraction.** e.g. Express 2.35 as a fraction.	
1C	**8. I can compare fractions.** e.g. Decide which is smaller: $\frac{11}{15}$ or $\frac{18}{25}$.	
1D	**9. I can add fractions.** e.g. Evaluate $\frac{5}{8} + \frac{1}{6}$.	
1D	**10. I can subtract fractions.** e.g. Evaluate $2\frac{2}{5} - 1\frac{3}{4}$.	
1D	**11. I can multiply fractions.** e.g. Evaluate $3\frac{1}{3} \times 1\frac{3}{5}$.	
1D	**12. I can divide fractions.** e.g. Evaluate $1\frac{4}{5} \div 1\frac{5}{7}$.	
1E	**13. I can simplify ratios.** e.g. Simplify $32:20$ and $0.3:0.12$.	
1E	**14. I can divide into a given ratio.** e.g. 350 cm is to be divided in the ratio $2:5$. Find the length of the larger portion.	
1E	**15. I can simplify rates.** e.g. Write \$140 every 4 hours in simplest form.	
1E	**16. I can determine a best buy.** e.g. What is better value: 2 kg of rice for \$3.60 or 5 kg of rice for \$17?	
1F	**17. I can convert between percentages, decimals and fractions.** e.g. Express 24.5% as a decimal and 8% as a fraction.	

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		✔
1F	**18. I can express a quantity as a percentage.** e.g. Express 34 out of 40 as a percentage.	
1F	**19. I can find a percentage of a quantity.** e.g. Find 36% of $60.	
1F	**20. I can find the original amount from a percentage.** e.g. Determine the original amount if 12% of the amount is $72.	
1G	**21. I can increase and decrease by a percentage.** e.g. Increase $30 by 8% and decrease $40 by 15%.	
1G	**22. I can find a percentage change.** e.g. Find the percentage change when a plant grows from 120 cm to 210 cm.	
1G	**23. I can find the original amount from a word problem.** e.g. The cost of petrol rises by 12% over a week to $1.54 a litre. What was the price at the start of the week? Round to the nearest cent.	
1H	**24. I can determine the selling price from a mark-up or discount.** e.g. Before school holidays an airline marks up all flights by 30%. If a flight costs $840, what will be its selling price for the holidays?	
1H	**25. I can calculate a sale saving.** e.g. A pair of jeans is discounted by 15%. If the sale price was $68, what was the original price?	
1I	**26. I can calculate wages and salaries.** e.g. Ryan earns an annual salary of $74 500. He works 48 weeks of the year and works 38-hour weeks. Determine his hourly rate of pay to the nearest cent.	
1I	**27. I can calculate overtime.** e.g. Ben's normal hourly rate of pay is $24.30. Calculate his earnings in a week if he works 12 hours at the normal rate, 6 hours at time and a half and 8 hours at double time.	
1I	**28. I can calculate commission.** e.g. A jeweller is paid a retainer of $250 per week and she also receives a commission of 6% on the value of goods she sells. If she sells $10 000 worth of jewellery during the week, how much will she earn?	
1I	**29. I can calculate tax.** e.g. Dara has an annual salary of $82 000. The tax she pays is $3572 plus 32.5c for each $1 over $37 000. Calculate Dara's net income for the year.	
1J	**30. I can calculate simple interest using the formula.** e.g. $2000 is invested at 4% p.a. simple interest for 3 years. Calculate the interest earned.	
1J	**31. I can find other unknowns using the simple interest formula.** e.g. An investment of $3000 at 4% p.a. simple interest earns $280 interest over a certain period of time. Calculate how long the money was invested for.	
1K	**32. I can calculate the balance using compound interest.** e.g. Find the total value of an investment if $12 000 is invested at 4% compounded annually for 5 years. Ext	
1K	**33. I can find the initial amount using compound interest.** e.g. After 4 years a loan increases to $120 300. If the interest was compounded annually at a rate of 7%, find the size of the initial loan to the nearest dollar. Ext	

Short-answer questions

1A **1** Evaluate the following.

- **a** $-4 \times (2 - (-3)) + 4$
- **b** $-3 - 4 \times (-2) + (-3)$
- **c** $(-8 \div 8 - (-1)) \times (-2)$
- **d** $\sqrt{25} \times \sqrt[3]{8}$
- **e** $(-2)^2 - 3^3$
- **f** $\sqrt[3]{1000} - (-3)^2$

1B **2** Round these numbers to three significant figures.

- **a** 21.483
- **b** 29 130
- **c** 0.15271
- **d** 0.002414

1B **3** Estimate the answer by first rounding each number to one significant figure.

- **a** $294 - 112$
- **b** 21.48×2.94
- **c** $1.032 \div 0.493$

1C **4** Write these fractions as decimals.

- **a** $2\frac{1}{8}$
- **b** $\frac{5}{6}$
- **c** $\frac{13}{7}$

1C **5** Write these decimals as fractions.

- **a** 0.75
- **b** 1.6
- **c** 2.55

1D **6** Simplify the following.

- **a** $\frac{5}{6} - \frac{1}{3}$
- **b** $1\frac{1}{2} + \frac{2}{3}$
- **c** $\frac{13}{8} - \frac{4}{3}$
- **d** $3\frac{1}{2} \times \frac{4}{7}$
- **e** $5 \div \frac{4}{3}$
- **f** $3\frac{3}{4} \div 1\frac{2}{5}$

1E **7** Simplify these ratios.

- **a** 30 : 12
- **b** 1.6 : 0.9
- **c** $7\frac{1}{2} : 1\frac{2}{5}$

1E **8** Divide 80 into the given ratio.

- **a** 5 : 3
- **b** 5 : 11
- **c** 1 : 2 : 5

1E **9** Dry dog food can be bought from store A for \$18 for 8 kg or from store B for \$14.19 for 5.5 kg.

- **a** Determine the cost per kilogram at each store and state which is the best buy (cheaper deal).
- **b** Determine to the nearest integer how many grams of each brand you get per dollar.

1F **10** Copy and complete the table at right.

Decimal	Fraction	Percentage
0.6		
	$\frac{1}{3}$	
		$3\frac{1}{4}\%$
	$\frac{3}{4}$	
1.2		
		200%

1F **11** Find:

- **a** 25% of \$310
- **b** 110% of 1.5

1G **12** Determine the original amount if:

- **a** 20% of the amount is 30
- **b** 72% of the amount is 18.

1G **13**

- **a** Increase 45 by 60%.
- **b** Decrease 1.8 by 35%.
- **c** Find the percentage change if \$150 is reduced by \$30.

1G **14** The mass of a cat increased by 12% to 14 kg over a 12-month period. What was its starting mass?

1H **15** Determine the discount given on a $15 000 car if it is discounted by 12%.

1H **16** The cost price of an article is $150 and its selling price is $175.

a Determine the profit made.

b Express the profit as a percentage of the cost price.

1I **17** Determine the hourly rate of pay for each case.

a A person with an annual salary of $36 062 working a 38-hour week

b A person who earns $429 working 18 hours at the hourly rate and 8 hours at time and a half

1I **18** Jo's monthly income is $5270; however, 20% of this is paid straight to the government in taxes. What is Jo's net yearly income?

1J **19** Find the simple interest earned on $1500 at 7% p.a. for 5 years.

1J **20** Rob invests $10 000 at 8% p.a. simple interest to produce $3600. For how long was the money invested?

1K Ext **21** Find the total value of an investment if $50 000 is invested at 4% compounded annually for 6 years. Round to the nearest cent.

1K Ext **22** After 8 years a loan grows to $75 210. If the interest was compounded annually at a rate of 8.5%, find the size of the initial loan to the nearest dollar.

Multiple-choice questions

1B **1** $\frac{2}{7}$ written as a decimal is:

A 0.29 **B** 0.286 **C** $0.28\dot{5}$ **D** $0.\overline{285714}$ **E** $0.28571\dot{4}$

1B **2** 3.0456 written to three significant figures is:

A 3.04 **B** 3.05 **C** 3.045 **D** 3.046 **E** 3.45

1C **3** 2.25 written as a fraction in simplest form is:

A $2\frac{1}{2}$ **B** $\frac{5}{4}$ **C** $\frac{9}{4}$ **D** $9\frac{1}{4}$ **E** $\frac{225}{100}$

1D **4** $1\frac{1}{2} - \frac{5}{6}$ is equal to:

A $\frac{2}{3}$ **B** $\frac{5}{6}$ **C** $-\frac{1}{2}$ **D** $\frac{2}{6}$ **E** $\frac{1}{2}$

ISBN 978-1-108-77288-4

1D **5** $\frac{2}{7} \times \frac{3}{4}$ is equivalent to:

A $\frac{8}{11}$ **B** $\frac{3}{7}$ **C** $\frac{5}{11}$ **D** $\frac{8}{12}$ **E** $\frac{3}{14}$

1D **6** $\frac{3}{4} \div \frac{5}{6}$ is equivalent to:

A $\frac{5}{8}$ **B** 1 **C** 21 **D** $\frac{4}{5}$ **E** $\frac{9}{10}$

1E **7** Simplifying the ratio 50 cm : 4 m gives:

A 50 : 4 **B** 8 : 1 **C** 25 : 2 **D** 1 : 8 **E** 5 : 40

1F **8** 28% as a fraction in its simplest form is:

A 0.28 **B** $\frac{28}{100}$ **C** $\frac{0.28}{100}$ **D** $\frac{2.8}{100}$ **E** $\frac{7}{25}$

1F **9** 15% of $1600 is equal to:

A 24 **B** 150 **C** $240 **D** $24 **E** 240

1I **10** Jane is paid a wage of $7.80 per hour. If she works 12 hours at this rate during a week plus 4 hours on a public holiday for which she gets paid at time and a half, her earnings for the week are:

A $140.40 **B** $124.80 **C** $109.20 **D** $156 **E** $62.40

1I **11** Simon earns a weekly retainer of $370 and 12% commission of any sales he makes. If he makes $2700 worth of sales in a particular week, he will earn:

A $595 **B** $652 **C** $694 **D** $738.40 **E** $649.60

1K Ext **12** $1200 is increased by 10% for two years with compound interest. The total balance at the end of the two years is:

A $252 **B** $1452 **C** $1450 **D** $240 **E** $1440

Extended-response questions

1 Pauline buys a formal dress at cost price from her friend Tila. Pauline paid $420 for the dress, which is normally marked up by 55%.

a How much did she save?

b What is the normal selling price of the dress?

c If Tila gets a commission of 15%:

  i how much commission did she get in dollars?

  ii how much commission did Tila lose by selling the dress at cost price rather than the normal selling price?

Ext 2 Matilda has two bank accounts with the given details.

- Investment: Principal $25 000, interest rate 6.5% compounded annually
- Loan: 11.5% compounded annually

a Find Matilda's investment account balance after:

  i 1 year

  ii 10 years (to the nearest cent).

b Find the total percentage increase in Matilda's investment account after 10 years correct to two decimal places.

c After 3 years Matilda's loan account has increased to $114 250. Find the initial loan amount to the nearest dollar.

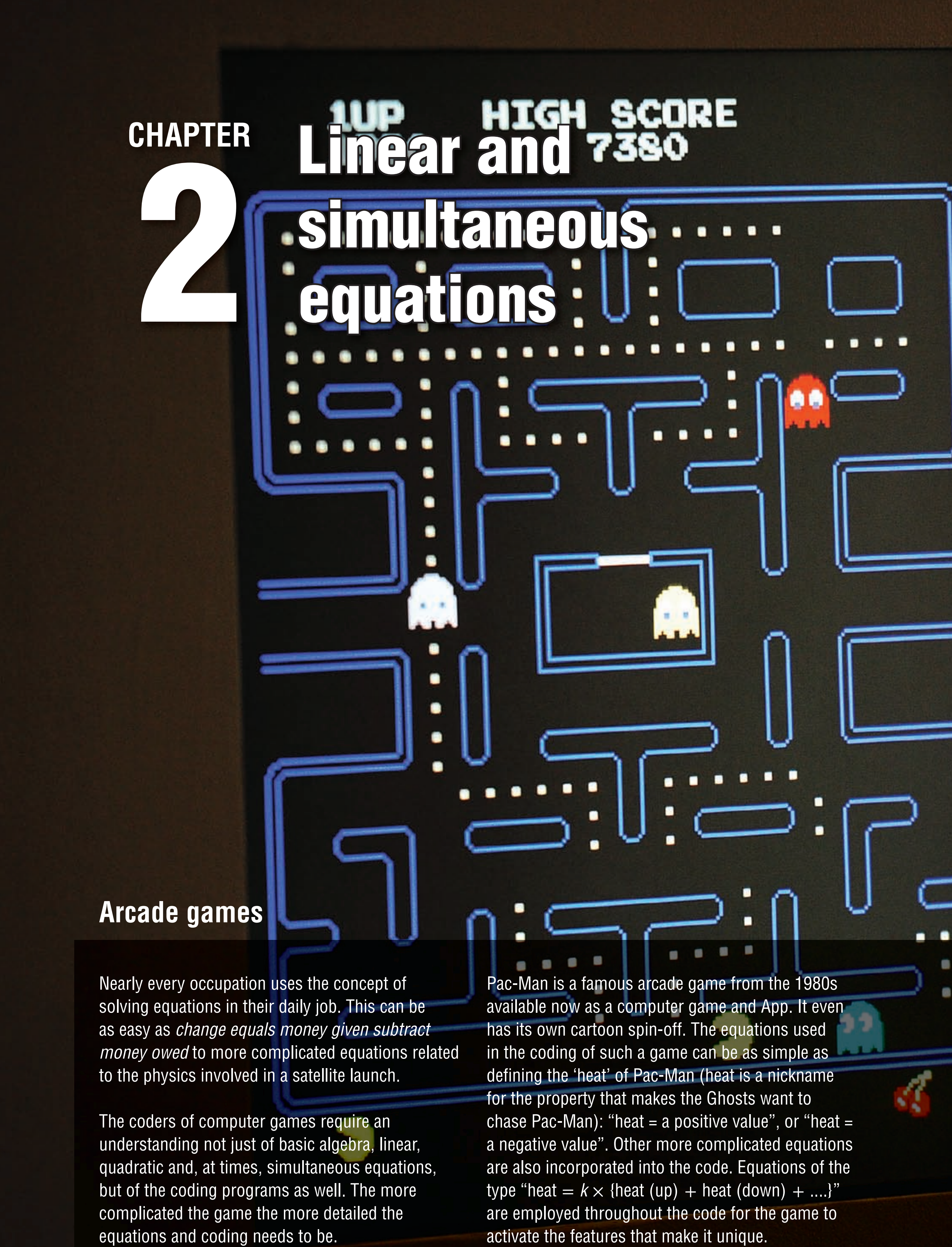
d Matilda reduces her $114 250 loan by $30 000. What is this reduction as a percentage to two decimal places?

e For Matilda's investment loan, what simple interest rate is equivalent to 5 years of the compounded interest rate of 6.5%? Round the answer to one decimal place.

CHAPTER

2

Linear and simultaneous equations



Arcade games

Nearly every occupation uses the concept of solving equations in their daily job. This can be as easy as *change equals money given subtract money owed* to more complicated equations related to the physics involved in a satellite launch.

The coders of computer games require an understanding not just of basic algebra, linear, quadratic and, at times, simultaneous equations, but of the coding programs as well. The more complicated the game the more detailed the equations and coding needs to be.

Pac-Man is a famous arcade game from the 1980s available now as a computer game and App. It even has its own cartoon spin-off. The equations used in the coding of such a game can be as simple as defining the 'heat' of Pac-Man (heat is a nickname for the property that makes the Ghosts want to chase Pac-Man): "heat = a positive value", or "heat = a negative value". Other more complicated equations are also incorporated into the code. Equations of the type "heat $= k \times$ {heat (up) $+$ heat (down) $+$ ....}" are employed throughout the code for the game to activate the features that make it unique.

ISBN 978-1-108-77288-4

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

In this chapter

2A Algebraic expressions (CONSOLIDATING)
2B Simplifying algebraic expressions (CONSOLIDATING)
2C Expanding algebraic expressions
2D Solving linear equations
2E Equations with brackets and pronumerals on both sides
2F Solving word problems
2G Inequalities
2H Using formulas
2I Simultaneous equations using substitution (EXTENDING)
2J Simultaneous equations using elimination (EXTENDING)
2K Applications of simultaneous equations (EXTENDING)

Australian Curriculum

NUMBER AND ALGEBRA
Patterns and algebra

Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate (ACMNA213)

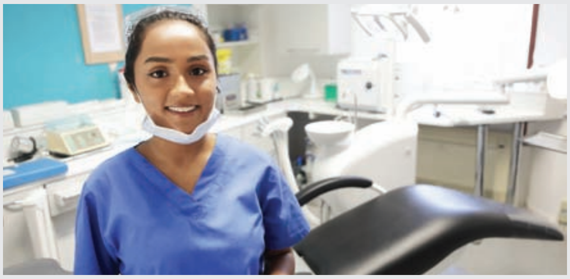
© ACARA

2A Algebraic expressions CONSOLIDATING

Learning intentions

- To review the definitions and conventions of algebra
- To know how to combine numbers and variables to write algebraic terms and expressions
- To be able to express a word problem as a mathematical expression using pronumerals and mathematical operations
- To be able to substitute values for pronumerals in expressions and evaluate

Algebra is central to the study of mathematics and is commonly used to solve problems in a vast array of theoretical and practical problems. Algebra involves the represention and manipulation of unknown or varying quantities in a mathematical context. Pronumerals (or variables) are used to represent these unknown quantities.



Dentists use an algebraic formula to calculate the quantity of local anaesthetic required for a procedure. The pronumerals in the formula represent the concentration of the anaesthetic being used and the patient's weight.

LESSON STARTER Remembering the vocabulary

State the parts of the expression $5x - 2xy + (4a)^2 - 2$ that match these words.

- pronumeral (or variable)
- term
- coefficient
- constant term
- squared term

KEY IDEAS

- In algebra, letters are used to represent numbers. These letters are called **pronumerals** or **variables**.
- An **expression** is a combination of numbers and pronumerals connected by any of the four operations $+, -, \times$ and $\div$. Brackets can also be used.

  For example: $5x^2 + 4y - 1$ and $3(x + 2) - \frac{y}{5}$

- A **term** is a combination of numbers and variables connected with only multiplication and/or division. Terms are separated with the operations + and −.
  For example: $5x + 7y$ is a two-term expression.
- **Coefficients** are the numbers being multiplied by pronumerals.
  For example: the 3 in $3x$, the -2 in $5 - 2x$ and the $\frac{1}{2}$ in $\frac{x^2}{2}$ are coefficients.
- **Constant terms** consist of a number only.
  For example: -2 is a constant in $x^2 + 4x - 2$ (The sign must be included.)
- Expressions can be **evaluated** by substituting a number for a pronumeral.
  For example: if $a = -2$ then $a + 6 = -2 + 6 = 4$.
- **Order of operations** should be followed when evaluating expressions:
  1. Brackets
  2. Powers
  3. Multiplication and division as they occur from left to right
  4. Addition and subtraction as they occur from left to right

BUILDING UNDERSTANDING

1 State the number of terms in these expressions.

a $5x + 2y$
b $1 + 2a^2$
c $b^2 + ca - 1$
d $\frac{x^2}{2}$

2 Match each item in the left column with an item in the right column.

a	Product	**A**	Division
b	Sum	**B**	Subtraction
c	Difference	**C**	Multiplication
d	Quotient	**D**	Addition
e	x^2	**E**	the reciprocal of a
f	$\frac{1}{a}$	**F**	the square of x

3 State the coefficient in these terms.

a $5xy$
b $-2a^2$
c $\frac{x}{3}$
d $-\frac{2a}{5}$

Example 1 Writing algebraic expressions from words

Write an algebraic expression for:

- **a** the number of tickets needed for 3 boys and r girls
- **b** the cost of P pies at \$3 each
- **c** the number of grams of peanuts for one child when 300 g of peanuts is shared equally among C children.

SOLUTION	EXPLANATION
a $3 + r$	3 tickets plus the number of girls
b $3P$	3 multiplied by the number of pies
c $\dfrac{300}{C}$	300 g divided into C parts

Now you try

Write an algebraic expression for:

- **a** the number of seats needed for a adults and 7 children
- **b** the cost of n apples at \$1.20 each
- **c** the number of pieces of chocolate for one person when 60 pieces is equally shared among p people.

Example 2 Converting words to expressions

Write an algebraic expression for:

- **a** five less than x
- **b** three more than twice x
- **c** the sum of a and b is divided by 4
- **d** the square of the sum of x and y.

SOLUTION	EXPLANATION
a $x - 5$	5 subtracted from x
b $2x + 3$	Twice x plus 3
c $\dfrac{a + b}{4}$	The sum of a and b is done first $(a + b)$ and the result divided by 4.
d $(x + y)^2$	The sum of x and y is done first and then the result is squared.

Now you try

Write an algebraic expression for:

a six more than x

b one less than three times x

c two less than a is divided by 3

d the square root of the sum of a and b.

Example 3 Substituting values into expressions

Evaluate these expressions when $a = 5$, $b = -2$ and $c = 3$.

a $7a - 2(a - c)$

b $b^2 - ac$

SOLUTION	EXPLANATION
a $7a - 2(a - c) = 7 \times 5 - 2(5 - 3)$ $= 35 - 2 \times 2$ $= 35 - 4$ $= 31$	Substitute the values for a and c. When using order of operations, evaluate brackets before moving to multiplication and division then addition and subtraction.
b $b^2 - ac = (-2)^2 - 5 \times 3$ $= 4 - 15$ $= -11$	Evaluate powers before the other operations: $(-2)^2 = -2 \times (-2) = 4$.

Now you try

Evaluate these expressions when $a = 3$, $b = -1$ and $c = -2$.

a $3(a - b) + 2c$

b $c^2 - 3ab$

Exercise 2A

FLUENCY	1, 2, 3–4(1/2)	2, 3–5(1/2)	3–4(1/2), 5

Example 1

1 Write an algebraic expression for:

a the number of seats needed for 5 adults and c children

b the cost of n bottles at \$2 each

c the number of litres of drink for one person if 10 litres is shared equally among p people.

Example 1

2 Write an algebraic expression for the following.

a The number of tickets required for:

- **i** 4 boys and r girls
- **ii** t boys and 2 girls
- **iii** b boys and g girls
- **iv** x boys, y girls and z adults.

b The cost, in dollars, of:

- **i** P pies at \$6 each
- **ii** 10 pies at \$$n$ each
- **iii** D drinks at \$2 each
- **iv** P pies at \$5 each and D drinks at \$2 each.

c The number of grams of lollies for one child when 500 g of lollies is shared equally among C children.

Example 2

3 Write an algebraic expression for each of the following.

- **a** The sum of 2 and x
- **b** The sum of ab and y
- **c** 5 less than x
- **d** The product of x and 3
- **e** The difference between $3x$ and $2y$
- **f** Three times the value of p
- **g** Four more than twice x
- **h** The sum of x and y is divided by 5
- **i** 10 less than the product of 4 and x
- **j** The square of the sum of m and n
- **k** The sum of the squares of m and n
- **l** The square root of the sum of x and y
- **m** The sum of a and its reciprocal
- **n** The cube of the square root of x

Example 3

4 Evaluate these expressions when $a = 4$, $b = -3$ and $c = 7$.

- **a** $b - ac$
- **b** $bc - a$
- **c** $a^2 - c^2$
- **d** $b^2 - ac$
- **e** $\frac{a + b}{2}$
- **f** $\frac{b^2 + c}{a}$
- **g** $\frac{1}{c} \times (a - b)$
- **h** $a^3 - bc$

5 Evaluate these expressions when $x = -2$, $y = -\frac{1}{2}$ and $z = \frac{1}{6}$.

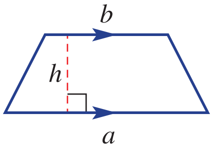
- **a** $xy + z$
- **b** $y^2 + x^2$
- **c** xyz
- **d** $\frac{xz + 1}{y}$

PROBLEM-SOLVING	6, 7	6, 7	7, 8

6 A rectangular garden bed is 12 m long and 5 m wide.

- **a** Find the area of the garden bed.
- **b** The length is increased by x m and the width is decreased by y m. Find the new length and width of the garden.
- **c** Write an expression for the area of the new garden bed.

7 The expression for the area of a trapezium is $\frac{1}{2}(a + b)h$ where a and b are the lengths of the two parallel sides and h is the distance between the two parallel sides.



- **a** Find the area of the trapezium with $a = 5$, $b = 7$ and $h = 3$.
- **b** A trapezium has $h = 4$ and area 12. If a and b are positive integers, what possible values can the variable a have?

8 The cost of 10 identical puzzles is $\$P$.

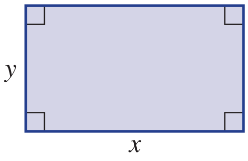
- **a** Write an expression for the cost of one puzzle.
- **b** Write an expression for the cost of n puzzles.

REASONING	9	9, 10	10, 11

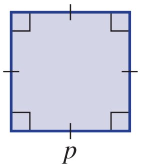
9 For each of these shapes, write an expression for:

- **i** the perimeter
- **ii** the area.

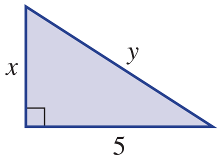
a



b



c



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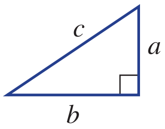
10 Decide if the following statements refer to the same or different expressions. If they are different, write an expression for each statement.

- **a** **A** Twice the sum of x and y
  - **B** The sum of $2x$ and y
- **b** **A** The difference between half of x and half of y
  - **B** Half of the difference between x and y

11 For a right-angled triangle with hypotenuse c and shorter sides a and b, Pythagoras' theorem states that $c^2 = a^2 + b^2$.



- **a** Which of these two descriptions also describes Pythagoras' theorem?
  - **A** The square of the hypotenuse is equal to the square of the sum of the two shorter sides.
  - **B** The square of the hypotenuse is equal to the sum of the squares of the two shorter sides.
- **b** For the incorrect description, write an equation to match.

ENRICHMENT: The sum of the first *n* positive integers	–	–	12

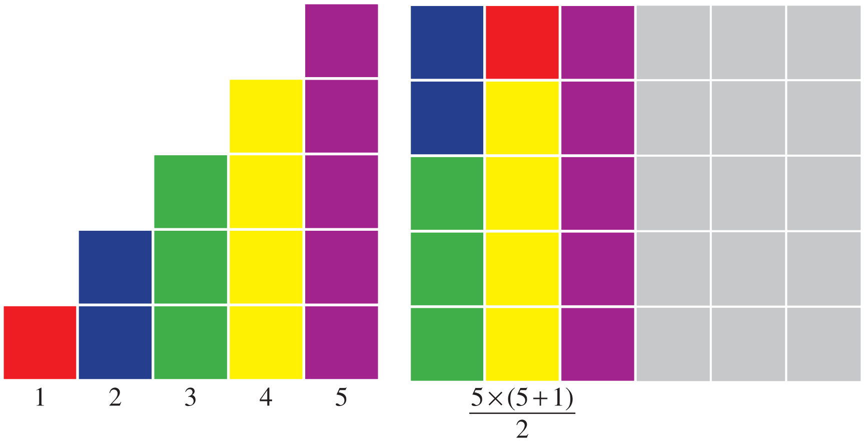
12 The rule for the sum of the first n positive integers is given by:

The product of n and one more than n all divided by 2.

- **a** Write an expression for the above description.
- **b** Test the expression to find these sums
  - **i** $1 + 2 + 3 + 4$ $(n = 4)$
  - **ii** $1 + 2 + 3 + \ldots + 10$ $(n = 10)$
- **c** Another way to describe the same expression is:

  The sum of half of the square of n and half of n.

  Write the expression for this description.
- **d** Check that your expressions in parts **a** and **c** are equivalent (the same) by testing $n = 4$ and $n = 10$.
- **e** $\frac{1}{2}(n^2 + n)$ is also equivalent to the above two expressions. Write this expression in words.



A diagram representing the sum of the first five positive integers arranged according to the expression in Question 12

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2B Simplifying algebraic expressions CONSOLIDATING

Learning intentions

- To review the conventions and processes for multiplication and division of algebraic terms
- To understand what is meant by like terms and that only like terms can be collected under addition and subtraction
- To be able to simplify algebraic expressions using addition and subtraction

Just as $2 + 2 + 2 + 2 = 4 \times 2$, so $x + x + x + x = 4 \times x$ or $4x$. We say that the expression $x + x + x + x$ is simplified to $4x$. Similarly, $3x + 5x = 8x$ and $8x - 3x = 5x$.

All these expressions have like terms and can be simplified to an expression with a smaller number of terms.

A single term such as $2 \times 5 \times x \div 10$ can also be simplified using multiplication and division, so $2 \times 5 \times x \div 10 = \dfrac{10x}{10} = x$.

Manufacturing companies, such as globe makers, find that an item's selling price, \$$x$, affects the number of sales and the revenue. Algebraic expressions often need to be simplified when developing formulas for a company's expenses, revenue and profit.

LESSON STARTER Are they equivalent?

All these expressions can be separated into two groups. Group them so that the expressions in each group are equivalent.

$2x$	$2x - y$	$4x - x - x$	$10x - y - 8x$
$\dfrac{24x}{12}$	$y + x - y + x$	$2 \times x - 1 \times y$	$-y + 2x$
$0 + \dfrac{1}{2} \times 4x$	$\dfrac{x}{\left(\frac{1}{2}\right)} + 0y$	$\dfrac{6x^2}{3x}$	$-1 \times y + \dfrac{x^2}{\frac{1}{2}x}$

KEY IDEAS

- The symbols for multiplication ($\times$) and division ($\div$) are usually not shown in simplified algebraic terms.
  For example: $5 \times a \times b = 5ab$ and $-7 \times x \div y^2 = -\dfrac{7x}{y^2}$
- When dividing algebraic expressions, common factors can be cancelled.
  For example: $\dfrac{7x}{14} = \dfrac{x}{2}$, $\quad \dfrac{a^2b}{a} = \dfrac{\cancel{a}^1 \times a \times b}{\cancel{a}^1} = ab$

$$\frac{7xy}{14y} = \frac{x}{2} \text{ and } \frac{15a^2b}{10a} = \frac{3 \times \cancel{5}^1 \times a \times \cancel{a}^1 \times b}{2 \times \cancel{5}^1 \times \cancel{a}^1} = \frac{3ab}{2}$$

- **Like terms** have the same pronumeral factors.
  - For example: $5x$ and $7x$ are like terms and $3a^2b$ and $-2a^2b$ are like terms.
  - Since $a \times b = b \times a$ then ab and ba are also like terms.
- The pronumeral part of a term is often written in alphabetical order.
- **Like terms** can be collected (added and subtracted) to form a single term.
  For example: $5ab + 8ab = 13ab$
  $4x^2y - 2yx^2 = 2x^2y$
- **Unlike terms** do not have the same pronumeral factors.
  For example: $5x$, x^2, xy and $\frac{4xyz}{5}$ are all unlike terms.

BUILDING UNDERSTANDING

1 State the missing word or expression.

a Like terms have the same __________ factors.
b $3x + 2x$ simplifies to __________.
c $3a^2b$ and $2a$ are __________ terms.

2 Simplify these fractions.

a $\frac{20}{8}$
b $\frac{30}{10}$
c $\frac{8}{40}$
d $-\frac{5}{100}$

3 Decide if the following pairs of terms are like terms.

a $4ab$ and $3ab$
b $2x$ and $7xy$
c 5 and $4m$
d $3t$ and $-6tw$
e $2mn$ and $9nm$
f $3x^2y$ and $7xy^2$

Example 4 Multiplying algebraic terms

Simplify the following.

a $3 \times 2b$
b $-2a \times 3ab$

SOLUTION	EXPLANATION
a $3 \times 2b = 3 \times 2 \times b$ $= 6b$	Multiply the coefficients.
b $-2a \times 3ab = -2 \times 3 \times a \times a \times b$ $= -6a^2b$	Multiply the coefficients and simplify.

Now you try

Simplify the following.

a $5 \times 7n$
b $-3x \times 6xy$

Example 5 Dividing algebraic terms

Simplify the following.

a $\frac{6ab}{18b}$ **b** $12a^2b \div (3ab)$

SOLUTION	EXPLANATION
a $\frac{{}^{1}\cancel{6}a\cancel{b}^{1}}{{}_{3}\cancel{18}\cancel{b}_{1}} = \frac{a}{3}$	Deal with numerals and promumerals separately, cancelling where possible.
b $12a^2b \div (3ab) = \frac{{}^{4}\cancel{12}a^{\cancel{2}\,1}\cancel{b}^{1}}{{}_{1}\cancel{3}\cancel{a}_{1}\cancel{b}_{1}}$ $= 4a$	Write as a fraction first. Cancel where possible, recall $a^2 = a \times a$.

Now you try

Simplify the following.

a $\frac{4xy}{8x}$ **b** $-20ab^2 \div (10ab)$

Example 6 Collecting like terms

Simplify the following by collecting like terms.

a $3x + 4 - 2x$ **b** $3x + 2y + 4x + 7y$ **c** $8ab^2 - 9ab - ab^2 + 3ba$

SOLUTION	EXPLANATION
a $3x + 4 - 2x = 3x - 2x + 4$ $= x + 4$	Collect like terms ($3x$ and $-2x$). The sign belongs to the term that follows. Combine their coefficients: $3 - 2 = 1$.
b $3x + 2y + 4x + 7y = 3x + 4x + 2y + 7y$ $= 7x + 9y$	Collect like terms and combine their coefficients.
c $8ab^2 - 9ab - ab^2 + 3ba$ $= 8ab^2 - ab^2 - 9ab + 3ab$ $= 7ab^2 - 6ab$	Collect like terms. Remember $ab = ba$ and $ab^2 = 1ab^2$. $8 - 1 = 7$ and $-9 + 3 = -6$

Now you try

Simplify the following by collecting like terms.

a $6a + 2 - a$ **b** $4m + n + 2m + 6n$ **c** $7a^2b + ab - 4a^2b - 3ba$

Exercise 2B

FLUENCY	1, 2–5($1/2$), 7–9($1/2$)	2–9($1/2$)	3–9($1/3$)

1 Simplify the following.

Example 4a
a **i** $5 \times 2m$ **ii** $2 \times 6b$ **iii** $3 \times 5p$

Example 4b
b **i** $4n \times 6n$ **ii** $-3q \times q$ **iii** $4b \times 2bc$

Example 4a
2 Simplify the following.

a $3 \times 6r$
b $4m \times 4n$
c $-2x \times 7y$
d $5m \times (-3n)$
e $-4c \times 3d$
f $2a \times 3b \times 5$
g $-4r \times 3 \times 2s$
h $5j \times (-4) \times 2k$

Example 4b
3 Simplify the following.

a $7a \times 3ab$
b $5mn \times (-3n)$
c $-3gh \times (-6h)$
d $3xy \times 4xy$
e $-4ab \times (-2ab)$
f $-2mn \times 3mn$

Example 5a
4 Simplify the following by cancelling.

a $\frac{8b}{2}$
b $-\frac{2a}{6}$
c $\frac{4ab}{6}$
d $\frac{3mn}{6n}$
e $-\frac{5xy}{20y}$
f $\frac{10st}{6t}$
g $\frac{u^2v}{u}$
h $\frac{5r^2s^2}{8rs}$
i $\frac{5ab^2}{9b}$

Example 5b
5 Simplify the following by first writing in fraction form.

a $2x \div 5$
b $-4 \div (-3a)$
c $11mn \div 3$
d $12ab \div 2$
e $-10 \div (2gh)$
f $8x \div x$
g $-3xy \div (yx)$
h $7mn \div (3m)$
i $-27pq \div (6p)$
j $24ab^2 \div (8ab)$
k $25x^2y \div (-5xy)$
l $9m^2n \div (18mn)$

6 Simplify the following.

a $x \times 4 \div y$
b $5 \times p \div 2$
c $6 \times (-a) \times b$
d $a \times (-3) \div (2b)$
e $-7 \div (5m) \times n$
f $5s \div (2t) \times 4$
g $6 \times 4mn \div (3m)$
h $8x \times 3y \div (8x)$
i $3ab \times 12bc \div (9abc)$
j $4x \times 3xy \div (2x)$
k $10m \times 4mn \div (8mn)$
l $3pq \times pq \div p$

Example 6a
7 Simplify the following by collecting like terms.

a $3a + 7a$
b $4n + 3n$
c $12y - 4y$
d $5x + 2x + 4x$
e $6ab - 2ab - ba$
f $7mn + 2mn - 2mn$
g $4y - 3y + 8$
h $7x + 5 - 4x$
i $6xy + xy + 4y$
j $5ab + 3 + 7ba$
k $2 - 5m - m$
l $4 - 2x + x$

Example 6b
8 Simplify the following by collecting like terms.

a $2a + 4b + 3a + 5b$
b $4x + 3y + 2x + 2y$
c $6t + 5 - 2t + 1$
d $5x + 1 + 6x + 3$
e $xy + 8x + 4xy - 4x$
f $3mn - 4 + 4nm - 5$
g $4ab + 2a + ab - 3a$
h $3st - 8ts + 2st + 3ts$

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Example 6b

9 Simplify the following by collecting like terms.

a $5xy^2 - 4xy^2$
b $3a^2b + 4ba^2$
c $8m^2n - 6nm^2 + m^2n$
d $7p^2q^2 - 2p^2q^2 - 4p^2q^2$
e $2x^2y - 4xy^2 + 5yx^2$
f $10rs^2 + 3rs^2 - 6r^2s$
g $x^2 - 7x - 3x^2$
h $a^2b - 4ab^2 + 3a^2b + b^2a$
i $10pq^2 - 2qp - 3pq^2 - 6pq$
j $12m^2n^2 - 2mn^2 - 4m^2n^2 + mn^2$

PROBLEM-SOLVING	10, 11	10–12	11–13

10 A farmer has x pigs and y chickens.

a Write an expression for the total number of heads.
b Write an expression for the total number of legs.

11 The length of a rectangle is three times its width x metres. Write an expression for:

a the rectangle's perimeter
b the rectangle's area.

12 A right-angled triangle has side lengths $5x$ cm, $12x$ cm and $13x$ cm. Write an expression for:

a the triangle's perimeter
b the triangle's area.

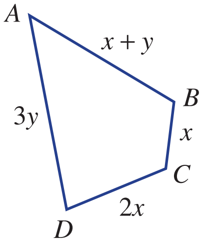
13 The average (mean) mark on a test for 20 students is x. Another student who scores 75 in the test is added to the list. Write an expression for the new average (mean).

REASONING	14	14	14, 15

14 Decide whether the following are always true for all real numbers.

a $a \times b = b \times a$
b $a \div b = b \div a$
c $a + b = b + a$
d $a - b = b - a$
e $a^2b = b^2a$
f $1 \div \frac{1}{a} = a \ (a \neq 0)$

15 The diagram shows the route taken by a salesperson who travels from A to D via B and C. All distances are in kilometres.



a If the salesperson then returns directly to A, write an expression (in simplest form) for the total distance travelled.
b If $y = x + 1$, write an expression for the total distance the salesperson travels in terms of x only. Simplify your expression.
c When $y = x + 1$, by how much would the distance have been reduced (in terms of x) if the salesperson had travelled directly from A to D and straight back to A?

ENRICHMENT: Higher powers	–	–	16(½)

16 For this question, note this example:

$$\frac{2a^3}{4a} = \frac{\cancel{2}_1 \times a \times a \times \cancel{a}_1}{{}^2\cancel{4} \times \cancel{a}_1}$$

$$= \frac{a^2}{2}$$

Simplify these expressions with higher powers.

a $\frac{a^4}{a}$
b $\frac{3b^3}{9b}$
c $\frac{4ab^3}{12ab}$
d $\frac{6a^4b^2}{16a^3b}$
e $-\frac{2a^5}{3a^2}$
f $-\frac{8a^5}{20a^7}$
g $\frac{4a^3}{10a^8}$
h $\frac{3a^3b}{12ab^4}$
i $\frac{15a^4b^2}{5ab}$
j $\frac{28a^3b^5}{7a^4b^2}$
k $\frac{2a^5b^2}{6a^2b^3}$
l $-\frac{5a^3b^7}{10a^2b^{10}}$

ISBN 978-1-108-77288-4

2C Expanding algebraic expressions

Learning intentions

- To understand how the distributive law is used to expand brackets
- To be able to expand brackets by multiplying through by the term outside the bracket
- To know to expand brackets first before collecting like terms when simplifying algebraic expressions

A mental technique to find the product of 5 and 23 might be to find 5×20 and add 5×3 to give 115. This technique uses the distributive law over addition.

$$\begin{aligned} \text{So } 5 \times 23 &= 5 \times (20 + 3) \\ &= 5 \times 20 + 5 \times 3 \end{aligned}$$

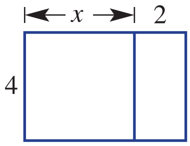
Since pronumerals (or variables) represent numbers, the same law applies for algebraic expressions.

$$5 \times (x + y) = 5 \times x + 5 \times y$$

Portable dance floors for hire come in 1 m by 1 m squares of two types, edge squares and inner squares. For a floor x m by y m, the number of edge squares $= 2x + 2(y - 2)$ and the number of inner squares $= (x - 2)(y - 2)$.

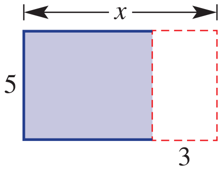
LESSON STARTER Rectangular distributions

This diagram shows two joined rectangles with the given dimensions.



- Find two different ways to write expressions for the combined area of the two rectangles.
- Compare your two expressions. Are they equivalent?

This diagram shows a rectangle of length x reduced by a length of 3.



- Find two different ways to write expressions for the remaining area (shaded).
- Compare your two expressions. Are they equivalent?

KEY IDEAS

- The **distributive law** is used to expand and remove brackets.
  - A term on the outside of the brackets is multiplied by each term inside the brackets.

    $a(b + c) = ab + ac$ or $a(b - c) = ab - ac$

    $-a(b + c) = -ab - ac$ or $-a(b - c) = -ab + ac$
  - If the number in front of the bracket is negative, the sign of each of the terms inside the brackets will change when expanded.

    For example: $-2(x - 3) = -2x + 6$ since $-2 \times x = -2x$ and $-2 \times (-3) = 6$.

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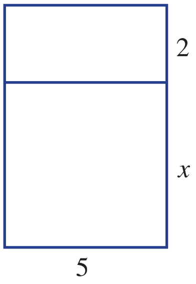
BUILDING UNDERSTANDING

1 This diagram shows two joined rectangles with the given dimensions.



- **a** State an expression for the area of:
  - **i** the larger rectangle (x by 5)
  - **ii** the smaller rectangle (2 by 5).
- **b** Use your answers from part **a** to find the combined area of both rectangles.
- **c** State an expression for the total side length of the side involving x.
- **d** Use your answer from part **c** to find the combined area of both rectangles.
- **e** State the missing terms in this statement: $5(x + 2) =$ _____ + _____

2
- **a** Substitute the value $x = 5$ into these expressions.
  - **i** $-2(x + 3)$
  - **ii** $-2x + 6$
- **b** Do your answers from part **a** suggest that $-2(x + 3) = -2x + 6$? When expanded what should $-2(x + 3)$ equal?
- **c** Substitute the value $x = 10$ into these expressions.
  - **i** $-3(x - 1)$
  - **ii** $-3x - 3$
- **d** Do your answers from part **c** suggest that $-3(x - 1) = -3x - 3$? When expanded what should $-3(x - 1)$ equal?

Example 7 Expanding simple expressions with brackets

Expand the following.

a $3(x + 4)$ **b** $5(x - 11)$ **c** $-2(x - 5)$

SOLUTION	EXPLANATION
a $3(x + 4) = 3x + 12$	$3 \times x = 3x$ and $3 \times 4 = 12$
b $5(x - 11) = 5x - 55$	$5 \times x = 5x$ and $5 \times (-11) = -55$
c $-2(x - 5) = -2x + 10$	$-2 \times x = -2x$ and $-2 \times (-5) = +10$

Now you try

Expand the following.

a $5(x + 2)$ **b** $3(x - 7)$ **c** $-3(x - 2)$

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Example 8 Expanding brackets and simplifying

Expand the following.

a $4(x + 3y)$

b $-2x(4x - 3)$

SOLUTION

a $4(x + 3y) = 4 \times x + 4 \times 3y$
$= 4x + 12y$

b $-2x(4x - 3) = -2x \times 4x + (-2x) \times (-3)$
$= -8x^2 + 6x$

EXPLANATION

Multiply each term inside the brackets by 4.
$4 \times x = 4x$ and $4 \times 3 \times y = 12y$.

Each term inside the brackets is multiplied by $-2x$.
$-2 \times 4 = -8$, $x \times x = x^2$ and $-2 \times (-3) = 6$

Now you try

Expand the following.

a $3(x + 7y)$

b $-3a(2a - 4)$

Example 9 Simplifying by removing brackets

Expand the following and collect like terms.

a $2 - 3(x - 4)$

b $3(x + 2y) - (3x + y)$

SOLUTION

a $2 - 3(x - 4) = 2 - (3x - 12)$
$= 2 - 3x + 12$
$= 14 - 3x$

b $3(x + 2y) - (3x + y)$
$= 3x + 6y - 3x - y$
$= 3x - 3x + 6y - y$
$= 5y$

EXPLANATION

$3(x - 4) = 3x - 12$
$-(3x - 12) = -1(3x - 12)$, so multiplying by negative 1 changes the sign of each term inside the brackets.

$-(3x + y) = -1(3x + y) = -3x - y$.
Collect like terms and simplify.

Now you try

Expand the following and collect like terms.

a $4 - 2(y - 1)$

b $3(x - 2y) - (2x + 3y)$

Exercise 2C

FLUENCY	1, 2–5(½)	2–6(½)	3–6(¼)

1 Expand the following.

Example 7a **a** **i** $2(x + 1)$ **ii** $5(x + 2)$

Example 7b **b** **i** $7(x - 4)$ **ii** $6(x - 3)$

Example 7c **c** **i** $-4(x - 2)$ **ii** $-2(x - 4)$

Example 7a,b **2** Expand the following.

a $2(x + 3)$ **b** $5(x + 12)$ **c** $2(x - 7)$ **d** $7(x - 9)$

e $3(2 + x)$ **f** $7(3 - x)$ **g** $4(7 - x)$ **h** $2(x - 6)$

Example 7c **3** Expand the following.

a $-3(x + 2)$ **b** $-2(x + 11)$ **c** $-5(x - 3)$ **d** $-6(x - 6)$

e $-4(2 - x)$ **f** $-13(5 + x)$ **g** $-20(9 + x)$ **h** $-300(1 - x)$

Example 8 **4** Expand the following.

a $2(a + 2b)$ **b** $5(3a - 2)$ **c** $3(4m - 5)$ **d** $-8(2x + 5)$

e $-3(4x + 5)$ **f** $-4x(x - 2y)$ **g** $-9t(2y - 3)$ **h** $a(3a + 4)$

i $d(2d - 5)$ **j** $-2b(3b - 5)$ **k** $2x(4x + 1)$ **l** $5y(1 - 3y)$

Example 9a **5** Expand the following and collect like terms.

a $3 + 2(x + 4)$ **b** $4 + 6(x - 3)$ **c** $2 + 5(3x - 1)$ **d** $5 + (3x - 4)$

e $3 + 4(x - 2)$ **f** $7 + 2(x - 3)$ **g** $2 - 3(x + 2)$ **h** $1 - 5(x + 4)$

i $5 - (x - 6)$ **j** $9 - (x - 3)$ **k** $5 - (3 + 2x)$ **l** $4 - (3x - 2)$

Example 9b **6** Expand the following and collect like terms.

a $2(x + 3) + 3(x + 2)$ **b** $2(x - 3) + 2(x - 1)$ **c** $3(2x + 1) + 5(x - 1)$

d $4(3x + 2) + 5(x - 3)$ **e** $-3(2x + 1) + (2x - 3)$ **f** $-2(x + 2) + 3(x - 1)$

g $2(4x - 3) - 2(3x - 1)$ **h** $-3(4x + 3) - 5(3x - 1)$ **i** $-(x + 3) - 3(x + 5)$

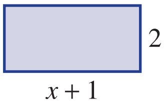
j $-2(2x - 4) - 3(3x + 5)$ **k** $3(3x - 1) - 2(2 - x)$ **l** $-4(5 - x) - (2x - 5)$

PROBLEM-SOLVING	7, 8	7, 8(½), 9	8(½), 9, 10

7 The length of a rectangle is 4 more than its width, x. Find an expanded expression for its area.

8 Find the area of these basic shapes in expanded form. All angles at vertices are 90°.

a



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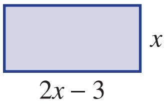

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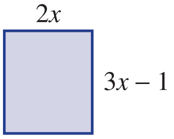

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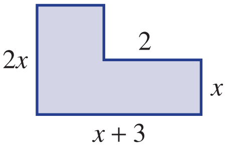

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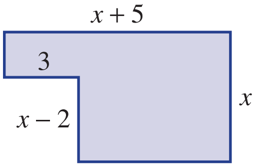
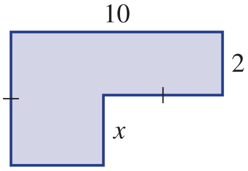


f



9 Gary gets a \$20 bonus for every computer he sells over and above his quota of 10 per week. If he sells n computers in a week and $n > 10$, write an expression for Gary's bonus in that week. (Give the answer in expanded form.)

10 Jill pays tax at 20c in the dollar for every dollar earned over \$10 000. If Jill earns \$$x$ and $x > 10\,000$, write an expression in expanded form for Jill's tax.

REASONING	11	11, 12(½)	11, 12(½)

11 Identify the errors in these expressions and then write the correct expansion.

a $2(x + 6) = 2x + 6$

b $x(x - 4) = 2x - 4x$

c $-3(x + 4) = -3x + 12$

d $-7(x - 7) = -7x - 49$

e $\begin{aligned} 5 - 2(x - 7) &= 5 - 2x - 14 \\ &= -9 - 2x \end{aligned}$

f $\begin{aligned} 4(x - 2) - 3(x + 2) &= 4x - 8 - 3x + 6 \\ &= x - 2 \end{aligned}$

12 In Years 7 and 8 we explored how to use the distributive law to find products mentally.

For example: $\begin{aligned} 7 \times 104 &= 7 \times (100 + 4) \\ &= 7 \times 100 + 7 \times 4 \\ &= 728 \end{aligned}$ and $\begin{aligned} 4 \times 298 &= 4 \times (300 - 2) \\ &= 4 \times 300 - 4 \times 2 \\ &= 1192 \end{aligned}$

Use the distributive law to evaluate these products mentally.

a 6×52 **b** 9×102 **c** 5×91 **d** 4×326

e 3×99 **f** 7×395 **g** 9×990 **h** 6×879

ENRICHMENT: Pronumerals and taxes	–	–	13

13 A progressive income tax system increases the tax rate for higher incomes. Here is an example.

Income	Tax
0–\$20 000	\$0
\$20 001–\$50 000	\$0 + 20% of income above \$20 000
\$50 001–\$100 000	a + 30% of income above \$50 000
\$100 000–	b + 50% of income above \$100 000

a Find the values of a and b in the above table.

b Find the tax payable for the following incomes.

- **i** \$35 000
- **ii** \$72 000
- **iii** \$160 000

c Find an expression for the tax payable for an income of \$$x$ if:

- **i** $0 \leqslant x \leqslant 20000$
- **ii** $20000 < x \leqslant 50000$
- **iii** $50000 < x \leqslant 100000$
- **iv** $x > 100000$

d Check that you have fully expanded and simplified your expressions for part **c**. Add steps if required.

e Use your expressions from parts **c** and **d** to check your answers to part **b** by choosing a particular income amount and checking against the table above.

2D Solving linear equations

Learning intentions

- To know the definition of an equation
- To understand that linear equations are solved by creating a series of equivalent equations
- To understand that equivalent equations are created by applying the same operation to both sides of the equation
- To be able to solve a linear equation by using inverse operations
- To know that an answer can be checked by substituting into the original equation

A mathematical statement containing an equals sign, a left-hand side and a right-hand side is called an equation. $5 = 10 \div 2$, $3x = 9$, $x^2 + 1 = 10$ and $\frac{1}{x} = \frac{x}{5}$ are examples of equations. Linear equations can be written in the form $ax + b = c$ where the power of x is 1. $4x - 1 = 6$, $3 = 2(x + 1)$ and $\frac{5x}{3} = \frac{2x + 1}{4}$ are all linear equations. Equations are solved by finding the value of the variable (or pronumeral) that makes the equation true. This can be done by inspection for very simple linear equations (for example, if $3x = 15$ then $x = 5$ since $3 \times 5 = 15$). More complex linear equations can be solved through a series of steps where each step produces an equivalent equation.

Computer programmers solve linear equations when writing software code for various applications, developing websites, troubleshooting network problems, calculating data upload times and adjusting security settings.

LESSON STARTER Why are they equivalent?

The following list of equations can be categorised into two groups. The equations in each group should be equivalent.

$$5x = 20 \qquad 2x - 1 = -3 \qquad x = 4 \qquad 1 - x = -3$$

$$7x = -7 \qquad 3 - 5x = -17 \qquad \frac{8x}{5} - \frac{3x}{5} = -1 \qquad x = -1$$

- Discuss how you divided the equations into the two groups.
- How can you check to see if the equations in each group are equivalent?

KEY IDEAS

- **Equivalent equations** are created by:
  - adding or subtracting the same number on both sides of the equation
  - multiplying or dividing both sides of the equation by the same number (not including 0).
- Solve a linear equation by creating equivalent equations using **inverse operations** (sometimes referred to as **backtracking**).
- The solution to an equation can be checked by substituting the solution into the original equation and checking that both sides are equal.

BUILDING UNDERSTANDING

① State the value of x that is the solution to these equations. No written working is required.

a $3x = 9$ **b** $\frac{x}{4} = 10$ **c** $x + 7 = 12$ **d** $x - 7 = -1$

② Use a 'guess and check' (trial and error) method to solve these equations. No written working is required.

a $2x + 1 = 7$ **b** $4 - x = 2$ **c** $2 + \frac{x}{3} = 6$ **d** $\frac{x+1}{7} = 1$

③ Which of the following equations are equivalent to $3x = 12$?

A $3x - 1 = 12$ **B** $-3x = -12$ **C** $\frac{3x}{4} = 3$ **D** $\frac{3x}{5} = 10$

Example 10 Solving simple linear equations

Solve each of the following equations.

a $2x + 3 = 4$ **b** $\frac{x}{4} - 3 = 7$ **c** $5 - 2x = 12$

d $\frac{2x}{3} + 5 = 7$ **e** $\frac{2x + 4}{9} = 2$

SOLUTION	EXPLANATION
a $2x + 3 = 4$	
$2x = 1$	Subtract 3 from both sides.
$x = \frac{1}{2}$	Divide both sides by 2.
Check: $2 \times \left(\frac{1}{2}\right) + 3 = 4$	Check the answer by substituting $x = \frac{1}{2}$ into the original equation.

Continued on next page

b $\frac{x}{4} - 3 = 7$	
$\frac{x}{4} = 10$	Add 3 to both sides.
$x = 40$	Multiply both sides by 4.
Check: $\frac{(40)}{4} - 3 = 10 - 3 = 7$	Check the answer by substituting $x = 40$ into $\frac{x}{4} - 3$. Since this equals 7, $x = 40$ is the solution.
c $5 - 2x = 12$	
$-2x = 7$	Subtract 5 from both sides.
$x = -\frac{7}{2}$	Divide both sides by -2.
Check: $5 - 2 \times \left(-\frac{7}{2}\right) = 5 + 7 = 12$	Check the answer.
d $\frac{2x}{3} + 5 = 7$	
$\frac{2x}{3} = 2$	Subtract 5 from both sides first and then multiply both sides by 3.
$2x = 6$	Divide both sides by 2.
$x = 3$	
Check: $\frac{2 \times (3)}{3} + 5 = 2 + 5 = 7$	Check the answer.
e $\frac{2x + 4}{9} = 2$	
$2x + 4 = 18$ $2x = 14$ $x = 7$	Multiply both sides by 9 first to eliminate the fraction. Solve the remaining equation by subtracting 4 from both sides and then dividing both sides by 2.
Check: $\frac{2 \times (7) + 4}{9} = \frac{18}{9} = 2$	Check the answer.

Now you try

Solve each of the following equations.

a $3x + 1 = 10$

b $\frac{x}{2} - 5 = 6$

c $4 - 3x = 7$

d $\frac{5x}{2} + 1 = 4$

e $\frac{4x + 2}{5} = 3$

Exercise 2D

FLUENCY	1, 2–5(1/2)	2–6(1/2)	2–6(1/3)

Example 10a
Example 10b

1 Solve each of the following equations.

a **i** $2x + 5 = 9$ **ii** $5a + 6 = 11$ **iii** $3m - 4 = 8$

b **i** $\frac{x}{4} + 3 = 5$ **ii** $\frac{x}{2} + 4 = 5$ **iii** $\frac{b}{3} + 5 = 9$

Example 10a

2 Solve each of the following equations. Check your answers using substitution.

a $2x - 4 = -6$ **b** $2n + 13 = 7$ **c** $2x + 5 = -7$

d $2b + 15 = 7$ **e** $3y - 2 = -13$ **f** $3a + 2 = 7$

g $4b + 7 = 25$ **h** $24x - 2 = 10$ **i** $6x - 5 = 3$

j $7y - 3 = -8$ **k** $2a + \frac{1}{2} = \frac{1}{4}$ **l** $5n - \frac{1}{4} = -1$

Example 10b

3 Solve each of the following equations.

a $\frac{t}{2} + 5 = 2$ **b** $\frac{a}{3} + 4 = 2$

c $\frac{y}{5} - 4 = 2$ **d** $\frac{x}{3} - 7 = -12$

e $\frac{s}{2} - 3 = -7$ **f** $\frac{x}{4} - 5 = -2$

g $\frac{m}{4} - 2 = 3$ **h** $1 - \frac{y}{5} = 2$

Example 10c

4 Solve each of the following equations.

a $12 - 2x = 18$ **b** $2 - 7x = 9$

c $15 - 5x = 5$ **d** $3 - 2x = -13$

e $2 - 5x = 9$ **f** $4 - 7x = 23$

g $5 - 8x = 2$ **h** $-3 - 4x = -10$

Example 10d

5 Solve these equations.

a $\frac{2b}{3} = 6$ **b** $\frac{3x}{2} = 9$ **c** $\frac{4x}{3} = -9$

d $\frac{2x}{5} = -3$ **e** $\frac{3x}{4} = \frac{1}{2}$ **f** $\frac{5n}{4} = -\frac{1}{5}$

g $\frac{2x}{3} - 1 = 7$ **h** $\frac{3x}{4} - 2 = 7$ **i** $3 + \frac{2x}{3} = -3$

j $5 + \frac{3d}{2} = -7$ **k** $11 - \frac{3f}{2} = 2$ **l** $3 - \frac{4z}{3} = 5$

Example 10e

6 Solve each of the following equations. Check your answers.

a $\frac{x+1}{3} = 4$ **b** $\frac{x+4}{2} = 5$ **c** $\frac{4+y}{3} = -2$

d $\frac{6+b}{2} = -3$ **e** $\frac{1-a}{2} = 3$ **f** $\frac{5-x}{3} = 2$

g $\frac{3m-1}{5} = 4$ **h** $\frac{2x+2}{3} = 4$ **i** $\frac{7x-3}{3} = 9$

j $\frac{3b-6}{2} = 5$ **k** $\frac{4-2y}{6} = 3$ **l** $\frac{9-5t}{3} = -2$

PROBLEM-SOLVING	7, 8	7, 8	7(½), 8, 9

7 For each of the following, write an equation and solve it to find the unknown value. Use x as the unknown value.

- **a** If 8 is added to a certain number, the result is 34.
- **b** Seven less than a certain number is 21.
- **c** I think of a number, double it and add 4. The result is 10.
- **d** I think of a number, halve it and subtract 4. The result is 10.
- **e** Four less than three times a number is 20.
- **f** A number is multiplied by 7 and the product is divided by 3. The final result is 8.

8 Five Easter eggs are added to my initial collection of Easter eggs. I share them between myself and two friends and each person gets exactly four. Find how many eggs there were initially by solving an appropriate equation.

9 My weekly pay is increased by \$200 per week. Half of my pay now goes to pay the rent and \$100 to buy groceries. If this leaves me with \$450, what was my original weekly pay?

REASONING	10	10	10, 11

10 Describe the error made in each of these incorrect solutions.

a
$$\begin{aligned} 2x - 1 &= 4 \\ x - 1 &= 2 \\ x &= 3 \end{aligned}$$

b
$$\begin{aligned} \frac{5x + 2}{3} &= 7 \\ \frac{5x}{3} &= 5 \\ 5x &= 15 \\ x &= 3 \end{aligned}$$

c
$$\begin{aligned} 5 - x &= 12 \\ x &= 7 \end{aligned}$$

d
$$\begin{aligned} \frac{x}{3} - 4 &= 2 \\ x - 4 &= 6 \\ x &= 10 \end{aligned}$$

11 An equation like $2(x + 3) = 8$ can be solved without expanding the brackets. The first step is to divide both sides by 2.

- **a** Use this approach to solve these equations.
  - **i** $3(x - 1) = 12$
  - **ii** $4(x + 2) = -4$
  - **iii** $7(5x + 1) = 14$
  - **iv** $5(1 - x) = -10$
  - **v** $-2(3x + 1) = 3$
  - **vi** $-5(1 - 4x) = 1$
- **b** By considering your solutions to the equations in part **a**, when do you think this method is most appropriate?

ENRICHMENT: Changing the subject	–	–	12(½)

12 Make a the subject of each of the following equations; that is, a should be on the left-hand side on its own.

- **a** $a - b = c$
- **b** $2a + b = c$
- **c** $c - ab = 2d$
- **d** $b = \frac{a}{c} - d$
- **e** $\frac{ab}{c} = -d$
- **f** $\frac{2a}{b} = \frac{1}{c}$
- **g** $\frac{2ab}{c} - 3 = -d$
- **h** $b - \frac{ac}{2d} = 3$
- **i** $\frac{a + b}{c} = d$
- **j** $\frac{b - a}{c} = -d$
- **k** $\frac{ad - 6c}{2b} = e$
- **l** $\frac{d - 4ac}{e} = 3f$

ISBN 978-1-108-77288-4

2E Equations with brackets and pronumerals on both sides

Learning intentions

- To know that equations involving brackets can be solved by first expanding the brackets
- To know to solve equations with variables on both sides by collecting variables on one side of the equation
- To be able to add or subtract terms to both sides of the equation to solve the equation

More complex linear equations may have pronumerals on both sides of the equation and/or brackets. Examples are $3x = 5x - 1$ or $4(x + 2) = 5x$. Brackets can be removed by expanding and equations with pronumerals on both sides can be solved by collecting like terms using addition and subtraction.

More complex equations require several steps completed in the correct order to maintain the balance and undo all the operations that have been applied around the pronumeral.

LESSON STARTER Steps in the wrong order

The steps to solve $3(2 - x) = -2(2x - 1)$ are listed here in the incorrect order.

$$3(2 - x) = -2(2x - 1)$$
$$x = -4$$
$$6 + x = 2$$
$$6 - 3x = -4x + 2$$

- Arrange them in the correct order working from top to bottom.
- By considering all the steps in the correct order, explain what has happened in each step.

KEY IDEAS

- Equations with brackets can be solved by first expanding the brackets. For example: $3(x + 1) = 2$ becomes $3x + 3 = 2$.
- If an equation has pronumerals on both sides, collect to one side by adding or subtracting one of the terms. For example: $3x + 4 = 2x - 3$ becomes $x + 4 = -3$ by subtracting $2x$ from both sides.

BUILDING UNDERSTANDING

1 State the result from expanding and simplifying these expressions.

a $3(x - 4) + x$ **b** $2(1 - x) + 2x$ **c** $3(x - 1) + 2(x - 3)$

2 State the next line of working for the given equations and instructions.

a $2(x + 3) = 5$ (expand the brackets)
b $5 + 2(x - 1) = 7$ (expand the brackets)
c $3x + 1 = x - 6$ (subtract x from both sides)
d $4x - 3 = 2x + 1$ (subtract $2x$ from both sides)

Example 11 Solving equations with brackets and pronumerals on both sides

Solve each of the following equations.

a $2(3x-4)=11$

b $2(x+3)-4x=8$

c $5x-2=3x-4$

d $3(2x+4)=8(x+1)$

SOLUTION	EXPLANATION
a $2(3x-4)=11$ $6x-8=11$ $6x=19$ $x=\frac{19}{6}$ or $3\frac{1}{6}$	Expand the brackets: $2(3x-4)=2\times 3x+2\times(-4)$. Add 8 to both sides and then divide both sides by 6, leaving your answer in fraction form.
b $2(x+3)-4x=8$ $2x+6-4x=8$ $-2x+6=8$ $-2x=2$ $x=-1$	Expand the brackets and collect any like terms, i.e. $2x-4x=-2x$. Subtract 6 from both sides. Divide by -2.
c $5x-2=3x-4$ $2x-2=-4$ $2x=-2$ $x=-1$	Collect x terms on one side by subtracting $3x$ from both sides. Add 2 to both sides and then divide both sides by 2.
d $3(2x+4)=8(x+1)$ $6x+12=8x+8$ $12=2x+8$ $4=2x$ $2=x$ $\therefore\ x=2$	Expand the brackets on each side. Subtract $6x$ from both sides, alternatively subtract $8x$ to end up with $-2x+12=8$. (Subtracting $6x$ keeps the x-coefficient positive.) Solve the equation and make x the subject.

Now you try

Solve each of the following equations.

a $3(2x-1)=4$

b $5(x+2)-2x=19$

c $7x-1=5x-7$

d $4(2x-3)=5(x+6)$

Exercise 2E

FLUENCY	1, 2–4($\frac{1}{2}$)	2–5($\frac{1}{2}$)	2–5($\frac{1}{4}$)

1 Solve each of the following equations.

Example 11a

a **i** $2(x + 3) = 11$ **ii** $5(a + 3) = 8$

Example 11b

b **i** $2(x + 4) + x = 14$ **ii** $2(x - 3) - 3x = 4$

Example 11a

2 Solve each of the following equations by first expanding the brackets.

a $3(m + 4) = 31$ **b** $5(y - 7) = -12$ **c** $4(p - 5) = -35$

d $2(k - 5) = 9$ **e** $4(5 - b) = 18$ **f** $2(1 - m) = 13$

g $5(3 - x) = 19$ **h** $7(2a + 1) = 8$ **i** $4(3x - 2) = 30$

j $3(3n - 2) = 0$ **k** $5(3 - 2x) = 6$ **l** $6(1 - 2y) = -8$

Example 11b

3 Expand and simplify, and then solve each of the following equations.

a $4(x - 1) + x - 1 = 0$ **b** $3(2x + 3) - 1 - 4x = 4$

c $3(x - 4) + 2(x + 1) = 15$ **d** $2(x + 1) - 3(x - 2) = 8$

e $6(x + 3) + 2x = 26$ **f** $3(x + 2) + 5x = 46$

g $3(2x - 3) + x = 12$ **h** $(3x + 1) + 3x = 19$

Example 11c

4 Solve each of the following equations.

a $5b = 4b + 1$ **b** $8a = 7a - 4$ **c** $4t = 10 - t$

d $3m - 8 = 2m$ **e** $5x - 3 = 4x + 5$ **f** $9a + 3 = 8a + 6$

g $12x - 3 = 10x + 5$ **h** $3y + 6 = 2 - y$ **i** $5m - 4 = 1 - 6m$

Example 11d

5 Solve each of the following equations.

a $5(x - 2) = 2x - 13$ **b** $3(a + 1) = a + 10$ **c** $3(y + 4) = y - 6$

d $2(x + 5) = x - 4$ **e** $5b - 4 = 6(b + 2)$ **f** $2(4m - 5) = 4m + 2$

g $3(2a - 3) = 5(a + 2)$ **h** $4(x - 3) = 3(3x + 1)$ **i** $3(x - 2) = 5(x + 4)$

j $3(n - 2) = 4(n + 5)$ **k** $2(a + 5) = -2(2a + 3)$ **l** $-4(x + 2) = 3(2x + 1)$

PROBLEM-SOLVING	6, 7	6($\frac{1}{2}$), 7	7, 8

6 Using x for the unknown number, write down an equation and then solve it to find the number.

a The product of 2 and 3 more than a number is 7.

b The product of 3 and 4 less than a number is -4.

c When 2 less than 3 lots of a number is doubled the result is 5.

d When 5 more than 2 lots of a number is tripled the result is 10.

e 2 more than 3 lots of a number is equivalent to 8 lots of the number.

f 2 more than 3 times the number is equivalent to 1 less than 5 times the number.

g 1 less than a doubled number is equivalent to 5 more than 3 lots of the number.

7 Since Tara started work her original hourly wage has been tripled, and then decreased by \$6. It is now to be doubled so that she gets \$18 an hour. Write an equation and solve it to find Tara's original hourly wage.

8 At the start of lunch, Jimmy and Jake each brought out a new bag of x marbles to play with their friends. By the end of lunch they were surprised to see they still had the same total number as before even though overall Jimmy had gained 5 marbles and Jake had ended up with double the result of 3 less than his original amount. How many marbles were originally in each bag?

REASONING	9, 10	9, 10	10, 11

9 Consider the equation $3(x - 2) = 9$.

- **a** Solve the equation by first dividing both sides by 3.
- **b** Solve the equation by first expanding the brackets.
- **c** Which of the above two methods is preferable and why?

10 Consider the equation $3(x - 2) = 7$.

- **a** Solve the equation by first dividing both sides by 3.
- **b** Solve the equation by first expanding the brackets.
- **c** Which of the above two methods is preferable and why?

11 Consider the equation $3x + 1 = 5x - 7$.

- **a** Solve the equation by first subtracting $3x$ from both sides.
- **b** Solve the equation by first subtracting $5x$ from both sides.
- **c** Which of the above two methods is preferable and why? Describe the differences.

ENRICHMENT: Literal solutions with factorisation	–	–	12(½)

12 Literal equations contain a variable (such as x) and other variables (or pronumerals) such as a, b and c. To solve such an equation for x, factorisation can be used as shown here.

$$\begin{aligned} ax &= bx + c & & \\ ax - bx &= c & & \text{Subtract } bx \text{ from both sides.} \\ x(a - b) &= c & & \text{Factorise by taking out } x. \\ x &= \frac{c}{a - b} & & \text{Divide both sides by } (a - b). \\ & & & (\text{Note: } a \neq b.) \end{aligned}$$

Solve each of the following for x in terms of the other pronumerals by using factorisation.

- **a** $ax = bx + d$
- **b** $ax + 1 = bx + 3$
- **c** $5ax = bx + c$
- **d** $3ax + 1 = 4bx - 5$
- **e** $ax - bc = xb - ac$
- **f** $a(x - b) = x - b$
- **g** $ax - bx - c = d + bd$
- **h** $a(x + b) = b(x - c) - x$

Using CAS and non-CAS calculators 2E: Solving equations

This activity is in the Interactive Textbook in the form of a printable PDF.

2F Solving word problems

Learning intentions

- To be able to identify the unknown in a word problem
- To know how to define a variable to represent the unknown
- To be able to interpret a question and write an equation to represent the scenario
- To know how to apply algebraic processes to solve the equation and answer the problem in words

Many types of problems can be solved by writing and solving linear equations. Often problems are expressed only in words. Reading and understanding the problem, defining a variable and writing an equation become important steps in solving the problem.

Lawyers can use linear equations when distributing inherited money. For example, if the first recipient receives $\$x$, the second person might receive \$5000 less and the third person could receive twice the second person's amount.

LESSON STARTER Too much television?

Three friends, Rick, Kate and Sue, compare how much television they watch in a week at home. Kate watches three times as much television as Rick and Sue watches 4 hours less television than Kate. In total they watch 45 hours of television. Find the number of hours of television watched by Rick.

- Let x hours be the number of hours of television watched by Rick.
- Write expressions for the number of hours of television watched by Kate and by Sue.
- Write an equation to represent the information above.
- Solve the equation.
- Answer the question in the original problem.

KEY IDEAS

- To solve a **word problem** using algebra:
  - Read the problem and find out what the question is asking for.
  - Define a variable and write a statement such as: 'Let x be the number of …'. The variable is often what you have been asked to find.
  - Write an equation using your defined variable to show the relationship between the facts in the question.
  - Solve the equation.
  - Answer the question in words.

BUILDING UNDERSTANDING

1. Give an equation for these situations.
   - **a** One less than twice a number is 11.
   - **b** Ben earns $\$x$ per week. \$100 plus two weeks' earnings gives \$2200.
   - **c** Maggie and Doris score 60 points between them and Doris scores 12 more points than Maggie.
   - **d** An engineer charges a total of \$480 including a \$100 up-front fee plus \$80 per hour for n hours work.

Example 12 Turning a word problem into an equation

Five less than a certain number is 9 less than three times the number. Write an equation and solve it to find the number.

SOLUTION	EXPLANATION
Let x be the number.	Define the unknown as a pronumeral.
$x - 5 = 3x - 9$ $-5 = 2x - 9$ $4 = 2x$ $x = 2$	5 less than x is $x - 5$ and this equals 9 less than three times x, i.e. $3x - 9$. Subtract x from both sides and solve the equation.
The number is 2.	Write the answer in words.

Now you try

Two more than a certain number is 3 less than double the number. Write an equation and solve it to find the number.

Example 13 Solving word problems

David and Mitch made 254 runs between them in a cricket match. If Mitch made 68 more runs than David, how many runs did each of them make?

SOLUTION	EXPLANATION
Let the number of runs for David be r.	Define the unknown value as a pronumeral.
Number of runs Mitch made is $r + 68$.	Write all other unknown values in terms of r.
$r + (r + 68) = 254$ $2r + 68 = 254$ $2r = 186$ $r = 93$	Write an equation: number of runs for David + number of runs for Mitch = 254. Subtract 68 from both sides and then divide both sides by 2.
David made 93 runs and Mitch made $93 + 68 = 161$ runs.	Express the answer in words.

Now you try

Flora and Jo raised \$426 between them in a fundraising effort. If Jo raised \$52 more than Flora, how much did each of them raise?

Exercise 2F

FLUENCY	1–7	2–8	3, 5, 6, 8–10

Example 12

1 For each of the following examples, make x the unknown number and write an equation.

- **a** Three less than a certain number is 9 less than four times the number.
- **b** Seven is added to a number and the result is then multiplied by 3. The result is 9.

Example 12

2 For each of the following, write an equation and solve it to find the number.

- **a** I think of a number, take away 9, then multiply the result by 4. This gives an answer of 12.
- **b** A number when doubled results in a number that is 5 more than the number itself.
- **c** Eight less than a certain number is 2 more than three times the number.

Example 13

3 Leonie and Emma scored 28 goals between them in a netball match. Leonie scored 8 more goals than Emma.

- **a** Define a variable for the number of goals scored by Emma.
- **b** Write the number of goals scored by Leonie in terms of the variable in part **a**.
- **c** Write an equation in terms of your variable to represent the problem.
- **d** Solve the equation in part **c** to find the unknown value.
- **e** How many goals did each of them score?

4 A rectangle is four times as long as it is wide and its perimeter is 560 cm.

- **a** Define a variable for the unknown width.
- **b** Write an expression for the length in terms of your variable in part **a**.
- **c** Write an equation involving your variable to represent the problem. Draw and label a rectangle to help you.
- **d** Solve the equation in part **c**.
- **e** What is the length and width of the rectangle?

5 Toni rented a car for a total cost of \$290. If the rental company charged \$40 per day, plus a hiring fee of \$50, for how many days did Toni rent the car?

6 Anthony walked a certain distance, and then ran twice as far as he walked. He then caught a bus for the last 2 km. If he travelled a total of 32 km, find how far Anthony walked and ran.

7 A prize of \$1000 is divided between Adele and Benita so that Adele receives \$280 more than Benita. How much did they each receive?

8 Kate is three times as old as her son. If Kate is 30 years older than her son, what are their ages?

9 A train station is between the towns Antville and Bugville. The station is four times as far from Bugville as it is from Antville. If the distance from Antville to Bugville is 95 km, how far is it from Antville to the station? (*Hint*: Draw a diagram to help you picture the problem.)

10 Andrew, Brenda and Cammi all work part-time at a supermarket. Cammi earns $20 more than Andrew and Brenda earns $30 less than twice Andrew's wage. If their total combined wage is $400, find how much each of these workers earns.

PROBLEM-SOLVING	11, 12, 14	11, 13, 14	11, 13–16

11 Macy bought a total of 12 fiction and non-fiction books. The fiction books cost $12 each and the non-fiction books cost $25 each. If she paid $248 altogether, how many of each kind of book did she purchase? Define the number of non-fiction books bought in terms of the number of fiction books bought.

12 If I multiply my age in six years' time by three, the resulting age is my mother's age now. If my mother is currently 48 years old, how old am I?

13 Twelve years ago Eric's father was seven times as old as Eric was. If Eric's father is now 54 years old, how old is Eric now?

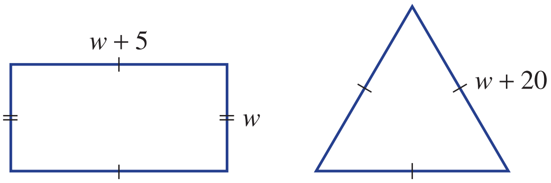
14 The second leg in a yacht race was half the length of the first leg, the third leg was two-thirds of the length of the second leg, and the last leg was twice the length of the second leg. If the total distance was 153 km, find the length of each leg.

15 The Ace Bicycle Shop charges a flat fee of $4, plus $1 per hour, for the hire of a bicycle. The Best Bicycle Shop charges a flat fee of $8, plus 50 cents per hour. Connie and her friends hire three bicycles from Ace, and David and his brother hire two bicycles from Best. After how many hours will their hire costs be the same?

16 Car A left Melbourne for Adelaide at 11:00 a.m. and travelled at an average speed of 70 km/h. Car B left Melbourne for Adelaide at 1:00 p.m. on the same day and travelled at an average speed of 90 km/h. At what time will Car B catch Car A?

REASONING	17	17, 18	18, 19

17 Two paddocks in the shapes shown below are to be fenced with wire. If the same total amount of wire is used for each paddock, what are the dimensions of each paddock in metres?



18 Consecutive integers can be represented algebraically as x, $x + 1$, $x + 2$ etc.

a Find three consecutive numbers that add to 84.

b **i** Write three consecutive even numbers starting with x.

ii Find three consecutive even numbers that add to 18.

c **i** Write three consecutive odd numbers starting with x.

ii Find three consecutive odd numbers that add to 51.

d **i** Write three consecutive multiples of 3 starting with x.

ii Find three consecutive multiples of 3 that add to 81.

19 Tedco produces a teddy bear which sells for \$24. Each teddy bear costs the company \$8 to manufacture and there is an initial start-up cost of \$7200.

a Write a rule for the total cost, \$$T$, of producing x teddy bears.

b If the cost of a particular production run was \$9600, how many teddy bears were manufactured in that run?

c Write a rule for the revenue, \$$R$, received by the company when x teddy bears are sold.

d How many teddy bears were sold if the revenue was \$8400?

e If they want to make an annual profit of \$54 000, how many teddy bears need to be sold?

ENRICHMENT: Worded challenges	–	–	20–22

20 An art curator was investigating the price trends of two art works that had the same initial value. The first painting, 'Green poles', doubled in value in the first year and then lost \$8000 in the second year. In the third year its value was three-quarters of that of the previous year.

The second painting, 'Orchids', added \$10 000 to its value in the first year. In the second year its value was only a third of that of the previous year. In the third year its value improved to double that of the previous year.

If the value of the two paintings was the same in the third year, write an equation and solve it to find the initial value of each painting.

21 Julia drove to her holiday destination over a period of five days. On the first day she travels a certain distance, on the second day she travels half that distance, on the third day a third of that distance, on the fourth day one-quarter of the distance and on the fifth day one-fifth of the distance. If her destination was 1000 km away, write an equation and solve it to find how far she travelled on the first day, to the nearest kilometre.

22 Anna King is x years old. Her brother Henry is two-thirds of her age and her sister Chloe is three times Henry's age. The twins who live next door are 5 years older than Anna. If the sum of the ages of the King children is equal to the sum of the ages of the twins, find the ages of all the children.

Progress quiz

2A **1** Write an algebraic expression for each of the following.

- **a** The number of tickets required for 5 boys, x girls and y adults
- **b** 15 less than the product of 4 and y
- **c** The sum of the squares of k and p
- **d** The square of the difference between m and n
- **e** The square root of 16 more than x

2A **2** Evaluate these expressions when $x = 4$ and $y = -2$.

a $x^2 - y^2$ **b** $5y - 3(x - y)$ **c** $\frac{x + y}{x}$ **d** $\frac{3xy}{2}$

2B **3** Simplify the following.

a $2a \times 7b$ **b** $5pq \times (-8p)$ **c** $\frac{12k^2}{3km}$ **d** $30xy^3 \div (-5xy)$

2B **4** Simplify the following by collecting like terms.

a $6xy + 7 - 4xy$ **b** $7 - 3mk + 4km + 4$

c $8xy - 3x - 3yx + 2y$ **d** $a^2b + ba + b^2a - ab + b$

2C **5** Expand the following.

a $4(a + 7)$ **b** $-3x(2x - 5)$

2C **6** Expand the following and collect like terms.

a $10 - 3(a - 2)$ **b** $4(2x + y) - 3(x + 2y)$

2D **7** Solve the following equations.

a $3x - 7 = 5$ **b** $\frac{a}{3} - 5 = 2$

c $9 - 2p = 16$ **d** $5 - \frac{x}{3} = 7$

e $\frac{3t}{5} - 4 = 8$ **f** $2k - \frac{2}{3} = \frac{1}{6}$

g $8 - \frac{2x}{3} = 14$ **h** $\frac{3k - 6}{3} = 5$

2E **8** Solve the following equations.

a $3(x - 4) = 7$ **b** $4(3a + 1) - 3a = 31$

c $7m - 4 = 3m - 12$ **d** $3(2y + 1) = 8(y - 2)$

2F **9** For each of the following examples, make x the unknown number and write an equation.

- **a** Five is added to a number and this is then multiplied by 2. The result is 14.
- **b** Nine less than a certain number is 6 more than four times the number.

2F **10** Andy's soccer team scores 14 more goals for the season than Jed's team. The two teams score a total of 30 goals for the season. Solve an appropriate equation to find the number of goals scored by Andy's team.

2F **11** If I multiply my age in five years' time by four, the resulting age is my grandfather's age now. If my grandfather is currently 88 years old, set up and solve an equation to determine how old I am now.

2G Inequalities

Learning intentions

- To be able to work with the symbols $>$, $\geqslant$, $<$, $\leqslant$
- To understand that an inequality represents an infinite set of values
- To be able to illustrate an inequality on a number line using known conventions
- To understand when an inequality sign needs to be reversed
- To be able to solve a linear inequality

An inequality (or inequation) is a mathematical statement which uses a $<$, $\leqslant$, $>$ or $\geqslant$ sign. Here are some examples of inequalities:

$$2 < 6, \quad 5 \geqslant -1, \quad 3x + 1 \leqslant 7 \quad \text{and} \quad 2x + \frac{1}{3} > \frac{x}{4}$$

Inequalities can represent an infinite set of numbers. For example, the inequality $2x < 6$ means that $x < 3$ and this is the infinite set of all real numbers less than 3.

The altitude, *h*, that passenger planes can successfully and economically fly at can be represented by an inequality: $0 < h \leqslant 40\,000$ feet or $0 < h \leqslant 12$ km.

LESSON STARTER Infinite solutions

Greg, Kevin and Greta think that they all have a correct solution to this inequality:

$$4x - 1 \geqslant x + 6$$

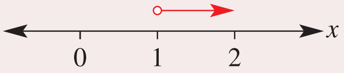
Greg says $x = 4$ is a solution.
Kevin says $x = 10$ is a solution.
Greta says $x = 100$ is a solution.

- Use substitution to show that they are all correct.
- Can you find the smallest whole number that is a solution to the inequality?
- Can you find the smallest number (including fractions) that satisfies the inequality? What method leads you to your answer?

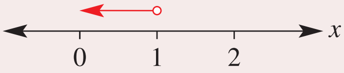
KEY IDEAS

- **Inequalities** can be illustrated using a number line because a line represents an infinite number of points.
  - Use an **open circle** when showing $>$ (greater than) or $<$ (less than).

For example: $x > 1$

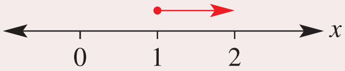


$x < 1$

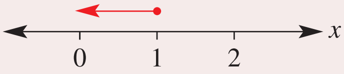


  - Use a **closed circle** when showing $\geqslant$ (greater than or equal to) or $\leqslant$ (less than or equal to).

For example: $x \geqslant 1$

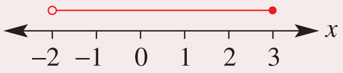


$x \leqslant 1$



- A **set** of numbers may have both an upper and lower bound.

  For example: $-2 < x \leqslant 3$



- Linear inequalities can be solved in a similar way to linear equations.
  - All the numbers that satisfy an inequality are called a **solution set**.
  - If we multiply or divide both sides of an inequality by a negative number, the inequality sign is reversed.
    For example: $5 < 8$ but $-5 > -8$, so if $-x > 1$ then $x < -1$.
  - If we swap the sides of an inequality, then the inequality sign is reversed.
    For example: $3 < 7$ but $7 > 3$ so if $2 > x$ then $x < 2$.

BUILDING UNDERSTANDING

① State the missing symbol $<$ or $>$ to make each statement true.

a 3 ☐ 2 **b** −1 ☐ 4 **c** −7 ☐ −3 **d** 5 ☐ −50

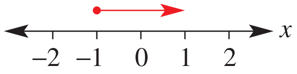
② Match the inequalities **a**, **b** and **c** with the number lines **A**, **B** and **C**.

a $x < 2$

b $x \geqslant -1$

c $-3 \leqslant x < 0$

A



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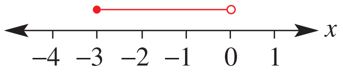
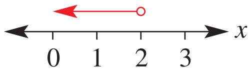


C



Example 14 Representing inequalities on a number line

Show each of the following examples on a number line.

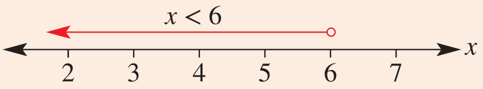
a x is less than 6, $(x < 6)$.

b x is greater than or equal to 2, $(x \geqslant 2)$.

c x is greater than −2 but less than or equal to 3, $(-2 < x \leqslant 3)$.

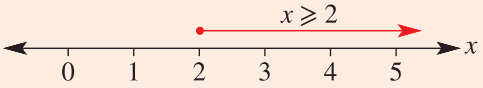
SOLUTION

a



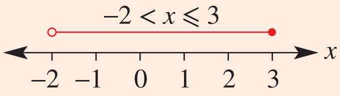
An open circle is used to indicate that 6 is not included.

b



A closed circle is used to indicate that 2 is included.

c



EXPLANATION

An open circle is used to indicate that −2 is not included and a closed circle is used to indicate that 3 is included.

Now you try

Show each of the following examples on a number line:

a x is less than 4, ($x < 4$).

b x is greater than or equal to -3, ($x \geqslant -3$).

c x is greater than -3 but less than or equal to 1, ($-3 < x \leqslant 1$).

Example 15 Solving inequalities

Find the solution set for each of the following inequalities.

a $x - 3 < 7$

b $5 - 2x > 3$

c $\dfrac{d}{4} - 3 \geqslant -11$

d $2a + 7 \leqslant 6a + 3$

SOLUTION	EXPLANATION
a $x - 3 < 7$ $x < 10$	Add 3 to both sides.
b $5 - 2x > 3$ $-2x > -2$ $x < 1$	Subtract 5 from both sides. Divide both sides by -2, and reverse the inequality sign.
c $\dfrac{d}{4} - 3 \geqslant -11$ $\dfrac{d}{4} \geqslant -8$ $d \geqslant -32$	Add 3 to both sides. Multiply both sides by 4; the inequality sign does not change.
d $2a + 7 \leqslant 6a + 3$ $7 \leqslant 4a + 3$ $4 \leqslant 4a$ $1 \leqslant a$ $a \geqslant 1$	Gather pronumerals on one side by subtracting $2a$ from both sides. Subtract 3 from both sides and then divide both sides by 4. Place the variable a on the left and reverse the inequality.

Now you try

Find the solution set for each of the following in equalities.

a $x - 2 < 9$

b $4 - 3x > 7$

c $\dfrac{a}{3} + 1 \geqslant -2$

d $3a - 1 \leqslant 5a + 6$

Exercise 2G

FLUENCY	1–5($\frac{1}{2}$)	2–7($\frac{1}{2}$)	2–7($\frac{1}{3}$)

Example 14

1 Show each of the following inequalities on a number line.

a $x > 2$ **b** $x \leqslant 4$ **c** $x \geqslant -1$

d $-1 < x < 1$ **e** $0 \leqslant x \leqslant 3$ **f** $-3 < x \leqslant 4$

Example 14

2 Show each of the following inequalities on a number line.

a $x \geqslant 1$ **b** $x < -2$ **c** $x < -8$

d $-2 \leqslant x \leqslant 2$ **e** $2 \leqslant x \leqslant 5$ **f** $-6 \leqslant x < -2$

Example 15a

3 Find the solution set for each of the following inequalities.

a $x + 5 < 8$ **b** $b - 2 > 3$ **c** $y - 8 > -2$

d $-12 + m < -7$ **e** $5x \geqslant 15$ **f** $4t > -20$

g $\frac{x}{3} \geqslant 4$ **h** $y + 10 \geqslant 0$ **i** $3m - 7 < 11$

j $4a + 6 \geqslant 12$ **k** $7x - 5 < 2$ **l** $2x - 7 > 9$

Example 15b

4 Find the solution set for each of the following inequalities.

a $4 - 3x > -8$ **b** $2 - 4n \geqslant 6$ **c** $4 - 5x \leqslant 1$

d $7 - a \leqslant 3$ **e** $5 - x \leqslant 11$ **f** $7 - x \leqslant -3$

g $-2x - 3 > 9$ **h** $-4t + 2 \geqslant 10$ **i** $-6m - 14 < 15$

Example 15c

5 Find the solution set for each of the following inequalities.

a $\frac{x}{2} - 5 \leqslant 3$ **b** $3 - \frac{x}{9} \geqslant 4$ **c** $\frac{2x}{5} \leqslant 8$

d $\frac{2x + 6}{7} < 4$ **e** $\frac{3x - 4}{2} > -6$ **f** $\frac{1 - 7x}{5} \leqslant 3$

6 Solve these inequalities involving brackets.

a $4(x + 2) < 12$ **b** $-3(a + 5) > 9$ **c** $5(3 - x) \geqslant 25$

d $2(3 - x) > 1$ **e** $5(y + 2) < -6$ **f** $-7(1 - x) < -11$

Example 15d

7 Find the solution set for each of the following inequalities.

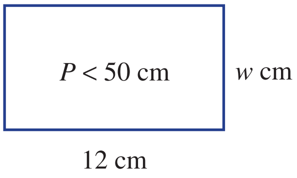
a $2x + 9 \leqslant 6x - 1$ **b** $6t + 2 > t - 1$ **c** $7y + 4 \leqslant 7 - y$

d $3a - 2 < 4 - 2a$ **e** $1 - 3m \geqslant 7 - 4m$ **f** $7 - 5b > -4 - 3b$

PROBLEM-SOLVING	8–10	9–11	10–12

8 Wendy is x years old and Jay is 6 years younger. The sum of their ages is less than 30. Write an inequality involving x and solve it. What can you say about Wendy's age?

9 The perimeter of a particular rectangle needs to be less than 50 cm. If the length of the rectangle is 12 cm and the width is w cm, write an inequality involving w and solve it. What width does the rectangle need to be?



10 How many integers satisfy both of the given equations?

a $2x + 1 \leqslant 5$ and $5 - 2x \leqslant 5$ **b** $7 - 3x > 10$ and $5x + 13 > -5$

c $\frac{x+1}{3} \geqslant -2$ and $2 - \frac{x}{3} > 3$ **d** $\frac{5x+1}{6} < 2$ and $\frac{x}{3} < 2x - 7$

11 The width of a rectangular area is 10 m and its height is $(2x - 4)$ m. If the area is less than 80 m^2, what are the possible integer values for x?

12 Two car rental companies have the following payment plans:
Carz: \$90 per week and 15c per kilometre
Renta: \$110 per week and 10c per kilometre
What is the maximum whole number of kilometres that can be travelled in one week with Carz if it is to cost less than it would with Renta?

REASONING	13	13, 14	14, 15

13 **a** Consider the inequality $2 > x$.

- **i** List five values of x between -1 and 2 which make the inequality true.
- **ii** What must be true about all the values of x if the inequality is true?

b Consider the inequality $-x < 5$.

- **i** List five values of x which make the inequality true.
- **ii** What must be true about all the values of x if the inequality is true?

c Complete these statements.

- **i** If $a > x$ then x __ __. **ii** If $-x < a$ then x __ __.

14 Consider the equation $9 - 2x > 3$.

a Solve the equation by first adding $2x$ to both sides, and then solve for x.

b Solve the equation by first subtracting 9 from both sides.

c What did you have to remember to do in part **b** to ensure that the answer is the same as in part **a**?

15 Combine all your knowledge from this chapter so far to solve these inequalities.

a $\frac{2(x+1)}{3} > x + 5$ **b** $2x + 3 \geqslant \frac{x-6}{3}$ **c** $\frac{2-3x}{2} < 2x - 1$

d $\frac{4(2x-1)}{3} \leqslant x + 3$ **e** $1 - x > \frac{7(2-3x)}{4}$ **f** $2(3 - 2x) \leqslant 4x$

ENRICHMENT: Literal inequalities	–	–	16(1/2)

16 Given a, b, c and d are positive numbers and that $1 < a < b$, solve each of the following for x.

a $ax - b > -c$ **b** $b - x \leqslant a$ **c** $\frac{x}{a} - b \leqslant c$

d $\frac{bx}{c} \leqslant a$ **e** $\frac{ax+b}{c} < d$ **f** $\frac{b-2x}{c} \leqslant d$

g $a(x + b) < c$ **h** $\frac{ax-b}{c} > -d$ **i** $a(b - x) > c$

j $ax + b \leqslant x - c$ **k** $ax + b > bx - 1$ **l** $b - ax \leqslant c - bx$

2H Using formulas

Learning intentions
- To understand what a formula represents and be able to identify the subject of a formula
- To be able to find an unknown in a formula by substituting values and evaluating or solving an equation
- To understand what it means to transpose a formula
- To be able to apply steps similar to solving equations to transpose a formula

A formula (or rule) is an equation that relates two or more variables. You can find the value of one of the variables if you are given the value of all others. Some common formulas contain squares, square roots, cubes and cube roots. The following are some examples of formulas.

- $A = \pi r^2$ is the formula for finding the area, A, of a circle given its radius, r.
- $F = \frac{9}{5}C + 32$ is the formula for converting degrees Celsius, C, to degrees Fahrenheit, F.
- $d = vt$ is the formula for finding the distance, d, given the velocity, v, and time, t.

A, F and d are said to be the subjects of the formulas given above.

Doctors and nurses calculate the amount of medicine, C, for a child using Young's formula: $C = \frac{a}{a + 12} \times D$, where a is the child's age in years, and D is the quantity of the adult dose.

LESSON STARTER Common formulas

As a class group, try to list at least 10 formulas that you know.

- Write down the formulas and describe what each variable represents.
- Which variable is the subject of each formula?

KEY IDEAS

- The **subject** of a **formula** is a variable that usually sits on its own on the left-hand side. For example, the C in $C = 2\pi r$ is the subject of the formula.
- A variable in a formula can be evaluated by substituting numbers for all other variables.
- A formula can be **transposed** (rearranged) to make another variable the subject.

  $C = 2\pi r$ can be transposed to give $r = \frac{C}{2\pi}$.

  - To transpose a formula use similar steps as you would for solving an equation, since variables represent numbers.
- Note that $\sqrt{a^2} = a$ if $a \geqslant 0$ and $\sqrt{a^2 + b^2} \neq a + b$ provided both a and b are not zero.

BUILDING UNDERSTANDING

1 State the letter that is the subject of these formulas.

a $A = \frac{1}{2}bh$
b $D = b^2 - 4ac$
c $M = \frac{a+b}{2}$
d $A = \pi r^2$

2 Describe the first step when attempting to transpose these formulas to make a the subject.

a $2a + 3 = b$
b $\frac{a-c}{4} = d$
c $\sqrt{4-a} = b$
d $b = \sqrt{a} + c$

3 State the steps to solve for x in these equations.

a **i** $3x + 5 = 12$ **ii** $ax + b = c$
b **i** $3x^2 = 75, x > 0$ **ii** $ax^2 = b, x > 0$

Example 16 Substituting values into formulas

Substitute the given values into the formula to evaluate the subject.

a $S = \frac{a}{1-r}$, when $a = 3$ and $r = 0.4$

b $E = \frac{1}{2}mv^2$, when $m = 4$ and $v = 5$

SOLUTION	EXPLANATION
a $S = \frac{a}{1-r}$	
$S = \frac{3}{1-0.4}$ $= \frac{3}{0.6}$ $= 5$	Substitute $a = 3$ and $r = 0.4$ and evaluate.
b $E = \frac{1}{2}mv^2$	
$E = \frac{1}{2} \times 4 \times 5^2$ $= \frac{1}{2} \times 4 \times 25$ $= 50$	Substitute $m = 4$ and $v = 5$ and evaluate. (Note: Square the value of v before multiplying by the value of m.)

Now you try

Substitute the given values into the formula to evaluate the subject.

a $a = \frac{t}{1-b}$, when $t = 5$ and $b = 0.5$

b $s = ut + \frac{1}{2}at^2$, when $u = 25$, $a = 10$ and $t = 4$

Example 17 Finding the unknown value in a formula

The area of a trapezium is given by $A = \frac{1}{2}(a + b)h$. Substitute $A = 12$, $a = 5$ and $h = 4$ to find the value of b.

SOLUTION	EXPLANATION
$A = \frac{1}{2}(a + b)h$	Write the formula and substitute the given values of A, a and h. Then solve for b.
$12 = \frac{1}{2} \times (5 + b) \times 4$ $12 = 2(5 + b)$ $6 = 5 + b$ $b = 1$	$\frac{1}{2} \times 4 = 2$ and divide both sides by 2 since 2 is a factor of 12. Alternatively, you can expand the brackets.

Now you try

The area of a trapezium is given by $A = \frac{1}{2}(a + b)h$. Substitute $A = 24$, $b = 3$ and $h = 2$ to find the value of a.

Example 18 Transposing formulas

Transpose each of the following to make b the subject.

a $c = a(x + b)$

b $c = \sqrt{a^2 + b^2} \quad (b > 0)$

SOLUTION	EXPLANATION
a $c = a(x + b)$	
$\frac{c}{a} = x + b$	Divide both sides by a.
$\frac{c}{a} - x = b$	Subtract x from both sides.
$b = \frac{c}{a} - x \quad \left(\text{or } b = \frac{c - ax}{a}\right)$	Make b the subject on the left side. An alternative answer has a common denominator, which will also be the answer format if you expand the brackets first.
b $c = \sqrt{a^2 + b^2} \quad (b > 0)$	
$c^2 = a^2 + b^2$	Square both sides to remove the square root.
$c^2 - a^2 = b^2$	Subtract a^2 from both sides.
$b^2 = c^2 - a^2$	Make b^2 the subject.
$b = \sqrt{c^2 - a^2}$	Take the square root of both sides, $b = \sqrt{c^2 - a^2}$ as b is positive.

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Now you try

Transpose each of the following to make a the subject.

a $b = 2(a - c)$

b $a^2 - b = c \quad (a > 0)$

Exercise 2H

FLUENCY	1, 2–4(½)	2–4(½)	2–4(⅓)

Example 16

1 Substitute the given values into the formula to evaluate the subject. Round to two decimal places where appropriate.

a $A = bh$, when $b = 3$ and $h = 7$

b $F = ma$, when $m = 4$ and $a = 6$

c $m = \dfrac{a + b}{4}$, when $a = 14$ and $b = -6$

d $t = \dfrac{d}{v}$, when $d = 18$ and $v = 3$

Example 16

2 Substitute the given values into the formula to evaluate the subject. Round to two decimal places where appropriate.

a $A = \pi r^2$, when $r = 12$

b $V = \dfrac{4}{3}\pi r^3$, when $r = 2$

c $c = \sqrt{a^2 + b^2}$, when $a = 12$ and $b = 22$

d $Q = \sqrt{2gh}$, when $g = 9.8$ and $h = 11.4$

e $I = \dfrac{MR^2}{2}$, when $M = 12.2$ and $R = 6.4$

f $x = ut + \dfrac{1}{2}at^2$, when $u = 0$, $t = 4$ and $a = 10$

Example 17

3 Substitute the given values into the formula and then solve the equations to determine the value of the unknown pronumeral. Round to two decimal places where appropriate.

a $m = \dfrac{F}{a}$, when $m = 12$ and $a = 3$

b $A = lw$, when $A = 30$ and $l = 6$

c $A = \dfrac{1}{2}(a + b)h$, when $A = 64$, $b = 12$ and $h = 4$

d $C = 2\pi r$, when $C = 26$

e $S = 2\pi r^2$, when $S = 72$

f $v^2 = u^2 + 2as$, when $v = 22$, $u = 6$ and $a = 12$

g $m = \sqrt{\dfrac{x}{y}}$, when $m = 8$ and $x = 4$

Example 18

4 Transpose each of the following formulas to make the pronumeral shown in brackets the subject.

a $A = 2\pi rh$ $[r]$

b $I = \dfrac{Prt}{100}$ $[r]$

c $p = m(x + n)$ $[n]$

d $d = \dfrac{a + bx}{c}$ $[x]$

e $V = \pi r^2 h \quad (r > 0)$ $[r]$

f $P = \dfrac{v^2}{R} \quad (v > 0)$ $[v]$

g $S = 2\pi rh + 2\pi r^2$ $[h]$

h $A = (p + q)^2$ $[p]$

i $T = 2\pi\sqrt{\dfrac{l}{g}}$ $[g]$

j $\sqrt{A} + B = 4C$ $[A]$

ISBN 978-1-108-77288-4

Updated September 2021

PROBLEM-SOLVING	5, 6	5–7	6–8

5 The formula $s = \frac{d}{t}$ gives the speed s km/h of a car which has travelled a distance of d km in t hours.

a Find the speed of a car which has travelled 400 km in 4.5 hours. Round to two decimal places.

b **i** Transpose the formula $s = \frac{d}{t}$ to make d the subject.

ii Find the distance covered if a car travels at 75 km/h for 3.8 hours.

6 The formula $F = \frac{9}{5}C + 32$ converts degrees Celsius, C, to degrees Fahrenheit, F.

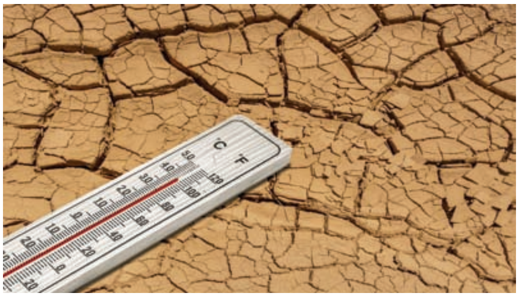
a Find what each of the following temperatures is in degrees Fahrenheit.

i 100°C **ii** 38°C

b Transpose the formula to make C the subject.

c Calculate each of the following temperatures in degrees Celsius. Round to one decimal place where necessary.

i 14°F **ii** 98°F



7 The velocity, v m/s, of an object is described by the rule $v = u + at$, where u is the initial velocity in m/s, a is the acceleration in m/s^2 and t is the time in seconds.

a Find the velocity after 3 seconds if the initial velocity is 5 m/s and the acceleration is 10 m/s^2.

b Find the time taken for a body to reach a velocity of 20 m/s if its acceleration is 4 m/s^2 and its initial velocity is 12 m/s.

8 The volume of water (V litres) in a tank is given by $V = 4000 - 0.1t$ where t is the time in seconds after a tap is turned on.

a Over time, does the water volume increase or decrease according to the formula?

b Find the volume after 2 minutes.

c Find the time it takes for the volume to reach 1500 litres. Round to the nearest minute.

d How long, to the nearest minute, does it take to completely empty the tank?

REASONING	9	9, 10	9($\frac{1}{2}$), 10

9 Write a formula for the following situations. Make the first listed variable the subject.

a \$$D$ given c cents

b d cm given e metres

c The discounted price \$$D$ that is 30% off the marked price \$$M$

d The value of an investment \$$V$ that is 15% more than the initial amount \$$P$

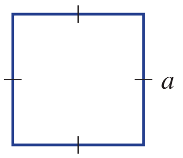
e The cost \$$C$ of hiring a car at \$50 upfront plus \$18 per hour for t hours

f The distance d km remaining in a 42 km marathon after t hours if the running speed is 14 km/h

g The cost \$$C$ of a bottle of soft drink if b bottles cost \$$c$

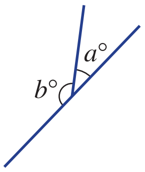
10 Write a formula for the value of a in these diagrams.

a

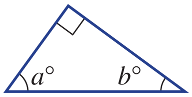


Perimeter $= P$

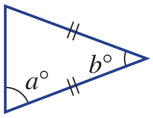
b



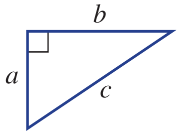
c



d

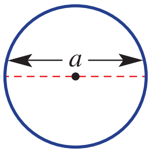


e



(Note: $a^2 + b^2 = c^2$.)

f



Area $= A$

ENRICHMENT: Basketball formulas

– – 11

11 The formula $T = 3x + 2y + f$ can be used to calculate the total number of points made in a basketball game where:

x = number of three-point goals
y = number of two-point goals
f = number of free throws made
T = total number of points

a Find the total number of points for a game in which 12 three-point goals, 15 two-point goals and 7 free throws were made.

b Find the number of three-point goals made if the total number of points was 36 and 5 two-point goals and 5 free throws were made.

c The formula $V = \left(p + \frac{3r}{2} + 2a + \frac{3s}{2} + 2b\right) - \frac{1.5t + 2f + m - o}{g}$ can be used to calculate the value, V, of a basketball player where:

p = points earned
r = number of rebounds
a = number of assists
s = number of steals
b = number of blocks
t = number of turnovers
f = number of personal fouls
m = number of missed shots
o = number of offensive rebounds
g = number of games played

Calculate the value of a player who earns 350 points and makes 2 rebounds, 14 assists, 25 steals, 32 blocks, 28 turnovers, 14 personal fouls, 24 missed shots, 32 offensive rebounds and plays 10 games.

2I Simultaneous equations using substitution EXTENDING

Learning intentions

- To understand that for two different linear equations with two unknowns there is at most one solution
- To know when the method of substitution can be used to solve simultaneous equations
- To be able to use the substitution technique to solve simultaneous equations

A linear equation with one unknown has one unique solution. For example, $x = 2$ is the only value of x that makes the equation $2x + 3 = 7$ true.

The linear equation $2x + 3y = 12$ has two unknowns and it has an infinite number of solutions. Each solution is a pair of x- and y-values that makes the equation true, for example $x = 0$ and $y = 4$ or $x = 3$ and $y = 2$ or $x = 4\frac{1}{2}$ and $y = 1$.

However, if we are told that $2x + 3y = 12$ and also that $y = 2x - 1$, we can find a single solution that satisfies both equations. Equations like this are called simultaneous linear equations, because we can find a pair of x- and y-values that satisfy both equations at the same time (simultaneously).

Civil engineers solve simultaneous equations to find the required strength of trusses that support bridges and towers. Linear equations are formed by equating the forces acting in opposite directions, horizontally and vertically, on each joint.

LESSON STARTER Multiple solutions

There is more than one pair of numbers x and y that satisfy the equation $x - 2y = 5$.

- Write down at least 5 pairs (x, y) that make the equation true.

A second equation is $y = x - 8$.

- Do any of your pairs that make the first equation true, also make the second equation true? If not, can you find the special pair of numbers that satisfies both equations simultaneously?

KEY IDEAS

- An algebraic method called **substitution** can be used to solve **simultaneous equations**. It is used when at least one of the equations has a single variable as the subject. For example, y is the subject in the equation $y = 3x + 1$.
- To solve simultaneous equations using substitution:
  1. Substitute one equation into the other, using brackets.
  2. Solve for the remaining variable.
  3. Substitute to find the value of the second variable.

$$\begin{aligned} 2x + 3y &= 8 \text{ and } y = x + 1 \\ 2x + 3(x + 1) &= 8 \\ 2x + 3x + 3 &= 8 \\ 5x + 3 &= 8 \\ 5x &= 5 \\ x &= 1 \\ \therefore y = (1) + 1 &= 2 \end{aligned}$$

BUILDING UNDERSTANDING

1 Find the value of x or y by substituting the known value.

a $y = 2x - 3 \ \ (x = 4)$ **b** $x = 5 - 2y \ \ (y = 4)$ **c** $2x + 4y = 8 \ \ (y = -3)$

2 Choose the correct option.

a When substituting $y = 2x - 1$ into $3x + 2y = 5$, the second equation becomes:

A $3x + 2(2x - 1) = 5$ **B** $3(2x - 1) + 2y = 5$ **C** $3x + 2y = 2x - 1$

b When substituting $x = 1 - 3y$ into $5x - y = 6$, the second equation becomes:

A $1 - 3x - y = 6$ **B** $5(1 - 3y) = 6$ **C** $5(1 - 3y) - y = 6$

3 Check whether $x = -2$ and $y = 2$ is a solution to each pair of simultaneous equations.

a $x - 2y = -6$ and $2x + y = 0$ **b** $2x + y = -2$ and $x = 4y - 10$

Example 19 Solving simultaneous equations using substitution

Solve each of the following pairs of simultaneous equations by using substitution.

a $x + y = 10$
$y = 4x$

b $4x - y = 6$
$y = 2x - 4$

c $3x + 2y = 19$
$y = 2x - 8$

SOLUTION	EXPLANATION
a $x + y = 10$ [1] $y = 4x$ [2]	Number the equations for reference.
$x + (4x) = 10$ $5x = 10$ $x = 2$	Substitute $y = 4x$ into [1]. Combine like terms and solve for x.
From [2] $y = 4x$ $= 4 \times (2)$ $= 8$	Substitute $x = 2$ into [2] to find the value of y.
$\therefore\ x = 2, y = 8$	
Check: $2 + 8 = 10$ and $8 = 4 \times 2$	Check answer by substituting $x = 2$ and $y = 8$ into [1] and [2].
b $4x - y = 6$ [1] $y = 2x - 4$ [2]	
$4x - (2x - 4) = 6$ $4x - 2x + 4 = 6$ $2x + 4 = 6$ $2x = 2$ $x = 1$	Substitute $y = 2x - 4$ into [1] using brackets. Use the distributive law and solve for x. $-(2x - 4) = -1(2x - 4)$ $= -1 \times 2x - 1 \times (-4)$ $= -2x + 4$
From [2] $y = 2x - 4$ $= 2 \times (1) - 4$ $= -2$	Substitute $x = 1$ into [2] to find the value of y.
$\therefore\ x = 1, y = -2$	
Check: $4 \times 1 - (-2) = 6$ and $-2 = 2 \times 1 - 4$	Check: substitute $x = 1$ and $y = -2$ into [1] and [2].

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c $3x + 2y = 19 \quad [1]$
$y = 2x - 8 \quad [2]$

$$\begin{aligned} 3x + 2(2x - 8) &= 19 \\ 3x + 4x - 16 &= 19 \\ 7x - 16 &= 19 \\ 7x &= 35 \\ x &= 5 \end{aligned}$$

Substitute $y = 2x - 8$ into [1].
Use the distributive law and solve for x.

From [2]
$$\begin{aligned} y &= 2x - 8 \\ &= 2 \times (5) - 8 \\ &= 2 \end{aligned}$$

Substitute $x = 5$ into [2] to find the value of y.

$\therefore\ x = 5, y = 2$

Check: $3 \times 5 + 2 \times 2 = 19$ and
$2 = 2 \times 5 - 8$

Check: substitute $x = 5$ and $y = 2$ into
[1] and [2].

Now you try

Solve each of the following pairs of simultaneous equations by using substitution.

a $x + y = 8$
$y = 3x$

b $3x - y = 9$
$y = 4x - 11$

c $5x + 3y = 7$
$y = x + 5$

Exercise 2I

FLUENCY	1, 3($^1/_2$), 4	2, 3–4($^1/_2$)	3–4($^1/_2$)

Example 19a

1 Solve each of the following pairs of simultaneous equations by using substitution.

a $x + y = 3$
$y = 2x$

b $x + y = 6$
$x = 5y$

c $x + 5y = 8$
$y = 3x$

Example 19a

2 Solve each of the following pairs of simultaneous equations by using substitution.

a $x - 5y = 3$
$x = 2y$

b $3x + 2y = 18$
$y = 3x$

c $x + 2y = 15$
$y = -3x$

Example 19b

3 Solve each of the following pairs of simultaneous equations by using substitution.

a $x + y = 12$
$y = x + 6$

b $2x + y = 1$
$y = x + 4$

c $5x + y = 5$
$y = 1 - x$

d $3x - y = 7$
$y = x + 5$

e $3x - y = 9$
$y = x - 1$

f $x + 2y = 6$
$x = 9 - y$

g $y - x = 14$
$x = 4y - 2$

h $3x + y = 4$
$y = 2 - 4x$

i $4x - y = 12$
$y = 8 - 6x$

Example 19c

4 Solve each of the following pairs of simultaneous equations by using substitution.

a $3x + 2y = 8$
$y = 4x - 7$

b $2x + 3y = 11$
$y = 2x + 1$

c $4x + y = 4$
$x = 2y - 8$

d $2x + 5y = -4$
$y = x - 5$

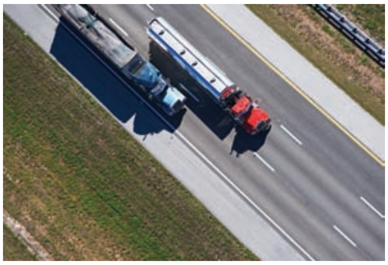
e $2x - 3y = 5$
$x = 5 - y$

f $3x + 2y = 5$
$y = 3 - x$

PROBLEM-SOLVING	5, 6	5, 6	6, 7

5 The sum of two numbers is 48 and the larger number is 14 more than the smaller number. Write two equations and solve them to find the two numbers.

6 The combined mass of two trucks is 29 tonnes. The heavier truck is 1 tonne less than twice the mass of the smaller truck. Write two equations and solve them to find the mass of each truck.



7 The perimeter of a rectangle is 11 cm and the length is 3 cm more than half the width. Find the dimensions of the rectangle.

REASONING	8	8, 9	8, 9

8 One of the common errors when applying the method of substitution is made in this working. Find the error and describe how to avoid it.

Solve $y = 3x - 1$ and $x - y = 7$.

$$\begin{aligned} x - 3x - 1 &= 7 \quad \text{(substituting } y = 3x - 1 \text{ into } x - y = 7\text{)} \\ -2x - 1 &= 7 \\ -2x &= 8 \\ x &= -4 \end{aligned}$$

9 If both equations have the same variable as the subject, substitution is still possible.

For example, solve $y = 3x - 1 \ldots [1]$ and $y = 2 - x \ldots [2]$

Substitute [1] into [2].

$$\begin{aligned} 3x - 1 &= 2 - x \\ 4x &= 3 \\ x &= \frac{3}{4} \text{ and } y = \frac{5}{4} \end{aligned}$$

Use this method to solve these simultaneous equations.

a $y = 4x + 1$
$y = 3 - 2x$

b $y = 3 - 4x$
$y = 2x + 8$

c $y = \frac{1}{2}x + 4$
$y = \frac{x + 1}{3}$

ENRICHMENT: Literally challenging	–	–	10

10 Use substitution to solve each of the following pairs of literal simultaneous equations for x and y in terms of a and b.

a $ax + y = b$
$y = bx$

b $ax + by = b$
$x = by$

c $x + y = a$
$x = y - b$

d $ax - by = a$
$y = x - a$

e $ax - y = a$
$y = bx + a$

f $ax - by = 2a$
$x = y - b$

Using CAS and non-CAS calculators 2I: Solving simultaneous equations

This activity is in the Interactive Textbook in the form of a printable PDF.

Applications and problem-solving

The following problems will investigate practical situations drawing upon knowledge and skills developed throughout the chapter. In attempting to solve these problems, aim to identify the key information, use diagrams, formulate ideas, apply strategies, make calculations and check and communicate your solutions.

IT consultants

1 Three IT consultants charge fees in different ways:

- Con: \$120 per hour
- Tracy: \$210 upfront plus \$90 per hour
- Rish: Undecided

Rish is investigating different upfront fees and hourly rates to compete with Con and Tracy in his IT consultancy business.

a What are the total fees for 5 hours of work for the following consultants?
i Con **ii** Tracy

b Write rules for the total fees $\$T$ for t hours work for:
i Con **ii** Tracy.

c Find the number of hours and total cost if Con's cost is equal to Tracy's cost.

d If Rish decides on an upfront fee of \$80 plus a \$100 hourly rate, find the time of hire if Rish's cost is equal to that for:
i Con **ii** Tracy.

e Rish decides on an upfront fee of \$105 and wants the total cost to be the same as Con and Tracy's for $t = 7$. What should Rish's hourly rate be?

f If Rish wants to be cheaper than both Con and Tracy for $t = 10$, give a possible upfront fee of more than \$100 and an hourly rate of more than \$80 that makes this possible.

Separating fact from fiction

2 Fiona explores the secondhand book stall at her school fair and notices that all fiction books are \$3 each and non-fiction books are \$4 each. She has a total of \$70 with her to spend.

Fiona is interested in the number of books of each type that she can buy for her \$70.

a If Fiona buys 10 of each type of book, does she spend all of her \$70?

b Can Fiona buy the following combinations of books without spending more than \$70?
i 9 fiction and 11 non-fiction
ii 12 fiction and 8 non-fiction

c Write an expression for the cost of x fiction and y non-fiction books.

d Fiona decides to buy exactly 21 books, spending exactly \$70 dollars.
  i Let x be the number of fiction books and y be the number of non-fiction books. Write a rule for y in terms of x.
  ii Find the number of each type of book she buys.
e Can Fiona spend exactly \$70 and buy 24 books? Show your working to justify your answers.
f In how many different ways can Fiona buy books if she spends exactly \$70? List the numbers of fiction and non-fiction books in each case.
g Is it possible for Fiona to spend exactly \$50 on books? If so give the combinations.

Kit homes

3 A German manufacturer of kit homes offers a basic essential module which is 10 m by 20 m. Additional bedrooms 5 m wide by x m long can be purchased, where x is an integer and $3 \leqslant x \leqslant 7$.

All bedrooms purchased must have the same dimensions and, when assembled, must share at least one entire wall with the essential module or another bedroom.

The manufacturer wishes to investigate the possible total areas of their kit homes and the perimeter of various arrangements.

a What are the possible lengths of the bedrooms?
b If three bedrooms are purchased:
  i find the total area of the kit home if $x = 5$ is chosen
  ii write an expression for the total area in terms of x
  iii find the value of x if the total area is 290 m^2
  iv find the maximum value of x if the total area is to be less than 250 m^2.
c If n bedrooms are purchased:
  i write an expression for the total area in terms of x and n
  ii find three combinations of x and n that give a total area of 260 m^2.
d If one bedroom is purchased, find:
  i the two possible perimeters of the total kit home if $x = 4$ is chosen
  ii the two possible perimeters of the total kit home in terms of x
  iii the value of x if the perimeter must be 66 m.
e If two bedrooms are purchased, find:
  i the minimum perimeter possible, stating your chosen value of x
  ii the maximum perimeter possible, stating your chosen value of x.

2J Simultaneous equations using elimination

EXTENDING

Learning intentions

- To understand that there are multiple methods for solving simultaneous equations
- To know that two equations can at times be added or subtracted to eliminate one of the variables
- To understand that a matching pair is required to eliminate a variable from two equations
- To know how to obtain a matching pair in simultaneous equations
- To know when to use addition and when to use subtraction to eliminate a variable
- To be able to use elimination to solve simultaneous equations

Another method used to solve simultaneous linear equations is called elimination. This involves the addition or subtraction of the two equations to eliminate one of the variables. We can then solve for the remaining variable and substitute to find the value of the second variable.

Traffic engineers solve simultaneous equations to model possible traffic flow in a network of roads. Using known and unknown flow rates, linear equations are formed by equating the number of vehicles entering and leaving intersections.

LESSON STARTER To add or subtract?

To use the method of elimination you need to decide if using addition or using subtraction will eliminate one of the variables.
Decide if the terms in these pairs should be added or subtracted to give the result of 0.

- $3x$ and $3x$
- $-x$ and x
- $2y$ and $-2y$
- $-7y$ and $-7y$

Describe under what circumstances addition or subtraction should be used to eliminate a pair of terms.

KEY IDEAS

- **Elimination** involves the addition or subtraction of two equations to remove one variable.
- Elimination is often used when both equations are of the form $ax + by = d$ or $ax + by + c = 0$.
- Add equations to eliminate terms of opposite sign:

$$\begin{array}{rr} & 3x - y = 4 \\ + & \underline{5x + y = 4} \\ & 8x = 8 \end{array}$$

- Subtract equations to eliminate terms of the same sign:

$$\begin{array}{rr} & 2x + 3y = 6 \\ - & \underline{2x - 5y = 7} \\ & 8y = -1 \end{array}$$

ISBN 978-1-108-77288-4

- If terms cannot be eliminated just by using addition or subtraction, first multiply one or both equations to form a matching pair.
  For example:

  matching pair

  1
  $$\begin{aligned} 3x - 2y &= 1 & &\longrightarrow & 3x - \boxed{2y} &= 1 & &\\ 2x + y &= 3 & &\xrightarrow{\times 2} & 4x + \boxed{2y} &= 6 & &\text{(Multiply both sides by 2.)}\end{aligned}$$

  matching pair

  2
  $$\begin{aligned} 7x - 2y &= 3 & &\xrightarrow{\times 4} & \boxed{28x} - 8y &= 12 & &\text{(Multiply both sides by 4.)}\\ 4x - 5y &= -6 & &\xrightarrow{\times 7} & \boxed{28x} - 35y &= -42 & &\text{(Multiply both sides by 7.)}\end{aligned}$$

BUILDING UNDERSTANDING

1 Choose a '+' or '−' sign for each statement to make them true.

a $3x \square 3x = 0$ **b** $-2y \square 2y = 0$

c $11y \square (-11y) = 0$ **d** $-4x \square (-4x) = 0$

2 **a** Decide whether addition or subtraction should be chosen to eliminate the variable x in these simultaneous equations.

i $x + 2y = 3$, $x - 5y = -4$

ii $-2x - y = -9$, $2x + 3y = 11$

iii $y - x = 0$, $3y - x = 8$

b Decide whether addition or subtraction will eliminate the variable y in these simultaneous equations.

i $4x - y = 6$, $x + y = 4$

ii $7x - 2y = 5$, $-3x - 2y = -5$

iii $10y + x = 14$, $-10y - 3x = -24$

Example 20 Solving simultaneous equations using elimination

Solve the following pairs of simultaneous equations by using elimination.

a $x - 2y = 1$, $-x + 5y = 2$

b $3x - 2y = 5$, $5x - 2y = 11$

SOLUTION

a
$$\begin{aligned} x - 2y &= 1 \quad [1]\\ \underline{-x + 5y} &\underline{= 2} \quad [2]\\ [1] + [2] \qquad 3y &= 3\\ y &= 1\end{aligned}$$

Add the two equations to eliminate x since $x + (-x) = 0$. Then solve for y.

$$\begin{aligned} \text{From } [1] \qquad x - 2y &= 1\\ x - 2 \times (1) &= 1\\ x - 2 &= 1\\ x &= 3\end{aligned}$$

Substitute $y = 1$ into equation [1] to find x.

$\therefore\ x = 3, y = 1$

Check: $3 - 2(1) = 1$
$-3 + 5(1) = 2$

Substitute $x = 3$ and $y = 1$ into the original equations to check.

Continued on next page

b	$3x - 2y = 5$ [1]	Subtract the two equations to eliminate y since they are the same sign, i.e. $-2y - (-2y) = -2y + 2y = 0$. Alternatively, could do [1] − [2] but [2] − [1] avoids negative coefficients.
	$\underline{5x - 2y = 11}$ [2]	
[2] − [1]	$2x = 6$	
	$x = 3$	
From [1]	$3x - 2y = 5$	Solve for x.
	$3 \times (3) - 2y = 5$	Substitute $x = 3$ into equation [1] to find y.
	$9 - 2y = 5$	
	$-2y = -4$	
	$y = 2$	
	$\therefore\ x = 3, y = 2$	
Check:	$3(3) - 2(2) = 5$ $5(3) - 2(2) = 11$	Substitute $x = 3$ and $y = 2$ into the original equations to check.

Now you try

Solve the following pairs of simultaneous equations by using elimination.

a $-x + y = 1$
$x - 2y = -3$

b $4x - 3y = -16$
$3x - 3y = -15$

Example 21 Solving simultaneous equations by creating a matching pair

Solve the following pairs of simultaneous equations by using elimination.

a $5x + 2y = -7$
$x + 7y = 25$

b $4x + 3y = 18$
$3x - 2y = 5$

SOLUTION		EXPLANATION
a	$5x + 2y = -7$ [1]	There are different numbers of x and y in each equation so multiply equation [2] by 5 to make the coefficient of x equal to that in equation [1].
	$x + 7y = 25$ [2]	
$5 \times (2)$	$5x + 35y = 125$ [3]	
	$\underline{5x + 2y = -7}$ [1]	
[3] − [1]	$33y = 132$	Subtract the equations to eliminate x.
	$y = 4$	
From [2]	$x + 7y = 25$	
	$x + 7 \times (4) = 25$	Substitute $y = 4$ in equation [2] to find x.
	$x + 28 = 25$	
	$x = -3$	
	$\therefore\ x = -3, y = 4$	Substitute $x = -3$ and $y = 4$ into the original equations to check.
b	$4x + 3y = 18$ [1]	Multiply equation [1] by 2 and equation [2] by 3 to make the coefficients of y equal in size but opposite in sign.
	$3x - 2y = 5$ [2]	
$2 \times [1]$	$8x + 6y = 36$ [3]	
$3 \times [2]$	$\underline{9x - 6y = 15}$ [4]	
[3] + [4]	$17x = 51$	Add the equations to eliminate y.
	$x = 3$	

ISBN 978-1-108-77288-4

From [1] $4x + 3y = 18$

$4 \times (3) + 3y = 18$

$12 + 3y = 18$

$3y = 6$

$y = 2$

Substitute $x = 3$ into equation [1] to find y.

$\therefore\ x = 3,\ y = 2$

Substitute $x = 3$ and $y = 2$ into the original equations to check.

Now you try

Solve the following pairs of simultaneous equations by using elimination.

a $7x + 2y = 12$, $x - 3y = 5$

b $3x + 2y = 5$, $5x - 3y = 21$

Exercise 2J

FLUENCY	1, 2, 3–5(1/2)	2, 3–5(1/2)	3–5(1/2)

Example 20a

1 Solve these simultaneous equations by first adding the equations.

a $x + 2y = 3$, $-x + 3y = 2$

b $x - 4y = 2$, $-x + 6y = 2$

c $-2x + y = 1$, $2x - 3y = -7$

Example 20a

2 Solve these simultaneous equations by first adding the equations.

a $3x - y = 2$, $2x + y = 3$

b $2x - 3y = -2$, $-5x + 3y = -4$

c $4x + 3y = 5$, $-4x - 5y = -3$

Example 20b

3 Solve these simultaneous equations by first subtracting the equations.

a $3x + y = 10$, $x + y = 6$

b $2x + 7y = 9$, $2x + 5y = 11$

c $2x + 3y = 14$, $2x - y = -10$

d $5x - y = -2$, $3x - y = 4$

e $-5x + 3y = -1$, $-5x + 4y = 2$

f $9x - 2y = 3$, $-3x - 2y = -9$

Example 21a

4 Solve the following pairs of simultaneous linear equations by using elimination.

a $4x + y = -8$, $3x - 2y = -17$

b $2x - y = 3$, $5x + 2y = 12$

c $-x + 4y = 2$, $3x - 8y = -2$

d $3x + 2y = 0$, $4x + y = -5$

e $4x + 3y = 13$, $x + 2y = -3$

f $3x - 4y = -1$, $6x - 5y = 10$

g $-4x - 3y = -5$, $7x - y = 40$

h $3x - 4y = -1$, $-5x - 2y = 19$

i $5x - 4y = 7$, $-3x - 2y = 9$

Example 21b

5 Solve the following pairs of simultaneous linear equations by using elimination.

a $3x + 2y = -1$, $4x + 3y = -3$

b $7x + 2y = 8$, $3x - 5y = 21$

c $6x - 5y = -8$, $-5x + 2y = -2$

d $2x - 3y = 3$, $3x - 2y = 7$

e $7x + 2y = 1$, $4x + 3y = 8$

f $5x + 7y = 1$, $3x + 5y = -1$

g $5x + 3y = 16$, $4x + 5y = 5$

h $3x - 7y = 8$, $4x - 3y = -2$

i $2x - 3y = 1$, $3x + 2y = 8$

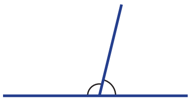
j $2x - 7y = 11$, $5x + 4y = -37$

k $3x + 5y = 36$, $7x + 2y = -3$

l $2x - 4y = 6$, $5x + 3y = -11$

PROBLEM-SOLVING	6, 7	6–8	7–9

6 The sum of two numbers is 30 and their difference is 12.
Write two equations and find the numbers.



7 Two supplementary angles differ by 24°. Write two equations and find the two angles.

8 The perimeter of a rectangular city block is 800 metres and the difference between the length and width is 123 metres. What are the dimensions of the city block?

9 A teacher collects a total of 17 mobile phones and iPads before a group of students heads off on a bushwalk. From a second group of students, 40 phones and iPads are collected. The second group had twice the number of phones and three times as many iPads as the first group. How many phones and how many iPads did the first group have?

REASONING	10	10, 11	11, 12

10 Consider the pair of simultaneous equations:

$$\begin{aligned} 2x + y &= 5 \ \ldots \quad [1] \\ 5x + y &= 11 \ \ldots \quad [2] \end{aligned}$$

a Solve the equations by first subtracting equation [2] from equation [1], i.e. [1] − [2].
b Now solve the equations by first subtracting equation [1] from equation [2], i.e. [2] − [1].
c Which method **a** or **b** is preferable and why?

11 To solve any of the pairs of simultaneous equations in this section using the method of substitution, what would need to be done first before the substitution is made?
Try these using substitution.

a $\begin{aligned} x + y &= 5 \\ 2x - y &= 7 \end{aligned}$

b $\begin{aligned} 3x - y &= -2 \\ x - 4y &= 3 \end{aligned}$

12 Find the solution to these pairs of simultaneous equations. What do you notice?

a $\begin{aligned} 2x + 3y &= 3 \\ 2x + 3y &= 1 \end{aligned}$

b $\begin{aligned} 7x - 14y &= 2 \\ y &= \frac{1}{2}x + 1 \end{aligned}$

ENRICHMENT: Literal elimination	–	–	13(1/2)

13 Use elimination to solve the following pairs of simultaneous equations to find the value of x and y in terms of the other pronumerals.

a $\begin{aligned} x + y &= a \\ x - y &= b \end{aligned}$

b $\begin{aligned} ax + y &= 0 \\ ax - y &= b \end{aligned}$

c $\begin{aligned} x - by &= a \\ -x - by &= 2a \end{aligned}$

d $\begin{aligned} 2ax + y &= b \\ x + y &= b \end{aligned}$

e $\begin{aligned} bx + 5ay &= 2b \\ bx + 2ay &= b \end{aligned}$

f $\begin{aligned} ax + 3y &= 14 \\ ax - y &= -10 \end{aligned}$

g $\begin{aligned} 2ax + y &= b \\ 3ax - 2y &= b \end{aligned}$

h $\begin{aligned} 2ax - y &= b \\ 3ax + 2y &= b \end{aligned}$

i $\begin{aligned} -x + ay &= b \\ 3x - ay &= -b \end{aligned}$

j $\begin{aligned} ax + 2y &= c \\ 2ax + y &= -c \end{aligned}$

k $\begin{aligned} ax - 4y &= 1 \\ x - by &= 1 \end{aligned}$

l $\begin{aligned} ax + by &= a \\ x + y &= 1 \end{aligned}$

m $\begin{aligned} ax + by &= c \\ -ax + y &= d \end{aligned}$

n $\begin{aligned} ax - by &= a \\ -x + y &= 2 \end{aligned}$

o $\begin{aligned} ax + by &= b \\ 3x - y &= 2 \end{aligned}$

p $\begin{aligned} ax - by &= b \\ cx - y &= 2 \end{aligned}$

q $\begin{aligned} ax + by &= c \\ dx - by &= f \end{aligned}$

r $\begin{aligned} ax + by &= c \\ dx + by &= f \end{aligned}$

2K Applications of simultaneous equations EXTENDING

Learning intentions

- To know that if there are two variables in a problem, two equations will be required to find the solution
- To be able to form a pair of equations from a word problem
- To be able to identify which technique is best to apply to solve the simultaneous equations
- To understand to check the solution by substituting into the original equations

Many problems can be described mathematically using a pair of simultaneous linear equations from which a solution can be obtained algebraically.

Weather forecasting uses algebraic equations to model changes over time of physical variables, such as air temperature and rainfall. Current observations are entered into supercomputers which simultaneously solve huge systems of equations, rapidly updating predictions.

LESSON STARTER The tyre store

In one particular week a total of 83 cars and motorcycles check into a garage to have their tyres changed. All the motorcycles change 2 tyres each and all the cars change 4 tyres each. The total number of tyres sold in the week is 284.

If you had to find the number of motorcycles and the number of cars that have their tyres changed in the week:

- What two variables should you define?
- What two equations can you write?
- Which method (substitution or elimination) would you use to solve the equations?
- What is the solution to the simultaneous equations?
- How would you answer the question in words?

KEY IDEAS

- To solve worded problems with simultaneous equations complete these steps.
  - Define two variables by writing down what they represent.
    For example: Let $\$C$ be the cost of …
    Let x be the number of …
  - Write a pair of simultaneous equations from the given information using your two variables.
  - Solve the equations simultaneously using substitution or elimination.
  - Check the solution by substituting into the original equations.
  - Express the answer in words.

BUILDING UNDERSTANDING

① The sum of two numbers is 42 and their difference is 6. Find the two numbers x and y by completing the following steps.

- **a** State a pair of simultaneous equations relating x and y.
- **b** Which method, substitution or elimination, would it be best to use to solve the pair of equations?

② The length l cm of a rectangle is 5 cm longer than its width w cm. If the perimeter is 84 cm, find the dimensions of the rectangle by completing the following steps.

- **a** State a pair of simultaneous equations relating l and w.
- **b** Which method, substitution or elimination, would it be best to use to solve the pair of equations?

③ A rectangular block of land has a perimeter of 120 m and the length l m of the block is three times the width w m. Find the dimensions of the block of land by completing the following steps.

- **a** State a pair of simultaneous equations relating l and w.
- **b** Which method, substitution or elimination, would it be best to use to solve the pair of equations?

Example 22 Solving word problems with simultaneous equations

Andrea bought two containers of ice-cream and three bottles of maple syrup for a total of $22. At the same shop, Bettina bought one container of the same ice-cream and two bottles of the same maple syrup for $13. How much does each container of ice-cream and each bottle of maple syrup cost?

SOLUTION	EXPLANATION
Let: $\$x$ be the cost of a container of ice-cream $\$y$ be the cost of a bottle of maple syrup	Define the unknowns. Ask yourself what you are being asked to find.
$2x + 3y = 22 \quad [1]$ $x + 2y = 13 \quad [2]$	2 containers of ice-cream and 3 bottles of maple syrup for a total of $22. 1 container of ice-cream and 2 bottles of maple syrup for $13.
$\begin{array}{lrl} 2 \times [2] & 2x + 4y = 26 & [3] \\ & \underline{2x + 3y = 22} & [1] \\ [3] - [1] & y = 4 & \end{array}$	Choose the method of elimination to solve. Multiply [2] by 2 to obtain a matching pair. Subtract equation [1] from [3].
$\begin{array}{lr} \text{From } [2] & x + 2y = 13 \\ & x + 2 \times (4) = 13 \\ & x + 8 = 13 \\ & x = 5 \end{array}$	Substitute $y = 4$ into [2]. Solve for x. Substitute $y = 4$ and $x = 5$ into original equations to check.
The cost of one container of ice-cream is $5 and the cost of one bottle of maple syrup is $4.	Answer the question in a sentence.

Now you try

Wally bought three identical hats and four identical scarfs for $70. From the same store on another day he bought five hats and two scarfs for $64.40. How much does each hat and each scarf cost?

Exercise 2K

FLUENCY	1–4	2–5	3–6

Example 22 **1** Mal bought three bottles of milk and four bags of chips for a total of $17. At the same shop, Barbara bought one bottle of milk and five bags of chips for $13. Find how much each bottle of milk and each bag of chips cost by completing the following steps.

- **a** Define two variables to represent the problem.
- **b** Write a pair of simultaneous equations relating the two variables.
- **c** Solve the pair of equations using substitution or elimination.
- **d** Write your answer in words.

Example 22 **2** Leonie bought seven lip glosses and two eye shadows for a total of $69 and Chrissie bought four lip glosses and three eye shadows for a total of $45. Find how much each lip gloss and each eye shadow costs by completing the following steps.

- **a** Define two variables to represent the problem.
- **b** Write a pair of simultaneous equations relating the two variables.
- **c** Solve the pair of equations using substitution or elimination.
- **d** Write your answer in words.

Example 22 **3** Steve bought five cricket balls and fourteen tennis balls for $130. Ben bought eight cricket balls and nine tennis balls for $141. Find the cost of a cricket ball and the cost of a tennis ball.

4 At a birthday party for 20 people each person could order a hot dog or a bucket of chips. If there were four times as many hot dogs ordered as buckets of chips, calculate how many hot dogs and how many buckets of chips were ordered.

5 The entry fee for a fun run is $10 for adults and $3 for children. A total of $3360 was collected from the 420 competitors. Find the number of adults and the number of children running.

6 Mila plants 820 hectares of potatoes and corn. To maximise his profit he plants 140 hectares more of potatoes than of corn. How many hectares of each does he plant?

ISBN 978-1-108-77288-4

Updated September 2021

PROBLEM-SOLVING	7, 8	7–9	8–10

7 Carrie has 27 coins in her purse. All the coins are 5 cent or 20 cent coins. If the total value of the coins is \$3.75, how many of each type does she have?

8 Michael is 30 years older than his daughter. In five years' time Michael will be 4 times as old as his daughter. How old is Michael now?

9 Jenny has twice as much money as Kristy. If I give Kristy \$250, she will have three times as much as Jenny. How much did they each have originally?

10 At a particular cinema the cost of an adult movie ticket is \$15 and the cost of a child's ticket is \$10. The seating capacity of the cinema is 240. For one movie session all seats are sold and \$3200 is collected from the sale of tickets. How many adult and how many children's tickets were sold?

REASONING	11	11, 12	12, 13

11 Wilfred and Wendy are in a long distance bike race. Wilfred rides at 20 km/h and has a 2 hour head start. Wendy travels at 28 km/h. How long does it take for Wendy to catch up to Wilfred? Use distance = speed × time.

12 Andrew travelled a distance of 39 km by jogging for 4 hours and cycling for 3 hours. He could have travelled the same distance by jogging for 7 hours and cycling for 2 hours. Find the speed at which he was jogging and the speed at which he was cycling.

13 Malcolm's mother is 27 years older than he is and their ages are both two-digit numbers. If Malcolm swaps the digits in his age he gets his mother's age.

- a How old is Malcolm if the sum of the digits in his age is 5?
- b What is the relationship between the digits in Malcolm's age if the sum of the digits is unknown.
- c If the sum of the digits in Malcolm's two-digit age is unknown, how many possible ages could he be? What are these ages?

ENRICHMENT: Digit swap	–	–	14, 15

14 The digits of a two-digit number sum to 10. If the digits swap places the number is 36 more than the original number. What is the original number? Can you show an algebraic solution?

15 The difference between the two digits of a two-digit number is 2. If the digits swap places the number is 18 less than the original number. What is the original number? Can you show an algebraic solution?

Movie theatre pricing

A small boutique movie theatre is opening soon and will be publishing adult and child ticket prices on its website. After some analysis, the owner decides to make the following assumptions:

- the sum of the cost of a child ticket and an adult ticket will be \$30
- 40 children and 25 adults are expected to attend each evening.

Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate.

Preliminary task

a If the cost of a child movie ticket is \$6, find the cost of an adult ticket.

b Find the revenue (total amount earned) for the movie theatre on one particular evening using the assumption that 40 children and 25 adults attend and a child ticket is \$6.

c Solve the following simultaneous equations.

$$a + c = 30$$
$$25a + 40c = 900$$

Modelling task

Formulate

a The problem is to determine a suitable price for the adult and child tickets to achieve a revenue of at least \$1000. Write down all the relevant information that will help solve this problem.

b Use the following facts to write two equations using \$$a$ for the cost of an adult ticket and \$$c$ for the cost of a child ticket.

- The sum of the cost of a child ticket and an adult ticket will be \$30.
- 40 children and 25 adults attend on one particular evening.
- The total revenue for the evening is \$930.

Solve

c Solve your simultaneous equations from part **b** to find the cost of a child ticket and the cost of an adult ticket.

d Repeat part **c** for the following total revenues.

i \$825 **ii** \$960

e Explore the following special cases for the total revenue earned and decide if they represent reasonable pricing structures.

i \$975 **ii** \$765

Evaluate and verity

f By considering your results from above, explain why it will be difficult for the theatre to earn more than \$1000 on each evening based on the assumptions made.

g If the sum of the cost of a child ticket and an adult ticket is changed to \$40, do you think it would be reasonable to expect that more than \$1000 revenue can be earned each evening. Show your choices and calculations to justify your answer.

Communicate

h Summarise your results and describe any key findings.

Extension questions

a By keeping the assumption that the cost of a child ticket and an adult ticket will be \$30, determine the maximum and minimum expected revenue for an evening at the cinema.

b Decide if the cinema owner should increase the price for the total cost of purchasing one adult and one child ticket to achieve a revenue of more than \$1000. Choose a suitable total cost and justify your choice showing all calculations.

Fire danger

In many countries fire indices have been developed to help predict the likelihood of fire occurring. One of the simplest fire-danger rating systems devised is the Swedish Angstrom Index. This index considers the relationship between relative humidity, temperature and the likelihood of fire danger.

The index, I, is given by: $I = \frac{H}{20} + \frac{27 - T}{10}$

where H is the percentage of relative humidity and T is the temperature in degrees Celsius. The table below shows the likelihood of a fire occurring for different index values.

Index	Likelihood of fire occurring
$I > 4.0$	Unlikely
$2.5 < I < 4.0$	Medium
$2.0 < I < 2.5$	High
$I < 2.0$	Very likely

Constant humidity

a If the humidity is 35% ($H = 35$), how hot would it have to be for the occurrence of fire to be

i very likely? **ii** unlikely?

Discuss your findings with regard to the range of summer temperatures for your capital city or nearest town.

b Repeat part **a** for a humidity of 40%.

c Describe how the 5% change in humidity affects the temperature at which fires become:

i very likely **ii** unlikely.

Constant temperature

a If the temperature was 30°C, investigate what humidity would make fire occurrence:

i very likely **ii** unlikely.

b Repeat part **a** for a temperature of 40°C.

c Determine how the ten-degree change in temperature affects the relative humidity at which fire occurrence becomes:

i very likely **ii** unlikely.

Reflection

Is this fire index more sensitive to temperature or to humidity? Explain your answer.

Investigate

Use the internet to investigate fire indices used in Australia. You can type in key words, such as Australia, fire danger and fire index.

Up for a challenge? If you get stuck on a question, check out the 'Working with unfamiliar problems' poster at the end of the book to help you.

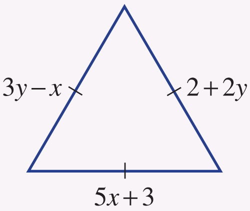
1 Transpose each of the following formulas to make the pronumeral shown in brackets the subject.

- **a** $wx - 4 = ax + k \quad [x]$
- **b** $\dfrac{a}{K} - y = \dfrac{b}{K} \quad [K]$
- **c** $\sqrt{\dfrac{a - w}{ak}} = m \quad [a]$

2 Five consecutive integers add to 195. Find the middle integer.

3 A group of office workers had some prize money to distribute amongst themselves. When all but one took \$9 each, the last person only received \$5. When they all took \$8 each there was \$12 left over. How much had they won?

4 The sides of an equilateral triangle have lengths $3y - x$, $5x + 3$ and $2 + 2y$, as shown. Find the length of the sides.



5 **a** If $a > b > 0$ and $c < 0$, insert an inequality sign to make a true statement.

- **i** $a + c$ ___ $b + c$
- **ii** ac ___ bc
- **iii** $a - b$ ___ 0
- **iv** $\dfrac{1}{a}$ ___ $\dfrac{1}{b}$

b Place a, b, c and d in order from smallest to largest given:

$$\begin{aligned} a &> b \\ a + b &= c + d \\ b - a &> c - d \end{aligned}$$

6 Find the values of x, y and z when:

$$\begin{aligned} x - 3y + 2z &= 17 \\ x - y + z &= 8 \\ y + z &= 3 \end{aligned}$$

7 Solve these equations for x.

- **a** $\dfrac{1}{x} + \dfrac{1}{a} = \dfrac{1}{b}$
- **b** $\dfrac{1}{2x} + \dfrac{1}{3x} = \dfrac{1}{4}$
- **c** $\dfrac{x - 1}{3} - \dfrac{x + 1}{4} = x$
- **d** $\dfrac{2x - 3}{4} - \dfrac{1 - x}{5} = \dfrac{x + 1}{2}$

ISBN 978-1-108-77288-4

Updated September 2021

Chapter summary

Linear and simultaneous equations

Substitution

The process of replacing a pronumeral with a given value

e.g. if $x = 3$, $2x + 4 = 2 \times 3 + 4$
$= 6 + 4$
$= 10$

e.g. if $x = 2$, $y = -4$, $xy = 2 \times (-4)$
$= -8$

Expanding brackets

e.g. $2(x + 3) = 2 \times x + 2 \times 3$
$= 2x + 6$
$-3(4 - 2x) = -12 + 6x$

Expressions

e.g. $\underset{\text{term}}{\boxed{2x}}\underset{\text{coefficient}}{-3}y + \underset{\text{constant}}{5}$

e.g. 2 more than 3 lots of m is $3m + 2$

Addition/subtraction

Only like terms can be combined under addition or subtraction.

Like terms: e.g. $3a$ and $7a$
$2ab$ and $5ab$
not $4a$ and $7ab$

e.g. $3x + 2y - x + 4y + 7$
$= 3x - x + 2y + 4y + 7$
$= 2x + 6y + 7$

Solving linear equations

Finding the value that makes an equation true

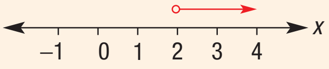
e.g. $2x + 5 = 9$
$2x = 4$ (subtract 5)
$x = 2$ (divide by 2)

e.g. $5(x - 1) = 2x + 7$
$5x - 5 = 2x + 7$ (expand)
$3x - 5 = 7$ (subtract $2x$)
$3x = 12$ (add 5)
$x = 4$ (divide by 3)

Inequalities

These can be represented using $>$, $<$, $\geqslant$, $\leqslant$ rather than $=$.

e.g. $x > 2$ 2 not included (open circle)



Solving inequalities uses the same steps as solving equations except when multiplying or dividing by a negative number. In this case, the inequality sign must be reversed.

e.g. $4 - 2x < 10$ (-4)
$-2x < 6$ $(\div -2)$
$x > -3$ (reverse sign)

Solving word problems

1 Define variable(s)
2 Set up equation(s)
3 Solve equation(s)
4 Check each answer and write in words

Formulas

Some common formulas

e.g. $A = \pi r^2$, $C = 2\pi r$

An unknown value can be found by substituting values for the other variables.

A formula can be transposed to make a different variable the subject.

e.g. make r the subject in $A = \pi r^2$

$A = \pi r^2$

$\frac{A}{\pi} = r^2$

$\sqrt{\frac{A}{\pi}} = r$

$\therefore r = \sqrt{\frac{A}{\pi}}$ where $r > 0$

Simultaneous equations (Ext)

Use substitution or elimination to find the solution that satisfies two equations.

Substitution

e.g. $2x + y = 12$ [1]
$y = x + 3$ [2]

In [1] replace y with [2]

$2x + (x + 3) = 12$
$3x + 3 = 12$
$x = 3$

Substitute $x = 3$ to find y

In [2] $y = (3) + 3 = 6$

Elimination

Ensure both equations have a matching pair.

Add 2 equations if matching pair has different sign; subtract if same sign.

e.g. $x + 2y = 2$ [1]
$2x + 3y = 5$ [2]
[1] × 2 $2x + 4y = 4$ [3]
[3] – [2] $y = -1$
In [1] $x + 2(-1) = 2$
$x - 2 = 2$
$\therefore x = 4$

ISBN 978-1-108-77288-4

Chapter checklist: Success criteria

		✔
2A	**1. I can write algebraic expressions from words.** e.g. Write an algebraic expression for the cost of t tickets at \$20 each.	
2A	**2. I can substitute values into expressions and evaluate.** e.g. Evaluate $a^2 + 3(ab - c)$ if $a = -2$, $b = 3$ and $c = -8$.	
2B	**3. I can multiply algebraic terms.** e.g. Simplify $4x \times 7xy$.	
2B	**4. I can divide algebraic terms.** e.g. Simplify $12xy \div (18x)$.	
2B	**5. I can collect like terms under addition and subtraction.** e.g. Simplify $4xy - 5x^2y - xy + 3x^2y$.	
2C	**6. I can expand brackets and simplify.** e.g. Expand $4x(3x - 5)$.	
2C	**7. I can simplify by removing brackets.** e.g. Expand and simplify $7 - 3(3x - 2)$.	
2D	**8. I can solve linear equations.** e.g. Solve $\frac{4 - x}{3} = 2$.	
2E	**9. I can solve linear equations with brackets.** e.g. Solve $2(4x + 3) = 7$.	
2E	**10. I can solve linear equations with pronumerals on both sides.** e.g. Solve $6x + 5 = 3x - 13$.	
2F	**11. I can turn a word problem into an equation.** e.g. Write an equation to represent that four more than a certain number is three less than two times the number.	
2F	**12. I can solve a word problem using an equation.** e.g. Ava and Elise have 27 soft toys between them. Ava has 5 more than Elise. Use an equation to find how many soft toys they each have.	
2G	**13. I can represent an inequality on a number line.** e.g. Show x is less than or equal to 3 ($x \leqslant 3$) on a number line.	
2G	**14. I can solve a linear inequality.** e.g. Find the solution set for $8 - 3x < 20$.	
2H	**15. I can substitute values into formulas.** e.g. Evaluate the formula $c = \sqrt{a^2 + b^2}$ when $a = 9$ and $b = 12$.	
2H	**16. I can find the unknown value in a formula.** e.g. In the formula $s = ut + \frac{1}{2}at^2$, find a if $s = 80$, $u = 10$ and $t = 4$.	
2H	**17. I can transpose formulas.** e.g. Transpose $V = \pi r^2 h$ to make r the subject.	
2I	**18. I can solve simultaneous equations using substitution.** e.g. Solve the following pair of simultaneous equations using substitution: $x + 2y = 4$ $y = 2x - 3$. Ext	
2J	**19. I can solve simultaneous equations using elimination.** e.g. Solve the following pair of simultaneous equations using elimination: $3x + 2y = 3$ $2x - y = -5$. Ext	
2K	**20. I can solve word problems with simultaneous equations.** e.g. At the football, 2 meat pies and 3 bottles of water cost \$21 while 3 meat pies and 5 bottles of water cost \$33. How much does each meat pie and each bottle of water cost? Ext	

Short-answer questions

2A 1 Write algebraic expressions to represent the following.

a The product of m and 7 **b** Twice the sum of x and y

c The cost of 3 movie tickets at m dollars each **d** n divided by 4 less 3

2A 2 Evaluate the following when $x = 2$, $y = -1$ and $z = 5$.

a $yz - x$ **b** $\dfrac{2z - 4y}{x}$ **c** $\dfrac{2z^2}{x} + y$ **d** $x(y + z)$

2B 3 Simplify.

a $2m \times 4n$ **b** $\dfrac{4x^2y}{12x}$ **c** $3ab \times 4b \div (2a)$

d $4 - 5b + 2b$ **e** $3mn + 2m - 1 - nm$ **f** $4p + 3q - 2p + q$

2C 4 Expand and simplify the following.

a $2(x + 7)$ **b** $-3(2x + 5)$ **c** $2x(3x - 4)$

d $-2a(5 - 4a)$ **e** $5 - 4(x - 2)$ **f** $4(3x - 1) - 3(2 - 5x)$

2D 5 Solve the following linear equations for x.

a $5x + 6 = 51$ **b** $\dfrac{x + 2}{4} = 7$ **c** $\dfrac{2x}{5} - 3 = 3$ **d** $\dfrac{2x - 5}{3} = -1$

e $7x - 4 = 10$ **f** $3 - 2x = 21$ **g** $1 - \dfrac{4x}{5} = 9$ **h** $2 - 7x = -3$

2D 6 Write an equation to represent each of the following and then solve it for the pronumeral.

a A number, n, is doubled and increased by 3 to give 21.

b The number of lollies, l, is decreased by 5 and then shared equally among three friends so that they each get 7 lollies.

c 5 less than the result of Toni's age, x, divided by 4 is 0.

2E 7 Solve the following linear equations.

a $2(x + 4) = 18$ **b** $3(2x - 3) = 2$ **c** $8x = 2x + 24$

d $5(2x + 4) = 7x + 5$ **e** $3 - 4x = 7x - 8$ **f** $1 - 2(2 - x) = 5(x - 3)$

2F 8 Nick makes an initial bid of \$$x$ in an auction for a signed cricket bat. By the end of the auction he has paid \$550, which is \$30 more than twice his initial bid. Set up and solve an equation to determine Nick's initial bid.

2G 9 Represent each of the following on a number line.

a $x > 1$ **b** $x \leqslant 7$ **c** $x \geqslant -4$

d $x < -2$ **e** $2 < x \leqslant 8$ **f** $-1 < x < 3$

2G 10 Solve the following inequalities.

a $x + 8 < 20$ **b** $2m - 4 > -6$ **c** $3 - 2y \leqslant 15$

d $\dfrac{7 - 2x}{3} > -9$ **e** $3a + 9 < 7(a - 1)$ **f** $-4x + 2 \leqslant 5x - 16$

2G **11** A car salesman earns \$800 per month plus a 10% commission on the value of sales he makes for the month. If he is aiming to earn a minimum of \$3200 a month, what is the possible value of sales that will enable this?

2H **12** Find the value of the unknown in each of the following formulas.

a $E = \sqrt{PR}$ when $P = 90$ and $R = 40$

b $v = u + at$ when $v = 20$, $u = 10$, $t = 2$

c $V = \frac{1}{3}Ah$ when $V = 20$, $A = 6$

2H **13** Rearrange the following formulas to make the variable in brackets the subject.

a $v^2 = u^2 + 2ax \quad [x]$

b $A = \frac{1}{2}r^2\theta \quad [\theta]$

c $P = RI^2, I > 0 \quad [I]$

d $S = \frac{n}{2}(a + l) \quad [a]$

2I/J Ext **14** Solve the following simultaneous equations using the substitution method for parts **a**–**c** and the elimination method for **d**–**f**.

a $x + 2y = 12$
$x = 4y$

b $2x + 3y = -6$
$y = x - 1$

c $7x - 2y = 6$
$y = 2x + 3$

d $x + y = 15$
$x - y = 7$

e $3x + 2y = -19$
$4x - y = -7$

f $3x - 5y = 7$
$5x + 2y = 22$

2K Ext **15** Billy went to the Show and spent \$78 on a combined total of 9 items including rides and showbags. If each showbag cost \$12 and each ride cost \$7, how many of each did Billy buy?

Multiple-choice questions

2A **1** The algebraic expression that represents 2 less than 3 lots of n is:

A $3(n - 2)$ **B** $2 - 3n$ **C** $3n - 2$ **D** $3 + n - 2$ **E** n

2B **2** The simplified form of $6ab + 14a \div 2 - 2ab$ is:

A $8ab$ **B** $8ab + 7a$ **C** $ab + 7a$ **D** $4ab + 7a$ **E** $4ab + \frac{7a}{2}$

2D **3** The solution to $\frac{x}{3} - 1 = 4$ is:

A $x = 13$ **B** $x = 7$ **C** $x = 9$ **D** $x = 15$ **E** $x = \frac{5}{3}$

2D **4** The result when a number is tripled and increased by 21 is 96. The original number is:

A 22 **B** 32 **C** 25 **D** 30 **E** 27

2E **5** The solution to the equation $3(x - 1) = 5x + 7$ is:

A $x = -4$ **B** $x = -5$ **C** $x = 5$ **D** $x = 3$ **E** $x = 1$

2F

6 $x is raised from a sausage sizzle. Once the $50 running cost is taken out, the money is shared equally amongst three charities so that they each get $120. An equation to represent this is:

A $\dfrac{x-50}{3}=120$ **B** $\dfrac{x}{3}-50=120$ **C** $\dfrac{x}{50}=360$

D $\dfrac{x}{3}=310$ **E** $3x+50=120$

2H

7 If $A=2\pi rh$ with $A=310$ and $r=4$, then the value of h is closest to:

A 12.3 **B** 121.7 **C** 24.7 **D** 38.8 **E** 10.4

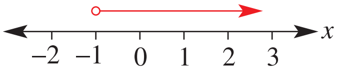
2H

8 The formula $d=\sqrt{\dfrac{a}{b}}$ transposed to make a the subject is:

A $a=\sqrt{bd}$ **B** $a=d\sqrt{b}$ **C** $a=\dfrac{d^2}{b^2}$ **D** $a=\dfrac{d}{\sqrt{b}}$ **E** $a=bd^2$

2G

9 The inequality representing the x-values on the number line below is:



A $x<-1$ **B** $x>-1$ **C** $x\leqslant-1$ **D** $x\geqslant-1$ **E** $-1<x<3$

2G

10 The solution to the inequality $1-2x>9$ is:

A $x<-4$ **B** $x<4$ **C** $x<-5$ **D** $x>-4$ **E** $x>5$

2I Ext

11 The solution to the simultaneous equations $x+2y=16$ and $y=x-4$ is:

A $x=4, y=0$ **B** $x=8, y=4$ **C** $x=6, y=2$

D $x=12, y=8$ **E** $x=5, y=1$

2J Ext

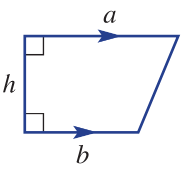
12 The solution to the simultaneous equations $2x+y=2$ and $2x+3y=10$ is:

A $x=0, y=2$ **B** $x=2, y=2$ **C** $x=2, y=-2$

D $x=-1, y=4$ **E** $x=-2, y=6$

Extended-response questions

1 The area of a trapezium is given by $A=\dfrac{1}{2}(a+b)h$. A new backyard deck in the shape of the trapezium shown is being designed.
Currently the dimensions are set such that $a=12$ m and $h=10$ m.



a What range of b values is required for an area of at most 110 m^2?

b Rearrange the area formula to make b the subject.

c Use your answer to part **b** to find the length b m required to give an area of 100 m^2.

d Rearrange the area formula to make h the subject.

e If b is set as 8 m, what does the width of the deck (h m) need to be reduced to for an area of 80 m^2?

2 Members of the Hayes and Thompson families attend the local Regatta by the Bay.

a The entry fee for an adult is \$18 and that for a student is \$8. The father and son from the Thompson family notice that after paying the entry fees and for 5 rides for the son and 3 for the adult, they have each spent the same amount. If the cost of a ride is the same for an adult and a student, write an equation and solve it to determine the cost of a ride.

Ext **b** For lunch each family purchases some buckets of hot chips and some drinks. The Hayes family buys 2 drinks and 1 bucket of chips for \$11 and the Thompson family buys 3 drinks and 2 buckets of chips for \$19. To determine how much each bucket of chips and each drink costs, complete the following steps.

- **i** Define two variables to represent the problem.
- **ii** Set up two equations relating the variables.
- **iii** Solve your equations in part **b ii** simultaneously.
- **iv** What is the cost of a bucket of chips and the cost of a drink?

CHAPTER

3

Pythagoras' theorem and trigonometry

UAVs: Unmanned aerial vehicles

UAVs, such as quadcopters, octocopters and drones, all use the same principles to navigate.

Before UAVs could be developed, the complex dynamics of their flight had to be represented mathematically. These equations were then coded to allow computer control. When in action, the flight-controlling program uses data input from several miniature electronic sensors including a GPS (for location), an accelerometer (for distance travelled) and a magnetometer (for direction of travel).

The GPS device receives time signals and position data from 4 of the 24 GPS satellites and calculates the UAV's distance from each satellite. Pythagoras' theorem and trigonometry is applied in three dimensions to compute the UAV's exact position on Earth giving its latitude, longitude and altitude.

Using data inputs from the GPS, magnetometer and accelerometer, the UAV's computer program applies trigonometry to calculate the direction and distance to fly to a given destination. Using these same

principles, the UAV can hold a position in windy conditions or automatically return to the start if its battery is running low or the control signal is lost.

UAVs can fly autonomously following a list of distances and bearings or GPS coordinates, and follow a variety of instructions such as where to photograph, turn, hover, orbit and land.

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

In this chapter

3A Pythagoras' theorem
3B Finding the length of the shorter sides
3C Applying Pythagoras' theorem
3D Pythagoras in three dimensions (EXTENDING)
3E Trigonometric ratios
3F Finding side lengths
3G Solving for the denominator
3H Finding an angle
3I Applying trigonometry (EXTENDING)
3J Bearings (EXTENDING)

Australian Curriculum

MEASUREMENT AND GEOMETRY
Pythagoras and trigonometry

Investigate Pythagoras' theorem and its application to solving simple problems involving right-angled triangles (ACMMG222)

Use similarity to investigate the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles (ACMMG223)

Apply trigonometry to solve right-angled triangle problems (ACMMG224)

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3A Pythagoras' theorem

Learning intentions

- To know that the hypotenuse is the name of longest side of a right-angled triangle
- To know Pythagoras' theorem and how it relates the side lengths of right-angled triangles
- To be able to use Pythagoras' theorem to find the length of the hypotenuse
- To be able to express a length as a decimal or an exact value written as a surd

Pythagoras was born on the Greek island of Samos in the 6th century BCE. He received a privileged education and travelled to Egypt and Persia where he developed his ideas in mathematics and philosophy. He settled in Crotone, Italy, where he founded a school. His many students and followers were called the Pythagoreans, and under the guidance of Pythagoras they lived a very structured life with strict rules. They aimed to be pure, self-sufficient and wise, where men and women were treated equally and all property was considered communal. They strove to perfect their physical and mental form, and made many advances in their understanding of the world through mathematics.

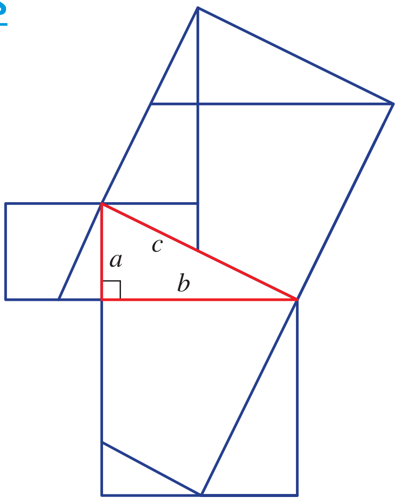
The Pythagoreans discovered the famous theorem, which is named after Pythagoras, and the existence of irrational numbers such as $\sqrt{2}$, which cannot be written down as a fraction or terminating decimal. Such numbers cannot be measured exactly with a ruler with fractional parts and were thought to be unnatural. The Pythagoreans called these numbers 'unutterable' numbers and it is believed that any member of the brotherhood who mentioned these numbers in public would be put to death.

Land surveyors measure the horizontal and vertical distance between two locations of different altitudes. The straight line distance of the slope, which forms a hypotenuse, is calculated using Pythagoras' theorem.

LESSON STARTER Matching the areas of squares

Look at this right-angled triangle and the squares drawn on each side. Each square is divided into smaller sections.

- Can you see how the parts of the two smaller squares would fit into the larger square?
- What is the area of each square if the side lengths of the right-angled triangle are a, b and c as marked?
- What do the answers to the above two questions suggest about the relationship between a, b and c?



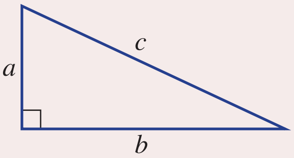
KEY IDEAS

- The longest side of a right-angled triangle is called the **hypotenuse** and is opposite the right angle.
- The **theorem of Pythagoras** says that the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.
  For the triangle shown, it is:

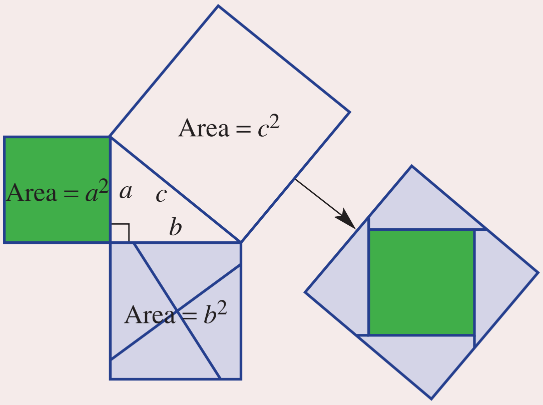
$$c^2 = a^2 + b^2$$

square of the hypotenuse

squares of the two shorter sides



- The theorem can be illustrated in a diagram like the one on the right. The sum of the areas of the two smaller squares ($a^2 + b^2$) is the same as the area of the largest square (c^2).

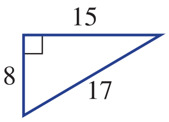


- Lengths can be expressed with **exact values** using **surds**. $\sqrt{2}$, $\sqrt{28}$ and $2\sqrt{3}$ are examples of surds.
  - When expressed as a decimal, a surd is an infinite non-recurring decimal with no pattern.
    For example: $\sqrt{2} = 1.4142135623\ldots$

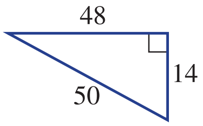
BUILDING UNDERSTANDING

① State the length of the hypotenuse (c units) in these right-angled triangles.

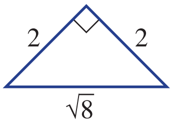
a



b

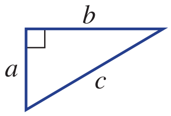


c

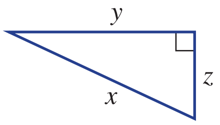


② State Pythagoras' theorem using the given pronumerals for these right-angled triangles. For example: $z^2 = x^2 + y^2$.

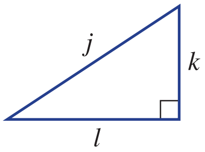
a



b



c



③ Evaluate the following, rounding to two decimal places in part **d**.

a 3.2^2 **b** $3^2 + 2^2$ **c** $\sqrt{64 + 36}$ **d** $\sqrt{24}$

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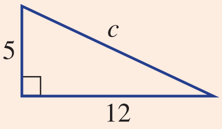
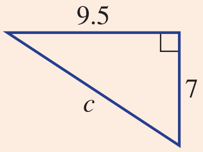
Example 1 Finding the length of the hypotenuse

Find the length of the hypotenuse in these right-angled triangles. Round to two decimal places in part **b**.

a



b

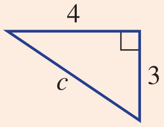


SOLUTION	EXPLANATION
a $c^2 = a^2 + b^2$ $= 5^2 + 12^2$ $= 169$	Write the rule and substitute the lengths of the two shorter sides.
$\therefore c = \sqrt{169}$ $= 13$	If $c^2 = 169$ then $c = \sqrt{169} = 13$.
b $c^2 = a^2 + b^2$ $= 7^2 + 9.5^2$ $= 139.25$	The order for a and b does not matter since $7^2 + 9.5^2 = 9.5^2 + 7^2$.
$\therefore c = \sqrt{139.25}$ $= 11.80$ (to 2 d.p.)	Round as required.

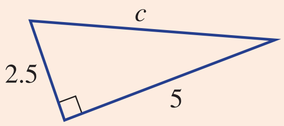
Now you try

Find the length of the hypotenuse in these right-angled triangles. Round to two decimal places in part **b**.

a

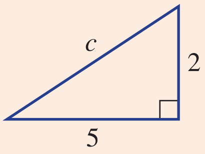


b



Example 2 Finding the length of the hypotenuse using exact values

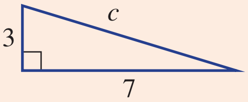
Find the length of the hypotenuse in this right-angled triangle, leaving your answer as an exact value.



SOLUTION	EXPLANATION
$c^2 = a^2 + b^2$ $= 5^2 + 2^2$ $= 29$	Apply Pythagoras' theorem to find the value of c.
$\therefore c = \sqrt{29}$	Express the answer exactly using a surd.

Now you try

Find the length of the hypotenuse in this right-angled triangle, leaving your answer as an exact value.



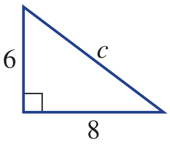
Exercise 3A

FLUENCY	1, 2–5($\frac{1}{2}$)	2–6($\frac{1}{2}$)	2–6($\frac{1}{3}$)

Example 1

1 Find the length of the hypotenuse in these right-angled triangles. Round to two decimal places in part **b**.

a



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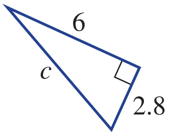
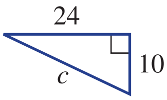


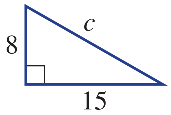
Example 1a

2 Find the length of the hypotenuse in each of the following right-angled triangles.

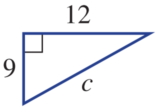
a



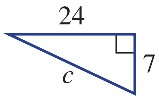
b



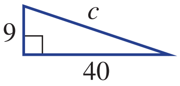
c



d



e



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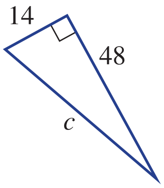

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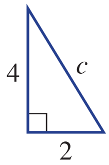
Example 1b

3 Find the length of the hypotenuse in each of these right-angled triangles, correct to two decimal places.

a



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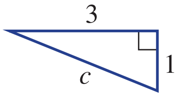
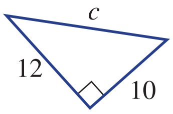
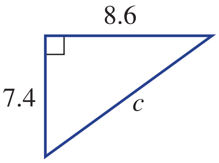


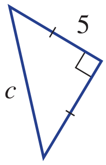
c



d



e



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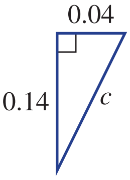
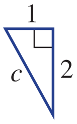


Example 2

4 Find the length of the hypotenuse in these triangles, leaving your answer as an exact value.

a



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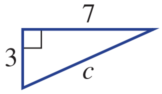

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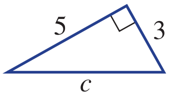

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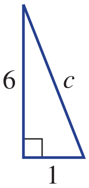

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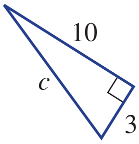

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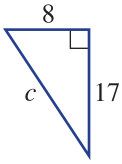
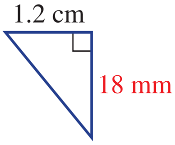


5 Find the length of the hypotenuse in each of these right-angled triangles, rounding to two decimal places where necessary. Convert to the units indicated in red.

a



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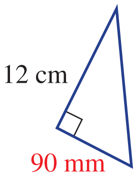

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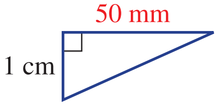

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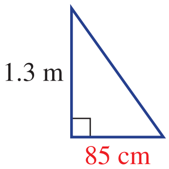

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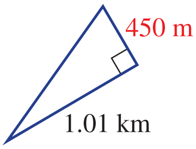

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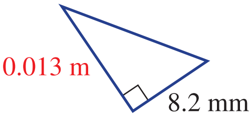
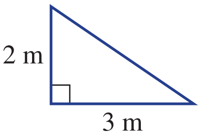


6 For each of these triangles, first calculate the length of the hypotenuse and then find the perimeter, correct to two decimal places.

a



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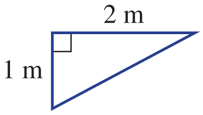

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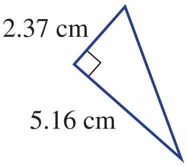

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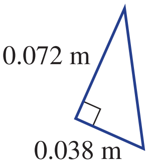

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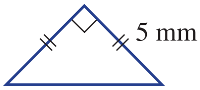

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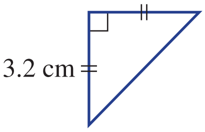
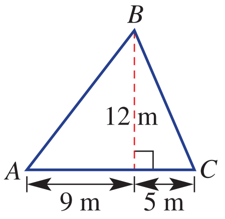
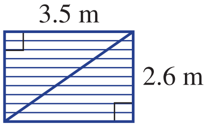


PROBLEM-SOLVING	7–9	7–9, 11	9–12

7 Find the perimeter of this triangle. (*Hint*: You will need to find *AB* and *BC* first.)

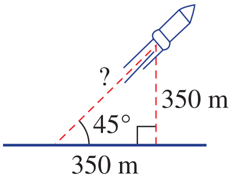


8 Find the length of the diagonal steel brace required to support a wall of length 3.5 m and height 2.6 m. Give your answer correct to one decimal place.

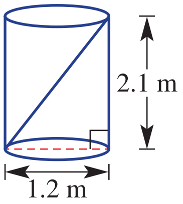


9 A helicopter hovers at a height of 150 m above the ground and is a horizontal distance of 200 m from a beacon on the ground. Find the direct distance of the helicopter from the beacon.

10 A miniature rocket blasts off at an angle of 45° and travels in a straight line. After a few seconds, it reaches a height of 350 m above the ground. At this point it has also covered a horizontal distance of 350 m. How far has the rocket travelled to the nearest metre?



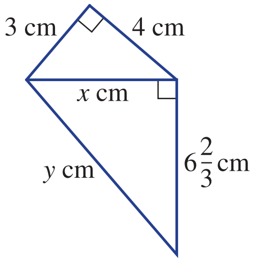
11 Find the length of the longest rod that will fit inside a cylinder of height 2.1 m and circular end surface of diameter 1.2 m. Give your answer correct to one decimal place.



12 For the shape on the right, find the value of:

a x

b y (as a fraction).

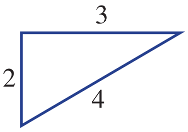


REASONING	13	13, 14(1/2)	14(1/2), 15

13 One way to check whether a four-sided figure is a rectangle is to ensure that both its diagonals are the same length. What should the length of the diagonals be if a rectangle has side lengths 3 m and 5 m? Answer to two decimal places.

14 We know that if the triangle has a right angle, then $c^2 = a^2 + b^2$. The converse of this is that if $c^2 = a^2 + b^2$ then the triangle must have a right angle. Test whether $c^2 = a^2 + b^2$ to see if these triangles have a right angle. They may not be drawn to scale.

a



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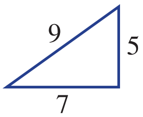

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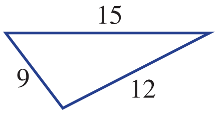
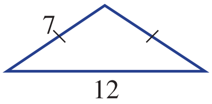


d



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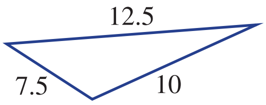

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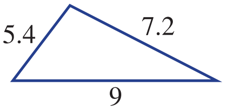
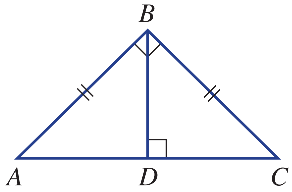


15 Triangle ABC is a right-angled isosceles triangle, and BD is perpendicular to AC. $DC = 4$ cm and $BD = 4$ cm.

a Find the length of BC correct to two decimal places.

b State the length of AB correct to two decimal places.

c Use Pythagoras' theorem and ΔABC to check that the length of AC is twice the length of DC.



ENRICHMENT: Kennels and kites	–	–	16, 17

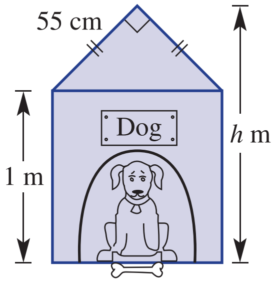
16 A dog kennel has the dimensions shown in the diagram on the right. Give your answers to each of the following correct to two decimal places.

a What is the width, in cm, of the kennel?

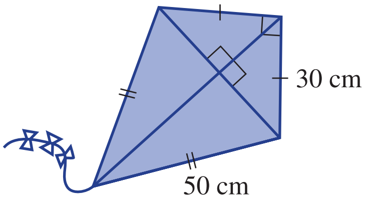
b What is the total height, h m, of the kennel?

c If the sloping height of the roof was to be reduced from 55 cm to 50 cm, what difference would this make to the total height of the kennel in centimetres? (Assume that the width is the same as in part **a**.)

d What is the length, in cm, of the sloping height of the roof of a new kennel if it is to have a total height of 1.2 m? (The height of the kennel without the roof is still 1 m and its width is unchanged.)



17 The frame of a kite is constructed with six pieces of timber dowel. The four pieces around the outer edge are two 30 cm pieces and two 50 cm pieces as shown. The top end of the kite is to form a right angle. Find the length of each of the diagonal pieces required to complete the construction. Answer to two decimal places.



3B Finding the length of the shorter sides

Learning intentions

- To know how to use Pythagoras' theorem when the unknown side is not the hypotenuse
- To be able to use Pythagoras' theorem to find the length of a shorter side

Throughout history, mathematicians have used known theorems to explore new ideas, discover new theorems and solve a wider range of problems. Similarly, Pythagoras knew that his right-angled triangle theorem could be manipulated so that the length of one of the shorter sides of a triangle can be found if the length of the other two sides are known.

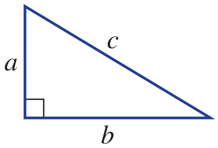
We know that the sum $7 = 3 + 4$ can be written as a difference $3 = 7 - 4$. Likewise, if $c^2 = a^2 + b^2$ then $a^2 = c^2 - b^2$ or $b^2 = c^2 - a^2$.

Applying this to a right-angled triangle means that we can now find the length of one of the shorter sides if the other two sides are known.

By knowing the vertical height between floors and an escalator's length, engineers can use Pythagoras' theorem to calculate the minimum horizontal width of floor space needed by an escalator.

LESSON STARTER True or false

Below are some mathematical statements relating to a right-angled triangle with hypotenuse c and the two shorter sides a and b.



Some of these mathematical statements are true and some are false. Can you sort them into true and false groups?

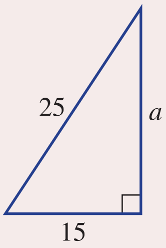
$a^2 + b^2 = c^2$	$a = \sqrt{c^2 - b^2}$	$c^2 - a^2 = b^2$	$a^2 - c^2 = b^2$
$c = \sqrt{a^2 + b^2}$	$b = \sqrt{a^2 - c^2}$	$c = \sqrt{a^2 - b^2}$	$c^2 - b^2 = a^2$

KEY IDEAS

- When finding the length of a side:
  - substitute known values into Pythagoras' theorem
  - solve this equation to find the unknown value.

  For example:
  - If $a^2 + 16 = 30$ then subtract 16 from both sides.
  - If $a^2 = 14$ then take the square root of both sides.
    - $a = \sqrt{14}$ is an **exact** answer (a surd).
    - $a = 3.74$ is a rounded decimal answer.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ 25^2 &= a^2 + 15^2 \\ 625 &= a^2 + 225 \\ 400 &= a^2 \\ a &= \sqrt{400} \\ a &= 20 \end{aligned}$$



BUILDING UNDERSTANDING

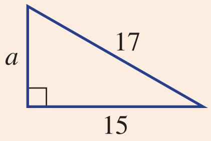
1. Find the value of a or b in these equations. (Both a and b are positive numbers.)
   a $a = \sqrt{196}$ **b** $a^2 = 144$ **c** $b^2 + 9 = 25$ **d** $36 + b^2 = 100$

2. Given that $a^2 + 64 = 100$, decide whether the following are true or false.
   a $a^2 = 100 - 64$ **b** $64 = 100 + a^2$ **c** $100 = \sqrt{a^2 + 64}$
   d $a = \sqrt{100 - 64}$ **e** $a = 6$ **f** $a = 10$

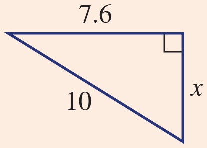
Example 3 Finding the length of a shorter side

In each of the following, find the value of the pronumeral. Round your answer in part **b** to two decimal places and give an exact answer to part **c**.

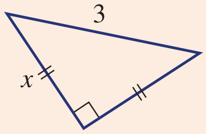
a



b



c



SOLUTION	EXPLANATION
a $a^2 + 15^2 = 17^2$	Write the rule and substitute the known sides.
$a^2 + 225 = 289$	Square 15 and 17.
$a^2 = 64$	Subtract 225 from both sides.
$a = \sqrt{64}$	Take the square root of both sides.
$a = 8$	
b $x^2 + 7.6^2 = 10^2$	Write the rule.
$x^2 + 57.76 = 100$	
$x^2 = 42.24$	Subtract 57.76 from both sides.
$x = \sqrt{42.24}$	Take the square root of both sides.
$x = 6.50$ (to 2 d.p.)	Round to two decimal places.

c $x^2 + x^2 = 3^2$

$$2x^2 = 9$$

$$x^2 = \frac{9}{2}$$

$$x = \sqrt{\frac{9}{2}}\left(= \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}\right)$$

Two sides are of length x.
Add like terms and then divide both sides by 2.

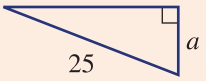
Take the square root of both sides. To express as an exact answer, do not round.

Different forms are possible.

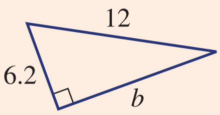
Now you try

In each of the following, find the value of the pronumeral. Round your answer in part **b** to two decimal places and give an exact answer to part **c**.

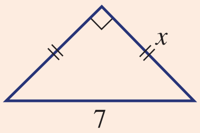
a



b



c



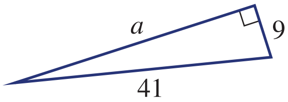
Exercise 3B

FLUENCY	1, 2–4(1/2), 5	2–4(1/2), 5	2–4(1/3), 5

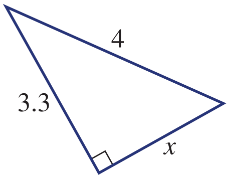
Example 3

1 In each of the following, find the value of the pronumeral. Round your answer in part **b** to two decimal places and give an exact answer to part **c**.

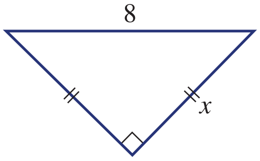
a



b



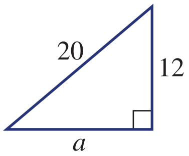
c



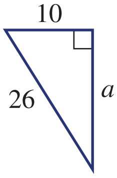
Example 3a

2 In each of the following, find the value of the pronumeral.

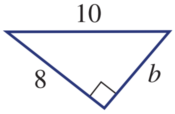
a



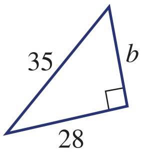
b



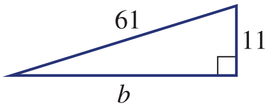
c



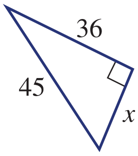
d



e



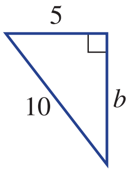
f



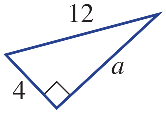
Example 3b

3 In each of the following, find the value of the pronumeral. Express your answers correct to two decimal places.

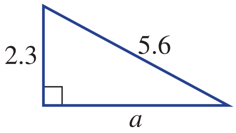
a



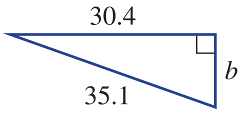
b



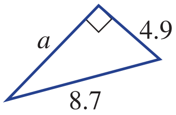
c



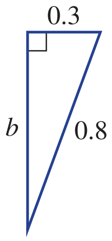
d



e

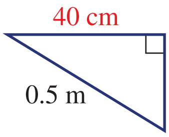


f

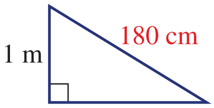


4 Find the length of the unknown side of each of these triangles, correct to two decimal places where necessary. Convert to the units shown in red.

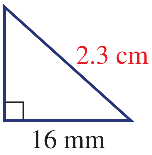
a



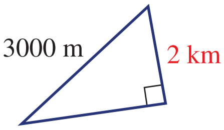
b



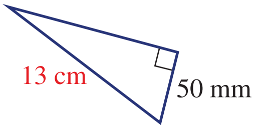
c



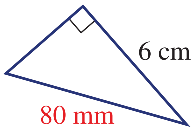
d



e



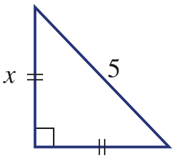
f



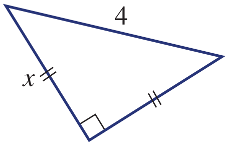
Example 3c

5 In each of the following, find the value of x as an exact answer.

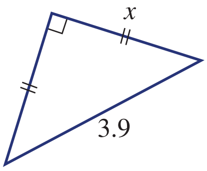
a



b



c

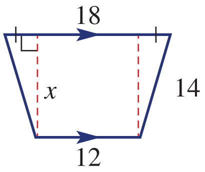


PROBLEM-SOLVING

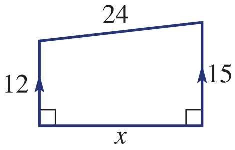
6–8 | 6, 7, 9 | 8–10

6 For each of the following diagrams, find the value of x. Give an exact answer each time.

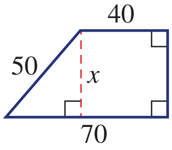
a



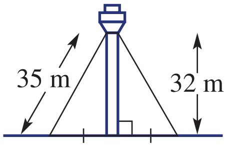
b



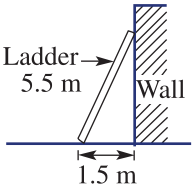
c



7 A 32 m communication tower is supported by 35 m cables stretching from the top of the tower to a position at ground level as shown. Find the distance from the base of the tower to the point where the cable reaches the ground, correct to one decimal place.

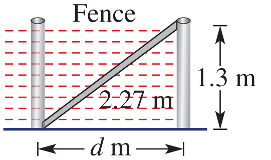


8 The base of a ladder leaning against a vertical wall is 1.5 m from the base of the wall. If the ladder is 5.5 m long, find how high the top of the ladder is above the ground, correct to one decimal place.



9 If a television has a screen size of 63 cm it means that the diagonal length of the screen is 63 cm. If the vertical height of a 63 cm screen is 39 cm, find the width of the screen to the nearest centimetre.

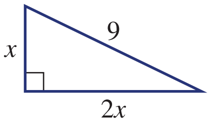
10 Two 1.3 m vertical fence posts are supported by a 2.27 m bar, as shown in the diagram on the right. Find the distance (d m) from the base of one post to the base of the second post. Give your answer correct to two decimal places.



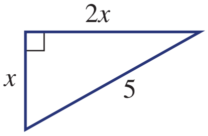
REASONING	11	11	11, 12

11 For these questions note that $(2x)^2 = 4x^2$ and $(3x)^2 = 9x^2$.
Find the value of x as an exact answer.

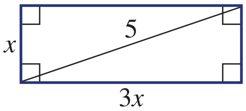
a



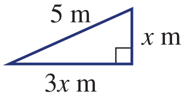
b



c



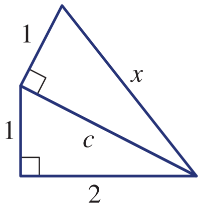
12 A right-angled triangle has a hypotenuse measuring 5 m. Find the lengths of the other sides if their lengths are in the given ratio. Give an exact answer. (*Hint*: You can draw a triangle like the one shown for part **a**.)



a 1 to 3 **b** 2 to 3 **c** 5 to 7

ENRICHMENT: The power of exact values	–	–	13, 14

13 Consider this diagram and the unknown length x.



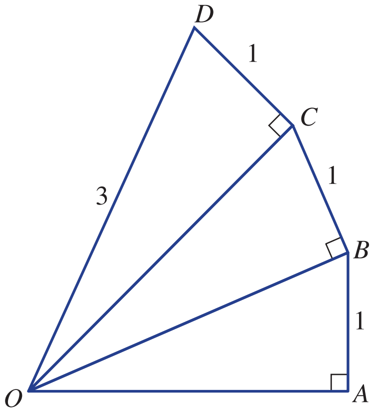
a Explain what needs to be found first before x can be calculated.

b Now try calculating the value x as an exact value.

c Was it necessary to calculate the value of c or was c^2 enough?

d What problems might be encountered if the value of c is calculated and rounded before the value of x is found?

14 In the diagram at right, $OD = 3$ and $AB = BC = CD = 1$.



a Using exact values find the length of:

i OC **ii** OB **iii** OA.

b Round your answer in part **a iii** to one decimal place and use that length to recalculate the lengths of OB, OC and OD (correct to two decimal places) starting with ΔOAB.

c Explain the difference between the given length $OD = 3$ and your answer for OD in part **b**.

d Investigate how changing the side length AB affects your answers to parts **a** to **c** above.

3C Applying Pythagoras' theorem

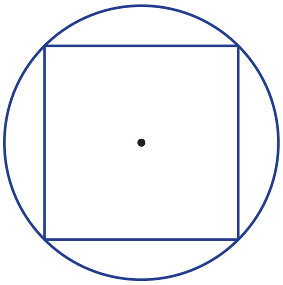
Learning intentions

- To be able to identify a right-angled triangle in a diagram and label known values
- To be able to apply Pythagoras' theorem to the triangle to find an unknown value

Initially it may not be obvious that Pythagoras' theorem can be used to help solve a particular problem. With further investigation, however, it may be possible to identify and draw a right-angled triangle that can help solve the problem. As long as two sides of the right-angled triangle are known, the length of the third side can be found.

Engineers and architects regularly use Pythagoras' theorem, for example, to calculate the length of each cable on the Anzac Bridge, Sydney.

LESSON STARTER The biggest square



Imagine trying to cut the largest square from a circle of a certain size and calculating the side length of the square. Drawing a simple diagram as shown does not initially reveal a right-angled triangle.

- If the circle has a diameter of 2 cm, can you find a good position to draw the diameter that also helps to form a right-angled triangle?
- Can you determine the side length of the largest square?
- What percentage of the area of a circle does the largest square occupy?

KEY IDEAS

- When applying Pythagoras' theorem, follow these steps.
  - Identify and draw a right-angled triangle that may help to solve the problem.
  - Label the sides with their lengths or with a letter (pronumeral) if the length is unknown.
  - Use Pythagoras' theorem to solve for the unknown.
  - Solve the problem by making any further calculations and answering in words.

BUILDING UNDERSTANDING

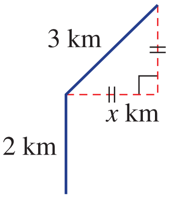
1 Match each problem (**a**, **b** or **c**) with both a diagram (**A**, **B** or **C**) and its solution (**I**, **II**, **III**).

a Two trees stand 20 m apart and they are 32 m and 47 m tall. What is the distance between the tops of the two trees?

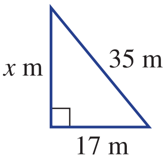
b A man walks due north for 2 km and then north-east for 3 km. How far north is he from his starting point?

c A kite is flying with a kite string of length 35 m. Its horizontal distance from its anchor point is 17 m. How high is the kite flying?

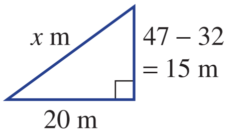
A



B



C



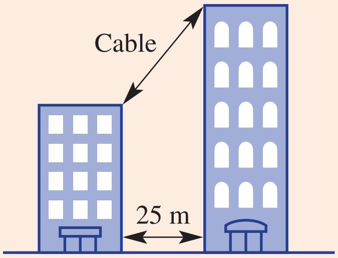
I The kite is flying at a height of 30.59 m.

II The distance between the top of the two trees is 25 m.

III The man has walked a total of 2 + 2.12 = 4.12 km north from his starting point.

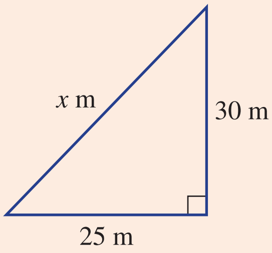
Example 4 Applying Pythagoras' theorem

Two skyscrapers are located 25 m apart and a cable links the tops of the two buildings. Find the length of the cable if the buildings are 50 m and 80 m in height. Give your answer correct to two decimal places.



SOLUTION

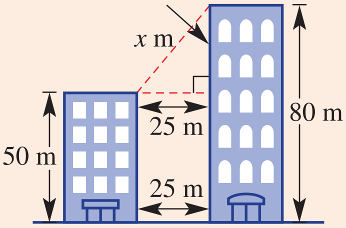
Let x m be the length of the cable.



EXPLANATION

Draw a right-angled triangle and label the measurements and pronumerals.

$80\text{ m} - 50\text{ m} = 30\text{ m}$



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ISBN 978-1-108-77288-4

Updated September 2021

$c^2 = a^2 + b^2$ $x^2 = 25^2 + 30^2$ $x^2 = 625 + 900$ $= 1525$ $x = \sqrt{1525}$ $= 39.05$ (to 2 d.p.)	Set up an equation using Pythagoras' theorem and solve for x.
The cable is 39.05 m long.	Answer the question in words.

Now you try

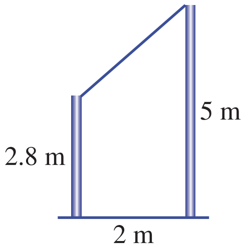
Two poles are located 40 m apart and a rope links the tops of the two poles. Find the length of the rope if the poles are 15 m and 21 m in height. Give your answer correct to two decimal places.

Exercise 3C

FLUENCY	1–4	2–5	3–5

Example 4

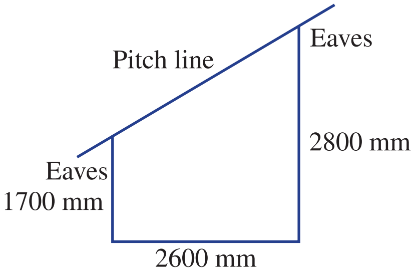
1 Two poles are located 2 m apart. A wire links the tops of the two poles. Find the length of the wire if the poles are 2.8 m and 5 m in height. Give your answer correct to one decimal place.



Example 4

2 Two skyscrapers are located 25 m apart and a cable of length 62.3 m links the tops of the two buildings. If the taller building is 200 metres tall, what is the height of the shorter building? Give your answer correct to one decimal place.

3 A garage is to be built with a skillion roof (a roof with a single slope). The measurements are given in the diagram. Calculate the pitch line length, to the nearest millimetre. Allow 500 mm for each of the eaves.



4 Two bushwalkers are standing on different mountain sides. According to their maps, one of them is at a height of 2120 m and the other is at a height of 1650 m. If the horizontal distance between them is 950 m, find the direct distance between the two bushwalkers. Give your answer correct to the nearest metre.

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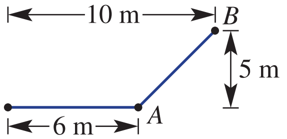
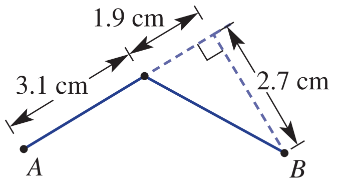
5 Find the direct distance between the points A and B in each of the following, correct to one decimal place.

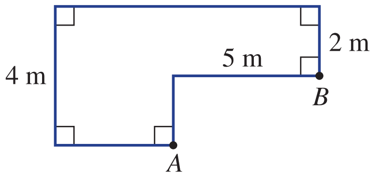
a



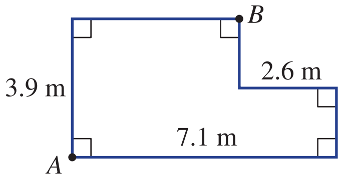
b



c



d

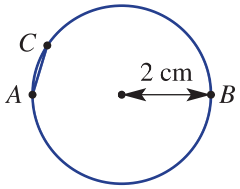


PROBLEM-SOLVING

6, 7	7, 8	7–9

6 A 100 m radio mast is supported by six cables in two sets of three cables. All six cables are anchored to the ground at an equal distance from the mast. The top set of three cables is attached at a point 20 m below the top of the mast. Each of the three lower cables is 60 m long and attached at a height of 30 m above the ground. If all the cables have to be replaced, find the total length of cable required. Give your answer correct to two decimal places.

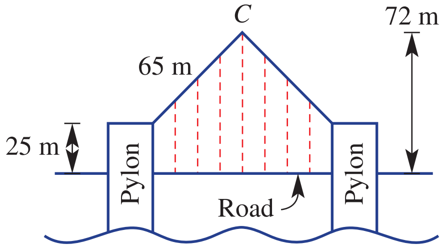
7 In a particular circle of radius 2 cm, AB is a diameter and C is a point on the circumference. Angle ACB is a right angle. The chord AC is 1 cm in length.



a Draw the triangle ABC as described, and mark in all the important information.

b Find the length of BC correct to one decimal place.

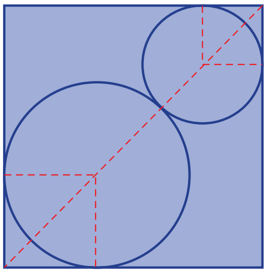
8 A suspension bridge is built with two vertical pylons and two straight beams of equal length that are positioned to extend from the top of the pylons to meet at a point C above the centre of the bridge, as shown in the diagram on the right.



a Calculate the vertical height of the point C above the tops of the pylons.

b Calculate the distance between the pylons; that is, the length of the span of the bridge, correct to one decimal place.

9 Two circles of radii 10 cm and 15 cm respectively are placed inside a square. Find the perimeter of the square to the nearest centimetre. (*Hint*: First find the diagonal length of the square using the diagram on the right.)



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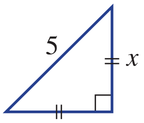
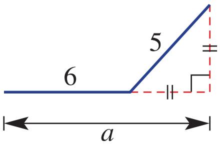
REASONING	10	10	10, 11

10 It is possible to find the length of the shorter sides of a right-angled isosceles triangle if only the hypotenuse length is known.

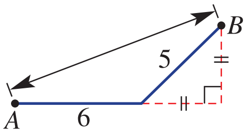
a Find the exact value of x in this right-angled isosceles triangle.



b Now find the exact value of a in this diagram.



c Finally, use your results from above to find the length of AB in this diagram correct to one decimal place.



11 Use the method outlined in Question **10** for this problem. In an army navigation exercise, a group of soldiers hiked due south from base camp for 2.5 km to a water hole. From there, they turned 45° to the left, to head south-east for 1.6 km to a resting point. When the soldiers were at the resting point, how far (correct to one decimal place):

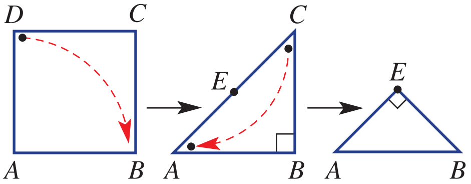
a east were they from the water hole?

b south were they from the water hole?

c were they in a straight line from base camp?

ENRICHMENT: Folding paper	–	–	12

12 A square piece of paper, $ABCD$, of side length 20 cm is folded to form a right-angled triangle ABC. The paper is folded a second time to form a right-angled triangle ABE as shown in the diagram below.



a Find the length of AC correct to two decimal places.

b Find the perimeter of each of the following, correct to one decimal place where necessary:

i square $ABCD$ **ii** triangle ABC **iii** triangle ABE.

c Use Pythagoras' theorem and your answer for part **a** to confirm that $AE = BE$ in triangle ABE.

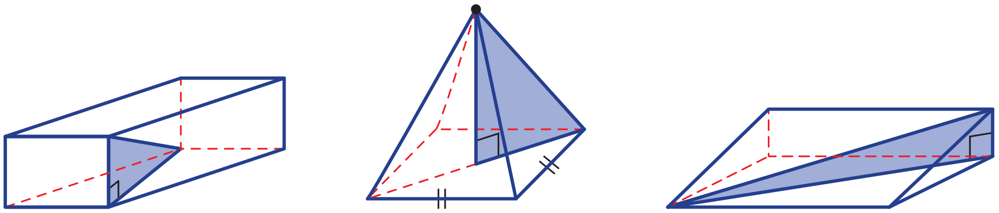
d Investigate how changing the initial side length changes the answers to the above.

3D Pythagoras in three dimensions EXTENDING

Learning intentions
- To understand how right-angled triangles can arise in 3D solids
- To be able to identify a right-angled triangle in a 3D solid
- To be able to apply Pythagoras' theorem in the identified triangle

If you cut a solid to form a cross-section, a two-dimensional shape is revealed. From that cross-section it may be possible to identify a right-angled triangle that can be used to find unknown lengths. These lengths can then tell us information about the three-dimensional solid.

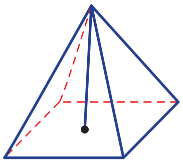
You can visualise right-angled triangles in all sorts of different solids.



Engineers, architects, builders and carpenters use Pythagoras' theorem to calculate the direct distance between two positions in three dimensions. For example, calculating the length of trusses that support the roof of this Sydney sports stadium.

LESSON STARTER How many triangles in a pyramid?

Here is a drawing of a square-based pyramid. By drawing lines from any vertex to the centre of the base and another point, how many different right-angled triangles can you visualise and draw? The triangles could be inside or on the outside surface of the pyramid.



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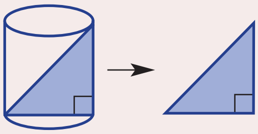
KEY IDEAS

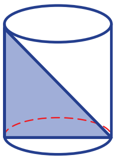
- Right-angled triangles can be identified in many three-dimensional solids.
- It is important to try to draw any identified right-angled triangle using a separate diagram.



BUILDING UNDERSTANDING

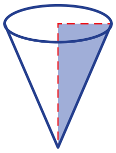
1 Decide if the following shaded regions would form right-angled triangles.

a



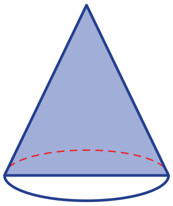
Right cylinder

b



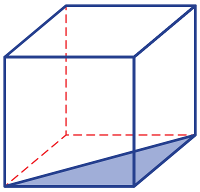
Right cone

c



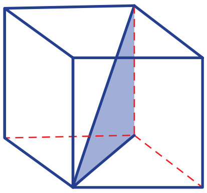
Cone

d



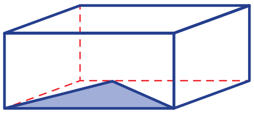
Cube

e



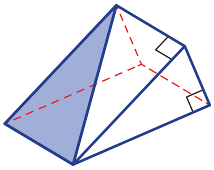
Cube

f



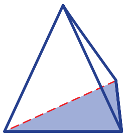
Rectangular prism

g



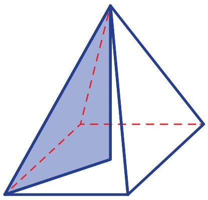
Triangular prism

h



Tetrahedron (regular triangular-based pyramid)

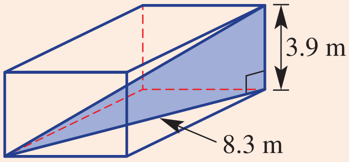
i

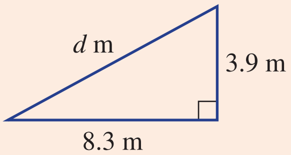


Right square-based pyramid (apex above centre of base)

Example 5 Using Pythagoras in 3D

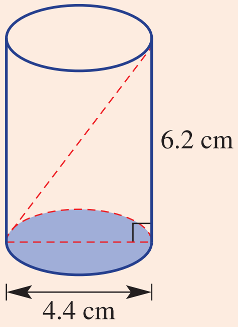
The length of the diagonal on the base of a rectangular prism is 8.3 m and the height of the rectangular prism is 3.9 m. Find the distance from one corner of the rectangular prism to the opposite corner. Give your answer correct to two decimal places.



SOLUTION	EXPLANATION
Let d m be the distance required.  	Draw a right-angled triangle and label all the measurements and pronumerals.
$d^2 = 3.9^2 + 8.3^2$ $= 84.1$ $\therefore\ d = 9.17$ (to 2 d.p.)	Use Pythagoras' theorem. Round $\sqrt{84.1}$ to two decimal places.
The distance from one corner of the rectangular prism to the opposite corner is approximately 9.17 m.	Write your answer in words.

Now you try

A cylinder has diameter 4.4 cm and height 6.2 cm. Find the distance from a point on the circumference of the base to a point on the opposite side at the top. Give your answer correct to two decimal places.

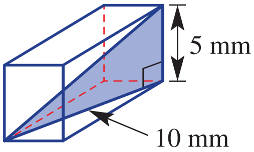


Exercise 3D

FLUENCY	1–5	2–6	2, 3, 5, 6

Example 5

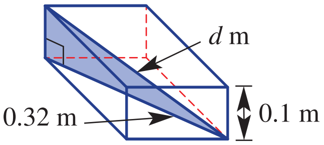
1 The length of the diagonal on the base of a rectangular prism is 10 mm and the height of the rectangular prism is 5 mm. Find the distance from one corner of the rectangular prism to the opposite corner. Give your answer correct to two decimal places.



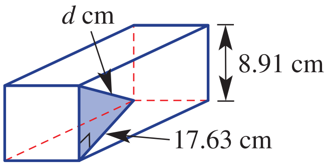
Example 5

2 Find the distance, d units, from one corner to the opposite corner in each of the following rectangular prisms. Give your answers correct to two decimal places.

a



b



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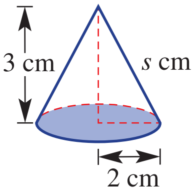
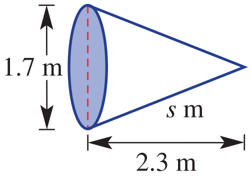
3 Find the slant height, s units, of each of the following cones. Give your answers correct to one decimal place.

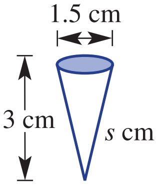
a



b



c

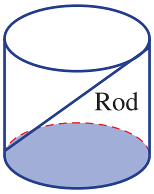


4 Find the length to the nearest millimetre of the longest rod that will fit inside a cylinder of the following dimensions.

a Diameter 10 cm and height 15 cm

b Radius 2.8 mm and height 4.2 mm

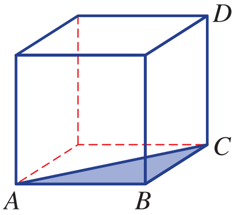
c Diameter 0.034 m and height 0.015 m



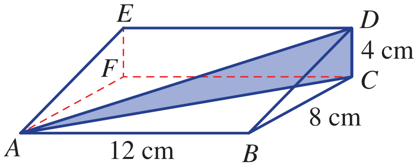
5 The cube in the diagram on the right has 1 cm sides.

a Find the length of AC as an exact value.

b Hence, find the length of AD correct to one decimal place.



6 Consider the shape shown.



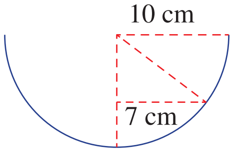
a Find the length of AC as an exact value.

b Hence, find the length of AD correct to one decimal place.

PROBLEM-SOLVING	7	7, 8	8, 9

7 A miner makes claim to a circular piece of land with a radius of 40 m from a given point, and is entitled to dig to a depth of 25 m. If the miner can dig tunnels at any angle, find the length of the longest straight tunnel that he can dig, to the nearest metre.

8 A bowl is in the shape of a hemisphere (half sphere) with radius 10 cm. The surface of the water in the container has a radius of 7 cm. How deep is the water? Give your answer to two decimal places.



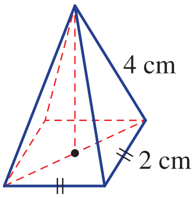
9 A cube of side length l sits inside a sphere of radius r so that the vertices of the cube sit on the sphere. Find the ratio $r : l$.

REASONING	10	10	10, 11

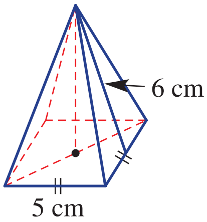
10 There are different ways to approach finding the height of a pyramid depending on what information is given. For each of the following square-based pyramids, find:

- **i** the exact length (using a surd) of the diagonal on the base
- **ii** the height of the pyramid correct to two decimal places.

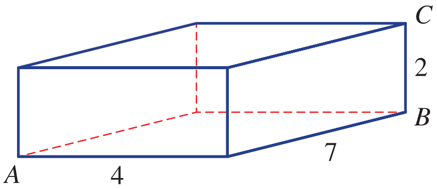
a



b



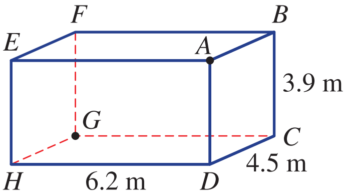
11 For this rectangular prism answer these questions.



- **a** Find the exact length AB.
- **b** Find AB correct to two decimal places.
- **c** Find the length AC using your result from part **a** and then round to two decimal places.
- **d** Find the length AC using your result from part **b** and then round to two decimal places.
- **e** How can you explain the difference between your results from parts **c** and **d** above?

ENRICHMENT: Spider crawl	–	–	12

12 A spider crawls from one corner, A, of the ceiling of a room to the opposite corner, G, on the floor. The room is a rectangular prism with dimensions as given in the diagram below.



- **a** Assuming the spider crawls in a direct line between points, find how far (correct to two decimal places) the spider crawls if it crawls from A to G via:
  - **i** B
  - **ii** C
  - **iii** D
  - **iv** F
- **b** Investigate other paths to determine the shortest distance that the spider could crawl in order to travel from point A to point G. (*Hint*: Consider drawing a net for the solid.)

3E Trigonometric ratios

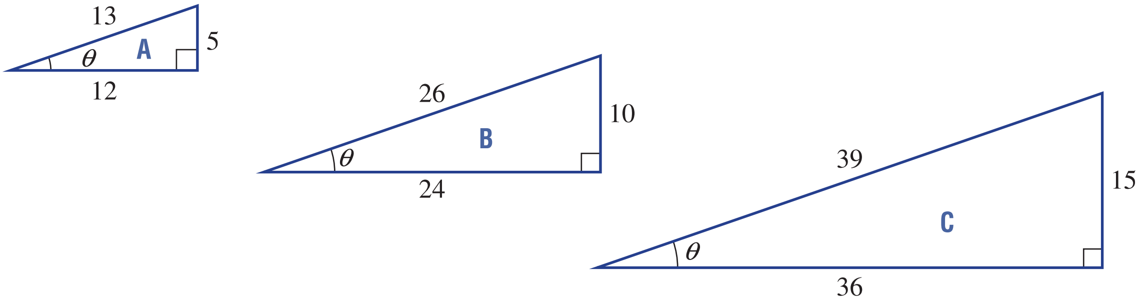
Learning intentions
- To know how to label the sides of a right-angled triangle with reference to an angle
- To know the three trigonometric ratios for right-angled triangles
- To understand that for any right-angled triangle with the same angles, the trigonometric ratios are the same
- To be able to write a trigonometric ratio for a right-angled triangle

The branch of mathematics called trigonometry deals with the relationship between the side lengths and angles in triangles. Trigonometry dates back to the ancient Egyptian and Babylonian civilisations in which a basic form of trigonometry was used in the building of pyramids and in the study of astronomy. The first table of values including chord and arc lengths on a circle for a given angle was created by Hipparchus in the 2nd century BCE in Greece. These tables of values helped to calculate the position of the planets. About three centuries later, Claudius Ptolemy advanced the study of trigonometry writing by13 books called the *Almagest*. Ptolemy also developed tables of values linking the sides and angles of a triangle, and produced many theorems which use the sine, cosine and tangent functions.

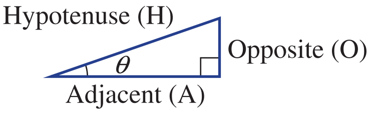
Everyday users of trigonometry include surveyors, engineers, architects, designers and builders of houses, carports, awnings and holiday cabins.

LESSON STARTER Constancy of sine, cosine and tangent

In geometry we would say that similar triangles have the same shape but are of different size. Here are three similar right-angled triangles. The angle θ (theta) is the same for all three triangles.



We will now calculate three special ratios – sine, cosine and tangent – for the angle θ in these triangles. We use the sides labelled Hypotenuse (H), Opposite (O) and Adjacent (A) as shown at right.



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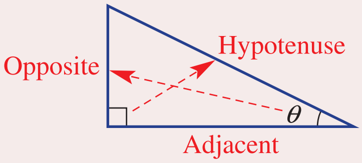
- Complete this table simplifying all fractions.
- What do you notice about the value of:
  a $\sin\theta \left(\text{i.e. } \frac{O}{H}\right)$ for all three triangles?
  b $\cos\theta \left(\text{i.e. } \frac{A}{H}\right)$ for all three triangles?
  c $\tan\theta \left(\text{i.e. } \frac{O}{A}\right)$ for all three triangles?
- Why are the three ratios ($\sin\theta$, $\cos\theta$ and $\tan\theta$) the same for all three triangles? Discuss.

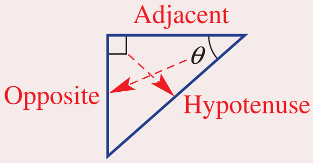
Triangle	$\frac{O}{H}$ **(sin θ)**	$\frac{A}{H}$ **(cos θ)**	$\frac{O}{A}$ **(tan θ)**
A	$\frac{5}{13}$		
B		$\frac{24}{26} = \frac{12}{13}$	
C			$\frac{15}{36} = \frac{5}{12}$

KEY IDEAS

■ For a right-angled triangle with another angle named θ:

- the **hypotenuse** is the longest side, opposite the 90° angle
- the **opposite** side is opposite θ
- the **adjacent** side is next to θ but is not the hypotenuse.





■ For a right-angled triangle with a given angle θ, the three ratios **sine (sin)**, **cosine (cos)** and **tangent (tan)** are given by:

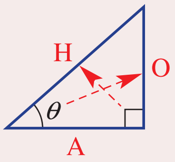
- sine of angle θ (or $\sin\theta$) $= \dfrac{\text{length of the opposite side}}{\text{length of the hypotenuse}}$
- cosine of angle θ (or $\cos\theta$) $= \dfrac{\text{length of the adjacent side}}{\text{length of the hypotenuse}}$
- tangent of angle θ (or $\tan\theta$) $= \dfrac{\text{length of the opposite side}}{\text{length of the adjacent side}}$

■ For any right-angled triangle with the same angles, these ratios are always the same.

■ The term **SOHCAHTOA** is useful when trying to remember the three ratios.

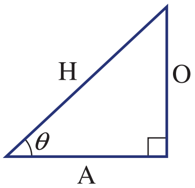
SOH CAH TOA

$\sin\theta = \dfrac{\text{Opposite}}{\text{Hypotenuse}}$ $\quad\cos\theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}}$ $\quad\tan\theta = \dfrac{\text{Opposite}}{\text{Adjacent}}$



BUILDING UNDERSTANDING

1 State the missing word in these sentences.



a H stands for the word ____________.

b O stands for the word ____________.

c A stands for the word ____________.

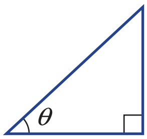
d $\sin\theta =$ ____________ $\div$ hypotenuse.

e $\cos\theta =$ adjacent $\div$ ____________.

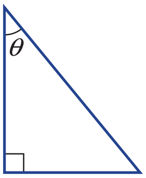
f $\tan\theta =$ opposite $\div$ ____________.

2 Decide which side of each triangle should be labelled as opposite to θ (O), adjacent to θ (A) or hypotenuse (H).

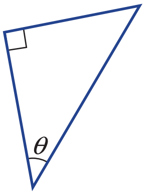
a



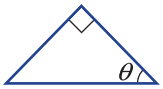
b



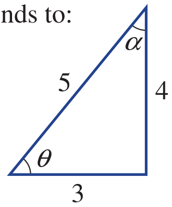
c



d



3 For the triangle shown, state the length of the side which corresponds to:



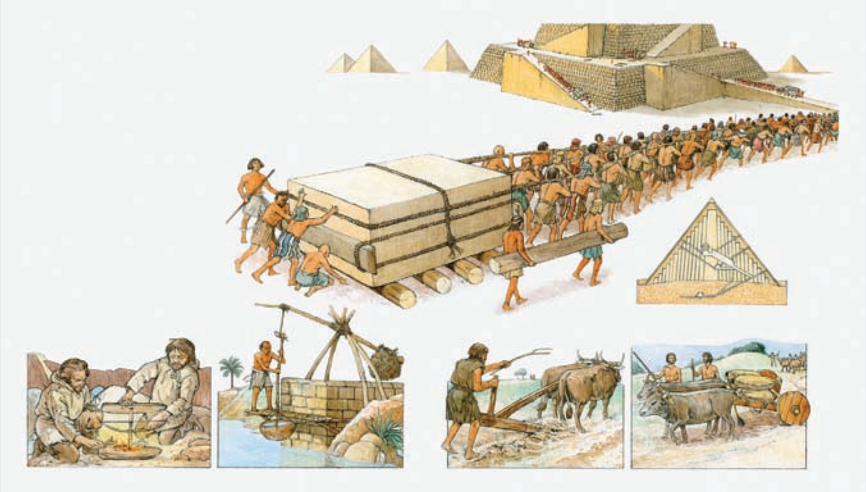
a the hypotenuse

b the side opposite angle θ

c the side opposite angle α

d the side adjacent to angle θ

e the side adjacent to angle α.

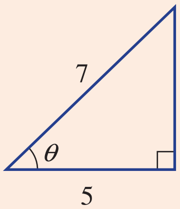


A basic form of trigonometry was used in the building of pyramids in ancient Egypt.

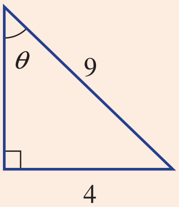
Example 6 Writing trigonometric ratios

Using the given sides, write a trigonometric ratio (in fraction form) for each of the following triangles.

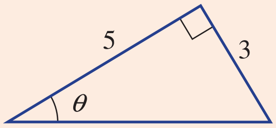
a



b



c

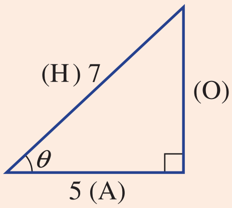


SOLUTION

EXPLANATION

a

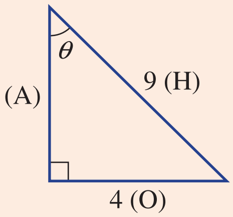
$$\begin{aligned}\cos\theta &= \frac{\text{A}}{\text{H}}\\ &= \frac{5}{7}\end{aligned}$$



Side length 7 is opposite the right angle so it is the hypotenuse (H). Side length 5 is adjacent to angle θ so it is the adjacent (A).

b

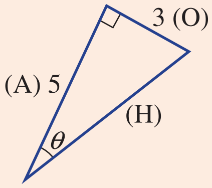
$$\begin{aligned}\sin\theta &= \frac{\text{O}}{\text{H}}\\ &= \frac{4}{9}\end{aligned}$$



Side length 9 is opposite the right angle so it is the hypotenuse (H). Side length 4 is opposite angle θ so it is the opposite (O).

c

$$\begin{aligned}\tan\theta &= \frac{\text{O}}{\text{A}}\\ &= \frac{3}{5}\end{aligned}$$

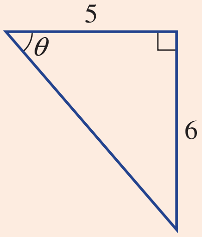


Side length 5 is the adjacent side to angle θ so it is the adjacent (A). Side length 3 is opposite angle θ so it is the opposite (O).

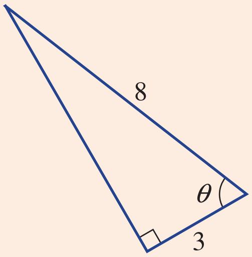
Now you try

Using the given sides, write a trigonometric ratio (in fraction form) for each of the following triangles.

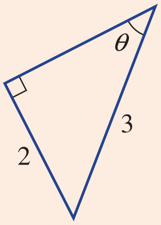
a



b



c

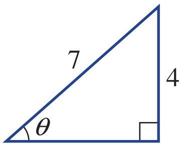


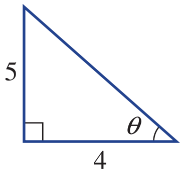
Exercise 3E

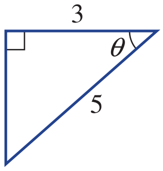
FLUENCY	1, 2($\frac{1}{2}$), 3, 4	2–5	3–5

Example 6

1 Using the given sides, write a trigonometric ratio (in fraction form) for each of the following triangles.

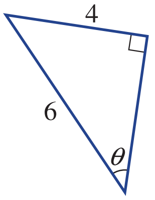
a


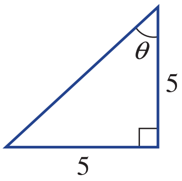
b


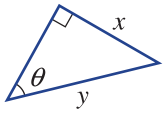
c


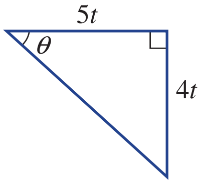
Example 6

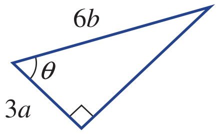
2 Using the given sides, write a trigonometric ratio (in fraction form) for each of the following triangles and simplify where possible.

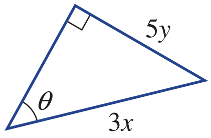
a


b


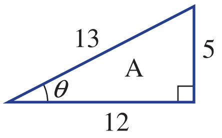
c


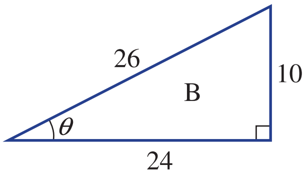
d


e


f


3 Here are two similar triangles A and B.



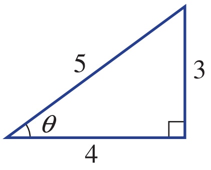


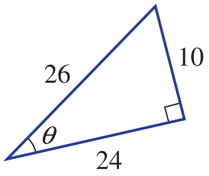
a **i** Write the ratio $\sin\theta$ (as a fraction) for triangle A.
  ii Write the ratio $\sin\theta$ (as a fraction) for triangle B.
  iii What do you notice about your two answers from parts **a i** and **ii** above?

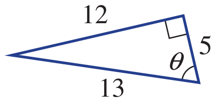
b **i** Write the ratio $\cos\theta$ (as a fraction) for triangle A.
  ii Write the ratio $\cos\theta$ (as a fraction) for triangle B.
  iii What do you notice about your two answers from parts **b i** and **ii** above?

c **i** Write the ratio $\tan\theta$ (as a fraction) for triangle A.
  ii Write the ratio $\tan\theta$ (as a fraction) for triangle B.
  iii What do you notice about your two answers from parts **c i** and **ii** above?

4 For each of these triangles, write a ratio (in simplified fraction form) for $\sin\theta$, $\cos\theta$ and $\tan\theta$.

a


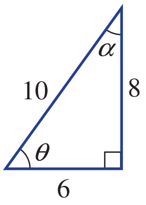
b


c


5 For the triangle shown on the right, write a ratio (in fraction form) for:

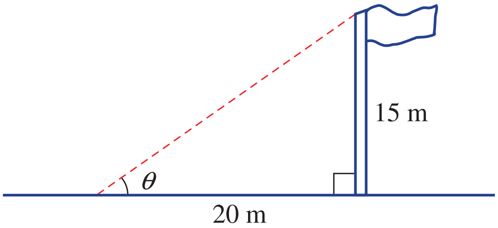
a $\sin\theta$ **b** $\sin\alpha$ **c** $\cos\theta$

d $\tan\alpha$ **e** $\cos\alpha$ **f** $\tan\theta$



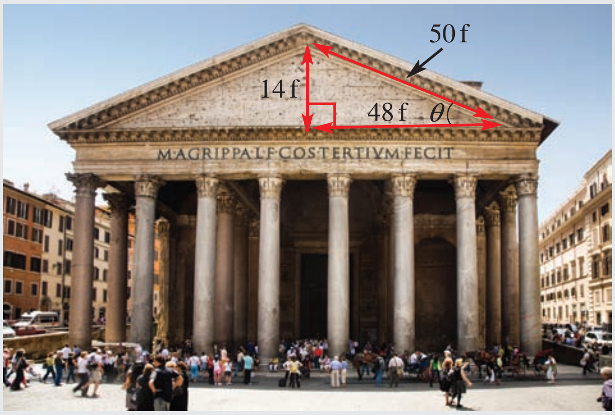
PROBLEM-SOLVING	6, 7	7, 8	8, 9

6 A vertical flag pole casts a shadow 20 m long. If the pole is 15 m high, find the ratio for $\tan\theta$.



7 The facade of a Roman temple has the given measurements in feet (f) below. Write down the ratio for:

a $\sin\theta$ **b** $\cos\theta$ **c** $\tan\theta$

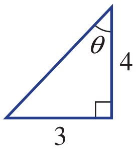


The Pantheon, a Roman temple that was built in 126 CE

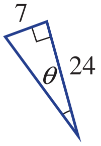
8 For each of the following:

- **i** use Pythagoras' theorem to find the unknown side
- **ii** find the ratios for $\sin\theta$, $\cos\theta$ and $\tan\theta$.

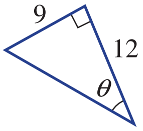
a



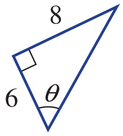
b



c



d



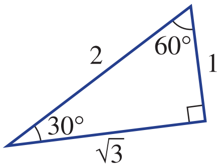
9 **a** Draw a right-angled triangle and mark one of the angles as θ. Mark in the length of the opposite side as 15 units and the length of the hypotenuse as 17 units.

b Using Pythagoras' theorem, find the length of the adjacent side.

c Determine the ratios for $\sin\theta$, $\cos\theta$ and $\tan\theta$.

REASONING	10	10, 11	10–12

10 This triangle has angles 90°, 60° and 30° and side lengths 1, 2 and $\sqrt{3}$.



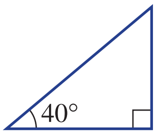
a Write a ratio for:

i sin 30° **ii** cos 30° **iii** tan 30°
iv sin 60° **v** cos 60° **vi** tan 60°

b What do you notice about the following pairs of ratios?

i cos 30° and sin 60° **ii** sin 30° and cos 60°

11 **a** Measure all the side lengths of this triangle to the nearest millimetre.



b Use your measurements from part **a** to find an approximate ratio for:

i cos 40° **ii** sin 40° **iii** tan 40°
iv sin 50° **v** tan 50° **vi** cos 50°

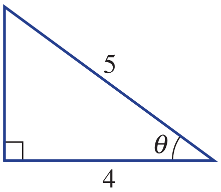
c Do you notice anything about the trigonometric ratios for 40° and 50°?

12 Decide if it is possible to draw a right-angled triangle with the given properties. Explain.

a $\tan \theta = 1$ **b** $\sin \theta = 1$
c $\cos \theta = 0$ **d** $\sin \theta > 1$ or $\cos \theta > 1$

ENRICHMENT: Pythagorean extensions	–	–	13

13 **a** Given that θ is acute and $\cos \theta = \frac{4}{5}$, find $\sin \theta$ and $\tan \theta$. (*Hint*: Use Pythagoras' theorem.)



b For each of the following, draw a right-angled triangle and then use it to find the other two trigonometric ratios.

i $\sin \theta = \frac{1}{2}$ **ii** $\cos \theta = \frac{1}{2}$ **iii** $\tan \theta = 1$

c Use your results from part **a** to calculate $(\cos \theta)^2 + (\sin \theta)^2$. What do you notice?

d Evaluate $(\cos \theta)^2 + (\sin \theta)^2$ for other combinations of $\cos \theta$ and $\sin \theta$. Research and describe what you have found.

3F Finding side lengths

Learning intentions

- To know how to evaluate a trigonometric value on a calculator
- To be able to set up and solve a trigonometric equation to find an unknown side length in the numerator

For similar triangles we know that the ratio of corresponding sides is always the same. This implies that the three trigonometric ratios for similar right-angled triangles are also constant if the internal angles are equal. Since ancient times, mathematicians have attempted to tabulate these ratios for varying angles. Here are the ratios for some angles in a right-angled triangle, correct to up to three decimal places.

Angle (θ)	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0	1	0
15°	0.259	0.966	0.268
30°	0.5	0.866	0.577
45°	0.707	0.707	1
60°	0.866	0.5	1.732
75°	0.966	0.259	3.732
90°	1	0	undefined

In modern times these values can be evaluated using calculators to a high degree of accuracy and can be used to help solve problems involving triangles with unknown side lengths.

A Segway's gyroscopes and tilt-sensors detect the angle and direction that a rider leans, and then the wheels roll to maintain balance. Its microprocessors apply complex motion-control algorithms, including trigonometry calculations using the angle of tilt.

LESSON STARTER Calculator start-up

All scientific or CAS calculators can produce accurate values of $\sin\theta$, $\cos\theta$ and $\tan\theta$.

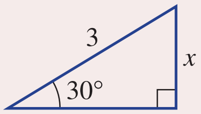
- Ensure that your calculator is in degree mode.
- Check the values in the above table to ensure that you are using the calculator correctly.
- Use trial and error to find (to the nearest degree) an angle θ which satisfies these conditions:
  - **a** $\sin\theta = 0.454$
  - **b** $\cos\theta = 0.588$
  - **c** $\tan\theta = 9.514$

ISBN 978-1-108-77288-4

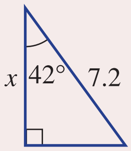
Updated September 2021

KEY IDEAS

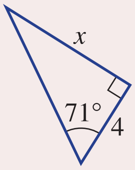
- If θ is in degrees, the ratios for $\sin\theta$, $\cos\theta$ and $\tan\theta$ can be found accurately using a calculator in **degree mode**.
- If the angles and one side length of a right-angled triangle are known, then the other side lengths can be found using the $\sin\theta$, $\cos\theta$ or $\tan\theta$ ratios.



$$\sin 30^\circ = \frac{x}{3}$$
$$\therefore\ x = 3 \times \sin 30^\circ$$



$$\cos 42^\circ = \frac{x}{7.2}$$
$$\therefore\ x = 7.2 \times \cos 42^\circ$$

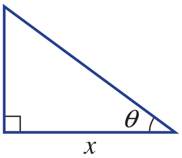


$$\tan 71^\circ = \frac{x}{4}$$
$$\therefore\ x = 4 \times \tan 71^\circ$$

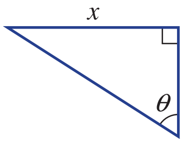
BUILDING UNDERSTANDING

① For the marked angle θ, decide if x represents the length of the opposite (O), adjacent (A) or hypotenuse (H) side.

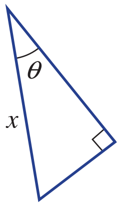
a



b

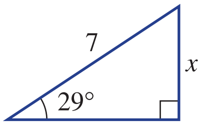


c

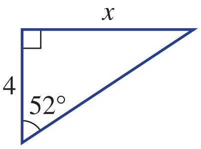


② Decide if you would use $\sin\theta = \frac{\text{O}}{\text{H}}$, $\cos\theta = \frac{\text{A}}{\text{H}}$ or $\tan\theta = \frac{\text{O}}{\text{A}}$ to help find the value of x in these triangles. Do not find the value of x, just state which ratio would be used.

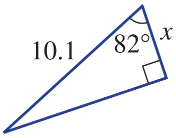
a



b



c



Example 7 Using a calculator

Use a calculator to evaluate the following, correct to two decimal places.

a sin 50° **b** cos 16° **c** tan 77°

SOLUTION	EXPLANATION
a sin 50° = 0.77 (to 2 d.p.)	sin 50° = 0.766044… the 3rd decimal place is greater than 4 so round up.

b $\cos 16^\circ = 0.96$ (to 2 d.p.)	$\cos 16^\circ = 0.961261\ldots$ the 3rd decimal place is less than 5 so round down.
c $\tan 77^\circ = 4.33$ (to 2 d.p.)	$\tan 77^\circ = 4.331475\ldots$ the 3rd decimal place is less than 5 so round down.

Now you try

Use a calculator to evaluate the following, correct to two decimal places.

a $\sin 70^\circ$ **b** $\cos 29^\circ$ **c** $\tan 54^\circ$

Example 8 Solving for x in the numerator of a trigonometric ratio

Find the value of x in the equation $\cos 20^\circ = \frac{x}{3}$, correct to two decimal places.

SOLUTION	EXPLANATION
$\cos 20^\circ = \frac{x}{3}$	
$x = 3 \times \cos 20^\circ$ $= 2.82$ (to 2 d.p.)	Multiply both sides of the equation by 3 and round as required.

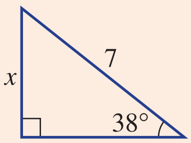
Now you try

Find the value of x in the equation $\sin 40^\circ = \frac{x}{5}$, correct to two decimal places.

Example 9 Finding side lengths

For each triangle, find the value of x correct to two decimal places.

a



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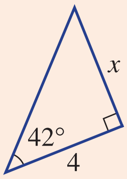

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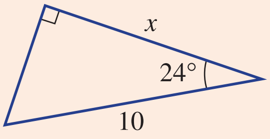

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SOLUTION

a $\sin 38° = \frac{O}{H}$

$\sin 38° = \frac{x}{7}$

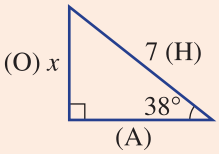
$x = 7 \sin 38°$

$= 4.31$ (to 2 d.p.)

EXPLANATION

As the opposite side (O) and the hypotenuse (H) are involved, the sin θ ratio must be used.

Multiply both sides by 7 and evaluate using a calculator.



b $\tan 42° = \frac{O}{A}$

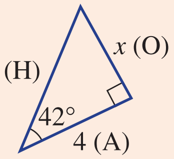
$\tan 42° = \frac{x}{4}$

$x = 4 \tan 42°$

$= 3.60$ (to 2 d.p.)

As the opposite side (O) and the adjacent side (A) are involved, the tan θ ratio must be used.

Multiply both sides by 4 and evaluate.



c $\cos 24° = \frac{A}{H}$

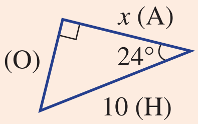
$\cos 24° = \frac{x}{10}$

$x = 10 \cos 24°$

$= 9.14$ (to 2 d.p.)

As the adjacent side (A) and the hypotenuse (H) are involved, the cos θ ratio must be used.

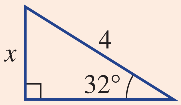
Multiply both sides by 10.



Now you try

For each triangle, find the value of x correct to two decimal places.

a



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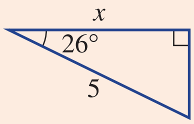

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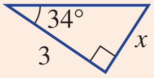

Exercise 3F

FLUENCY	1, 2, 3–4(1/2)	2, 3–4(1/3)	2, 3–4(1/4)

Example 7

1 Use a calculator to evaluate the following, correct to two decimal places.

a $\sin 20°$ **b** $\cos 37°$ **c** $\tan 64°$

Example 7

2 Use a calculator to evaluate the following, correct to two decimal places.

a $\sin 47°$ **b** $\cos 84°$ **c** $\tan 14.1°$ **d** $\sin 27.4°$

Example 8

3 For each of the following, find the value of x correct to two decimal places.

a $\sin 50° = \frac{x}{4}$ **b** $\tan 81° = \frac{x}{3}$ **c** $\cos 33° = \frac{x}{6}$

d $\cos 75° = \frac{x}{3.5}$ **e** $\sin 24° = \frac{x}{4.2}$ **f** $\tan 42° = \frac{x}{10}$

g $\frac{x}{7.1} = \tan 18.4°$ **h** $\frac{x}{5.3} = \sin 64.7°$ **i** $\frac{x}{12.6} = \cos 52.9°$

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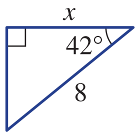
Example 9

4 For the triangles given below, find the value of x correct to two decimal places.

a


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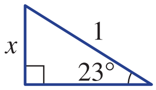

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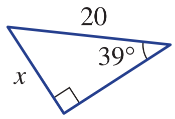
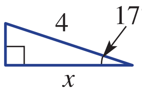


d


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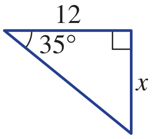

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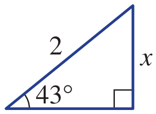

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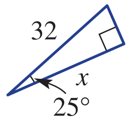

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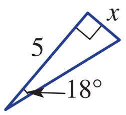

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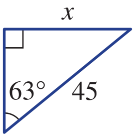

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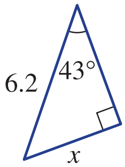

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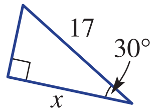

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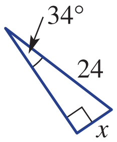
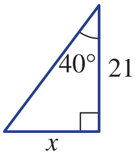


m


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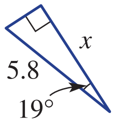

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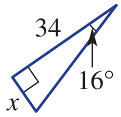

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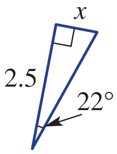
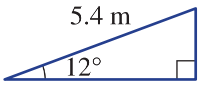
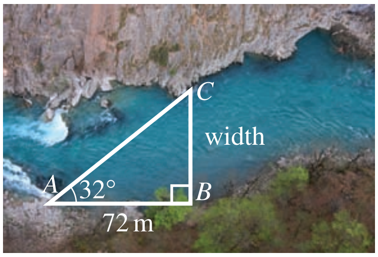


PROBLEM-SOLVING	5, 6	5–7	6–8

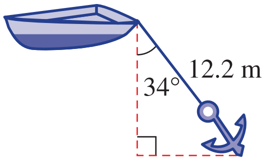
5 Amy walks 5.4 m up a ramp that is inclined at 12° to the horizontal. How high (correct to two decimal places) is she above her starting point?



6 Kane wanted to measure the width of a river. He placed two markers, A and B, 72 m apart along the bank. C is a point directly opposite marker B. Kane measured angle CAB to be 32°. Find the width of the river correct to two decimal places.



7 One end of a 12.2 m rope is tied to a boat. The other end is tied to an anchor, which is holding the boat steady in the water. If the anchor is making an angle of 34° with the vertical, how deep is the water? Give your answer correct to two decimal places.



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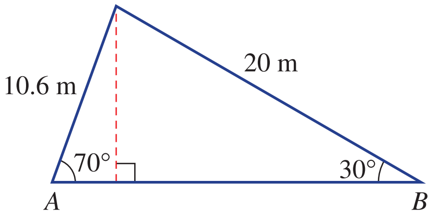
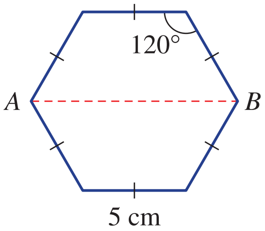
8 Find the length AB in these diagrams. Round to two decimal places where necessary.

a

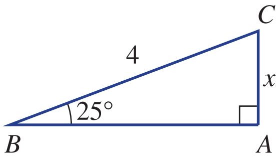


b



REASONING	9	9	9, 10

9 Consider the right-angled triangle shown.



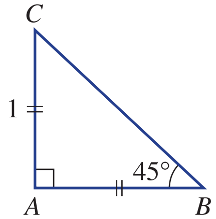
- **a** Find the size of $\angle C$.
- **b** Calculate the value of x correct to three decimal places using the sine ratio.
- **c** Calculate the value of x correct to three decimal places using the cosine ratio.
- **d** Comment on your answers to parts **b** and **c**.

10 Complementary angles sum to 90°.

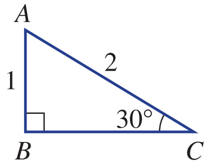
- **a** Find the complementary angles to these angles.
  - **i** 10°
  - **ii** 28°
  - **iii** 54°
  - **iv** 81°
- **b** Evaluate:
  - **i** sin 10° and cos 80°
  - **ii** sin 28° and cos 62°
  - **iii** cos 54° and sin 36°
  - **iv** cos 81° and sin 9°
- **c** What do you notice in part **b**?
- **d** Complete the following.
  - **i** sin 20° = cos ________
  - **ii** sin 59° = cos ________
  - **iii** cos 36° = sin ________
  - **iv** cos 73° = sin ________

ENRICHMENT: Exact values	–	–	11

11 $\sqrt{2}$, $\sqrt{3}$ and $\frac{1}{\sqrt{2}}$ are examples of exact values.



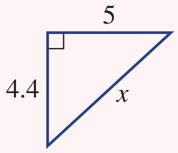
- **a** For the triangle shown (right), use Pythagoras' theorem to find the exact length BC.
- **b** Use your result from part **a** to write down the exact value of:
  - **i** sin 45°
  - **ii** cos 45°
  - **iii** tan 45°
- **c** For the second triangle (below right), use Pythagoras' theorem to find the exact length BC.
- **d** Use your result from part **c** to write down the exact value of:
  - **i** sin 30°
  - **ii** cos 30°
  - **iii** tan 30°
  - **iv** sin 60°
  - **v** cos 60°
  - **vi** tan 60°



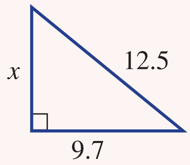
3A/B

1 Find the length of the missing side in these right-angled triangles. Round to two decimal places.

a



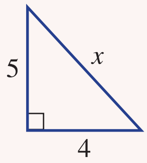
b



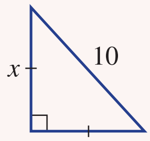
3A/B

2 Find the exact value of x in these right-angled triangles.

a



b



3C

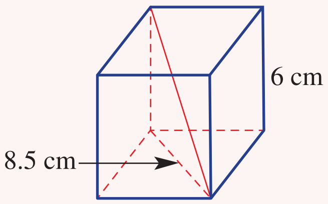
3 A ladder 230 cm long is placed 50 cm from the edge of a building, how far up the side of the building will this ladder reach? Round to one decimal place.

3D

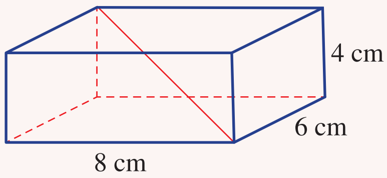
Ext

4 Find the length of the diagonals of these prisms, correct to one decimal place.

a



b



3E

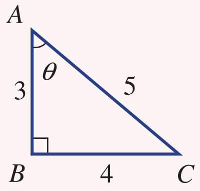
5 Consider the triangle ABC.

a Name the hypotenuse.

b Name the side adjacent to angle ACB.

c Write the ratio for $\cos\theta$.

d Write the ratio for $\tan\theta$.



3F

6 Solve for x, correct to two decimal places.

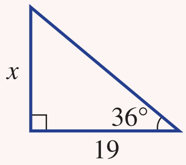
a $x = 12.7\cos 54^\circ$

b $\tan 30^\circ = \frac{x}{12}$

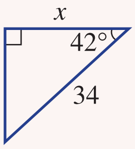
3F

7 Find the values of the pronumerals, correct to two decimal places.

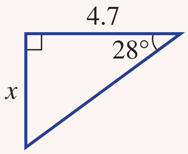
a



b



c

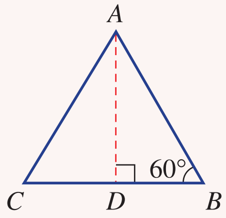


3A–F

8 Triangle ABC is equilateral with a perimeter of 12 cm.

a Find the height AD using any suitable method, correct to three decimal places.

b Calculate the area of the triangle ABC, correct to one decimal place.



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3G Solving for the denominator

Learning intentions

- To know how to rearrange an equation to make a pronumeral in the denominator of a fraction the subject
- To be able to solve a trigonometric equation with an unknown side length in the denominator

So far we have constructed trigonometric ratios using a pronumeral that has always appeared in the numerator.

For example: $\frac{x}{5} = \sin 40°$

This makes it easy to solve for x as both sides of the equation can be multiplied by 5.

If, however, the pronumeral appears in the denominator, there are a number of algebraic steps that can be taken to find the solution.

Trigonometry is an essential tool for builders and carpenters. Given the roof pitch (i.e. angle) and its horizontal span, the lengths of the rafters and trusses are calculated using trigonometry.

LESSON STARTER Solution steps

Three students attempt to solve $\sin 40° = \frac{5}{x}$ for x.

Nick says $x = 5 \times \sin 40°$.

Sharee says $x = \frac{5}{\sin 40°}$.

Dori says $x = \frac{1}{5} \times \sin 40°$.

- Which student has the correct solution?
- Can you show the algebraic steps that support the correct answer?

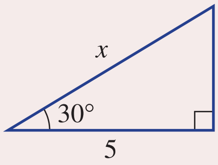
KEY IDEAS

- If the unknown value of a trigonometric ratio is in the **denominator**, you need to rearrange the equation to make the pronumeral the subject.

  For example: For the triangle shown, $\cos 30° = \frac{5}{x}$

  Multiply both sides by x. $x \times \cos 30° = 5$

  Divide both sides by cos 30°. $x = \frac{5}{\cos 30°}$



BUILDING UNDERSTANDING

1 Solve these simple equations for x.

a $\frac{4}{x} = 2$

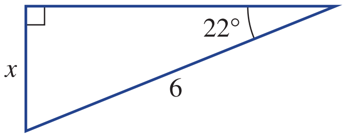
b $\frac{20}{x} = 4$

c $5 = \frac{35}{x}$

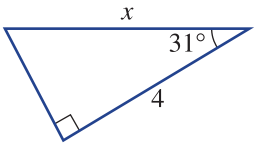
d $\frac{10}{x} = 2.5$

2 Decide whether the pronumeral x will sit in the numerator (N) or denominator (D) for the trigonometric ratios for these diagrams.

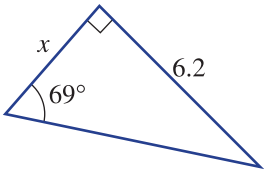
a



b



c



Example 10 Solving for a pronumeral in the denominator

Solve for x in the equation $\cos 35° = \frac{2}{x}$, correct to two decimal places.

SOLUTION	EXPLANATION
$\cos 35° = \frac{2}{x}$	
$x \cos 35° = 2$	Multiply both sides of the equation by x.
$x = \frac{2}{\cos 35°}$	Divide both sides of the equation by cos 35°.
$= 2.44$ (to 2 d.p.)	Evaluate and round to two decimal places.

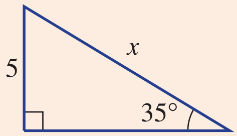
Now you try

Solve for x in the equation $\sin 62° = \frac{3}{x}$, correct to two decimal places.

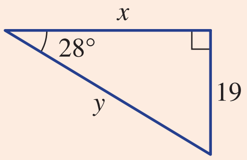
Example 11 Finding side lengths

Find the values of the pronumerals, correct to two decimal places.

a



b



SOLUTION

a

$$\begin{aligned}\sin 35^\circ &= \frac{O}{H}\\ \sin 35^\circ &= \frac{5}{x}\\ x\sin 35^\circ &= 5\\ x &= \frac{5}{\sin 35^\circ}\\ &= 8.72 \text{ (to 2 d.p.)}\end{aligned}$$

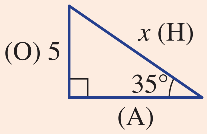
b

$$\begin{aligned}\tan 28^\circ &= \frac{O}{A}\\ \tan 28^\circ &= \frac{19}{x}\\ x\tan 28^\circ &= 19\\ x &= \frac{19}{\tan 28^\circ}\\ &= 35.73 \text{ (to 2 d.p.)}\end{aligned}$$

$$\begin{aligned}\sin 28^\circ &= \frac{19}{y}\\ y\sin 28^\circ &= 19\\ y &= \frac{19}{\sin 28^\circ}\\ &= 40.47 \text{ (to 2 d.p.)}\end{aligned}$$

EXPLANATION

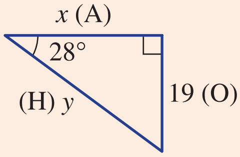
As the opposite side (O) is given and we require the hypotenuse (H), use $\sin\theta$.



Multiply both sides of the equation by x and then divide both sides of the equation by $\sin 35^\circ$.

Evaluate on a calculator and round to two decimal places.

As the opposite side (O) is given and the adjacent (A) is required, use $\tan\theta$.

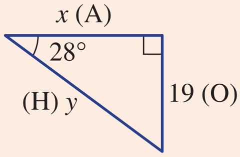


Multiply both sides of the equation by x.

Divide both sides of the equation by $\tan 28^\circ$ and round the answer to two decimal places.

y can be found by using $\sin\theta$.
Alternatively, Pythagoras' theorem could be used: $x^2 + 19^2 = y^2$,

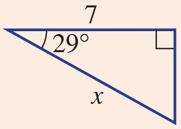
where $x = \frac{19}{\tan 28^\circ}$



Now you try

Find the values of the pronumerals, correct to two decimal places.

a



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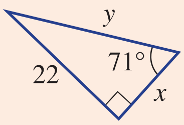


Exercise 3G

FLUENCY	1, 2–4(1/2)	2–4(1/2)	2–4(1/2)

Example 10

1 For each of the following equations, find the value of x correct to two decimal places.

a $\cos 43^\circ = \frac{3}{x}$

b $\sin 36^\circ = \frac{4}{x}$

c $\tan 9^\circ = \frac{6}{x}$

Example 10

2 For each of the following equations, find the value of x correct to two decimal places.

a $\tan 64^\circ = \frac{2}{x}$

b $\cos 67^\circ = \frac{5}{x}$

c $\sin 12^\circ = \frac{3}{x}$

d $\sin 38.3^\circ = \frac{5.9}{x}$

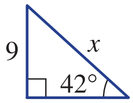
e $\frac{45}{x} = \tan 21.4^\circ$

f $\frac{18.7}{x} = \cos 32^\circ$

Example 11a

3 Find the value of x correct to two decimal places using the sine, cosine or tangent ratios.

a



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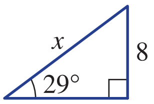
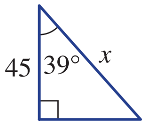


c



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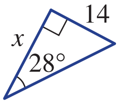

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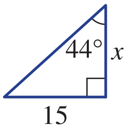

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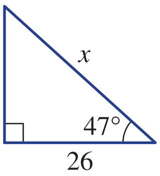

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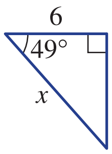

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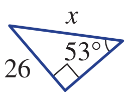
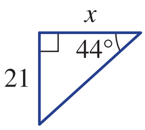


i



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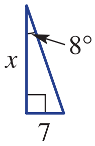

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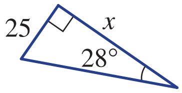

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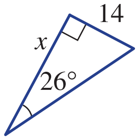
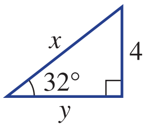


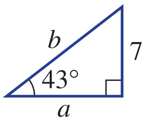
Example 11b

4 Find the value of each pronumeral correct to one decimal place.

a



b



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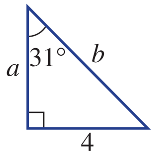

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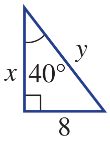
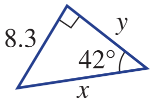
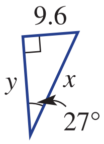


e



f



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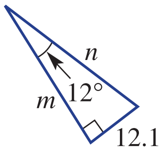

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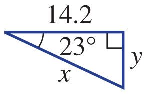
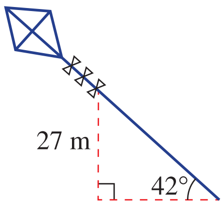


PROBLEM-SOLVING	5, 6	5–7	6, 7

5 A kite is flying at a height of 27 m above the anchor point. If the string is inclined at an angle of 42° to the horizontal, find the length of the string, correct to the nearest metre.



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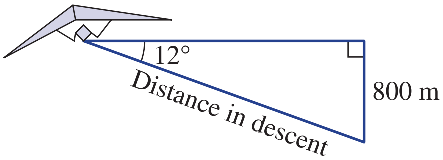
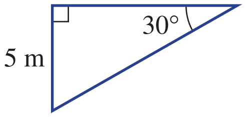
6 A paraglider flying at a height of 800 m descends at an angle of 12° to the horizontal. How far (to the nearest metre) does it travel as it descends to the ground?

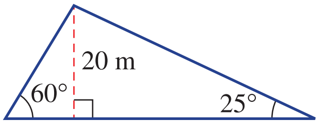


7 Find the perimeter of these triangles, correct to one decimal place.

a

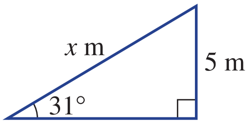


b



REASONING 8 8 8

8 In calculating the value of x for this triangle, two students come up with these answers, correct to two decimal places.



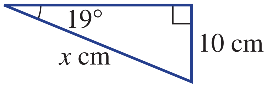
A $x = \frac{5}{\sin 31°} = \frac{5}{0.52} = 9.62$ **B** $x = \frac{5}{\sin 31°} = 9.71$

a Which of the two answers is more accurate and why?

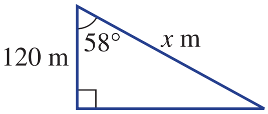
b What advice would you give to the student whose answer is not accurate?

c Find the difference in the answers for these triangles when methods A and B are used to calculate the value of x correct to two decimal places.

i



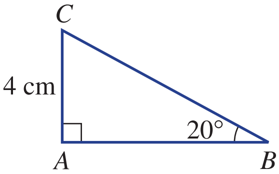
ii



ENRICHMENT: Linking tan θ to sin θ and cos θ – – 9

9 **a** For this triangle, find the length of these sides correct to three decimal places.

i AB **ii** BC

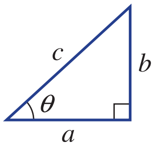


b Calculate these ratios to two decimal places.

i sin 20° **ii** cos 20° **iii** tan 20°

c Evaluate $\frac{\sin 20°}{\cos 20°}$ using your results from part **b**. What do you notice?

d For this triangle with side lengths a, b and c, find an expression for:



i $\sin\theta$ **ii** $\cos\theta$ **iii** $\tan\theta$ **iv** $\frac{\sin\theta}{\cos\theta}$

e Simplify your expression for part **d iv**. What do you notice?

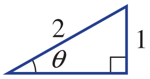
Using CAS and non-CAS calculators 3G: Trigonometry

This activity is in the Interactive Textbook in the form of a printable PDF.

3H Finding an angle

Learning intentions

- To understand that the inverse trigonometric ratios are used to find angles in right-angled triangles
- To know that two side lengths are required to find angles in right-angled triangles
- To know how to use a calculator to find a value using an inverse trigonometric ratio
- To be able to find the value of an angle in a right-angled triangle given two side lengths



Logically, if you can use trigonometry to find a side length of a right-angled triangle given one angle and one side, you should be able to find an angle if you are given two sides.

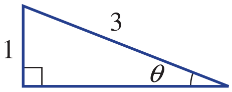
We know that $\sin 30^\circ = \frac{1}{2}$, so if we were to determine θ when $\sin\theta = \frac{1}{2}$, the answer would be $\theta = 30^\circ$.

We write this as $\theta = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$ and we say that the inverse sine of $\frac{1}{2}$ is 30°.

Calculators can be used to help solve problems using inverse sine ($\sin^{-1}$), inverse cosine ($\cos^{-1}$) and inverse tangent ($\tan^{-1}$). For angles in degrees, ensure your calculator is in degree mode.

LESSON STARTER Trial and error can be slow

We know that for this triangle, $\sin\theta = \frac{1}{3}$.

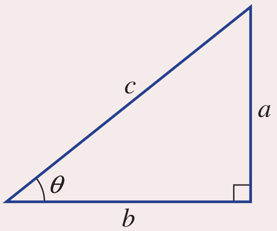


- Guess the angle θ.
- For your guess use a calculator to see if
  $\sin\theta = \frac{1}{3} = 0.333\ldots$
- Update your guess and use your calculator to check once again.
- Repeat this trial-and-error process until you think you have the angle θ correct to three decimal places.
- Now evaluate $\sin^{-1}\left(\frac{1}{3}\right)$ and check your guess.

The International Space Station has a 16.94 m robotic arm with four segments that is used on spacewalks, and for capturing and releasing visiting spacecraft. Trigonometry is used to calculate the required angle between segments for grappling specific spacecraft.

KEY IDEAS

- **Inverse sine** ($\sin^{-1}$), **inverse cosine** ($\cos^{-1}$) and **inverse tangent** ($\tan^{-1}$) can be used to find angles in right-angled triangles.
  - $\sin\theta = \frac{a}{c}$ means $\theta = \sin^{-1}\left(\frac{a}{c}\right)$
  - $\cos\theta = \frac{b}{c}$ means $\theta = \cos^{-1}\left(\frac{b}{c}\right)$
  - $\tan\theta = \frac{a}{b}$ means $\theta = \tan^{-1}\left(\frac{a}{b}\right)$



- Note that $\sin^{-1} x$ does *not* mean $\frac{1}{\sin x}$.

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Updated September 2021

BUILDING UNDERSTANDING

① Use a calculator to evaluate the following, rounding the answer to two decimal places.

a $\sin^{-1}(0.2)$ **b** $\cos^{-1}(0.75)$ **c** $\tan^{-1}(0.5)$

② State the missing number.

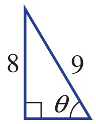
a If $\sin 30^\circ = \frac{1}{2}$ then $30^\circ = \sin^{-1}$ (_____).

b If $\cos 50^\circ = 0.64$ then _____ $= \cos^{-1}(0.64)$.

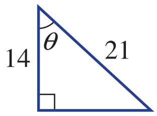
c If $\tan 45^\circ = 1$ then _____ $= \tan^{-1}$ (_____).

③ Which trigonometric ratio should be used to solve for θ?

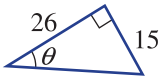
a



b



c



Example 12 Using inverse trigonometric ratios

Find the value of θ to the level of accuracy indicated.

a $\sin\theta = 0.3907$ (nearest degree)

b $\tan\theta = \frac{1}{2}$ (one decimal place)

SOLUTION	EXPLANATION
a $\sin\theta = 0.3907$	
$\theta = \sin^{-1}(0.3907)$	Use the $\sin^{-1}$ key on your calculator.
$= 23^\circ$ (to nearest degree)	Round to the nearest whole number.
b $\tan\theta = \frac{1}{2}$	
$\theta = \tan^{-1}\left(\frac{1}{2}\right)$	Use the $\tan^{-1}$ key on your calculator and round the answer to one decimal place.
$= 26.6^\circ$ (to 1 d.p.)	

Now you try

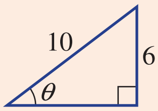
Find the value of θ to the level of accuracy indicated.

a $\cos\theta = 0.416$ (nearest degree)

b $\tan\theta = \frac{5}{3}$ (one decimal place)

Example 13 Finding an angle

Find the value of θ to the nearest degree.



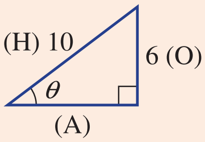
SOLUTION

$$\sin\theta = \frac{O}{H}$$
$$= \frac{6}{10}$$

$$\theta = \sin^{-1}\left(\frac{6}{10}\right)$$
$$= 37^\circ \text{ (to the nearest degree)}$$

EXPLANATION

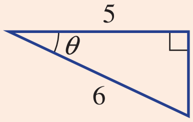
As the opposite side (O) and the hypotenuse (H) are given, use $\sin\theta$.



Use the $\sin^{-1}$ key on your calculator and round as required.

Now you try

Find the value of θ to the nearest degree.



Exercise 3H

FLUENCY	1, 2–4(1/2)	2–4(1/2)	2–4(1/2)

Example 12a

1 Find the value of θ.
- a $\sin\theta = 0.5$
- b $\cos\theta = 0.5$
- c $\tan\theta = 1$

Example 12a

2 Find the value of θ to the nearest degree.
- a $\cos\theta = 0.8660$
- b $\sin\theta = 0.7071$
- c $\tan\theta = 0.5774$
- d $\sin\theta = 1$
- e $\tan\theta = 1.192$
- f $\cos\theta = 0$

Example 12b

3 Find the angle θ correct to two decimal places.
- a $\sin\theta = \frac{4}{7}$
- b $\sin\theta = \frac{1}{3}$
- c $\sin\theta = \frac{9}{10}$
- d $\cos\theta = \frac{1}{4}$
- e $\cos\theta = \frac{4}{5}$
- f $\cos\theta = \frac{7}{9}$
- g $\tan\theta = \frac{3}{5}$
- h $\tan\theta = \frac{8}{5}$
- i $\tan\theta = 12$

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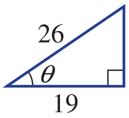
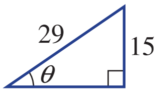
Example 13

4 Find the value of θ to the nearest degree.

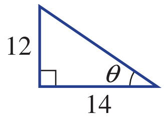
a



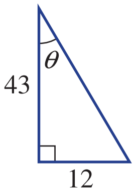
b



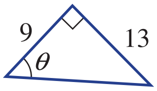
c



d



e



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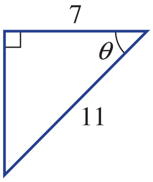

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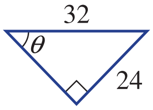

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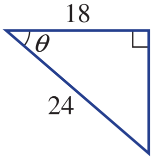
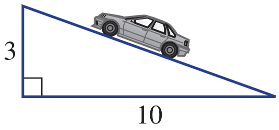


PROBLEM-SOLVING

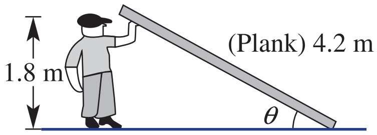
5–7 6–8 7–9

5 A road rises at a grade of 3 in 10. Find the angle (to the nearest degree) the road makes with the horizontal.

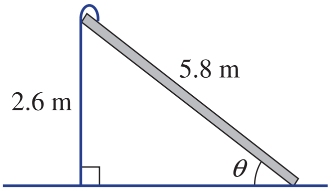


6 When a 2.8 m long seesaw is at its maximum height it is 1.1 m off the ground. What angle (correct to two decimal places) does the seesaw make with the ground?

7 Adam, who is 1.8 m tall, holds up a plank of wood that is 4.2 m long. Find the angle that the plank makes with the ground, correct to one decimal place.

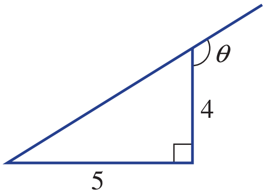


8 A children's slide has a length of 5.8 m. The vertical ladder is 2.6 m high. Find the angle the slide makes with the ground, correct to one decimal place.

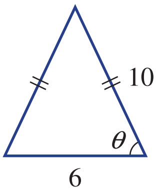


9 Find the value of θ, correct to one decimal place.

a



b



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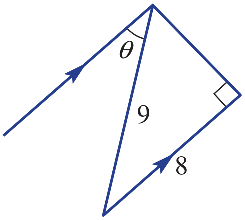


REASONING	10	10, 11	11, 12

10 Find all the angles to the nearest degree in right-angled triangles with these side lengths.

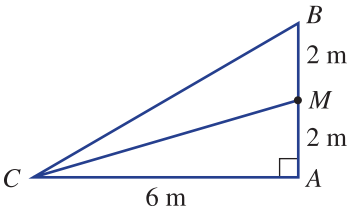
a 3, 4, 5

b 5, 12, 13

c 7, 24, 25

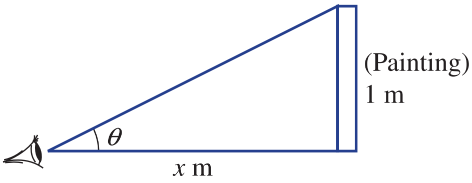
11 For what value of θ is $\sin \theta = \cos \theta$?

12 If M is the midpoint of AB, decide whether $\angle ACM$ is exactly half of angle $\angle ACB$. Investigate and explain.



ENRICHMENT: Viewing angle	–	–	13

13 Jo has forgotten her glasses and is trying to view a painting in a gallery. Her eye level is at the same level as the base of the painting and the painting is 1 metre tall. Answer the following to the nearest degree for angles and to two decimal places for lengths.



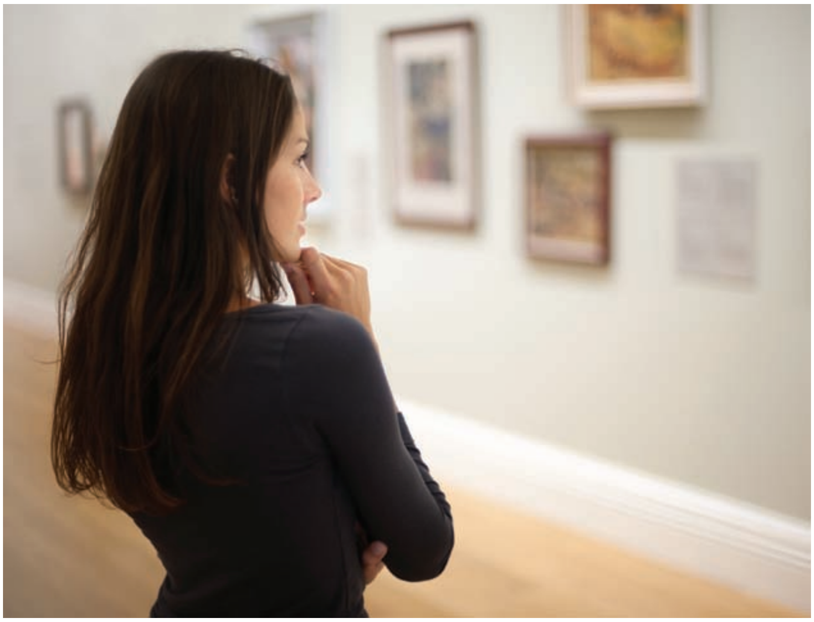
a If $x = 3$, find the viewing angle θ.

b If $x = 2$, find the viewing angle θ.

c If Jo can stand no closer than 1 metre from the painting, what is Jo's largest viewing angle?

d When the viewing angle is 10°, Jo has trouble seeing the painting. How far is she from the painting at this viewing angle?

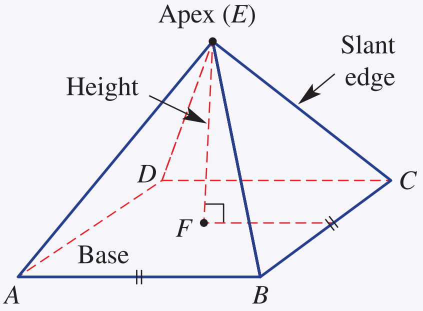
e Theoretically, what would be the largest viewing angle if Jo could go as close as she would like to the painting?



Applications and problem-solving

The following problems will investigate practical situations drawing upon knowledge and skills developed throughout the chapter. In attempting to solve these problems, aim to identify the key information, use diagrams, formulate ideas, apply strategies, make calculations and check and communicate your solutions.

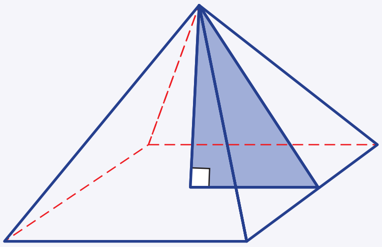
Solving the pyramid



1 Richard is studying ancient pyramids and notices that many of them are in fact right square-based pyramids.
Such structures have the following properties:

- The base is a square.
- The apex is directly above the centre of the base.

Richard is interested in the properties of such a solid and how Pythagoras' theorem can be used to find various lengths. Given two lengths, Richard is looking to 'solve the pyramid' by finding the lengths of all the other edges, heights or diagonals.



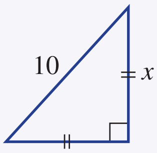
a There are many right-angled triangles that can be drawn inside or on the surface of a right, square-based pyramid. Make three copies of the pyramid and shade in a different right-angled triangle for each one. An example is shown at right.

b Richard is interested in finding the length of a slant edge. A square-based pyramid has base length 100 m and height 50 m. Use various right-angled triangles to find the following lengths. Round each answer correct to one decimal place, but try not to accumulate errors by using rounded answers for subsequent calculations. (Use stored answers in a calculator.) Refer to the diagram at the top of the page.

i AC **ii** CF **iii** CE (slant edge)

c A pyramid has base length 100 m and slant edge 80 m. Calculate the height of the pyramid correct to one decimal place.

d A pyramid has slant edge 80 m and height 50 m.



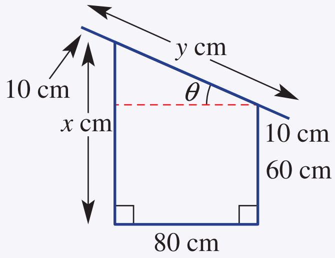
i First check that you can solve for an unknown in the triangle shown here. Round to one decimal place.

ii Now find the base length of the pyramid described above. Round to one decimal place.

Rabbit roof

2 Sophie is building a hutch for her pet rabbit and wants to include a sloping roof to keep the rain off. A cross-section of the hutch is shown here.

The minimum height at one side is 60 cm and a 10 cm roof overhang is required at each side. The width of the hutch is 80 cm, but the taller side, x cm, can vary depending on the slope of the roof (θ).



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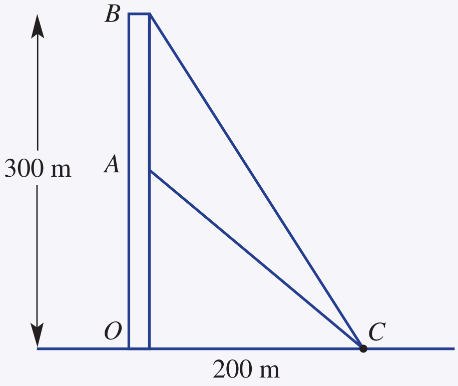
Sophie is investigating the relationship between the height of the hutch (x cm) and the roof span (y cm). The angle or slope of the roof is also of interest as these measurements affect lengths and the costs of materials.

- **a** If Sophie chooses the angle for the roof to be 30°, find these lengths correct to one decimal place.
  - **i** Wall height, x cm
  - **ii** Roof span, y cm
- **b** If Sophie chooses the wall height, x cm, to be 90 cm, find these measurements correct to one decimal place.
  - **i** Roof span, y cm
  - **ii** Roof angle θ
- **c** If Sophie chooses the roof span, y cm, to be 120 cm, find these measurements correct to one decimal place where necessary.
  - **i** Wall height, x cm
  - **ii** Roof angle θ
- **d** Write a rule for each of the following:
  - **i** y in terms of θ
  - **ii** y in terms of x
  - **iii** x in terms of θ
  - **iv** x in terms of y.
- **e** Given the size of the available materials, Sophie will save money if the roof span, y cm, is less than 110 cm, but she feels that the angle θ needs to be larger than 25° for sufficient water run-off. Investigate to see if this is achievable and give reasons.

Communications cabling

3 A 300 m high communications tower is to be supported by six cables, including three reaching to the very top and three reaching to a point somewhere up the tower. Each of the three anchor points on the ground is shared by two cables as shown. The anchor points are 200 m from the base of the tower.

One pair of cables only



The construction designers are interested in the total length of cable required to support the tower, as well as the position of point A for the shorter cable. They are exploring the distance between points A and B and the angles the cables make with the ground.

Round all answers in this word problem to two decimal places where necessary.

- **a** For each long cable that joins to the tower at the very top, find:
  - **i** its length
  - **ii** its angle of elevation from ground level.
- **b** If point A is positioned halfway up the tower, find:
  - **i** the angle of elevation of the shorter cables
  - **ii** the total length of cable required for all six cables.
- **c** If 2000 m of cable is used in total for the six cables, find the position at which the shorter cables connect to the tower (point A).
- **d** The designers decide that the distance AB on a second 300 m high tower should be 100 m. They are hoping to use less than 1900 m of cabling. Will this be possible? Give reasons.

3I Applying trigonometry EXTENDING

Learning intentions
- To know the meaning of angle of elevation and angle of depression
- To be able to draw a diagram from information in a word problem and identify any right-angled triangles in the diagram
- To know how to use alternate angles to mark equal angles in diagrams with parallel lines
- To be able to apply trigonometry to find unknown side lengths and angles

In many situations, angles are measured up or down from the horizontal. These are called angles of elevation and depression. Combined with the mathematics of trigonometry, these angles can be used to solve problems, provided right-angled triangles can be identified. The line of sight to a helicopter 100 m above the ground, for example, creates an angle of elevation inside a right-angled triangle.

LESSON STARTER Illustrate the situation

For the situation described below, draw a detailed diagram showing these features:

- an angle of elevation
- an angle of depression
- any given lengths
- a right-angled triangle that will help to solve the problem.

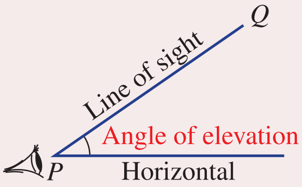
A cat and a bird eye each other from their respective positions. The bird is 20 m up a tree and the cat is on the ground 30 m from the base of the tree. Find the angle their line of sight makes with the horizontal.

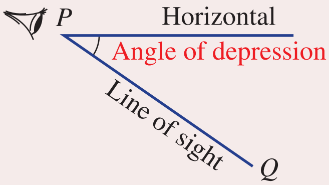
By using trigonometry and higher-level mathematics, it can be shown that the angle of elevation from an observer to a rainbow's highest point (red part) is always 42°. In a fainter secondary rainbow, the highest point (violet part) always has a 51° angle of elevation.

Compare your diagram with those of others in your class. Is there more than one triangle that could be drawn and used to solve the problem?

KEY IDEAS

- To solve application problems involving trigonometry follow these steps.
  1. Draw a diagram and label the key information.
  2. Identify and draw the appropriate right-angled triangles separately.
  3. Solve using trigonometry to find the missing measurements.
  4. Express your answer in words.
- The **angle of elevation** or **depression** of a point, Q, from another point, P, is given by the angle the line PQ makes with the horizontal.





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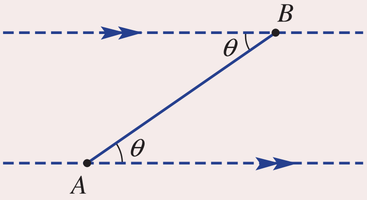
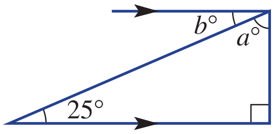
- Angles of elevation or depression are always measured from the horizontal.
- In this diagram the angle of elevation of B from A is equal to the angle of depression of A from B. They are equal alternate angles in parallel lines.



BUILDING UNDERSTANDING

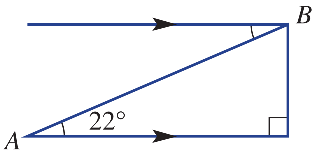
1 State the values of the pronumerals in this diagram.



2 For this diagram:

a what is the angle of elevation of B from A?

b what is the angle of depression of A from B?



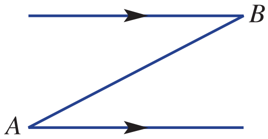
3 Complete these tasks for the diagram at right.

a State where you would mark in the following:

  i the angle of elevation (θ) of B from A

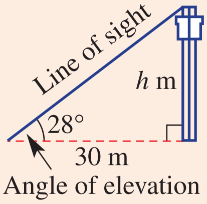
  ii the angle of depression (α) of A from B.

b Is $\theta = \alpha$ in your diagram? Why?



Example 14 Using angles of elevation

The angle of elevation of the top of a tower from a point on the ground 30 m away from the base of the tower is 28°. Find the height of the tower to the nearest metre.



SOLUTION

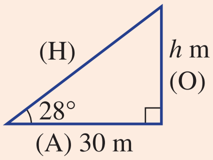
Let the height of the tower be h m.

$$\tan 28^\circ = \frac{\text{O}}{\text{A}}$$
$$= \frac{h}{30}$$
$$h = 30 \tan 28^\circ$$
$$= 15.951\ldots$$

The height is 16 m, to the nearest metre.

EXPLANATION

As the opposite side (O) is required and the adjacent (A) is given, use $\tan\theta$.



Multiply both sides by 30 and evaluate. Round to the nearest metre and write the answer in words.

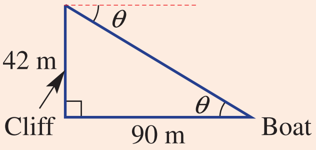
Continued on next page

Now you try

The angle of elevation to the top of a tree from a point 50 m away from the base of the tree is 32°. Find the height of the tree to the nearest metre.

Example 15 Finding an angle of depression

From the top of a vertical cliff Andrea spots a boat out at sea. If the top of the cliff is 42 m above sea level and the boat is 90 m away from the base of the cliff, find the angle of depression from Angela to the boat to the nearest degree.

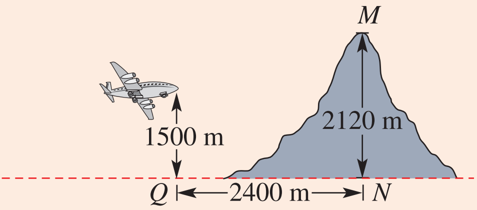
SOLUTION	EXPLANATION
	Draw a diagram and label all the given measurements. Use alternate angles in parallel lines to mark θ inside the triangle.
$\tan\theta = \frac{\text{O}}{\text{A}}$ $= \frac{42}{90}$	As the opposite (O) and adjacent sides (A) are given, use $\tan\theta$.
$\theta = \tan^{-1}\left(\frac{42}{90}\right)$ $\theta = 25.0168\ldots^\circ$	Use the $\tan^{-1}$ key on your calculator.
The angle of depression is 25°, to the nearest degree.	Round to the nearest degree and express the answer in words.

Now you try

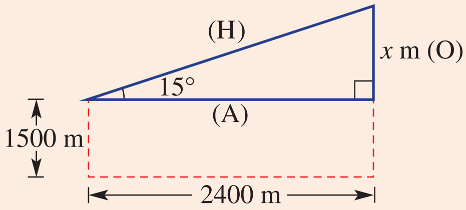
A drone films a concert on a stage from the air. If the drone is 20 m above the level of the stage and a horizontal distance of 50 m from the stage, find the angle of depression from the drone to the stage to the nearest degree.

Example 16 Applying trigonometry

An aeroplane flying at an altitude of 1500 m starts to climb at an angle of 15° to the horizontal when the pilot sees a mountain peak 2120 m high, 2400 m away from him horizontally. Will the aeroplane clear the mountain?



SOLUTION



$$\tan 15^\circ = \frac{x}{2400}$$

$$x = 2400\ \tan 15^\circ$$

$$= 643.078\ldots$$

x needs to be greater than $2120 - 1500 = 620$

Since $x > 620$ m the aeroplane will clear the mountain peak.

EXPLANATION

Draw a diagram, identifying and labelling the right-angled triangle to help solve the problem. The aeroplane will clear the mountain if the opposite (O) is greater than $(2120 - 1500)\ \text{m} = 620\ \text{m}$

Set up the trigonometric ratio using tan.

Multiply by 2400 and evaluate.

Answer the question in words.

Now you try

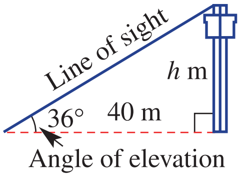
A submarine initially at sea level dives at an angle of 11° to the horizontal. At a horizontal distance of 1200 m there is an uncharted reef at a depth of 240 m. Will the submarine pass over the reef at that point?

Exercise 3I

FLUENCY	1–6	2–7	3, 5, 7, 8

Example 14

1 The angle of elevation of the top of a tower from a point on the ground 40 m from the base of the tower is 36°. Find the height of the tower to the nearest metre.

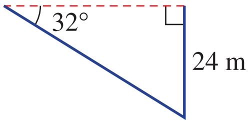


Example 14

2 The angle of elevation of the top of a castle wall from a point on the ground 25 m from the base of the wall is 32°. Find the height of the castle wall to the nearest metre.

3 From a point on the ground, Emma measures the angle of elevation of an 80 m tower to be 27°. Find how far Emma is from the base of the tower, correct to the nearest metre.

4 From a pedestrian overpass, Chris spots a landmark at an angle of depression of 32°. How far away (to the nearest metre) is the landmark from the base of the 24 m high overpass?



5 From a lookout tower, David spots a bushfire at an angle of depression of 25°. If the lookout tower is 42 m high, how far away (to the nearest metre) is the bushfire from the base of the tower?

Example 15

6 From the top of a vertical cliff, Josh spots a swimmer out at sea. If the top of the cliff is 38 m above sea level and the swimmer is 50 m from the base of the cliff, find the angle of depression from Josh to the swimmer, to the nearest degree.

Example 15

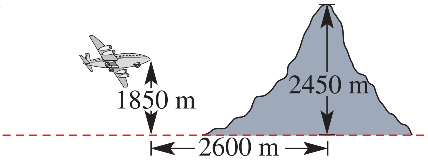
7 From a ship, a person is spotted floating in the sea 200 m away. If the viewing position on the ship is 20 m above sea level, find the angle of depression from the ship to the person in the sea. Give your answer to the nearest degree.

8 A power line is stretched from a pole to the top of a house. The house is 4.1 m high and the power pole is 6.2 m high. The horizontal distance between the house and the power pole is 12 m. Find the angle of elevation of the top of the power pole from the top of the house, to the nearest degree.

PROBLEM-SOLVING	9, 10	9–11	9, 12, 13

Example 16

9 An aeroplane flying at 1850 m starts to climb at an angle of 18° to the horizontal to clear a mountain peak 2450 m high and a horizontal distance of 2600 m away. Will the aeroplane clear the mountain?

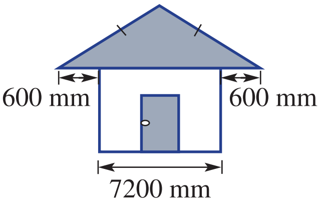


10 A road has a steady gradient of 1 in 10.

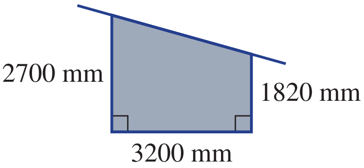
a What angle does the road make with the horizontal? Give your answer to the nearest degree.

b A car starts from the bottom of the inclined road and drives 2 km along the road. How high vertically has the car climbed? Use your rounded answer from part **a** and give your answer correct to the nearest metre.

11 A house is to be built using the design shown on the right. The eaves are 600 mm and the house is 7200 mm wide, excluding the eaves. Calculate the length (to the nearest mm) of a sloping edge of the roof, which is pitched at 25° to the horizontal.



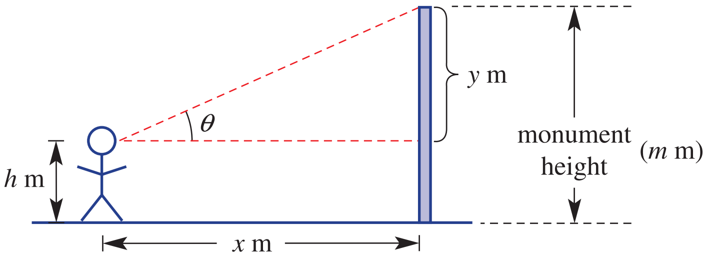
12 A garage is to be built with measurements as shown in the diagram on the right. Calculate the sloping length and pitch (angle) of the roof if the eaves extend 500 mm on each side. Give your answers correct to the nearest unit.



13 The chains on a swing are 3.2 m long and the seat is 0.5 m off the ground when it is in the vertical position. When the swing is pulled as far back as possible, the chains make an angle of 40° with the vertical. How high off the ground, to the nearest cm, is the seat when it is at this extreme position?

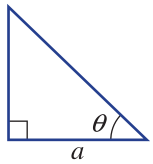
REASONING	14	14	14, 15

14 A person views a vertical monument *x* metres away as shown.



- **a** If $h = 1.5$, $x = 20$ and $\theta = 15°$, find the height of the monument to two decimal places.
- **b** If $h = 1.5$, $x = 20$ and $y = 10$, find θ correct to one decimal place.
- **c** Let the height of the monument be m metres. Write expressions for:
  - **i** m using (in terms of) y and h
  - **ii** y using (in terms of) x and θ
  - **iii** m using (in terms of) x, θ and h.

15 Find an expression for the area of this triangle using a and θ.



ENRICHMENT: Plane trigonometry	–	–	16–18

16 An aeroplane takes off and climbs at an angle of 20° to the horizontal, at 190 km/h a speed of along its flight path for 15 minutes.

- **a** **i** Find the distance the aeroplane travels in 15 minutes.
  - **ii** Calculate the height of the aeroplane after 15 minutes correct to two decimal places.
- **b** If the angle at which the aeroplane climbs is twice the original angle but its speed is halved, will it reach a greater height after 15 minutes? Explain.
- **c** If the aeroplane's speed is doubled and its climbing angle is halved, will the aeroplane reach a greater height after 15 minutes? Explain.

17 The residents of Skeville live 12 km from an airport. They maintain that any aeroplane flying at a height lower than 4 km disturbs their peace. Each Sunday they have an outdoor concert from 12:00 noon till 2:00 p.m.

- **a** Will an aeroplane taking off from the airport at an angle of 15° over Skeville disturb the residents?
- **b** When the aeroplane in part **a** is directly above Skeville, how far (to the nearest metre) has it flown?
- **c** If the aeroplane leaves the airport at 11:50 a.m on Sunday and travels at an average speed of 180 km/h, will it disturb the start of the concert?
- **d** Investigate what average speed (correct to the nearest km/h) the aeroplane can travel at so that it does not disturb the concert. Assume it leaves at 11:50 a.m.

18 Peter observes an aeroplane flying directly overhead at a height of 820 m. Twenty seconds later, the angle of elevation of the aeroplane from Peter is 32°. Assume the aeroplane flies horizontally.

- **a** How far (to the nearest metre) did the aeroplane fly in 20 seconds?
- **b** What is the aeroplane's speed in km/h, correct to the nearest km/h?

3J Bearings EXTENDING

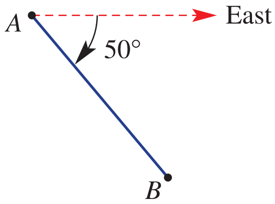
Learning intentions

- To understand how true bearings are measured and written
- To be able to state the true bearing of one point from another
- To know that opposite directions differ by 180°
- To be able to use compass directions to draw diagrams and form right-angled triangles labelled with angles from bearings
- To be able to use trigonometry to solve problems involving bearings
- To be able to calculate bearings from recorded measurements

Bearings are used to indicate direction and therefore are commonly used to navigate the sea or air in ships or planes. Bushwalkers use bearings with a compass to help follow a map and navigate bushland. The most common type of bearing is the true bearing measured clockwise from north.

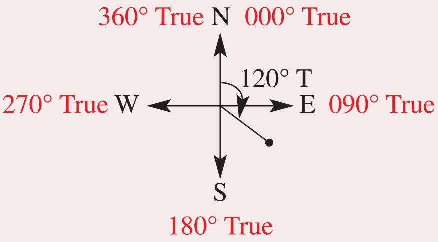
LESSON STARTER Opposite directions

Marg at point A and Jim at point B start walking toward each other. Marg knows that she has to face 50° south of due east.



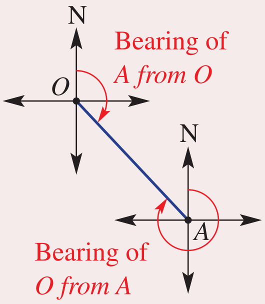
- Measured clockwise from north, can you help Marg determine the true compass bearing that she should walk on?
- Can you find the bearing Jim should walk on?
- Draw a detailed diagram that supports your answers above.

KEY IDEAS



- A **true bearing** is an angle measured clockwise from north.
  - True bearings are written using three digits.

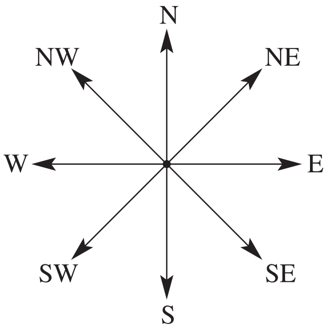
  For example: 008° T, 032° T or 120° T.
- To describe the true bearing of an object positioned at A from an object positioned at O, we need to start at O, face north then turn clockwise through the required angle to face the object at A.
- Opposite directions differ by 180°.
  - $\theta + 180°$ is the opposite bearing if $0° \leqslant \theta < 180°$.
  - $\theta - 180°$ is the opposite bearing if $180° \leqslant \theta < 360°$.
- When solving problems with bearings, draw a diagram including four point compass directions (N, E, S, W) at each point.



BUILDING UNDERSTANDING

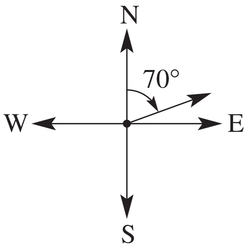
① Give the true bearings for these common directions.

a North (N)
b North-east (NE)
c East (E)
d South-east (SE)
e South (S)
f South-west (SW)
g West (W)
h North-west (NW)

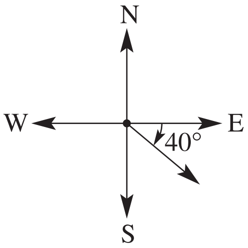


② State the true bearings shown in these diagrams. Use three digits, for example, 045° T.

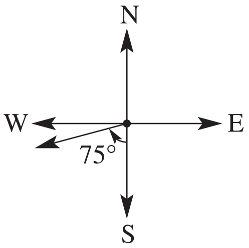
a



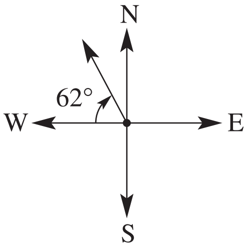
b



c



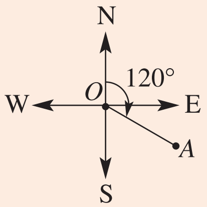
d



Example 17 Stating true bearings

For the diagram shown give:

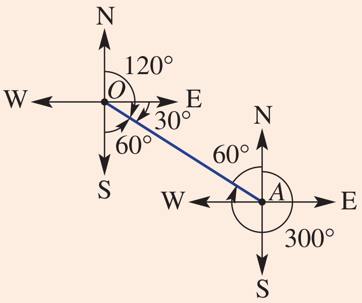
a the true bearing of *A* from *O*
b the true bearing of *O* from *A*.



SOLUTION	EXPLANATION
a The bearing of *A* from *O* is 120° T.	Start at *O*, face north and turn clockwise until you are facing *A*.

Continued on next page

b



Start at A, face north and turn clockwise until you are facing O. Mark in a compass at A and use alternate angles in parallel lines to mark a 60° angle.

The bearing of O from A is:
$(360 - 60)^\circ\text{ T} = 300^\circ\text{ T}$

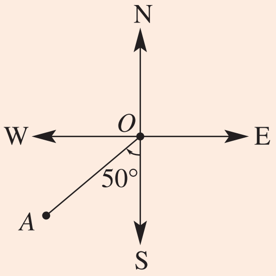
The true bearing is then 60° short of 360°. Alternatively, add 180° to 120° to get the opposite direction.

Now you try

For the diagram shown give:

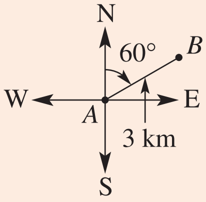
a the true bearing of A from O

b the true bearing of O from A.



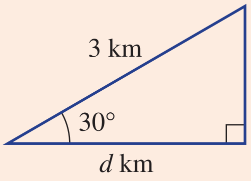
Example 18 Using bearings with trigonometry

A bushwalker walks 3 km on a true bearing of 060° from point A to point B. Find how far east (correct to one decimal place) point B is from point A.



SOLUTION

Let the distance travelled towards the east be d km.



EXPLANATION

Define the distance required and draw and label the right-angled triangle.

As the adjacent (A) is required and the hypotenuse (H) is given, use $\cos\theta$.

$$\cos 30° = \frac{d}{3}$$

$$\begin{aligned} d &= 3\cos 30° \\ &= 2.6 \text{ (to 1 d.p.)} \end{aligned}$$

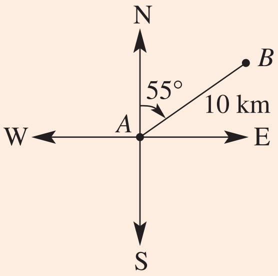
Multiply both sides of the equation by 3 and evaluate, rounding to one decimal place.

$\therefore$ The distance east is 2.6 km.

Express the answer in words.

Now you try

A ship sails 10 km on a true bearing of 055° from point A to point B.
Find how far north (correct to one decimal place) point *B* is from point *A*.

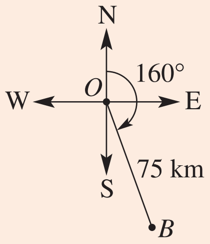


Example 19 Calculating a bearing

A fishing boat starts from point *O* and sails 75 km on a bearing of 160° T to point *B*.

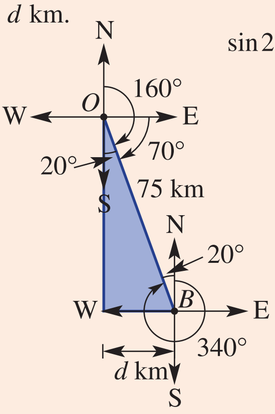
a How far east (to the nearest kilometre) of its starting point is the boat?

b What is the true bearing of *O* from *B*?



SOLUTION

a Let the distance travelled towards the east be *d* km.



$$\sin 20° = \frac{d}{75}$$

$$\begin{aligned} d &= 75\sin 20° \\ &= 25.651\ldots \end{aligned}$$

The boat has travelled 26 km to the east of its starting point, to the nearest kilometre.

EXPLANATION

Draw a diagram and label all the given measurements. Mark in a compass at *B* and use alternate angles to label extra angles.

Set up a trigonometric ratio using sine and solve for *d*.

Round to the nearest kilometre and write the answer in words.

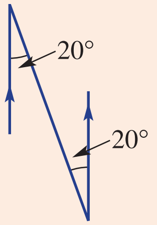
Continued on next page

b The bearing of O from B is $(360 - 20)°\text{ T} = 340°\text{ T}$

Start at B, face north then turn clockwise to face O.

Alternatively, add 180° to 160° to get the opposite direction.

Alternate angle = 20°

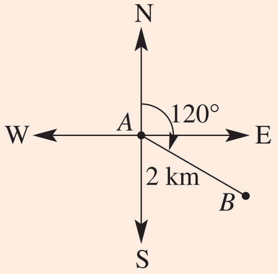


Now you try

A hiker walks 2 km on a true bearing of 120° from point A to point B.

a Find how far east (correct to one decimal place) point B is from point A.

b What is the bearing of A from B?



Exercise 3J

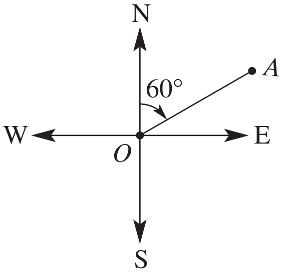
FLUENCY	1, 2(½), 3–5	2(½), 3, 4, 6	2(½), 4–6

Example 17

1 For each diagram shown, write:

- **i** the true bearing of A from O
- **ii** the true bearing of O from A.

a



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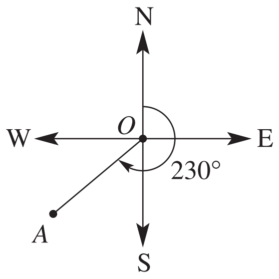
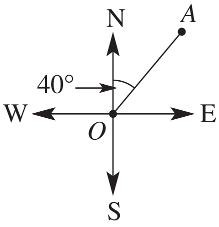


Example 17

2 For each diagram shown, write:

- **i** the true bearing of A from O
- **ii** the true bearing of O from A.

a



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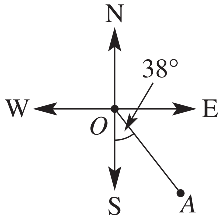
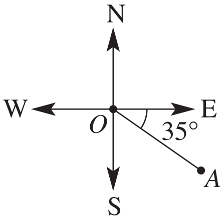
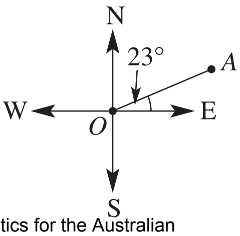


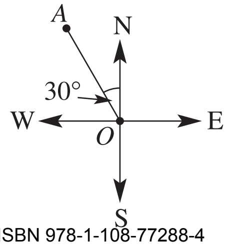
c



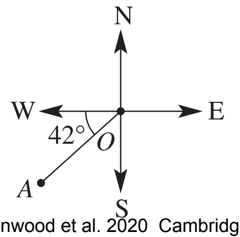
d



e

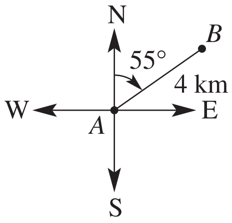


f



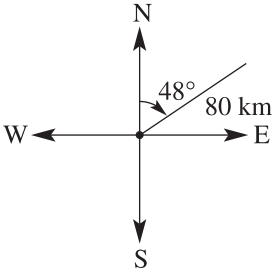
Example 18

3 A bushwalker walks 4 km on a true bearing of 055° from point A to point B. Find how far east point B is from point A, correct to two decimal places.

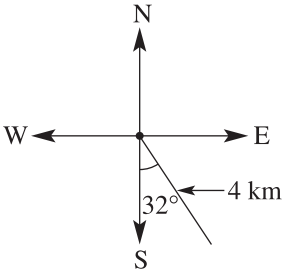


Example 18

4 A speed boat travels 80 km on a true bearing of 048°. Find how far east of its starting point the speed boat is, correct to two decimal places.



5 After walking due east, then turning and walking due south, a hiker is 4 km, 148° T from her starting point. Find how far she walked in a southerly direction, correct to one decimal place.



6 A four-wheel drive vehicle travels for 32 km on a true bearing of 200°. How far west (to the nearest kilometre) of its starting point is it?

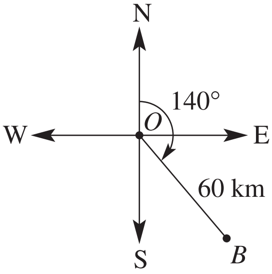
PROBLEM-SOLVING	7–9	8–10	8–11

Example 19

7 A fishing boat starts from point O and sails 60 km on a true bearing of 140° to point B.

a How far east of its starting point is the boat, to the nearest kilometre?

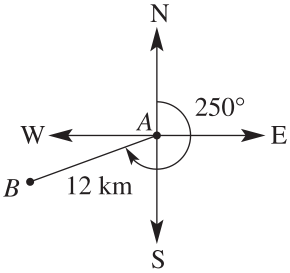
b What is the bearing of O from B?



8 Two towns, A and B, are 12 km apart. The true bearing of B from A is 250°.

a How far west of A is B, correct to one decimal place?

b Find the bearing of A from B.



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9 A helicopter flies on a true bearing of 140° for 210 km and then flies due east for 175 km. How far east (to the nearest kilometre) has the helicopter travelled from its starting point?

10 Christopher walks 5 km south and then walks on a true bearing of 036° until he is due east of his starting point. How far is he from his starting point, to one decimal place?

11 Two cyclists leave from the same starting point. One cyclist travels due west while the other travels on a true bearing of 202°. After travelling for 18 km, the second cyclist is due south of the first cyclist. How far (to the nearest metre) has the first cyclist travelled?

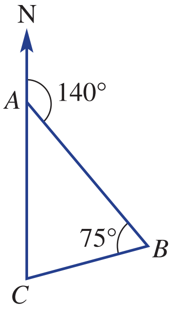
REASONING	12	12	12, 13

12 A true bearing is $a°$. Write an expression for the true bearing of the opposite direction of $a°$ if:

a a is between 0 and 180

b a is between 180 and 360.

13 A hiker walks on a triangular pathway starting at point A, walking to point B then C, then A again as shown.



a Find the bearing from B to A.

b Find the bearing from B to C.

c Find the bearing from C to B.

d If the initial bearing was instead 133° and $\angle ABC$ is still 75°, find the bearing from B to C.

e If $\angle ABC$ was 42°, with the initial bearing of 140°, find the bearing from B to C.

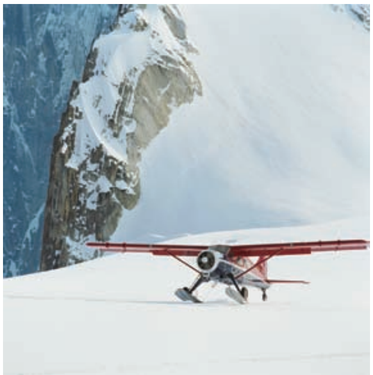
ENRICHMENT: Speed trigonometry	–	–	14, 15

14 An aeroplane flies on a true bearing of 168° for two hours at an average speed of 310 km/h. How far (to the nearest kilometre):

a has the aeroplane travelled?

b south of its starting point is the aeroplane?

c east of its starting point is the aeroplane?



15 A pilot intends to fly directly to Anderly, which is 240 km due north of his starting point. The trip usually takes 50 minutes. Due to a storm, the pilot changes course and flies to Boxleigh on a true bearing of 320° for 150 km, at an average speed of 180 km/h.

a Find (to the nearest kilometre) how far:

  i north the plane has travelled from its starting point

  ii west the plane has travelled from its starting point.

b How many kilometres is the plane from Anderly?

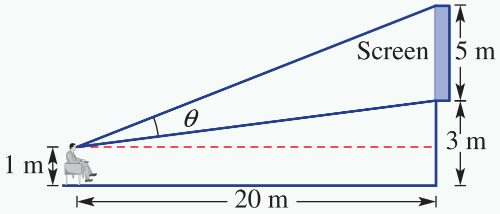
c From Boxleigh the pilot flies directly to Anderly at 240 km/h.

  i Compared to the usual route, how many extra kilometres (to the nearest kilometre) has the pilot travelled in reaching Anderly?

  ii Compared to the usual trip, how many extra minutes (correct to one decimal place) did the trip to Anderly take?

Viewing angle

Lucas is keen to maximise his experience at the outdoor cinema and usually sits at a distance of 20 metres horizontally from the screen. The base of the screen is 3 metres above the ground and the screen is 5 metres in height. Lucas' eye level is 1 metre above ground level as shown.



Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate. Use two decimal places for all rounding.

Preliminary task

- **a** What is the vertical distance between Lucas' eye level and the base of the screen?
- **b** Find the angle of elevation from Lucas' eye level to the base of the screen.
- **c** Find the angle of elevation from Lucas' eye level to the top of the screen.
- **d** Find Lucas' viewing angle (θ).

Modelling task

Formulate

- **a** The problem is to determine how close Lucas should sit to maximise his viewing angle. Write down all the relevant information that will help solve this problem with the aid of a diagram.

Solve

- **b** Determine Lucas' viewing angle if the 20 metre horizontal distance is changed to:
  - **i** 14 metres
  - **ii** 8 metres
  - **iii** 2 metres.
- **c** Choose integer values for the horizontal distance between Lucas and the screen and determine which integer horizontal distance delivers the largest viewing angle and state the corresponding viewing angle.

Evaluate and verify

- **d** Find the viewing angle for the following horizontal distances:
  - **i** your answer from part **c** plus 0.1 metres
  - **ii** your answer from part **c** minus 0.1 metres.
- **e** Do your results from part **d** confirm that you have found a distance which maximises the viewing angle? Explain why.
- **f** Refine your choices for the horizontal distance and determine a distance correct to one decimal place that maximises Lucas' viewing angle.
- **g** Draw a scale diagram illustrating where Lucas should sit to maximise his viewing angle. Decide if this position for Lucas is reasonable.

Communicate

- **h** Summarise your results and describe any key findings.

Extension questions

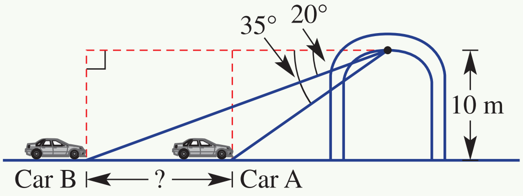
- **a** Investigate how varying the height of the base of the screen above the ground (previously 3 metres) changes the position that Lucas should sit to maximise the viewing angle. Show your calculations and make drawings to suit.
- **b** Investigate how varying the height of the screen (previously 5 metres) changes the position that Lucas should sit to maximise the viewing angle. Show your calculations and make drawings to suit.

Constructing triangles to solve problems

Illustrations for some problems may not initially look as if they include right-angled triangles. A common mathematical problem-solving technique is to construct right-angled triangles so that trigonometry can be used.

Car gap

Two cars are observed in the same lane from an overpass bridge 10 m above the road. The angles of depression to the cars are 20° and 35°.



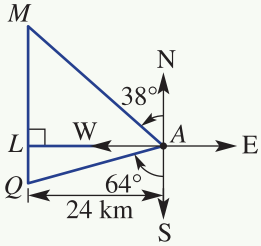
a Find the horizontal distance from car A to the overpass. Show your diagrams and working.
b Find the horizontal distance from car B to the overpass.
c Find the distance between the fronts of the two cars.

Now try these

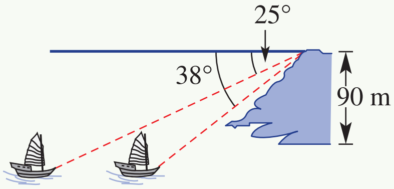
Solve these similar types of problems. You will need to draw detailed diagrams and split the problem into parts. Refer to the problem above if you need help.

a An observer is 50 m horizontally from a hot air balloon. The angle of elevation to the top of the balloon is 60° and to the bottom of the balloon's basket is 40°. Find the total height of the balloon (to the nearest metre) from the base of the basket to the top of the balloon.

b A ship (at A) is 24 km due east of a lighthouse (L). The captain takes bearings from two landmarks, M and Q, which are due north and due south of the lighthouse respectively. The true bearings of M and Q from the ship are 322° and 244° respectively. How far apart are the two landmarks?



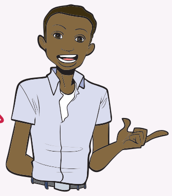
c From the top of a 90 m cliff the angles of depression of two boats in the water, both directly east of the lighthouse, are 25° and 38° respectively. What is the distance between the two boats to the nearest metre?



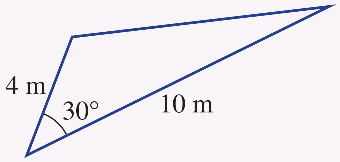
d A person on a boat 200 m out to sea views a 40 m high castle wall on top of a 32 m high cliff. Find the viewing angle between the base and top of the castle wall from the person on the boat.

1 A right-angled isosceles triangle has area of 4 square units. Determine the exact perimeter of the triangle.

Up for a challenge? If you get stuck on a question, check out the 'Working with unfamiliar problems' poster at the end of the book to help you.

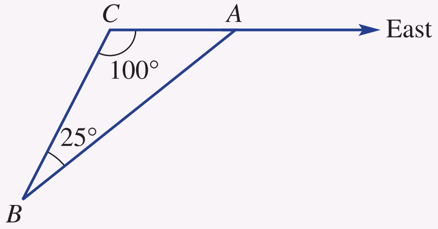


2 Find the area of the triangle below using trigonometry. (*Hint*: Insert a line showing the height of the triangle.)



3 A rectangle $ABCD$ has sides $AB = CD = 34$ cm. E is a point on CD such that $CE = 9$ cm and $ED = 25$ cm. AE is perpendicular to EB. What is the length of BC?

4 Find the bearing from B to C in this diagram.



5 Which is a better fit? A square peg in a round hole or a round peg in a square hole. Use area calculations and percentages to investigate.

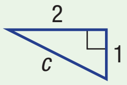
6 Boat A is 20 km from port on a true bearing of 025° and boat B is 25 km from port on a true bearing of 070°. Boat B is in distress. What bearing (to the nearest degree) should boat A travel on to reach boat B?

7 For positive integers m and n such that $n < m$, the Pythagorean triples (e.g. 3, 4, 5) can be generated using $a = m^2 - n^2$ and $b = 2mn$, where a and b are the two shorter sides of the right-angled triangle.

- **a** Using these formulas and Pythagoras' theorem to calculate the third side, generate the Pythagorean triples for these values of m and n.
  - **i** $m = 2, n = 1$
  - **ii** $m = 3, n = 2$
- **b** Using the expressions for a and b and Pythagoras' theorem, find a rule for c (the hypotenuse) in terms of n and m.

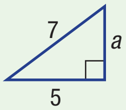
Finding the hypotenuse

$$c^2 = 2^2 + 1^2$$
$$= 5$$
$$c = \sqrt{5}$$
$$= 2.24 \text{ (to 2 decimal places)}$$

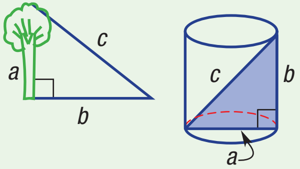


Shorter sides

$$a^2 + 5^2 = 7^2$$
$$a^2 = 7^2 - 5^2$$
$$= 24$$
$$\therefore a = \sqrt{24}$$

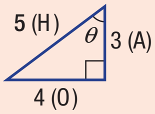


Applications

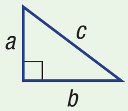


SOHCAHTOA

$$\sin\theta = \frac{O}{H} = \frac{4}{5}$$
$$\cos\theta = \frac{A}{H} = \frac{3}{5}$$
$$\tan\theta = \frac{O}{A} = \frac{4}{3}$$



Pythagoras' theorem

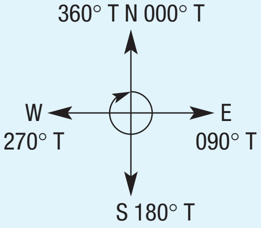


$$c^2 = a^2 + b^2$$

Pythagoras' theorem and trigonometry

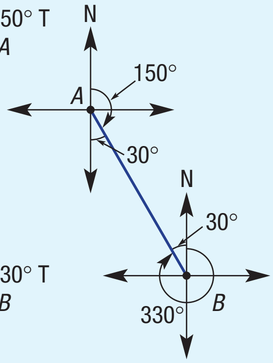
Bearings (Ext)

True bearings are measured clockwise from north.

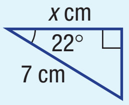


B is 150° T from A

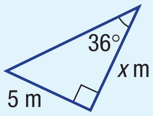
A is 330° T from B



Finding side lengths

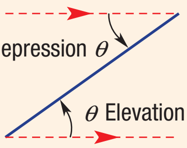


$$\cos 22° = \frac{x}{7}$$
$$x = 7\cos 22°$$
$$= 6.49$$
(to 2 d.p.)

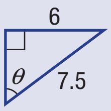


$$\tan 36° = \frac{5}{x}$$
$$x \times \tan 36° = 5$$
$$x = \frac{5}{\tan 36°}$$
$$= 6.88 \text{ (to 2 d.p.)}$$

Elevation and depression (Ext)

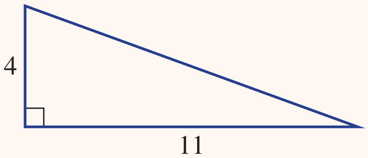
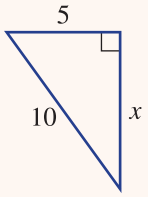
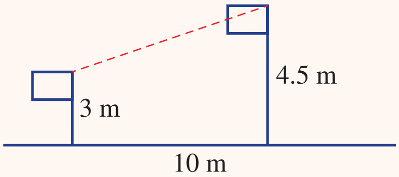
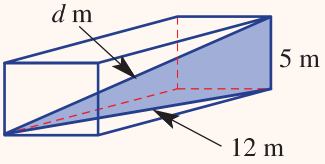
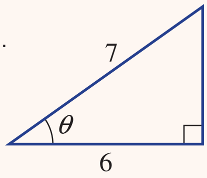
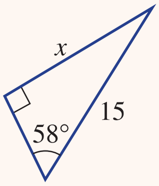
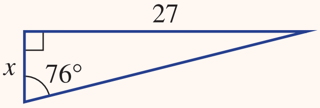


Finding angles



$$\sin\theta = \frac{6}{7.5}$$
$$\theta = \sin^{-1}\left(\frac{6}{7.5}\right)$$
$$= 53.13° \text{ (to 2 d.p.)}$$

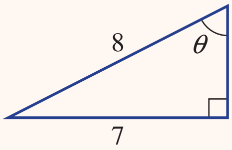
Chapter checklist: Success criteria

		✔
3A	**1. I can find the length of the hypotenuse using Pythagoras' theorem.** e.g. Find the length of the hypotenuse as an exact value and rounded to one decimal place.  	
3B	**2. I can find the length of a shorter side using Pythagoras' theorem.** e.g. Find the value of the pronumeral correct to one decimal place.  	
3C	**3. I can apply Pythagoras' theorem.** e.g. Two flag poles are located 10 m apart. The flag poles are 4.5 m and 3 m high. A cable links the top of the two flag poles. Find the length of the cable correct to one decimal place.  	
3D	**4. I can use Pythagoras in 3D.** e.g. Find the distance, d m, from one corner to the opposite corner in the rectangular prism shown.  Ext	
3E	**5. I can write a trigonometric ratio.** e.g. Using the given sides, write a trigonometric ratio for the triangle shown.  	
3F	**6. I can find a side length using a trigonometric ratio.** e.g. Find the value of x, correct to one decimal place.  	
3G	**7. I can find a side length using a trigonometric ratio with the unknown in the denominator.** e.g. Find the value of x, correct to one decimal place.  	
3H	**8. I can use an inverse trigonometric ratio.** e.g. Find the value of θ in $\cos\theta = 0.674$ correct to one decimal place.	

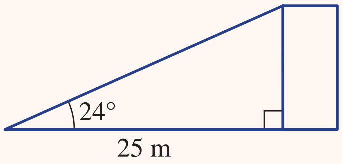
Chapter checklist

✔

3H **9. I can find an angle using trigonometric ratios.**
e.g. Find the value of θ, correct to the nearest degree.

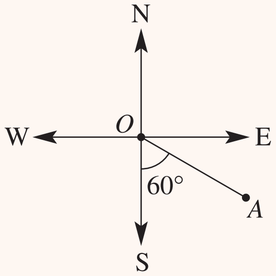


3I **10. I can apply trigonometry using an angle of elevation.** Ext
e.g. The angle of elevation of the top of a building from a point on the ground 25 m from the base of the building is 24°. Find the height of the building to the nearest metre.

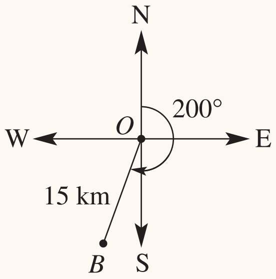


3I **11. I can apply trigonometry using an angle of depression.** Ext
e.g. A duck on a lake spots a fish below the surface. The fish is 8 m below water level and a horizontal direct distance of 6 m from the duck. Find the angle of depression of the fish from the duck to the nearest degree.

3J **12. I can state true bearings.** Ext
e.g. For the diagram shown, give the true bearing of *A* from *O* and the true bearing of *O* from *A*.



3J **13. I can use bearings with trigonometry.** Ext
e.g. A yacht sails from point *O* for 15 km on a bearing of 200° T to point *B*. How far west is the yacht from its starting point (to the nearest km) and what is the bearing of *O* from *B*?

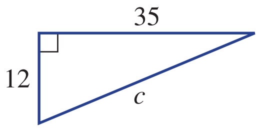


Short-answer questions

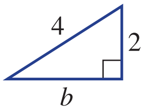
3A/B

1 Find the unknown length in these triangles. Give an exact answer.

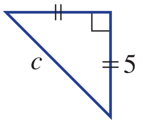
a



b

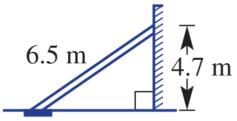


c



3B

2 A steel support beam of length 6.5 m is connected to a wall at a height of 4.7 m above the ground. Find the distance (to the nearest centimetre) between the base of the building and the point where the beam is joined to the ground.

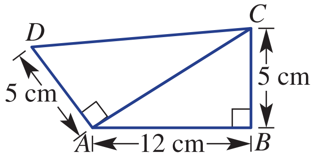


3A

3 For this double triangle, find:

a AC

b CD (correct to two decimal places).



3C

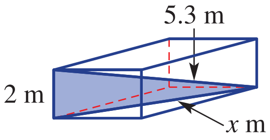
4 Two cafés on opposite sides of an atrium in a shopping centre are respectively 10 m and 15 m above the ground floor. If the cafés are linked by a 20 m escalator, find the horizontal distance (to the nearest metre) across the atrium, between the two cafés.

3D

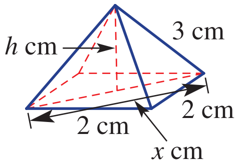
Ext

5 Find the values of the pronumerals in the three-dimensional objects shown below. Give the answer correct to two decimal places.

a



b



3F

6 Find the following values correct to two decimal places.

a sin 40°

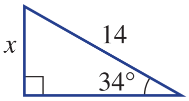
b tan 66°

c cos 44°

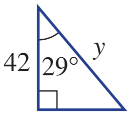
3F/G/H

7 Find the value of each pronumeral correct to two decimal places.

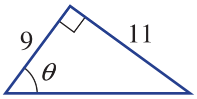
a



b



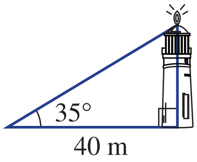
c



3I

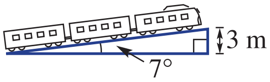
Ext

8 The angle of elevation of the top of a lighthouse from a point on the ground 40 m from its base is 35°. Find the height of the lighthouse to two decimal places.



3I Ext

9 A train travels up a slope that makes an angle of 7° with the horizontal. When the train is at a height of 3 m above its starting point, what is the distance it has travelled up the slope? Give the answer to the nearest metre.

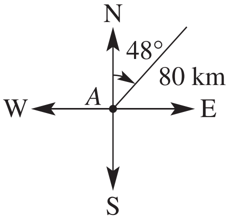


3J Ext

10 A yacht sails 80 km on a true bearing of 048°.

a How far east of its starting point is the yacht, correct to two decimal places?

b How far north of its starting point is the yacht, correct to two decimal places?



3I Ext

11 From a point on the ground, Geoff measures the angle of elevation of a 120 m tall tower to be 34°. How far is Geoff from the base of the tower, correct to two decimal places?

3J Ext

12 A ship leaves Coffs Harbour and sails 320 km east. It then changes direction and sails 240 km due north to its destination. What will the ship's true bearing be from Coffs Harbour when it reaches its destination, correct to two decimal places?

3I Ext

13 From the roof of a skyscraper, Aisha spots a car at an angle of depression of 51°. If the skyscraper is 78 m high, how far away is the car from the base of the skyscraper, correct to one decimal place?

3F

14 Penny wants to measure the width of a river. She places two markers, *A* and *B*, 10 m apart along one bank. *C* is a point directly opposite marker *B*. Penny measures angle *BAC* to be 28°. Find the width of the river to one decimal place.

3I Ext

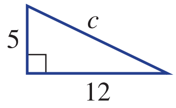
15 An aeroplane takes off and climbs at an angle of 15° to the horizontal, at a speed of 210 km/h along its flight path for 15 minutes.

a Find the distance the aeroplane travels.

b Find the height the aeroplane reaches, correct to two decimal places.

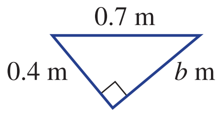
Multiple-choice questions

3A **1** For the right-angled triangle shown, the length of the hypotenuse can be found from the equation:



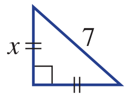
A $c^2 = 5^2 + 12^2$
B $c^2 = 5^2 - 12^2$
C $c^2 = 12^2 - 5^2$
D $c^2 = 5^2 \times 12^2$
E $(5 + 12)^2$

3B **2** For the right-angled triangle shown, the value of b is given by:



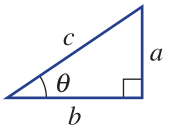
A $\sqrt{0.7^2 + 0.4^2}$
B $\sqrt{0.7^2 - 0.4^2}$
C $\sqrt{0.4^2 - 0.7^2}$
D $\sqrt{0.7^2 \times 0.4^2}$
E $\sqrt{(0.7 - 0.4^2)^2}$

3B **3** For the right-angled triangle shown:



A $x^2 = \frac{49}{2}$
B $7x^2 = 2$
C $x^2 = \frac{7}{2}$
D $x^2 + 7^2 = x^2$
E $x^2 = \frac{2}{7}$

3E **4** For the triangle shown:

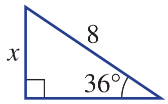


A $\sin\theta = \frac{a}{b}$
B $\sin\theta = \frac{c}{a}$
C $\sin\theta = \frac{a}{c}$
D $\sin\theta = \frac{b}{c}$
E $\sin\theta = \frac{c}{b}$

3F **5** The value of cos 46° correct to four decimal places is:

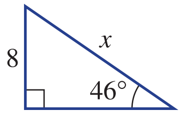
A 0.7193
B 0.6947
C 0.594
D 0.6532
E 1.0355

3F **6** In the diagram the value of x, correct to two decimal places, is:



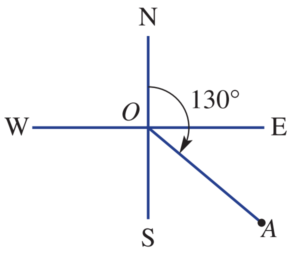
A 40
B 13.61
C 4.70
D 9.89
E 6.47

3G **7** The length of x in the triangle is given by:



A 8 sin 46°
B 8 cos 46°
C $\frac{8}{\cos 46°}$
D $\frac{8}{\sin 46°}$
E $\frac{\cos 46°}{8}$

3J Ext **8** The true bearing of A from O is 130°. The true bearing of O from A is:



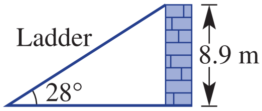
A 050°
B 220°
C 310°
D 280°
E 170°

ISBN 978-1-108-77288-4

Updated September 2021

3G

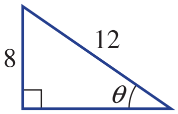
9 A ladder is inclined at an angle of 28° to the horizontal. If the ladder reaches 8.9 m up the wall, the length of the ladder correct to the nearest metre is:



A 19 m **B** 4 m **C** 2 m
D 10 m **E** 24 m

3H

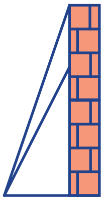
10 The value of θ in the diagram, correct to two decimal places, is:



A 0.73° **B** 41.81° **C** 48.19°
D 33.69° **E** 4.181°

Extended-response questions

1 An extension ladder is initially placed so that it reaches 2 m up a wall. The foot of the ladder is 80 cm from the base of the wall.



a Find the length of the ladder, to the nearest centimetre, in its original position.

b Without moving the foot, the ladder is extended so that it reaches 1 m further up the wall. How far (to the nearest centimetre) has the ladder been extended?

c The ladder is placed so that its foot is now 20 cm closer to the base of the wall.

- **i** How far up the wall can the extended ladder length found in part **b** reach? Round to two decimal places.
- **ii** Is this further than the distance in part **b**?

Ext

2 From the top of a 100 m cliff, Skevi sees a boat out at sea at an angle of depression of 12°.

a Draw a diagram for this situation.

b Find how far out to sea the boat is to the nearest metre.

c A swimmer is 2 km away from the base of the cliff and in line with the boat. What is the angle of depression to the swimmer, to the nearest degree?

d How far away is the boat from the swimmer, to the nearest metre?

Ext **3** A pilot takes off from Amber Island and flies for 150 km at 040° T to Barter Island where she unloads her first cargo. She intends to fly to Dream Island, but a bad thunderstorm between Barter and Dream islands forces her to fly off-course for 60 km to Crater Atoll on a bearing of 060° T . She then turns on a bearing of 140° T and flies for 100 km until she reaches Dream Island, where she unloads her second cargo. She then takes off and flies 180 km on a bearing of 055° T to Emerald Island.

a How many extra kilometres did she fly trying to avoid the storm? Round to the nearest kilometre.

b From Emerald Island she flies directly back to Amber Island. How many kilometres did she travel on her return trip? Round to the nearest kilometre.

CHAPTER 4

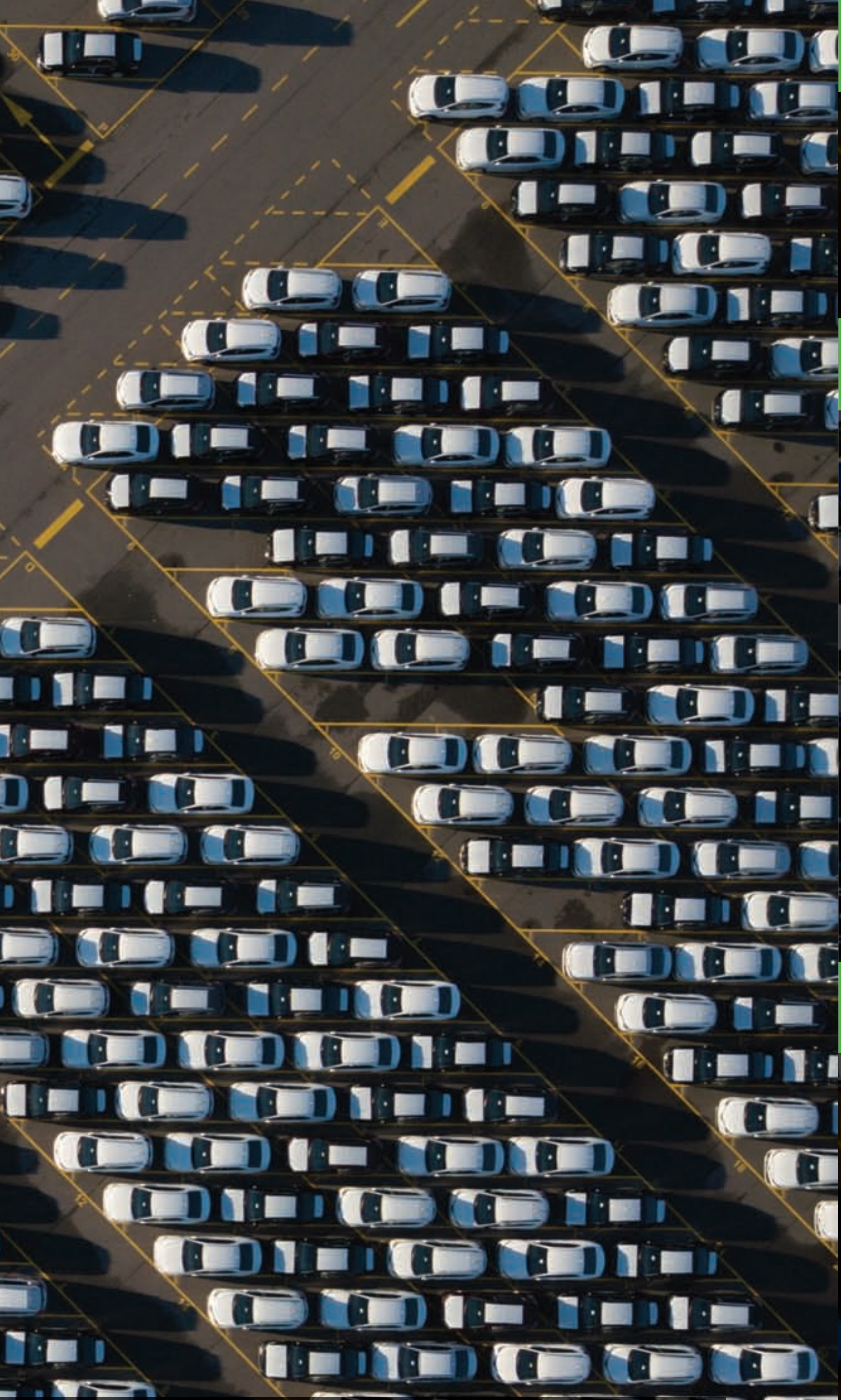
Linear relations

Supply and demand

A business can use linear relations to find the most suitable selling price to make both profit and sales.

For example, consider the production and sales of a hatchback car. If the selling price of this hatchback car is too low, there is no profit made. A high selling price brings profit, which enables the production of more cars. But of the customers who prefer this model, how many are willing to pay a higher price? Fewer sales also mean less profit. The selling price needs to be suitable for both the customer and the manufacturer.

Linear equations for Supply (i.e. production) and Demand (i.e. sales) relate P, the price of one car and n, the number of cars. The Supply graph has a positive slope since as P increases, n also increases. The Demand graph has a negative slope because as P increases, n decreases. The intersection point of these two lines occurs at the



selling price at which the number of cars produced equals the number of customers who will buy at that price.

Applying mathematical techniques to achieve minimal waste and maximum profit are skills used by an Operational Research Analyst. Areas of work include manufacturing, finance, retail, the airline industry, computer services, healthcare, transportation, mining and the military.

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

In this chapter

4A Introduction to linear relations (CONSOLIDATING)
4B Graphing straight lines using intercepts
4C Lines with one intercept
4D Gradient
4E Gradient and direct proportion
4F Gradient–intercept form
4G Finding the equation of a line
4H Midpoint and length of a line segment
4I Perpendicular and parallel lines (EXTENDING)
4J Linear modelling
4K Graphical solutions to simultaneous equations

Australian Curriculum

NUMBER AND ALGEBRA
Real numbers

Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems (ACMNA208)

Linear and non-linear relationships

Find the distance between two points located on a Cartesian plane using a range of strategies, including graphing software (ACMNA214)

Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software (ACMNA294)

Sketch linear graphs using the coordinates of two points and solve linear equations (ACMNA215)

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4A Introduction to linear relations CONSOLIDATING

Learning intentions

- To review the features of the Cartesian plane
- To understand that a linear relation is a set of ordered pairs that form a straight line when graphed
- To understand that the rule for a straight line graph gives the relationship between the x- and y-coordinates in the ordered pair
- To be able to use a linear rule to construct a table of ordered pairs and plot these on a Cartesian plane to form a straight line graph
- To be able to decide if a point is on a line using the line's rule
- To know what the x- and y-intercepts are and to be able to identify them on a graph or table

If two variables are related in some way, we can use mathematical rules to precisely describe this relationship. The most simple kind of mathematical relationship is one that can be illustrated with a straight line graph. These are called linear relations. The volume of petrol in your car at a service bowser, for example, might initially be 10 L and then be increasing by 1.2 L per second after that. This is an example of a linear relationship between *volume* and *time* because the volume is increasing at a constant rate of 1.2 L/s.

A car's annual running costs can be \$4000 p.a., including fuel, repairs, registration and insurance. If a car costs \$17 000, the total cost, C, for n years could be expressed as the linear relation: $C = 4000n + 17\,000$.

LESSON STARTER Is it linear?

Here are three rules linking x and y.

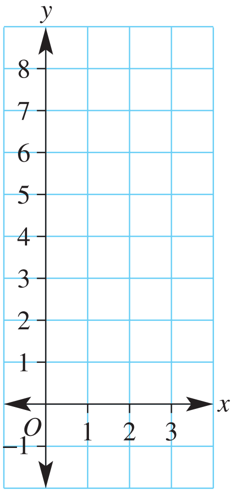
1 $y_1 = \frac{2}{x} + 1$

2 $y_2 = x^2 - 1$

3 $y_3 = 3x - 4$

First complete this simple table and graph.

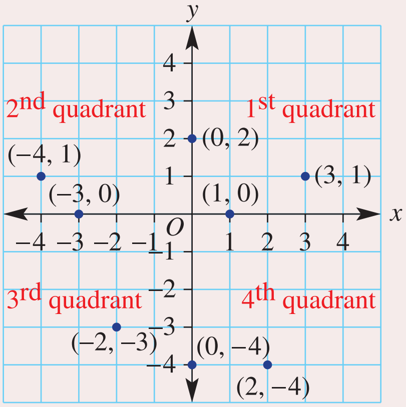
x	1	2	3
y_1			
y_2			
y_3			



- Which of the three rules do you think is linear?
- How do the table and graph help you decide if it is linear?

KEY IDEAS

- **Coordinate geometry** provides a link between geometry and algebra.
- The **Cartesian plane** (or number plane) consists of two axes which divide the number plane into four **quadrants**.
  - The horizontal x-axis and vertical y-axis intersect at the **origin** $(0, 0)$, titled O, at right angles.
  - A point is precisely positioned on a Cartesian plane using the **coordinate pair** (x, y) where x describes the horizontal position and y describes the vertical position of the point from the origin.

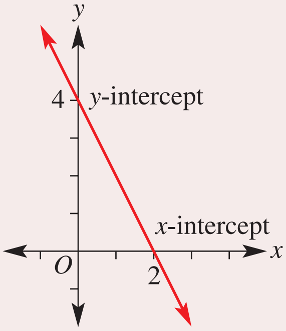


- A **linear relation** is a set of ordered pairs (x, y) that when graphed give a straight line.
- The x-intercept is the x-coordinate at the point where the graph cuts the x-axis.
- The y-intercept is the y-coordinate at the point where the graph cuts the y-axis.

x-intercept

x	−2	−1	0	1	2	3
y	8	6	4	2	0	−2

y-intercept



BUILDING UNDERSTANDING

① Refer to the Cartesian plane shown.

a Give the coordinates of all the points A–L.

b State which points lie on:

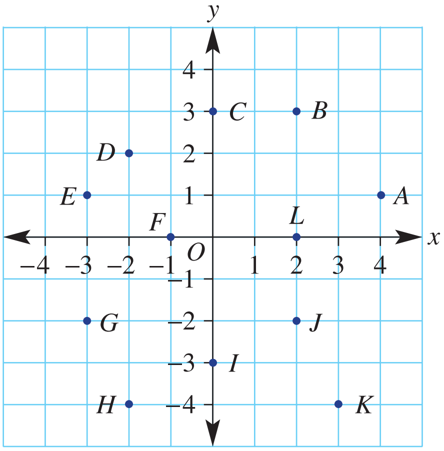
i the x-axis

ii the y-axis.

c State which points lie inside the:

i 2nd quadrant

ii 4th quadrant.



② For the rule $y = 3x - 4$, find the value of y for these x-values.

a $x = 2$

b $x = -1$

③ For the rule $y = -2x + 1$, find the value of y for these x-values.

a $x = 0$

b $x = -10$

④ Give the coordinates of three pairs of points on the line $y = x + 3$.

ISBN 978-1-108-77288-4

Example 1 Plotting points to graph straight lines

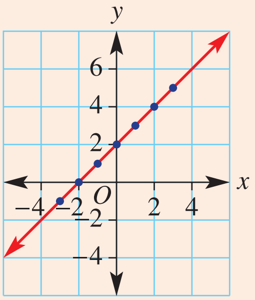
Using $-3 \leqslant x \leqslant 3$, construct a table of values and plot a graph for these linear relations.

a $y = x + 2$ **b** $y = -2x + 2$

SOLUTION

a

x	−3	−2	−1	0	1	2	3
y	−1	0	1	2	3	4	5



EXPLANATION

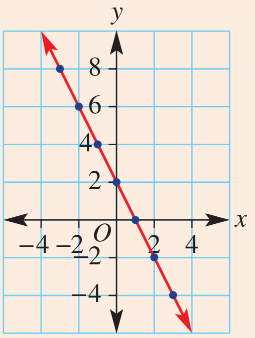
Use $-3 \leqslant x \leqslant 3$ as instructed and substitute each value of x into the rule $y = x + 2$.

The coordinates of the points are read from the table, i.e. $(-3, -1)$, $(-2, 0)$, etc.

Plot each point and join to form a straight line. Extend the line to show it continues in either direction.

b

x	−3	−2	−1	0	1	2	3
y	8	6	4	2	0	−2	−4



Use $-3 \leqslant x \leqslant 3$ as instructed and substitute each value of x into the rule $y = -2x + 2$.

For example:

$$\begin{aligned} x = -3,\ y &= -2 \times (-3) + 2 \\ &= 6 + 2 \\ &= 8 \end{aligned}$$

Plot each point and join to form a straight line. Extend the line beyond the plotted points.

Now you try

Using $-3 \leqslant x \leqslant 3$, construct a table of values and plot a graph for these linear relations.

a $y = x - 1$ **b** $y = -3x + 1$

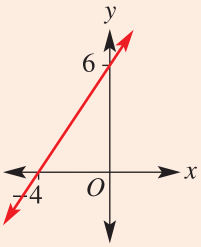
Example 2 Reading off the x-intercept and y-intercept

Write down the x-intercept and y-intercept from this table and graph.

a

x	–2	–1	0	1	2	3
y	6	4	2	0	–2	– 4

b



SOLUTION

a The x-intercept is 1.

The x-intercept is at the point where $y = 0$ (on the x-axis).

The y-intercept is 2.

The y-intercept is at the point where $x = 0$ (on the y-axis).

b The x-intercept is -4.

The x-intercept is at the point where $y = 0$ (on the x-axis).

The y-intercept is 6.

The y-intercept is at the point where $x = 0$ (on the y-axis).

EXPLANATION

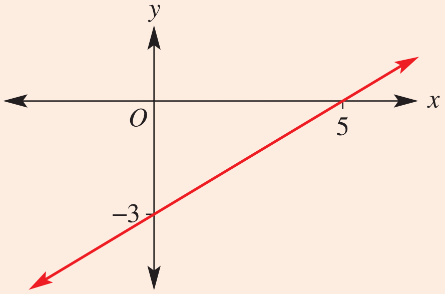
Now you try

Write down the x-intercept and y-intercept from this table and graph.

a

x	–3	–2	–1	0	1	2
y	4	3	2	1	0	–1

b



Example 3 Deciding if a point is on a line

Decide if the point $(-2, 4)$ is on the line with the given rules.

a $y = 2x + 10$ **b** $y = -x + 2$

SOLUTION	**EXPLANATION**
a $y = 2x + 10$	
Substitute $x = -2$ $y = 2(-2) + 10$ $= 6$	Find the value of y by substituting $x = -2$ into the rule for y.
$\therefore$ the point $(-2, 4)$ is not on the line.	The y-value is not 4, so $(-2, 4)$ is not on the line.
b $y = -x + 2$	
Substitute $x = -2$ $y = -(-2) + 2$ $= 4$	By substituting $x = -2$ into the rule for the line, y is 4.
$\therefore$ the point $(-2, 4)$ is on the line.	So $(-2, 4)$ is on the line.

Now you try

Decide if the point $(-1, 6)$ is on the line with the given rules.

a $y = -2x + 4$ **b** $y = 3x + 5$

Exercise 4A

FLUENCY 1, 2–4(½) 2–5(½) 2–5(½)

Example 1 **1** Using $-3 \leqslant x \leqslant 3$, construct a table of values and plot a graph for these linear relations.

a $y = x + 3$ **b** $y = -2x - 1$

Example 1 **2** Using $-3 \leqslant x \leqslant 3$, construct a table of values and plot a graph for these linear relations.

a $y = x - 1$ **b** $y = 2x - 3$ **c** $y = -x + 4$ **d** $y = -3x$

Example 2 **3** Write down the x- and y-intercepts for the following tables and graphs.

a

x	–3	–2	–1	0	1	2	3
y	4	3	2	1	0	1	2

b

x	–3	–2	–1	0	1	2	3
y	–1	0	1	2	3	4	5

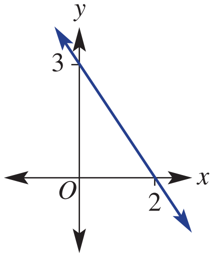
c

x	–1	0	1	2	3	4
y	10	8	6	4	2	0

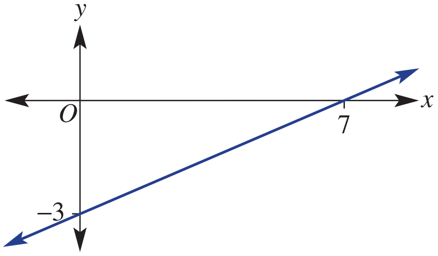
d

x	–5	–4	–3	–2	–1	0
y	0	2	4	6	8	10

e



f



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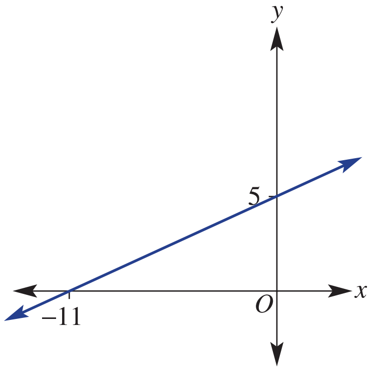

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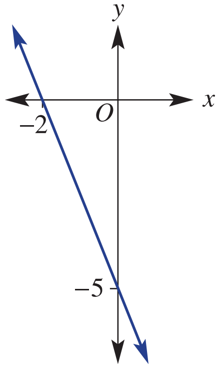


Example 3

4 Decide if the point (1, 2) is on the line with the given rule.

a $y = x + 1$ **b** $y = 2x - 1$ **c** $y = -x + 3$

d $y = -2x + 4$ **e** $y = -x + 5$ **f** $y = -\frac{1}{2}x + \frac{1}{2}$

Example 3

5 Decide if the point (–3, 4) is on the line with the given rule.

a $y = 2x + 8$ **b** $y = x + 7$ **c** $y = -x - 1$

d $y = \frac{1}{3}x + 6$ **e** $y = -\frac{1}{3}x + 3$ **f** $y = \frac{5}{6}x + \frac{11}{2}$

PROBLEM-SOLVING 6 6, 7 6(½), 7

6 Find a rule in the form $y = mx + c$ (e.g. $y = 2x - 1$) that matches these tables of values.

a

x	0	1	2	3	4
y	2	3	4	5	6

b

x	–1	0	1	2	3
y	–2	0	2	4	6

c

x	–2	–1	0	1	2	3
y	–3	–1	1	3	5	7

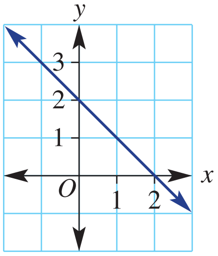
d

x	–3	–2	–1	0	1	2
y	5	4	3	2	1	0

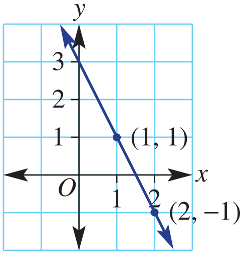
7 Match rules **a**, **b**, **c** and **d** to the graphs **A**, **B**, **C** and **D**.

a $y = 2x + 1$ **b** $y = -x - 1$ **c** $y = -2x + 3$ **d** $x + y = 2$

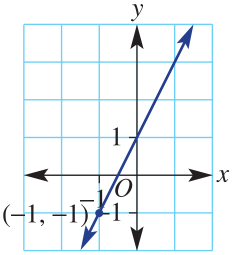
A



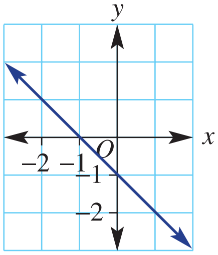
B



C



D



REASONING

8	8, 9	9, 10

8 Decide if the following equations are true or false.

a $\frac{2x + 4}{2} = x + 4$

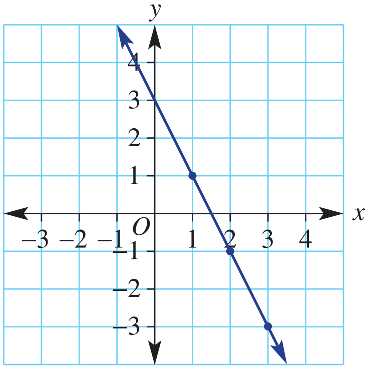
b $\frac{3x - 6}{3} = x - 2$

c $\frac{1}{2}(x - 1) = \frac{1}{2}x - \frac{1}{2}$

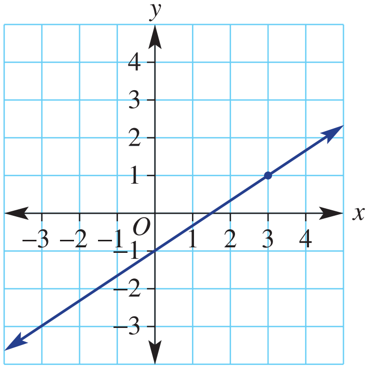
d $\frac{2}{3}(x - 6) = \frac{2}{3}x - 12$

9 Give reasons why the x-intercept on these graphs is $\frac{3}{2}$.

a



b



10 Decide if the following rules are equivalent.

a $y = 1 - x$ and $y = -x + 1$

b $y = 1 - 3x$ and $y = 3x - 1$

c $y = -2x + 1$ and $y = -1 - 2x$

d $y = -3x + 1$ and $y = 1 - 3x$

ENRICHMENT: Tough rule finding

–	–	11

11 Find the linear rule linking x and y in these tables.

a

x	−1	0	1	2	3
y	5	7	9	11	13

b

x	−2	−1	0	1	2
y	22	21	20	19	18

c

x	0	2	4	6	8
y	−10	−16	−22	−28	−34

d

x	−5	−4	−3	−2	−1
y	29	24	19	14	9

e

x	1	3	5	7	9
y	1	2	3	4	5

f

x	−14	−13	−12	−11	−10
y	$5\frac{1}{2}$	5	$4\frac{1}{2}$	4	$3\frac{1}{2}$

4B Graphing straight lines using intercepts

Learning intentions

- To know that at most two points are required to sketch a straight line graph
- To know how to find the *x*- and *y*-intercepts from the rule for a linear graph
- To be able to sketch a linear graph by finding intercepts and joining in a line

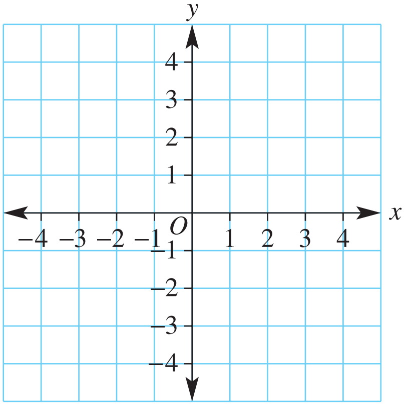
When linear rules are graphed, all the points lie in a straight line. It is therefore possible to graph a straight line using only two points. Two critical points that help draw these graphs are the *x*-intercept and *y*-intercept introduced in the previous section.

Businesses claim a tax deduction for the annual decrease in the value, *V*, of equipment over *n* years. A straight line segment joining the intercepts (0, *V*) and (*n*, 0) shows that the rate of decrease per year is $\frac{V}{n}$.

LESSON STARTER Two key points

Consider the relation $y = \frac{1}{2}x + 1$ and complete this table and graph.

x	–4	–3	–2	–1	0	1	2
y							

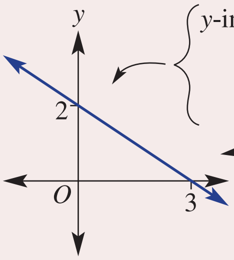


- What are the coordinates of the point where the line crosses the *y*-axis? That is, state the coordinates of the *y*-intercept.
- What are the coordinates of the point where the line crosses the *x*-axis? That is, state the coordinates of the *x*-intercept.
- Discuss how you might find the coordinates of the *x*- and *y*-intercepts without drawing a table and plotting points. Explain your method.

KEY IDEAS

- Two points are required to sketch a straight line graph. Often these points are the axes intercepts.
- The ***y*-intercept** is the *y*-value at the point on the *y*-axis where $x = 0$.
  - Substitute $x = 0$ to find the *y*-intercept.
- The ***x*-intercept** is the *x*-value at the point on the *x*-axis where $y = 0$.
  - Substitute $y = 0$ to find the *x*-intercept.

Example: $2x + 3y = 6$



y-intercept ($x = 0$)
$$2(0) + 3y = 6$$
$$3y = 6$$
$$y = 2$$

x-intercept ($y = 0$)
$$2x + 3(0) = 6$$
$$2x = 6$$
$$x = 3$$

BUILDING UNDERSTANDING

1 Mark the following x- and y-intercepts on a set of axes and join in a straight line.

a x-intercept $= -2$, y-intercept $= 3$ **b** x-intercept $= 4$, y-intercept $= 6$

2 **a** Find the value of y in these equations.

i $2y = 6$ **ii** $y = 3 \times 0 + 4$ **iii** $-2y = 12$

b Find the value of x in these equations.

i $-4x = -40$ **ii** $0 = 2x - 2$ **iii** $\frac{1}{2}x = 3$

3 For these equations find the y-intercept by letting $x = 0$.

a $x + y = 4$ **b** $x - y = 5$ **c** $2x + 3y = 9$

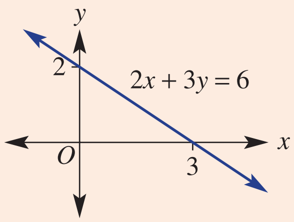
4 For these equations find the x-intercept by letting $y = 0$.

a $2x - y = -4$ **b** $4x - 3y = 12$ **c** $y = 3x - 6$

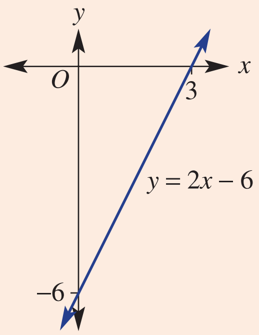
Example 4 Sketching with intercepts

Sketch the graph of the following, showing the x- and y-intercepts.

a $2x + 3y = 6$ **b** $y = 2x - 6$

SOLUTION	EXPLANATION
a $2x + 3y = 6$ y-intercept (let $x = 0$): $2(0) + 3y = 6$ $3y = 6$ $y = 2$	Only two points are required to generate a straight line. For the y-intercept, substitute $x = 0$ into the rule and solve for y by dividing each side by 3.
$\therefore$ the y-intercept is 2.	State the y-intercept. The coordinates are (0, 2).
x-intercept (let $y = 0$): $2x + 3(0) = 6$ $2x = 6$ $x = 3$	Similarly to find the x-intercept, substitute $y = 0$ into the rule and solve for x.
$\therefore$ the x-intercept is 3.	State the x-intercept. The coordinates are (3, 0).
	Mark and label the intercepts on the axes and sketch the graph by joining the two intercepts.

b $y = 2x - 6$ y-intercept (let $x = 0$):	
$y = 2(0) - 6$ $y = -6$	Substitute $x = 0$ for the y-intercept. Simplify to find the y-coordinate.
$\therefore$ the y-intercept is -6.	The coordinates are $(0, -6)$.
x-intercept (let $y = 0$): $0 = 2x - 6$ $6 = 2x$ $x = 3$	Substitute $y = 0$ for the x-intercept. Solve the remaining equation for x by adding 6 to both sides and then divide both sides by 2.
$\therefore$ the x-intercept is 3.	The coordinates are $(3, 0)$.
	Mark in the two intercepts and join to sketch the graph.



Now you try

Sketch the graph of the following, showing the x- and y-intercepts.

a $3x + 5y = 15$ **b** $y = 3x - 6$

Exercise 4B

FLUENCY	1, 2–3($\frac{1}{2}$)	2–4($\frac{1}{2}$)	2–4($\frac{1}{3}$)

Example 4 **1** Sketch the graph of the following, showing the x- and y-intercepts.

a $x + y = 5$ **b** $y = 2x - 3$

Example 4a **2** Sketch the graph of the following relations, by finding the x- and y-intercepts.

a $x + y = 2$ **b** $x - y = -2$ **c** $2x + y = 4$
d $3x - y = 9$ **e** $4x - 2y = 8$ **f** $3x + 2y = 6$
g $y - 3x = 12$ **h** $-5y + 2x = -10$ **i** $-x + 7y = 21$

Example 4b **3** Sketch the graph of the following relations, showing the x- and y-intercepts.

a $y = 3x + 3$ **b** $y = 2x + 2$ **c** $y = x - 5$
d $y = -x - 6$ **e** $y = -2x - 2$ **f** $y = -2x + 4$

4 Sketch the graph of each of the following mixed linear relations.

a $x + 2y = 8$ **b** $3x - 5y = 15$ **c** $y = 3x - 6$
d $3y - 4x = 12$ **e** $2x - y - 4 = 0$ **f** $2x - y + 5 = 0$

PROBLEM-SOLVING	5, 6	5, 6	6, 7

5 The distance d metres of a vehicle from an observation point after t seconds is given by the rule $d = 8 - 2t$.

a Find the distance from the observation point initially (at $t = 0$).

b Find after what time t the distance d is equal to 0 (substitute $d = 0$).

c Sketch a graph of d versus t between the d and t intercepts.

6 The height, h metres, of a lift above the ground after t seconds is given by $h = 100 - 8t$.

a How high is the lift initially (at $t = 0$)?

b How long does it take for the lift to reach the ground ($h = 0$)?

7 Find the x- and y-axis intercepts of the graphs with the given rules. Write answers using fractions.

a $3x - 2y = 5$
b $x + 5y = -7$
c $y - 2x = -13$
d $y = -2x - 1$
e $2y = x - 3$
f $-7y = 1 - 3x$

REASONING	8	8, 9	9, 10

8 Use your algebra and fraction skills to help sketch graphs for these relations by finding x- and y-intercepts.

a $\frac{x}{2} + \frac{y}{3} = 1$
b $y = \frac{8 - x}{4}$
c $\frac{y}{2} = \frac{2 - 4x}{8}$

9 Explain why the graph of the equation $ax + by = 0$ must pass through the origin for any values of the constants a and b.

10 Write down the rule for the graph with these axes intercepts. Write the rule in the form $ax + by = d$.

a $(0, 4)$ and $(4, 0)$

b $(0, 2)$ and $(2, 0)$

c $(0, -3)$ and $(3, 0)$

d $(0, 1)$ and $(-1, 0)$

e $(0, k)$ and $(k, 0)$

f $(0, -k)$ and $(-k, 0)$

ENRICHMENT: Intercept families	–	–	11

11 Find the x- and y-intercepts in terms of the constants a, b and c for these relations.

a $ax + by = c$
b $y = \frac{a}{b}x + c$
c $\frac{ax - by}{c} = 1$
d $ay - bx = c$
e $ay = bx + c$
f $a(x + y) = bc$

Using CAS and non-CAS calculators 4B: Sketching straight lines

This activity is in the Interactive Textbook in the form of a printable PDF.

4C Lines with one intercept

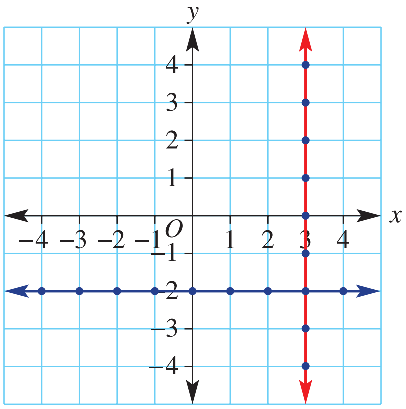
Learning intentions

- To know that vertical and horizontal lines and lines through the origin have only one intercept
- To know the equation form of both vertical and horizontal lines
- To be able to sketch vertical and horizontal lines
- To know the form of a linear equation that passes through the origin
- To understand that a second point is needed to sketch lines that pass through the origin
- To be able to sketch lines that pass through the origin

Lines with one intercept include vertical lines, horizontal lines and lines that pass through the origin.

LESSON STARTER What rule satisfies all points?

Here is one vertical and one horizontal line.



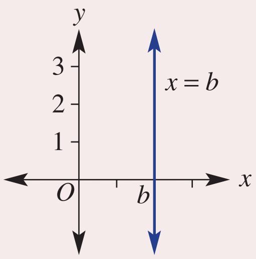
Linear relationships with graphs through (0, 0) include:

- fertiliser in kg = usage rate in kg/acre × number of acres;
- annual interest = interest rate p.a. × amount invested;
- weekly pay = pay rate in $/h × number of hours.

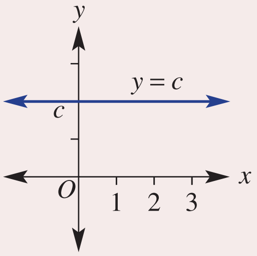
- For the vertical line shown, write down the coordinates of all the points shown as dots.
- What is always true for each coordinate pair?
- What simple equation describes every point on the line?
- For the horizontal line shown, write down the coordinates of all the points shown as dots.
- What is always true for each coordinate pair?
- What simple equation describes every point on the line?
- Where do the two lines intersect?

KEY IDEAS

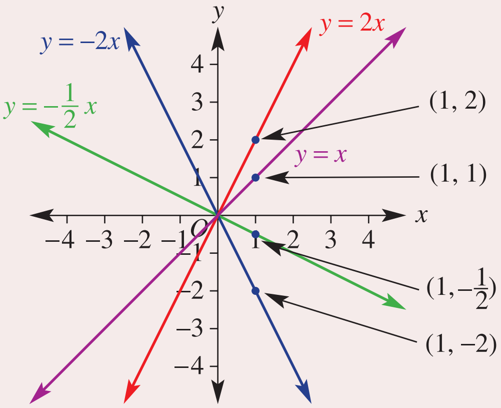
- Vertical line: $x = b$
  - Parallel to the y-axis
  - Equation of the form $x = b$, where b is a constant
  - x-intercept is b



- Horizontal line: $y = c$
  - Parallel to the x-axis
  - Equation of the form $y = c$, where c is a constant
  - y-intercept is c



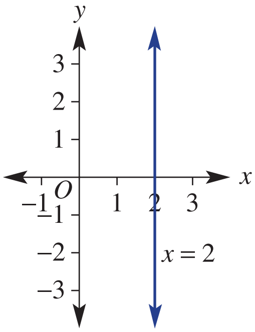
- Lines through the origin (0, 0): $y = mx$
  - y-intercept is 0
  - x-intercept is 0
  - Substitute $x = 1$ or any other value of x to find a second point



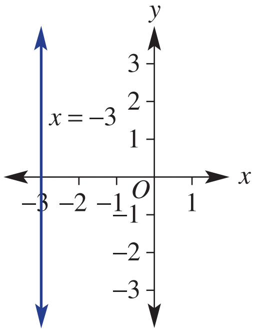
BUILDING UNDERSTANDING

① State the x-intercept for these graphs.

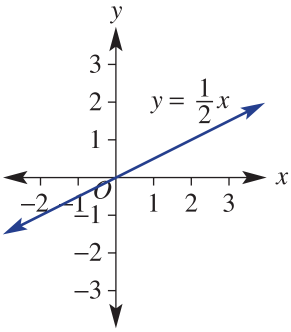
a



b

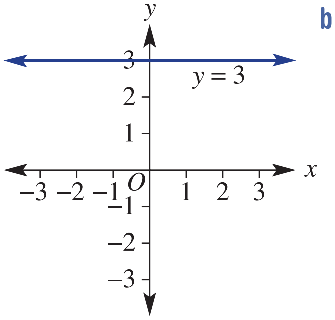


c



② State the y-intercept for these graphs.

a



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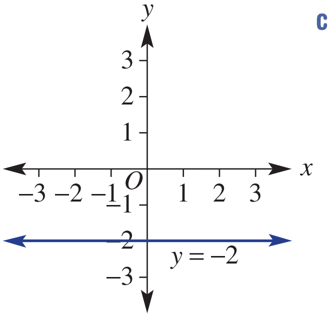
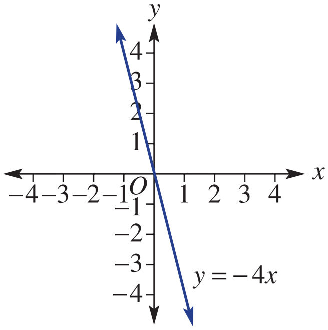


c



③ Find the value of y if $x = 1$ using these rules.

a $y = 5x$ **b** $y = \frac{1}{3}x$ **c** $y = -4x$ **d** $y = -0.1x$

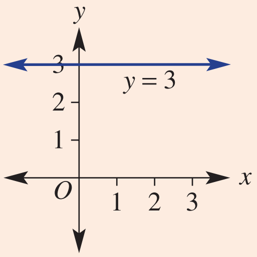
Example 5 Graphing vertical and horizontal lines

Sketch the graph of the following horizontal and vertical lines.

a $y = 3$ **b** $x = -4$

SOLUTION

a

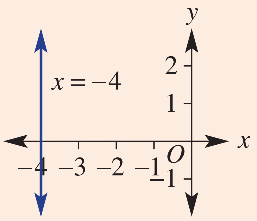


EXPLANATION

y-intercept is 3.

Sketch a horizontal line through all points where $y = 3$.

b



x-intercept is -4.

Sketch a vertical line through all points where $x = -4$.

Now you try

Sketch the graph of the following horizontal and vertical lines.

a $y = -1$ **b** $x = 3$

Example 6 Sketching lines that pass through the origin

Sketch the graph of $y = 3x$.

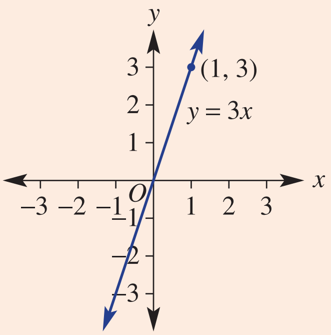
SOLUTION

The x- and the y-intercept are both 0.

Another point (let $x = 1$):

$$y = 3 \times (1)$$
$$y = 3$$

Another point is at $(1, 3)$.



EXPLANATION

The equation is of the form $y = mx$.

As two points are required to generate the straight line, find another point by substituting $x = 1$.

Other x-values could also be used.

Plot and label both points and sketch the graph by joining the points in a straight line.

Now you try

Sketch the graph of $y = -2x$.

Exercise 4C

FLUENCY	1, 2–5(1/2)	2–5(1/2)	2–5(1/2)

Example 5

1 Sketch the graph of the following horizontal and vertical lines.

a $y = 4$ **b** $x = -5$

Example 5

2 Sketch the graph of the following horizontal and vertical lines.

a $x = 2$ **b** $x = 5$ **c** $y = 4$ **d** $y = 1$

e $x = -3$ **f** $x = -2$ **g** $y = -1$ **h** $y = -3$

Example 6

3 Sketch the graph of the following linear relations that pass through the origin.

a $y = 2x$ **b** $y = 5x$ **c** $y = 4x$ **d** $y = x$

e $y = -4x$ **f** $y = -3x$ **g** $y = -2x$ **h** $y = -x$

4 Sketch the graphs of these special lines all on the same set of axes and label with their equations.

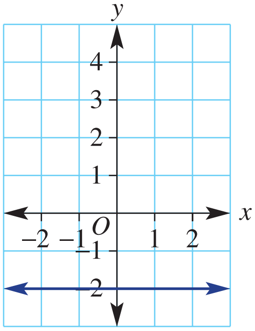
a $x = -2$ **b** $y = -3$ **c** $y = 2$ **d** $x = 4$

e $y = 3x$ **f** $y = -\frac{1}{2}x$ **g** $y = -1.5x$ **h** $x = 0.5$

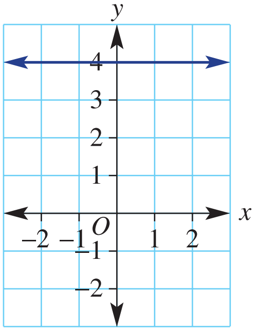
i $x = 0$ **j** $y = 0$ **k** $y = 2x$ **l** $y = 1.5x$

5 Give the equation of each of the following graphs.

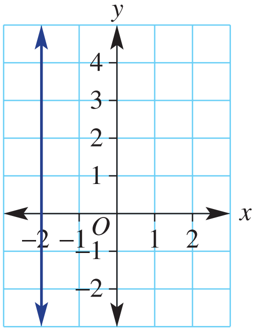
a



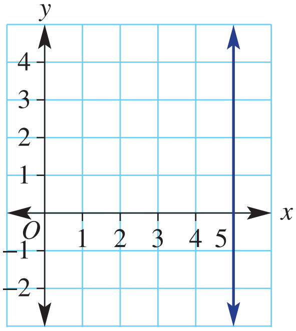
b



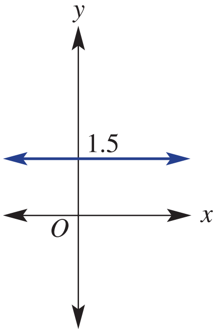
c



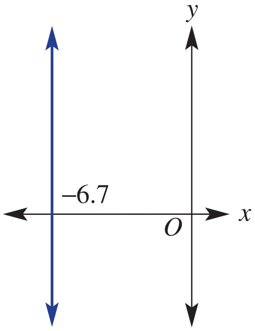
d



e



f



PROBLEM-SOLVING	6, 7	6(1/2), 7, 8(1/2), 9	6(1/2), 8(1/2), 9, 10

6 Find the equation of the straight line that is:

a parallel to the x-axis and passes through the point (1, 3)

b parallel to the y-axis and passes through the point (5, 4)

c parallel to the y-axis and passes through the point (−2, 4)

d parallel to the x-axis and passes through the point (0, 0).

7 If the surface of the sea is represented by the x-axis, state the equation of the following paths.

- **a** A plane flies horizontally at 250 m above sea level. One unit is 1 metre.
- **b** A submarine travels horizontally 45 m below sea level. One unit is 1 metre.

8 The graphs of these pairs of equations intersect at a point. Find the coordinates of the point.

- **a** $x = 1, y = 2$
- **b** $x = -3, y = 5$
- **c** $x = 0, y = -4$
- **d** $x = 4, y = 0$
- **e** $y = -6x, x = 0$
- **f** $y = 3x, x = 1$
- **g** $y = -9x, x = 3$
- **h** $y = 8x, y = 40$

9 Find the area of the rectangle contained within the following four lines.

- **a** $x = 1, x = -2, y = -3, y = 2$
- **b** $x = 0, x = 17, y = -5, y = -1$

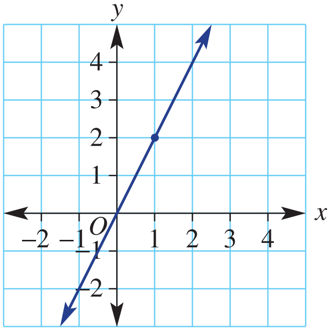
10 The lines $x = -1$, $x = 3$ and $y = -2$ form three sides of a rectangle. Find the possible equation of the fourth line if:

- **a** the area of the rectangle is:
  - **i** 12 square units
  - **ii** 8 square units
  - **iii** 22 square units.
- **b** the perimeter of the rectangle is:
  - **i** 14 units
  - **ii** 26 units
  - **iii** 31 units.

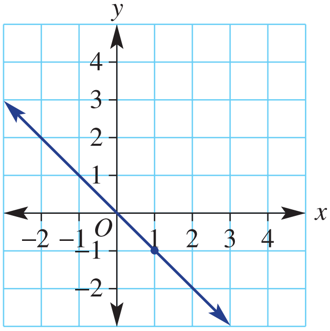
REASONING	11	11–12($\frac{1}{2}$)	11–13($\frac{1}{2}$)

11 The rules of the following graphs are of the form $y = mx$. Use the points marked with a dot to find m and hence state the equation.

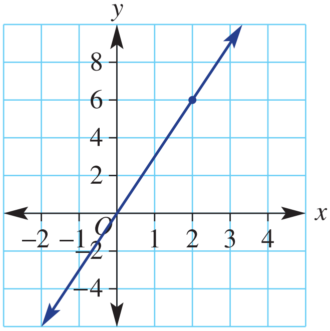
a



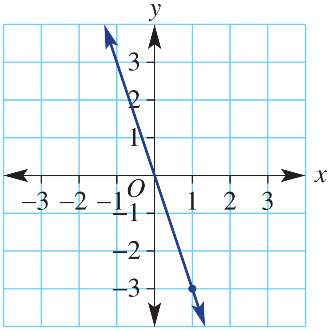
b



c



d



12 Find the equation of the line that passes through the origin and the given point.

a $(1, 3)$ **b** $(1, 4)$ **c** $(1, -5)$ **d** $(1, -2)$

13 Sketch the graph of each of the following by first making y or x the subject.

a $y - 8 = 0$ **b** $x + 5 = 0$

c $x + \frac{1}{2} = 0$ **d** $y - 0.6 = 0$

e $y + 3x = 0$ **f** $y - 5x = 0$

g $2y - 8x = 0$ **h** $5y + 7x = 0$

ENRICHMENT: Trisection	–	–	14–16

14 A vertical line, horizontal line and another line that passes through the origin all intersect at $(-1, -5)$. What are the equations of the three lines?

15 The lines $y = c$, $x = b$ and $y = mx$ all intersect at one point.

a State the coordinates of the intersection point.

b Find m in terms of c and b.

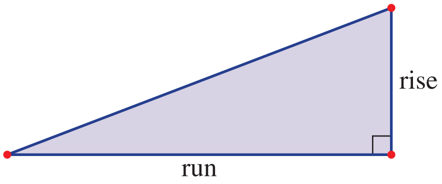
16 The area of a triangle formed by $x = 4$, $y = -2$ and $y = mx$ is 16 square units. Find the value of m given $m > 0$.

4D Gradient

Learning intentions

- To understand that the gradient is the ratio of the vertical change of a graph to its horizontal change between two points
- To understand that the gradient of a straight line is constant
- To know that the gradient can be positive, negative, zero or undefined
- To be able to find the gradient of a line using a graph or two given points

The gradient of a line is a measure of its slope. It is a number that describes the steepness of a line and is calculated by considering how far a line rises or falls between two points within a given horizontal distance. The horizontal distance between two points is called the *run* and the vertical distance is called the *rise*.

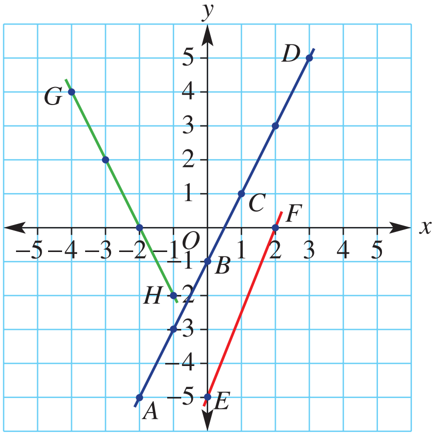


The gradient of a wheelchair ramp is vital for safety and ease of access. The Australian building code for wheelchair ramps requires a maximum gradient of $\frac{1}{14}$, i.e. a 1 m rise for every 14 m of horizontal run.

LESSON STARTER Which line is the steepest?

The three lines shown below right connect the points *A*–*H*.

- Calculate the rise and run (working from left to right) and also the fraction $\frac{\text{rise}}{\text{run}}$ for these segments.
  - **a** *AB*
  - **b** *BC*
  - **c** *BD*
  - **d** *EF*
  - **e** *GH*
- What do you notice about the fractions $\frac{\text{rise}}{\text{run}}$ for parts **a**, **b** and **c**?
- How does the $\frac{\text{rise}}{\text{run}}$ for *EF* compare with the $\frac{\text{rise}}{\text{run}}$ for parts **a**, **b** and **c**? Which of the two lines is the steeper?
- Your $\frac{\text{rise}}{\text{run}}$ for *GH* should be negative. Why is this the case?
- Discuss whether or not *GH* is steeper than *AD*.

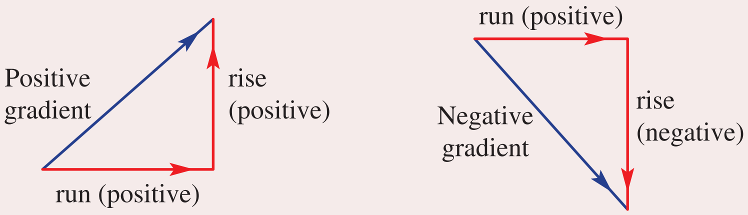


Use computer software (interactive geometry) to produce a set of axes and grid.

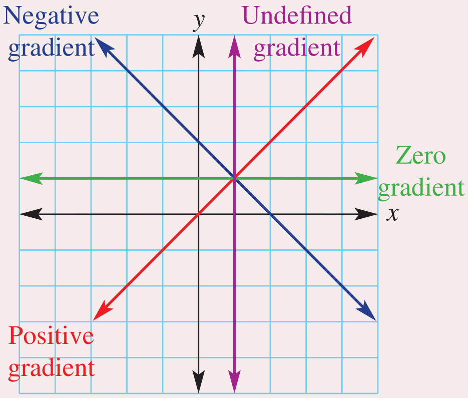
- Construct a line segment with endpoints on the grid. Show the coordinates of the endpoints.
- Calculate the rise (vertical distance between the endpoints) and the run (horizontal distance between the endpoints).
- Calculate the gradient as the *rise* divided by the *run*.
- Now drag the endpoints and explore the effect on the gradient.
- Can you drag the endpoints but retain the same gradient value? Explain why this is possible.
- Can you drag the endpoints so that the gradient is zero or undefined? Describe how this can be achieved.

KEY IDEAS

- **Gradient** $(m) = \frac{\text{rise}}{\text{run}}$



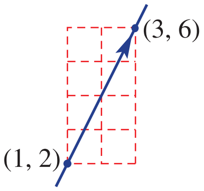
- When we work from left to right, the run is always positive.
- The gradient can be positive, negative, zero or undefined.
- A vertical line has an undefined gradient.



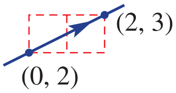
BUILDING UNDERSTANDING

① Calculate the gradient using $\frac{\text{rise}}{\text{run}}$ for these lines. Remember to give a negative answer if the line is sloping downwards from left to right.

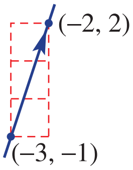
a



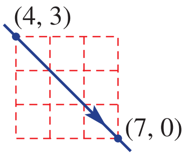
b



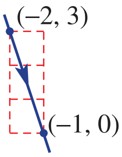
c



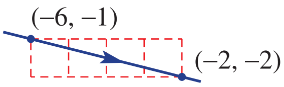
d



e



f



② Use the words: positive, negative, zero or undefined to complete each sentence.

a The gradient of a horizontal line is ___________.

b The gradient of the line joining (0, 3) with (5, 0) is ___________.

c The gradient of the line joining (−6, 0) with (1, 1) is ___________.

d The gradient of a vertical line is ___________.

③ Use the gradient $m\left(=\frac{\text{rise}}{\text{run}}\right)$ to find the missing value.

a $m = 4$, rise $= 8$, run $=$?

b $m = 6$, rise $= 3$, run $=$?

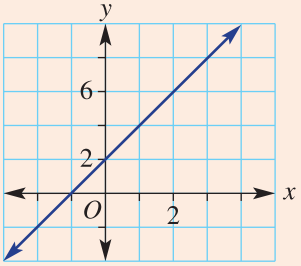
c $m = \frac{3}{2}$, run $= 4$, rise $=$?

d $m = -\frac{2}{5}$, run $= 15$, rise $=$?

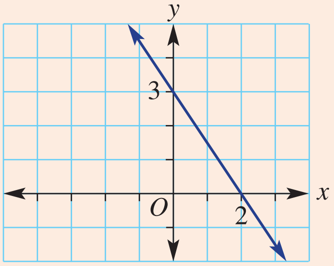
Example 7 Finding the gradient of a line

For each graph, state whether the gradient is positive, negative, zero or undefined, and then find the gradient where possible.

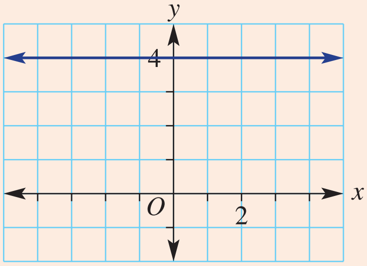
a



b



c



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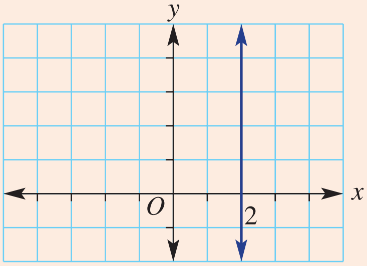
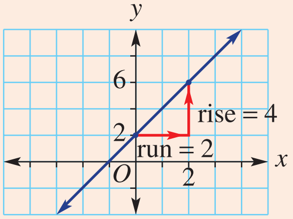


SOLUTION

EXPLANATION

a The gradient is positive.

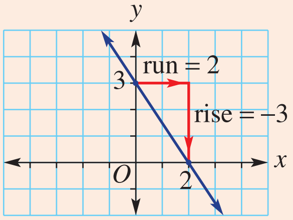


$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{4}{2} = 2$$

By inspection, the gradient will be positive since the graph rises from left to right. Select any two points and create a right-angled triangle to determine the rise and run.

Substitute rise = 4 and run = 2.

b The gradient is negative.



$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{-3}{2} = -\frac{3}{2}$$

By inspection, the gradient will be negative since y-values decrease from left to right.

Rise = −3 and run = 2.

c The gradient is 0.

The line is horizontal.

d The gradient is undefined.

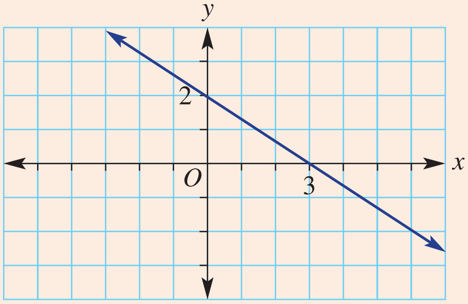
The line is vertical.

Continued on next page

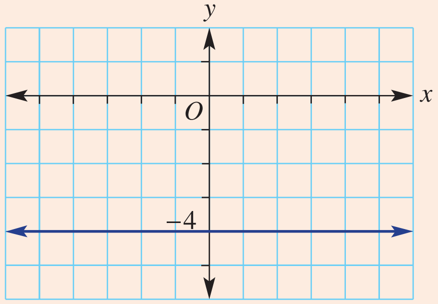
Now you try

For each graph, state whether the gradient is positive, negative, zero or undefined, then find the gradient where possible.

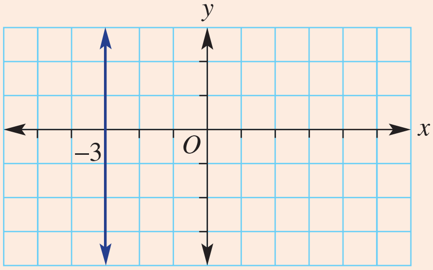
a



b



c



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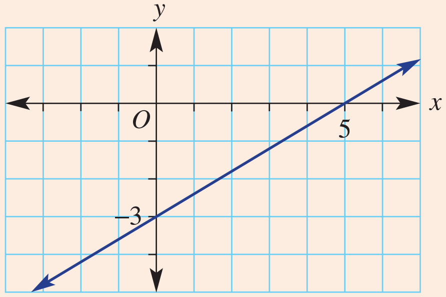


Example 8 Finding the gradient between two points

Find the gradient (m) of the line joining the given points.

a $A(3, 4)$ and $B(5, 6)$

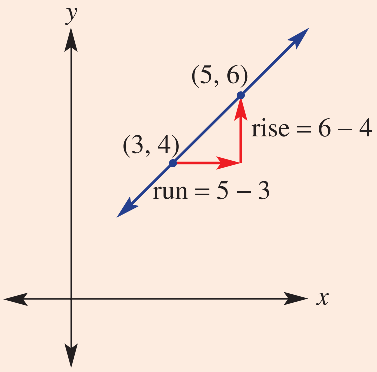
b $A(-3, 6)$ and $B(1, -3)$

SOLUTION

a
$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{6-4}{5-3} \\ &= \frac{2}{2} \\ &= 1 \end{aligned}$$

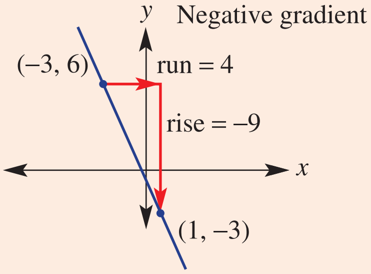
EXPLANATION

Plot points to see a positive gradient and calculate rise and run.



b $m = \frac{\text{rise}}{\text{run}}$

$= \frac{-9}{4}$ or $-\frac{9}{4}$ or -2.25



Now you try

Find the gradient (m) of the line joining the given points.

a $A(2, 6)$ and $B(3, 8)$ **b** $A(-5, 1)$ and $B(2, -3)$

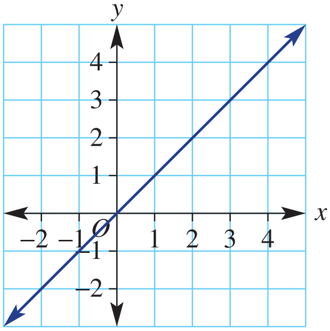
Exercise 4D

FLUENCY	1, 2, 3(1/2)	2, 3(1/2)	2–3(1/2), 4

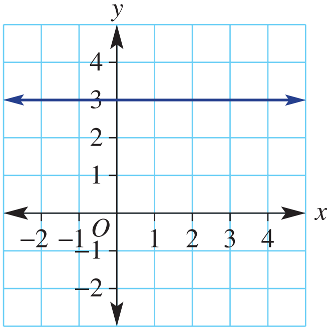
Example 7

1 For each graph, state whether the gradient is positive, negative, zero or undefined, and then find the gradient where possible.

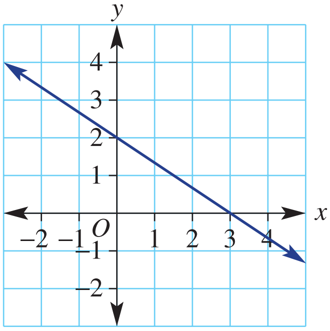
a



b



c



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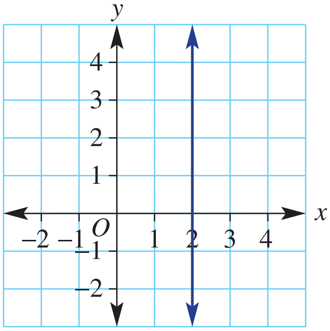
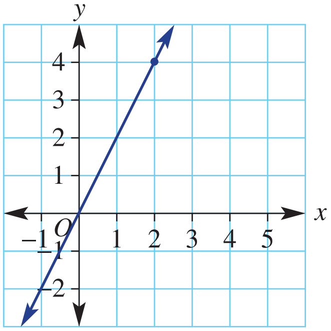


Example 7

2 For each graph state whether the gradient is positive, negative, zero or undefined, and then find the gradient where possible.

a



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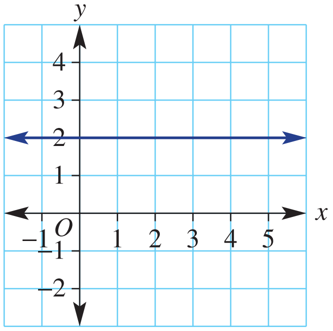

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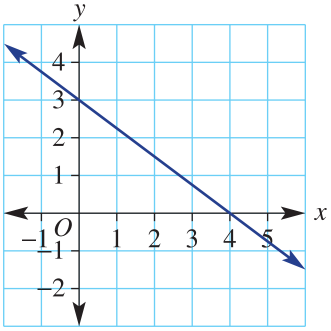

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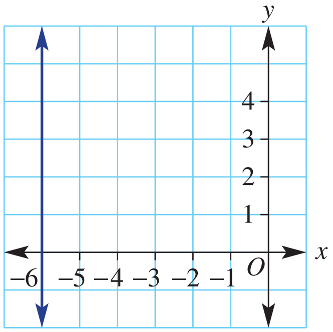
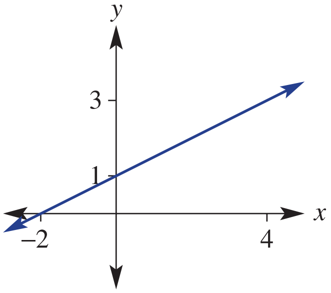
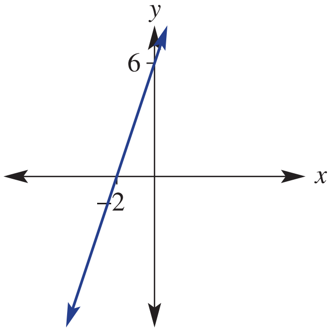


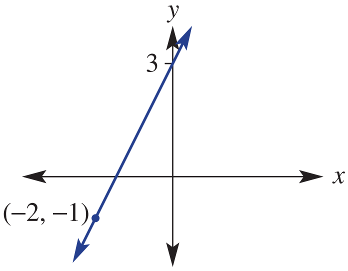
e



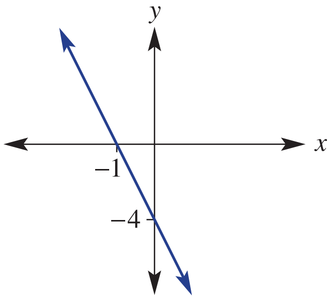
f



g



h

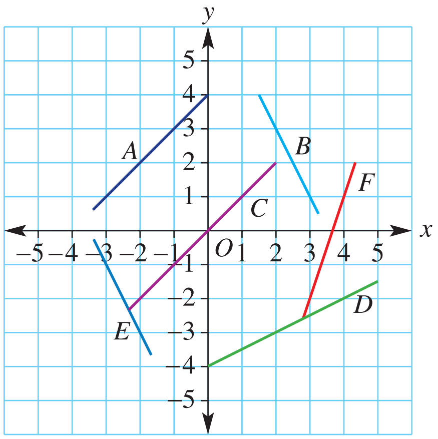


Example 8

3 Find the gradient of the lines joining the following pairs of points.

- **a** $A(2, 3)$ and $B(3, 5)$
- **b** $C(-2, 6)$ and $D(0, 8)$
- **c** $E(-4, 1)$ and $F(4, -1)$
- **d** $G(2, 1)$ and $H(5, 5)$
- **e** $A(1, 5)$ and $B(2, 7)$
- **f** $C(-2, 4)$ and $D(1, -2)$
- **g** $E(-3, 4)$ and $F(2, -1)$
- **h** $G(-1, 5)$ and $H(1, 6)$
- **i** $A(-2, 1)$ and $B(-4, -2)$
- **j** $C(3, -4)$ and $D(1, 1)$
- **k** $E(3, 2)$ and $F(0, 1)$
- **l** $G(-1, 1)$ and $H(-3, -4)$

4 Find the gradient of the lines A–F on this graph and grid.



PROBLEM-SOLVING			
	5, 6	5–7	6–8

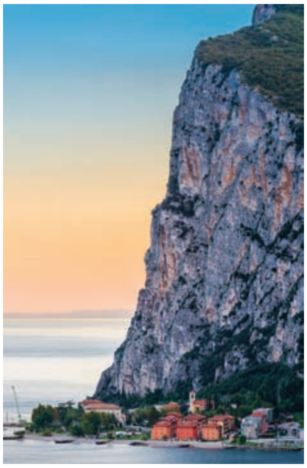
5 Find the gradient corresponding to the following slopes.

a A road falls 10 m for every 200 horizontal metres.

b A cliff rises 35 metres for every 2 metres horizontally.

c A plane descends 2 km for every 10 horizontal kilometres.

d A submarine ascends 150 m for every 20 horizontal metres.



6 Find the missing number.

a The gradient joining the points (0, 2) and (1, ?) is 4.

b The gradient joining the points (?, 5) and (1, 9) is 2 .

c The gradient joining the points (−3, ?) and (0, 1) is −1.

d The gradient joining the points (−4, −2) and (?, −12) is −4.

7 A train climbs a slope with gradient 0.05. How far horizontally has the train travelled after rising 15 metres?

8 Complete this table showing the gradient, x-intercept and y-intercept for straight lines.

	A	B	C	D	E	F
Gradient	3	−1	$\frac{1}{2}$	$-\frac{2}{3}$	0.4	−1.25
x-intercept	−3			6	1	
y-intercept		−4	$\frac{1}{2}$			3

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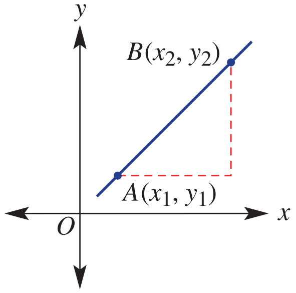
REASONING	9	9, 10	9, 10

9 Give a reason why a line with gradient $\frac{7}{11}$ is steeper than a line with gradient $\frac{3}{5}$.

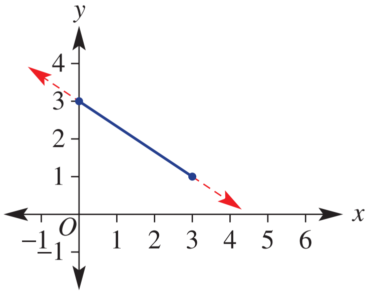
10 The two points A and B shown here have coordinates (x_1, y_1) and (x_2, y_2).



- **a** Write a rule for the run using x_1 and x_2.
- **b** Write a rule for the rise using y_1 and y_2.
- **c** Write a rule for the gradient m using x_1, x_2, y_1 and y_2.
- **d** Use your rule to find the gradient between these pairs of points.
  - **i** (1, 1) and (3, 4)
  - **ii** (0, 2) and (4, 7)
  - **iii** (−1, 2) and (2, −3)
  - **iv** (−4, −6) and (−1, −2)
- **e** Does your rule work for points that include negative coordinates? Explain why.

ENRICHMENT: Where does it hit?	–	–	11

11 The line here has gradient $-\frac{2}{3}$ which means that it falls 2 units for every 3 across. The y-intercept is 3.



- **a** Use the gradient to find the y-coordinate on the line where:
  - **i** $x = 6$
  - **ii** $x = 9$
- **b** What will be the x-intercept?
- **c** What would be the x-intercept if the gradient was changed to the following?
  - **i** $m = -\frac{1}{2}$
  - **ii** $m = -\frac{5}{4}$
  - **iii** $m = -\frac{7}{3}$
  - **iv** $m = -\frac{2}{5}$

4E Gradient and direct proportion

Learning intentions

- To understand what it means for two variables to be directly proportional
- To know the form of the equation that links two variables that are directly proportional
- To understand that the gradient of the graph equals the rate of change of one variable with respect to the other
- To be able to use a constant rate of change in a word problem to sketch a graph and form a linear rule

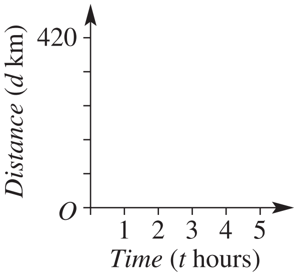
The connection between gradient, rate problems and direct proportion can be illustrated by the use of linear rules and graphs. If two variables are directly related, then the rate of change of one variable with respect to the other is constant. This implies that the rule linking the two variables is linear and can be represented as a straight line graph passing through the origin. The amount of water spraying from a sprinkler, for example, is directly proportional to the time since the sprinkler was turned on. The gradient of the graph of *water volume* versus *time* will equal the rate at which water is spraying from the sprinkler.

A bicycle's speed, v m/min, is in direct proportion to the number of pedal revolutions, P. For example, $v = 6P$ m/min for a wheel of circumference 2 m and gear ratio 3 : 1. If $P = 90$, $v = 540$ m/min, which is a speed of 32.4 km/h.

LESSON STARTER Average speed

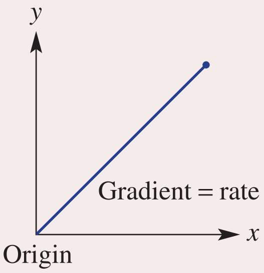
Over 5 hours, Sandy travels 420 km.

- What is Sandy's average speed for the trip?
- Is speed a rate? Discuss.
- Draw a graph of distance versus time, assuming a constant speed.
- Where does your graph intersect the axes and why?
- Find the gradient of your graph. What do you notice?
- Find a rule linking distance (d) and time (t).

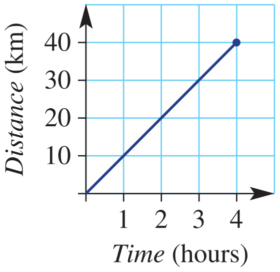


KEY IDEAS

- If two variables are **directly proportional**:
  - the rate of change of one variable with respect to the other is constant
  - the graph is a straight line passing through the origin
  - the rule is of the form $y = mx$
  - the gradient (m) of the graph equals the rate of change of y with respect to x.



BUILDING UNDERSTANDING



1 This graph shows how far a cyclist travels over 4 hours.

a State how far the cyclist has travelled after:
  - **i** 1 hour
  - **ii** 2 hours
  - **iii** 3 hours.

b State the speed of the cyclist (rate of change of distance over time).

c Find the gradient of the graph.

d What do you notice about your answers from parts **b** and **c**?

2 The rule linking the height of a plant and time is given by $h = 5t$ where h is in millimetres and t is in days.

a Find the height of the plant after 3 days.

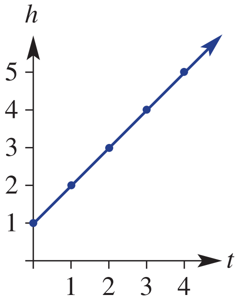
b Find the time for the plant to reach:
  - **i** 30 mm
  - **ii** 10 cm.

c State the missing values for h in this table.

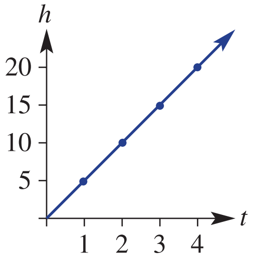
t	0	1	2	3	4
h					

d Choose the correct graph that matches the information above.

A



B



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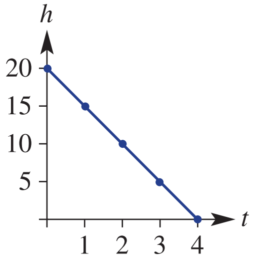

e Find the gradient of the graph.

Example 9 Exploring direct proportion

Water is poured into an empty tank at a constant rate. It takes 3 hours to fill the tank with 6000 litres.

a What is the rate at which water is poured into the tank?

b Draw a graph of volume (V litres) vs time (t hours) using $0 \leqslant t \leqslant 3$.

c Find:
  - **i** the gradient of your graph
  - **ii** the rule for V.

d Use your rule to find:
  - **i** the volume after 1.5 hours
  - **ii** the time to fill 5000 litres.

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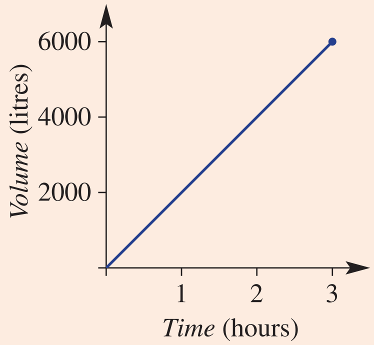
SOLUTION	EXPLANATION
a 6000 L in 3 hours = 2000 L/h	6000 L per 3 hours = 2000 L per 1 hour
b 	Plot the two endpoints (0, 0) and (3, 6000) and then join with a straight line.
c **i** gradient $= \frac{6000}{3} = 2000$	The gradient is the same as the rate.
ii $V = 2000t$	2000 L are filled for each hour.
d **i** $V = 2000t$ $= 2000 \times (1.5)$ $= 3000$ litres	Substitute $t = 1.5$ into your rule.
ii $V = 2000t$ $5000 = 2000t$ $2.5 = t$ $\therefore$ it takes $2\frac{1}{2}$ hours.	Substitute $V = 5000$ into the rule and solve for t.

Now you try

On a long-distance journey a car travels 450 km in 5 hours.

a What is the rate of change of distance over time (i.e. speed)?

b Draw a graph of distance (d km) versus time (t hours) using $0 \leqslant t \leqslant 5$.

c Find:

- **i** the gradient of your graph
- **ii** the rule for d.

d Use your rule to find:

- **i** the distance after 3.5 hours
- **ii** the time to travel 135 km.

Exercise 4E

FLUENCY

1–3	1–4	2–4

Example 9

1 A 300 litre fish tank takes 3 hours to fill from a hose.

a What is the rate at which water is poured into the tank?

b Draw a graph of volume (V litres) vs time (t hours) using $0 \leqslant t \leqslant 3$.

c Find:

i the gradient of your graph

ii the rule for V.

d Use your rule to find:

i the volume after 1.5 hours

ii the time to fill 200 litres.

Example 9

2 A solar-powered car travels 100 km in 4 hours.

a What is the rate of change of distance over time (i.e. speed)?

b Draw a graph of distance (d km) versus time (t hours) using $0 \leqslant t \leqslant 4$.

c Find:

i the gradient of your graph

ii the rule for d.

d Use your rule to find:

i the distance after 2.5 hours

ii the time to travel 40 km.

3 Write down a rule linking the given variables.

a I travel 600 km in 12 hours. Use d for distance in km and t for time in hours.

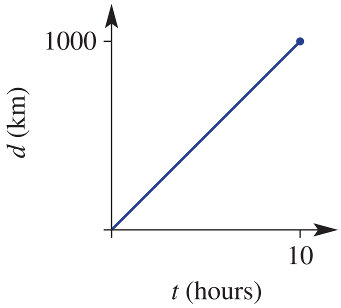
b A calf grows 12 cm in 6 months. Use g for growth height in cm and t for time in months.

c The cost of petrol is \$100 for 80 litres. Use \$$C$ for cost and n for the number of litres.

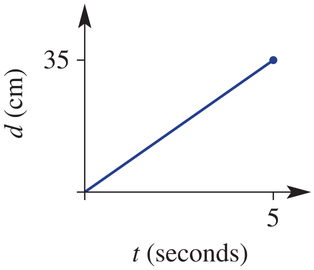
d The profit is \$10 000 for 500 tonnes. Use \$$P$ for profit and t for the number of tonnes.

4 Use the gradient to find the rate of change of distance over time (speed) for these graphs. Use the units given on each graph.

a



b



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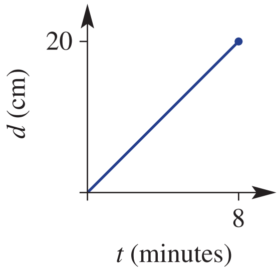

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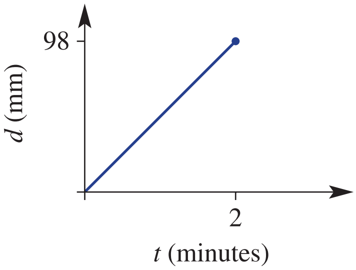


PROBLEM-SOLVING	5, 6	5–7	7, 8

5 The trip computer for a car shows that the fuel economy for a trip is 8.5 L per 100 km.

a How many litres would be used for 120 km?

b How many litres would be used for 850 km?

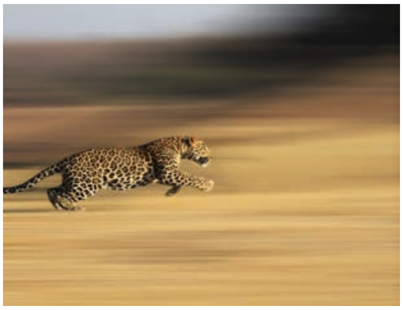
c How many kilometres could be travelled if the capacity of the car's petrol tank was 68 L?

6 Who is travelling the fastest?

- Mick runs 120 m in 20 seconds.
- Sally rides 700 m in 1 minute.
- Udhav jogs 2000 m in 5 minutes.

7 Which animal is travelling the slowest?

- A leopard runs 200 m in 15 seconds.
- A jaguar runs 2.5 km in 3 minutes.
- A panther runs 60 km in 1.2 hours.



8 An investment fund starts at \$0 and grows at a rate of \$100 per month. Another fund starts at \$4000 and reduces by \$720 per year. After how long will the funds have the same amount of money?

REASONING	9	9, 10	10–12

9 The circumference of a circle (given by $C = 2\pi r$) is directly proportional to its radius.

a Find the circumference of a circle with the given radius. Give an exact answer, e.g. 6π.

i $r = 0$ ii $r = 2$ iii $r = 6$

b Draw a graph of C against r for $0 \leqslant r \leqslant 6$. Use exact values for C.

c Find the gradient of your graph. What do you notice?

10 Is the area of a circle directly proportional to its radius? Give a reason.

11 The base length of a triangle is 4 cm but its height h cm is variable.

a Write a rule for the area of this triangle.

b What is the rate at which the area changes with respect to height h?

12 Over a given time interval, is the speed of an object directly proportional to the distance travelled? Give a rule for speed (s) in terms of distance (d) if the time taken is 5 hours.

ENRICHMENT: Rate challenge	–	–	13, 14

13 Hose A can fill a bucket in 2 minutes and hose B can fill the same bucket in 3 minutes. How long would it take to fill the bucket if both hoses were used at the same time?

14 A river is flowing downstream at a rate of 2 km/h. Murray can swim at a rate of 3 km/h. Murray jumps in and swims downstream for a certain distance and then turns around and swims upstream back to the start. In total it takes 30 minutes. How far did Murray swim downstream?

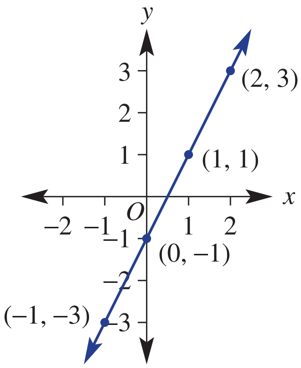
4F Gradient–intercept form

Learning intentions

- To know the gradient–intercept form of a straight line graph
- To be able to determine the gradient and y-intercept from the rule of a linear graph
- To be able to rearrange linear equations
- To be able to use the y-intercept and gradient to sketch a straight line graph

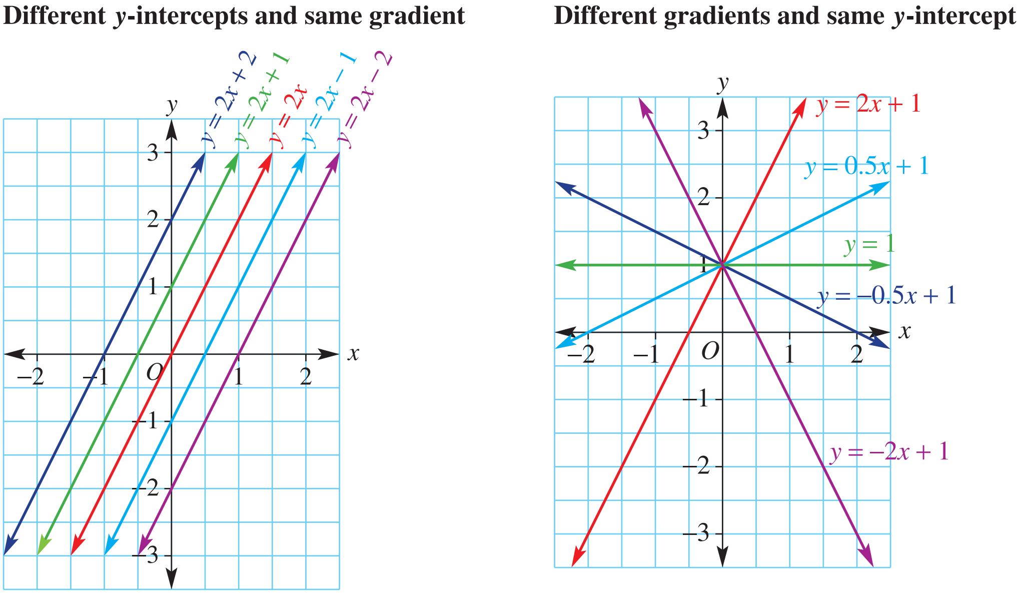
Shown here is the graph of the rule $y = 2x - 1$. It shows a gradient of 2 and a y-intercept of -1. The fact that these two numbers correspond to numbers in the rule is no coincidence. This is why rules written in this form are called gradient–intercept form. Here are other examples of rules in this form:

$$y = -5x + 2,\ y = \frac{1}{2}x - 0.5 \text{ and } y = \frac{x}{5} + 20$$



LESSON STARTER Family traits

The graph of a linear relation can be sketched easily if you know the gradient and the y-intercept. If one of these is kept constant, we create a family of graphs.



- For the first family, discuss the relationship between the y-intercept and the given rule for each graph.
- For the second family, discuss the relationship between the gradient and the given rule for each graph.

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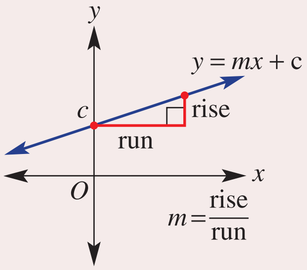
KEY IDEAS

- The **gradient–intercept form** of a straight line equation:
  $y = mx + c$ (or $y = mx + b$ depending on preference)
  m = gradient y-intercept = c
- If the y-intercept is zero, the equation becomes $y = mx$ and these graphs will therefore pass through the origin.
- To sketch a graph using the **gradient–intercept method**, locate the y-intercept and use the gradient to find a second point.
  For example, if $m = \frac{2}{5}$, move 5 across and 2 up.



BUILDING UNDERSTANDING

1. State a rule (in gradient–intercept form) for a straight line with the given properties.
   - **a** gradient = 2, y-intercept = 5
   - **b** gradient = −2, y-intercept = 3
   - **c** gradient = −1, y-intercept = −2
   - **d** gradient = $-\frac{1}{2}$, y-intercept = −10
2. Substitute $x = -3$ to find the value of y for these rules.
   - **a** $y = x + 4$
   - **b** $y = 2x + 1$
   - **c** $y = -2x + 3$
3. Rearrange to make y the subject.
   - **a** $y - x = 7$
   - **b** $x + y = 3$
   - **c** $2y - 4x = 10$

Example 10 Stating the gradient and y-intercept

State the gradient and the y-intercept for the graphs of the following relations.

a $y = 2x + 1$ **b** $y = -3x$

SOLUTION	EXPLANATION
a $y = 2x + 1$	The rule is given in gradient-intercept form.
gradient = 2	The gradient is the coefficient of x.
y-intercept = 1	The constant term is the y-intercept.
b $y = -3x$	
gradient = −3	The gradient is the coefficient of x including the negative sign.
y-intercept = 0	The constant term is not present, so the y-intercept = 0.

Now you try

State the gradient and the y-intercept for the graphs of the following relations.

a $y = 5x - 2$ **b** $y = 6x$

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Example 11 Rearranging linear equations

Rearrange these linear equations into the form shown in the brackets.

a $4x + 2y = 10$ $(y = mx + c)$

b $y = 4x - 7$ $(ax + by = d)$

SOLUTION	EXPLANATION
a $4x + 2y = 10$ $2y = -4x + 10$	Subtract $4x$ from both sides. Here $10 - 4x$ is better written as $-4x + 10$.
$y = -2x + 5$	Divide both sides by 2.
b $y = 4x - 7$	
$y - 4x = -7$	Subtract $4x$ from both sides
or $-4x + y = -7$ or $4x - y = 7$	Multiply both sides by -1 to convert between forms.

Now you try

Rearrange these linear equations into the form shown in the brackets.

a $10x + 2y = 20$ $(y = mx + c)$

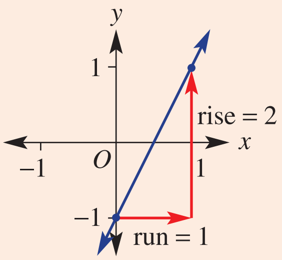
b $y = -2x + 3$ $(ax + by = d)$

Example 12 Sketching linear graphs using the gradient and *y*-intercept

Find the value of the gradient and *y*-intercept for these relations and sketch their graphs.

a $y = 2x - 1$

b $x + 2y = 6$

SOLUTION	EXPLANATION
a $y = 2x - 1$ $y\text{-intercept} = -1$ $\text{gradient} = 2 = \frac{2}{1}$	The rule is in gradient–intercept form so we can read off the gradient and the *y*-intercept.
	Label the *y*-intercept at $(0, -1)$. From the gradient $\frac{2}{1}$, for every 1 across, move 2 up. From $(0, -1)$ this gives a second point at $(1, 1)$. Mark and join the points to form a line.

b $x + 2y = 6$

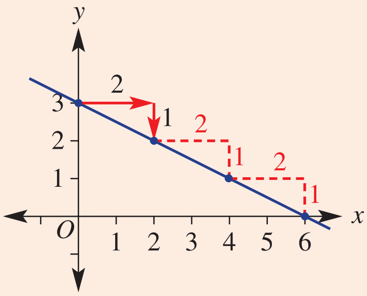
$$2y = -x + 6$$

$$y = \frac{-x + 6}{2}$$

$$= -\frac{1}{2}x + 3$$

so the y-intercept is 3

$$m = -\frac{1}{2} = \frac{-1}{2}$$



Rewrite in the form $y = mx + c$ to read off the gradient and y-intercept.

Make y the subject by subtracting x from both sides and then dividing both sides by 2.

$\frac{-x}{2}$ is the same as $-\frac{1}{2}x$ and $6 \div 2 = 3$.

Link the negative sign to the rise (-1) so the run is positive $(+2)$

Mark the y-intercept at $(0, 3)$ and then from this point move 2 right and 1 down to give a second point at $(2, 2)$.

Note that the x-intercept will be 6. If the gradient is $\frac{-1}{2}$ then a run of 6 gives a fall of 3.

Now you try

Find the value of the gradient and y-intercept for these relations and sketch their graphs.

a $y = 3x - 2$

b $2x - 5y = 10$

Exercise 4F

FLUENCY	1, 2–4(1/2)	2–5(1/2)	2–5(1/3)

Example 10

1 State the gradient and the y-intercept for the graphs of the following relations.

a $y = 7x + 3$

b $y = -2x$

Example 10

2 State the gradient and y-intercept for the graphs of the following relations.

a $y = 3x - 4$

b $y = -5x - 2$

c $y = -2x + 3$

d $y = \frac{1}{3}x + 4$

e $y = -4x$

f $y = 2x$

g $y = x$

h $y = -0.7x$

Example 11

3 Rearrange these linear equations into the form shown in the brackets.

a $2x + y = 3$ $(y = mx + c)$

b $-3x + y = -1$ $(y = mx + c)$

c $6x + 2y = 4$ $(y = mx + c)$

d $-3x + 3y = 6$ $(y = mx + c)$

e $y = 2x - 1$ $(ax + by = d)$

f $y = -3x + 4$ $(ax + by = d)$

g $3y = x - 1$ $(ax + by = d)$

h $7y - 2 = 2x$ $(ax + by = d)$

Example 12a **4** Find the gradient and y-intercept for these relations and sketch their graphs.

a $y = x - 2$
b $y = 2x - 1$
c $y = \frac{1}{2}x + 1$
d $y = -\frac{1}{2}x + 2$
e $y = -3x + 3$
f $y = \frac{3}{2}x + 1$
g $y = -\frac{4}{3}x$
h $y = \frac{5}{3}x - \frac{1}{3}$

Example 12b **5** Find the gradient and y-intercept for these relations and sketch their graphs. Rearrange each equation first.

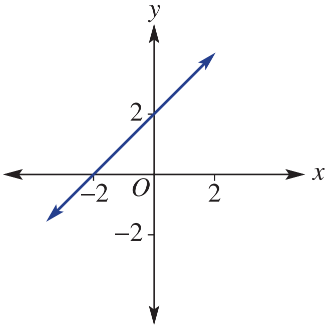
a $x + y = 4$
b $x - y = 6$
c $x + 2y = 6$
d $x - 2y = 8$
e $2x - 3y = 6$
f $4x + 3y = 12$
g $x - 3y = -4$
h $2x + 3y = 6$
i $3x - 4y = 12$
j $x + 4y = 0$
k $x - 5y = 0$
l $x - 2y = 0$

PROBLEM-SOLVING	6	6, 7–8(1/2)	7–8(1/2)

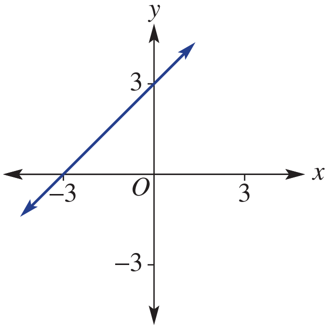
6 Match the following equations to the straight lines shown.

a $y = 3$
b $y = -2x - 1$
c $y = -2x - 4$
d $y = x + 3$
e $x = 2$
f $y = x + 2$

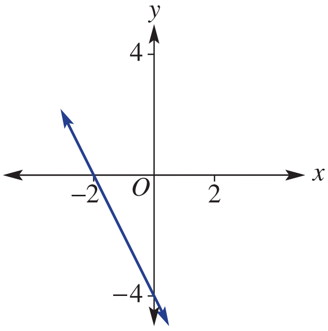
A



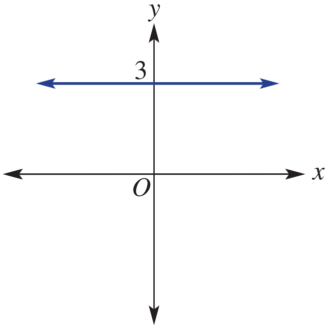
B



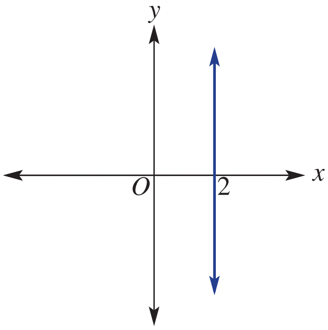
C



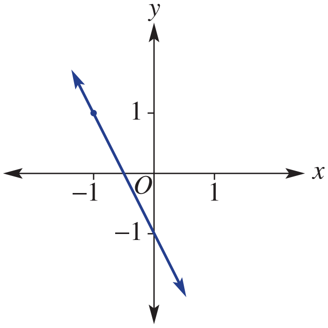
D



E



F



7 Sketch the graph of each of the following linear relations, by first finding the gradient and y-intercept.

a $5x - 2y = 10$ **b** $y = 6$ **c** $x + y = 0$ **d** $y = 5 - x$

e $y = \frac{x}{2} - 1$ **f** $4y - 3x = 0$ **g** $4x + y - 8 = 0$ **h** $2x + 3y - 6 = 0$

8 Which of these linear relations have a gradient of 2 and y-intercept of -3?

A $y = 2(x - 3)$ **B** $y = 3 - 2x$ **C** $y = \frac{3 - 2x}{-1}$ **D** $y = 2(x - 1.5)$

E $y = \frac{2x - 6}{2}$ **F** $y = \frac{4x - 6}{2}$ **G** $2y = 4x - 3$ **H** $-2y = 6 - 4x$

REASONING	9	9, 10	10,11

9 Jeremy says that the graph of the rule $y = 2(x + 1)$ has gradient 2 and y-intercept 1.

a Explain his error.

b What can be done to the rule to help show the y-intercept?

10 A horizontal line has gradient 0 and y-intercept at $(0, k)$. Using gradient–intercept form, write the rule for the line.

11 Write the rule $ax + by = d$ in gradient–intercept form. Then state the gradient m and the y-intercept.

ENRICHMENT: The missing y-intercept	–	–	12

12 This graph shows two points $(-1, 3)$ and $(1, 4)$ with a gradient of $\frac{1}{2}$. By considering the gradient, the y-intercept can be calculated to be 3.5 (or $\frac{7}{2}$) so $y = \frac{1}{2}x + \frac{7}{2}$.

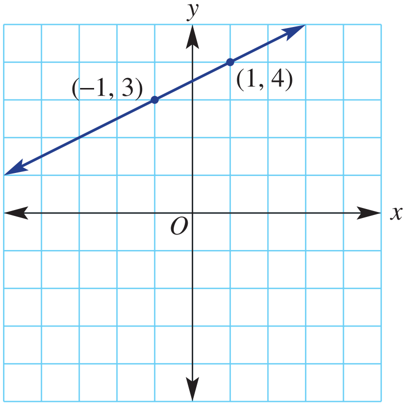
Use this approach to find the rule of the line passing through these points.

a $(-1, 1)$ and $(1, 5)$

b $(-2, 4)$ and $(2, 0)$

c $(-1, -1)$ and $(2, 4)$

d $(-3, 1)$ and $(2, -1)$



The equation of this straight section of mountain railway could be expressed in the form $y = mx + c$.

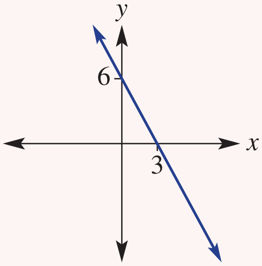
Progress quiz

4A **1** Read off the x- and y-intercepts from this table and graph.

a

x	–2	–1	0	1	2	3
y	–8	–6	–4	–2	0	2

b



4A **2** Decide if the point $(-1, 3)$ is on the line with the given rules.

a $y = 2x - 1$ **b** $y = 2x + 5$

4B **3** Sketch the graph of the following relations, by first finding the x- and y-intercepts.

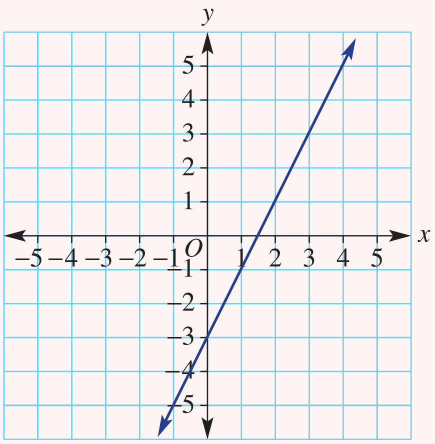
a $3x + 2y = 6$ **b** $y = \frac{3x}{4} + 3$

4C **4** Sketch the graphs of these special lines all on the same set of axes and label with their equations.

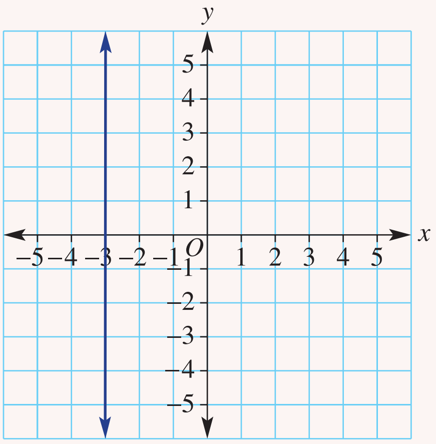
a $y = 2$ **b** $x = 4$ **c** $y = 2x$

4D **5** For each graph state whether the gradient is positive, negative, zero or undefined, and then find the gradient where possible.

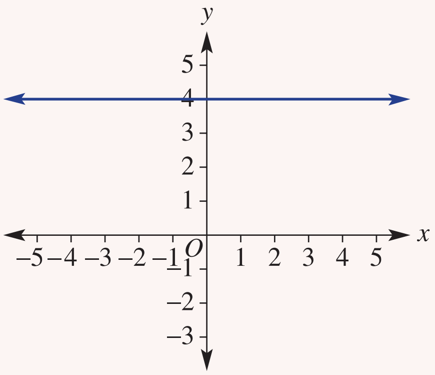
a



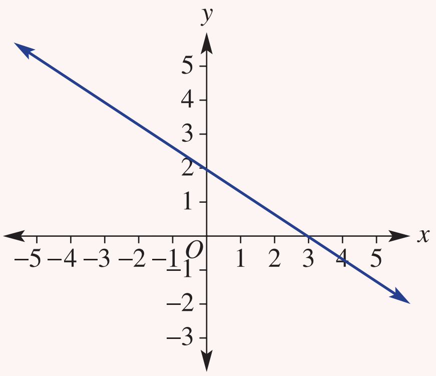
b



c



d



4D 6 Find the gradient (m) of the line joining the given points.

- a $P(2, 5)$ and $Q(4, 9)$
- b $A(-3, 2)$ and $B(1, -10)$

4E 7 An empty tank on a water truck is being filled. It takes 4 hours to fill the tank with 6000 litres.

- a What is the rate at which water is poured into the tank?
- b Draw a graph of volume (V litres) vs time (t hours) using $0 \leqslant t \leqslant 4$.
- c Find:
  - i the gradient of your graph
  - ii the rule for V.
- d Use your rule to find:
  - i the volume after 3 hours
  - ii the time to fill 5250 litres.

4F 8 Rearrange these linear equations into the form shown in the brackets.

- a $5x + y = 8 \ (y = mx + c)$
- b $y = 3x - 5 \ (ax + by = d)$

4F 9 Find the value of the gradient and y-intercept for these relations and sketch their graphs.

- a $y = 3x - 2$
- b $2x + 3y = 9$

4G Finding the equation of a line

Learning intentions

- To know that all straight line equations can be written in the form $y = mx + c$
- To know how to use the y-intercept and gradient to form a straight line equation
- To be able to find the equation of a line given two points

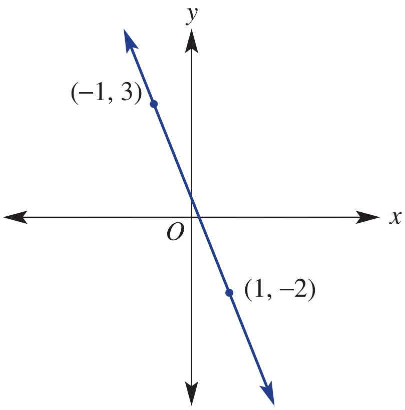
Using the gradient–intercept form, the rule (or equation) of a line can be found by calculating the value of the gradient and the y-intercept. Given a graph, the gradient can be calculated using two points. If the y-intercept is known, then the value of the constant in the rule is obvious. If it is not known, another point can be used to help find its value.

Engineers use linear equations to algebraically model straight supports, such as those in the Madrid airport. Using the slope of the support structure and a location in three dimensions, its linear equation can be found.

LESSON STARTER But we don't know the y-intercept!

A line with the rule $y = mx + c$ passes through two points $(-1, 3)$ and $(1, -2)$.

- Using the information given, is it possible to find the value of m? If so, calculate its value.
- The y-intercept is not given on the graph. Discuss what information could be used to find the value of the constant c in the rule. Is there more than one way you can find the y-intercept?
- Write the rule for the line.



KEY IDEAS

- To find the equation of a line in gradient–intercept form $y = mx + c$, you need to find:
  - the value of the gradient (m) using $m = \frac{\text{rise}}{\text{run}}$
  - the value of the constant (c), by observing the y-intercept or by substituting another point.

BUILDING UNDERSTANDING

1 Substitute the given values of m and c into $y = mx + c$ to find the rule.

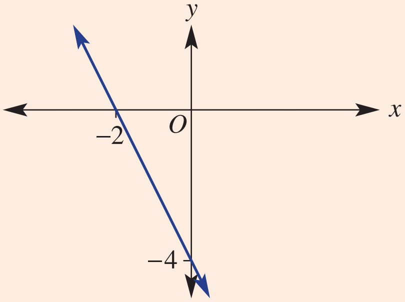
a $m = 2, c = 5$ **b** $m = 4, c = -1$ **c** $m = -2, c = 5$

2 Substitute the point into the given rule and solve to find the value of c. For example, using $(3, 4)$, substitute $x = 3$ and $y = 4$ into the rule.

a $(3, 4), y = x + c$ **b** $(-2, 3), y = 3x + c$ **c** $(3, -1), y = -2x + c$

Example 13 Finding the equation of a line given the y-intercept and another point

Determine the equation of the straight line shown here.



SOLUTION

$$y = mx + c$$

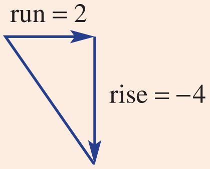
$$m = \frac{\text{rise}}{\text{run}} = \frac{-4}{2} = -2$$

y-intercept $= -4$.

$\therefore\ y = -2x - 4$

EXPLANATION

Write down the general straight line equation.

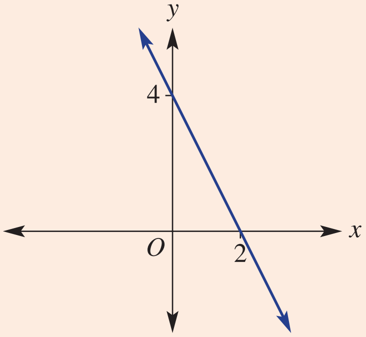


The y-intercept is -4.

Substitute $m = -2$ and the y-intercept into the general equation.

Now you try

Determine the equation of the straight line shown here.



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Example 14 Finding the equation of a line given the gradient and a point

Find the equation of the line which has a gradient m of $\frac{1}{3}$ and passes through the point $(9, 2)$.

SOLUTION	EXPLANATION
$y = mx + c$	
$y = \frac{1}{3}x + c$	Substitute $m = \frac{1}{3}$ into $y = mx + c$.
$2 = \frac{1}{3}(9) + c$	Since $(9, 2)$ is on the line, it must satisfy the equation $y = \frac{1}{3}x + c$; hence, substitute the point $(9, 2)$ where $x = 9$ and $y = 2$ to find c. Simplify and solve for c.
$2 = 3 + c$	
$-1 = c$	
$\therefore\ y = \frac{1}{3}x - 1$	Write the equation in the form $y = mx + c$.

Now you try

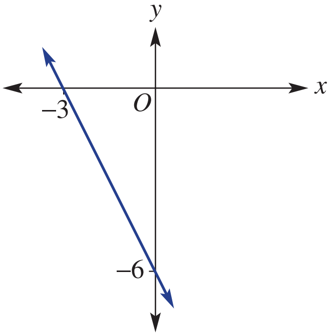
Find the equation of the line which has a gradient of $\frac{2}{3}$ and passes through the point $(-3, 4)$.

Exercise 4G

FLUENCY	1, 2, 3–4(½)	2–4(½)	2(½), 3–4(⅓)

Example 13

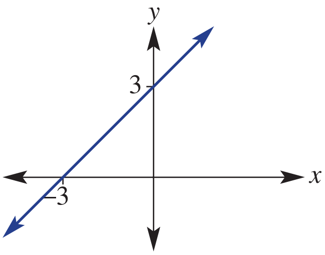
1 Determine the equation of the straight line shown here.



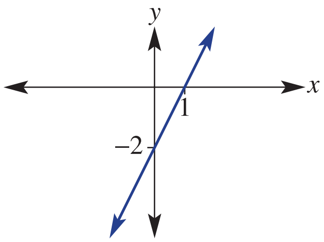
Example 13

2 Determine the equation of the following straight lines.

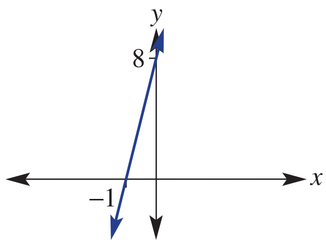
a



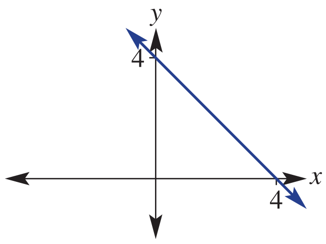
b



c



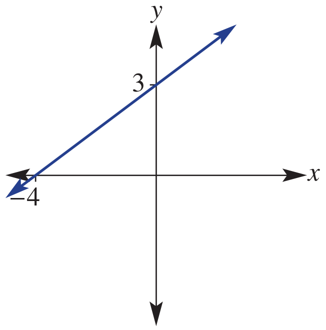
d



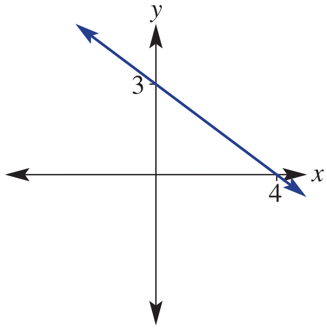
Example 13

3 Find the equation of these straight lines, which have fractional gradients.

a



b



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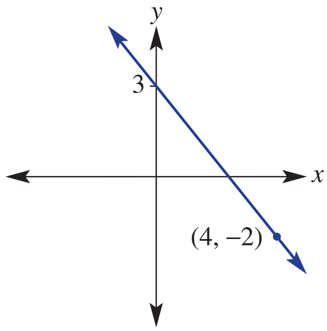

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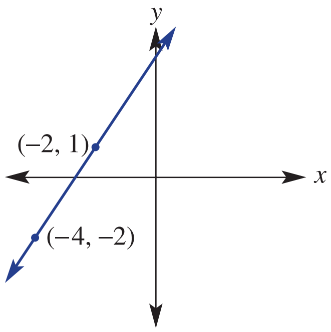
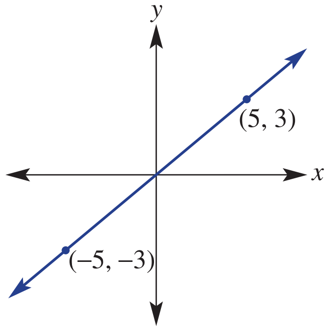
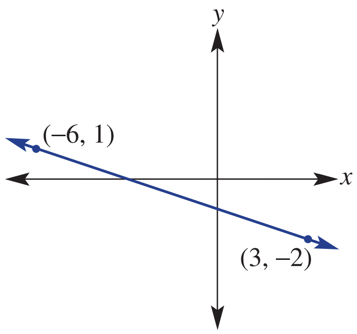


e



f



Example 14

4 Find the equation of the line for which the gradient and one point are given.

a $m = 3$, point $(1, 8)$
b $m = -2$, point $(2, -5)$
c $m = -3$, point $(2, 2)$
d $m = 1$, point $(1, -2)$
e $m = -3$, point $(-1, 6)$
f $m = 5$, point $(2, 9)$
g $m = -1$, point $(4, 4)$
h $m = -3$, point $(3, -3)$
i $m = -2$, point $(-1, 4)$
j $m = -4$, point $(-2, -1)$

PROBLEM-SOLVING	5(½), 6	5(½), 6–8	5(½), 7–9

5 For the line connecting the following pairs of points:

i find the gradient
ii find the equation.

a $(2, 6)$ and $(4, 10)$
b $(-3, 6)$ and $(5, -2)$
c $(1, 7)$ and $(3, -1)$
d $(-4, -8)$ and $(1, -3)$

6 A line has gradient -2 and y-intercept 5. Find its x-intercept.

7 A line passes through the points $(-1, -2)$ and $(3, 3)$. Find its x- and y-intercepts.

8 The tap on a tank has been left on and water is running out. The volume of water in the tank after 1 hour is 100 L and after 5 hours the volume is 20 L. Assuming the relationship is linear, find a rule and then state the initial volume of water in the tank.

9 The coordinates $(0, 0)$ mark the take-off point for a rocket constructed as part of a science class. The positive x direction from $(0, 0)$ is considered to be east.

a Find the equation of the rocket's path if the rocket rises at a rate of 5 m vertically for every 1 m in an easterly direction.

b A second rocket is fired from 2 m vertically above the launch site of the first rocket. It rises at a rate of 13 m for every 2 m in an easterly direction. Find the equation describing its path.

REASONING	10	10(½), 11	11, 12

10 A line has equation $y = mx - 2$. Find the value of m if the line passes through:

a $(2, 0)$
b $(1, 6)$
c $(-1, 4)$
d $(-2, -7)$

11 A line with rule $y = 2x + c$ passes through $(1, 5)$ and $(2, 7)$.

a Find the value of c using the point $(1, 5)$.
b Find the value of c using the point $(2, 7)$.
c Does it matter which point you use? Explain.

12 A line passes through the origin and the point (a, b). Write its equation in terms of a and b.

ENRICHMENT: The general rule	–	–	13

13 To find the equation of a line between two points (x_1, y_1) and (x_2, y_2) some people use the rule:

$$y - y_1 = m(x - x_1) \text{ where } m = \frac{y_2 - y_1}{x_2 - x_1}$$

Use this rule to find the equation of the line passing through these pairs of points. Write your answer in the form $y = mx + c$.

a $(1, 2)$ and $(3, 6)$
b $(0, 4)$ and $(2, 0)$
c $(-1, 3)$ and $(1, 7)$
d $(-4, 8)$ and $(2, -1)$
e $(-3, -2)$ and $(4, 3)$
f $(-2, 5)$ and $(1, -8)$

4H Midpoint and length of a line segment

Learning intentions

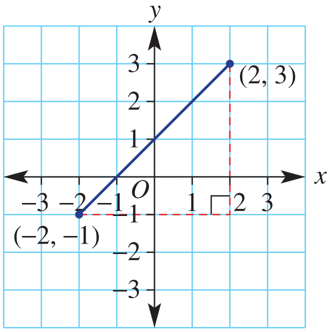
- To understand that a line segment has a midpoint and a length
- To know how to find the midpoint of a line segment between two points
- To understand how Pythagoras' theorem can be used to find the distance between two points
- To be able to find the length of a line segment

A line segment (or line interval) has a defined length and therefore must have a midpoint. Both the midpoint and length can be found by using the coordinates of the endpoints.

Industrial robots are programmed with the length algorithm to calculate the shortest straight line distance between two points in three dimensions, each located by coordinates (x, y, z).

LESSON STARTER Choosing a method

This graph shows a line segment between the points at $(-2, -1)$ and $(2, 3)$.



- What is the horizontal distance between the two points?
- What is the vertical distance between the two points?
- What is the x-coordinate of the point halfway along the line segment?
- What is the y-coordinate of the point halfway along the line segment?
- Discuss and explain a method for finding the midpoint of a line segment.
- Discuss and explain a method for finding the length of a line segment.

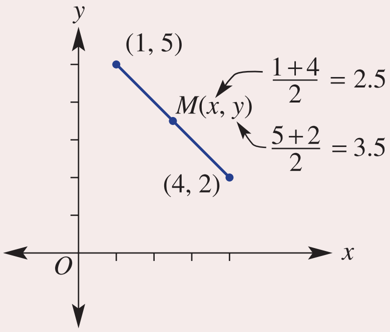
Using graphing software or interactive geometry software, produce a line segment like the one shown above. Label the coordinates of the endpoints and the midpoint. Also find the length of the line segment. Now drag one or both of the endpoints to a new position.

- Describe how the coordinates of the midpoint relate to the coordinates of the endpoints. Is this true for all positions of the endpoints that you choose?
- Now use your software to calculate the vertical distance and the horizontal distance between the two endpoints. Then square these lengths. Describe these squared lengths compared to the square of the length of the line segment. Is this true for all positions of the endpoints that you choose?

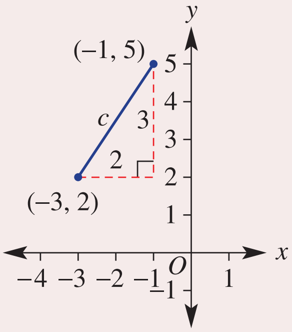
ISBN 978-1-108-77288-4

KEY IDEAS

- **The midpoint** (M) of a line segment is the halfway point between the two endpoints.
  - The x-coordinate of the midpoint is the average (mean) of the x-coordinates of the two endpoints.
  - The y-coordinate of the midpoint is the average (mean) of the y-coordinates of the two endpoints.
  - $M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$



- The **length of a line segment** (or line interval) is found using Pythagoras' theorem. This gives the distance between any two points.
  - The line segment is the hypotenuse (longest side) of a right-angled triangle.
  - Find the horizontal distance by subtracting the lower x-coordinate from the upper x-coordinate.
  - Find the vertical distance by subtracting the lower y-coordinate from the upper y-coordinate.



$$\begin{aligned}\text{horizontal distance} &= -1 - (-3)\\ &= 2\end{aligned}$$

$$\begin{aligned}\text{vertical distance} &= 5 - 2\\ &= 3\end{aligned}$$

$$\begin{aligned}c^2 &= 2^2 + 3^2\\ &= 13\\ \therefore\ c &= \sqrt{13}\end{aligned}$$

BUILDING UNDERSTANDING

1 Find the number that is 'halfway' between these pairs of numbers.

a 1, 7 **b** 5, 11 **c** −2, 4 **d** −6, 0

2 Find the average (mean) of these pairs of numbers.

a 4, 7 **b** 0, 5 **c** −3, 0 **d** −4, −1

3 Evaluate c correct to two decimal places.

a $c^2 = 1^2 + 2^2$ **b** $c^2 = 5^2 + 7^2$ **c** $c^2 = 10^2 + 2^2$

Example 15 Finding a midpoint

Find the midpoint $M(x, y)$ of the line segment joining these pairs of points.

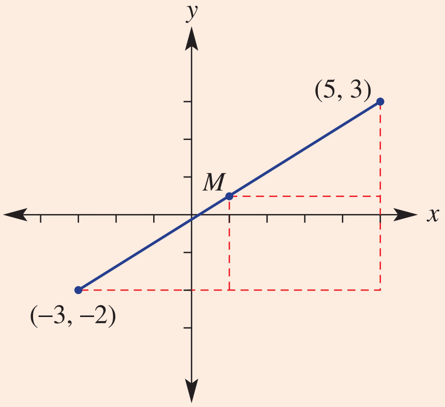
a (1, 0) and (4, 4) **b** (−3, −2) and (5, 3)

SOLUTION	EXPLANATION
a $x = \dfrac{1 + 4}{2} = 2.5$ $y = \dfrac{0 + 4}{2} = 2$ $\therefore\ M = (2.5, 2)$	Find the average (mean) of the x-coordinates and the y-coordinates of the two points.

b $$x = \frac{-3+5}{2} = 1$$

$$y = \frac{-2+3}{2} = 0.5$$

$$\therefore\ M = (1, 0.5)$$



Now you try

Find the midpoint $M(x, y)$ of the line segment joining these pairs of points.

a (0, 2) and (4, 5)

b (−4, 1) and (2, −4)

Example 16 Finding the length of a line segment

Find the length of the segment joining (−2, 2) and (4, −1), correct to two decimal places.

SOLUTION

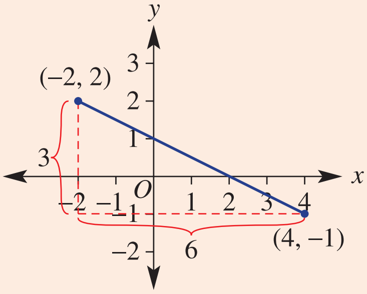
$$\begin{aligned}\text{Horizontal length} &= 4 - (-2)\\ &= 6\end{aligned}$$

$$\begin{aligned}\text{Vertical length} &= 2 - (-1)\\ &= 3\end{aligned}$$

$$\begin{aligned}c^2 &= 6^2 + 3^2\\ &= 45\\ \therefore\ c &= \sqrt{45}\end{aligned}$$

$\therefore$ length = 6.71 units (to 2 d.p.)

EXPLANATION



Apply Pythagoras' theorem $c^2 = a^2 + b^2$.

Round as required.

Now you try

Find the length of the segment joining (−1, 5) and (3, −2), correct to two decimal places.

Exercise 4H

FLUENCY

1, 2–3(1/2) | 2–3(1/2) | 2–3(1/3)

Example 15

1 Find the midpoint $M(x, y)$ of the line segment joining these pairs of points.

a	(0, 2) and (2, 8)	**b**	(−1, 5) and (6, −1)

Example 15

2 Find the midpoint $M(x, y)$ of the line segment joining these pairs of points.

a	(0, 0) and (6, 6)	**b**	(0, 0) and (4, 4)
c	(3, 0) and (5, 2)	**d**	(−2, 0) and (0, 6)
e	(−4, −2) and (2, 0)	**f**	(1, 3) and (2, 0)
g	(−3, 7) and (4, −1)	**h**	(−2, −4) and (−1, −1)
i	(−7, −16) and (1, −1)	**j**	(−4, −3) and (5, −2)

Example 16

3 Find the length of the segment joining these pairs of points correct to two decimal places.

a	(1, 1) and (2, 6)	**b**	(1, 2) and (3, 4)	**c**	(0, 2) and (5, 0)
d	(−2, 0) and (0, −4)	**e**	(−1, 3) and (2, 1)	**f**	(−2, −2) and (0, 0)
g	(−1, 7) and (3, −1)	**h**	(−4, −1) and (2, 3)	**i**	(−3, −4) and (3, −1)

PROBLEM-SOLVING

4, 5 | 4, 6 | 5–7

4 Find the missing coordinates in this table. M is the midpoint of points A and B.

A	*B*	*M*
(4, 2)		(6, 1)
	(0, –1)	(–3, 2)
	(4, 4)	(–1, 6.5)

5 A circle has centre (2, 1). Find the coordinates of the endpoint of a diameter if the other endpoint has these coordinates.

a (7, 1)

b (3, 6)

c (−4, −0.5)

6 Find the perimeter of these shapes correct to one decimal place.

a A triangle with vertices (−2, 0), (−2, 5) and (1, 3)

b A trapezium with vertices (−6, −2), (1, −2), (0, 4) and (−5, 4)

7 Find the coordinates of the four points which have integer coordinates and are a distance of $\sqrt{5}$ from the point (1, 2). (*Hint*: $5 = 1^2 + 2^2$.)

REASONING

8 | 8, 9 | 8, 9

8 A line segment has endpoints (x_1, y_1) and (x_2, y_2) and midpoint $M(x, y)$.

a Write a rule for x, the x-coordinate of the midpoint.

b Write a rule for y, the y-coordinate of the midpoint.

c Test your rule to find the coordinates of M if $x_1 = -3$, $y_1 = 2$, $x_2 = 5$ and $y_2 = -3$.

9 A line segment has endpoints (x_1, y_1) and (x_2, y_2). Assume $x_2 > x_1$ and $y_2 > y_1$.

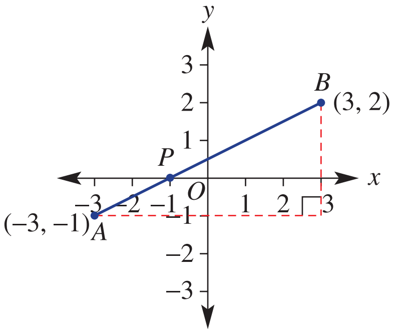
a Write a rule for:

- **i** the horizontal distance between the endpoints
- **ii** the vertical distance between the endpoints
- **iii** the length of the segment.

b Use your rule to show that the length of the segment joining $(-2, 3)$ with $(1, -3)$ is $\sqrt{45}$.

ENRICHMENT: Division by ratio	–	–	10

10 Looking from left to right, this line segment shows the point $P(-1, 0)$, which divides the segment in the ratio 1 : 2.



a What fraction of the horizontal distance between the endpoints is P from A?

b What fraction of the vertical distance between the endpoints is P from A?

c Find the coordinates of point P on the segment AB if it divides the segment in these ratios.

- **i** 2 : 1
- **ii** 1 : 5
- **iii** 5 : 1

d Find the coordinates of point P, which divides the segments with the given endpoints in the ratio 2 : 3.

- **i** $A(-3, -1)$ and $B(2, 4)$
- **ii** $A(-4, 9)$ and $B(1, -1)$
- **iii** $A(-2, -3)$ and $B(4, 0)$
- **iv** $A(-6, -1)$ and $B(3, 8)$

4I Perpendicular and parallel lines EXTENDING

Learning intentions

- To understand what is meant by lines being parallel or perpendicular
- To know the relationship between the gradients of lines that are either parallel or perpendicular
- To be able to find the gradient of a perpendicular line
- To be able to find the equation of a line given a point and a line to which it is parallel or perpendicular

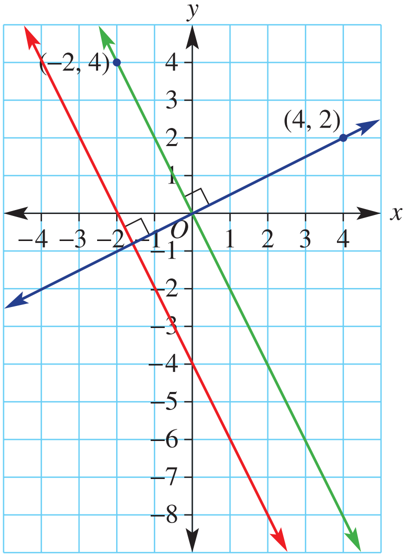
Perpendicular and parallel lines are commonplace in mathematics and in the world around us. Using parallel lines in buildings, for example, ensures that beams or posts point in the same direction. Perpendicular beams help to construct rectangular shapes, which are central in the building of modern structures.

The architect who designed this building has used linear equations to model the sets of parallel lines on its exterior. Shapes with parallel sides are arranged to emphasise the many parallel lines.

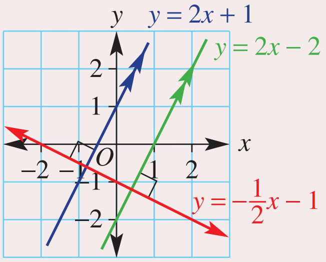
LESSON STARTER How are they related?

This graph shows a pair of parallel lines and a line perpendicular to the other two. Find the equation of all three lines.

- What do you notice about the equations for the pair of parallel lines?
- What do you notice about the gradient of the line that is perpendicular to the other two lines?
- Write down the equations of three other lines that are parallel to $y = -2x$.
- Write down the equations of three other lines that are perpendicular to $y = -2x$.



KEY IDEAS



- If two lines are **parallel** then they have the same gradient.
- If two **perpendicular** lines (at right angles) have gradients m_1 and m_2 then:

  $$m_1 \times m_2 = -1 \text{ or } m_2 = -\frac{1}{m_1}$$

  - The **negative reciprocal** of m_1 gives m_2.

    For example: If $m_1 = 4$ then $m_2 = -\frac{1}{4}$.

    $$\text{If } m_1 = -\frac{2}{3} \text{ then } m_2 = \frac{-1}{\left(-\frac{2}{3}\right)} = -1 \times \left(-\frac{3}{2}\right) = \frac{3}{2}.$$

BUILDING UNDERSTANDING

1 Decide if the pairs of lines with these equations are parallel (have the same gradient).

a $y = 3x - 1$ and $y = 3x + 4$

b $y = 7x - 2$ and $y = 2x - 7$

c $y = x + 4$ and $y = x - 3$

d $y = \frac{1}{2}x - 1$ and $y = -\frac{1}{2}x + 2$

2 Two perpendicular lines with gradients m_1 and m_2 are such that $m_2 = -\frac{1}{m_1}$. Find m_2 for the given values of m_1.

a $m_1 = 5$

b $m_1 = -3$

3 Decide if the pairs of lines with these equations are perpendicular (i.e. whether $m_1 \times m_2 = -1$).

a $y = 4x - 2$ and $y = -\frac{1}{4}x + 3$

b $y = 2x - 3$ and $y = \frac{1}{2}x + 4$

c $y = -\frac{1}{2}x + 6$ and $y = -\frac{1}{2}x - 2$

d $y = -\frac{1}{5}x + 1$ and $y = 5x + 2$

Example 17 Finding the equation of a parallel line

Find the equation of a line which is parallel to $y = 3x - 1$ and passes through $(0, 4)$.

SOLUTION	EXPLANATION
$y = mx + c$ $m = 3$	Since it's parallel to $y = 3x - 1$, the gradient is the same so $m = 3$.
$c = 4$ $\therefore\ y = 3x + 4$	The y-intercept is given in the question, so $c = 4$.

Now you try

Find the equation of a line which is parallel to $y = 4x + 3$ and passes through $(0, -2)$.

Example 18 Finding the equation of a perpendicular line

Find the equation of a line which is perpendicular to the line $y = 2x - 3$ and passes through $(0, -1)$.

SOLUTION	EXPLANATION
$y = mx + c$	
$m = -\frac{1}{2}$	Since it is perpendicular to $y = 2x - 3$, $m_2 = -\frac{1}{m_1} = -\frac{1}{2}$.
$c = -1$	The y-intercept is given.
$y = -\frac{1}{2}x - 1$	

Now you try

Find the equation of a line which is perpendicular to the line $y = -3x + 1$ and passes through $(0, 4)$.

Exercise 4I

FLUENCY	1, 2, 3(½)	2, 3(½), 4	2–3(½), 4

Example 17

1 Find the equation of the line that is parallel to the given line and passes through the point.

a $y = 2x + 3, (0, 1)$
b $y = -x + 3, (0, 5)$

Example 17

2 Find the equation of the line that is parallel to the given line and passes through the point.

a $y = 4x + 2, (0, 8)$
b $y = -2x - 3, (0, -7)$
c $y = \frac{2}{3}x + 6, (0, -5)$
d $y = -\frac{4}{5}x - 3, \left(0, \frac{1}{2}\right)$

Example 18

3 Find the equation of the line that is perpendicular to the given line and passes through the point.

a $y = 3x - 2, (0, 3)$
b $y = 5x - 4, (0, 7)$
c $y = -2x + 3, (0, -4)$
d $y = -x + 7, (0, 4)$
e $y = -7x + 2, \left(0, -\frac{1}{2}\right)$
f $y = x - \frac{3}{2}, \left(0, \frac{5}{4}\right)$

4 a Write the equation of the line parallel to $y = 4$ which passes through these points.
 i $(0, 1)$
 ii $(-3, -2)$

b Write the equation of the line parallel to $x = -2$ which passes through these points.
 i $(3, 0)$
 ii $(-3, -3)$

c Write the equation of the line perpendicular to $y = -3$ which passes through these points.
 i $(2, 0)$
 ii $(0, 0)$

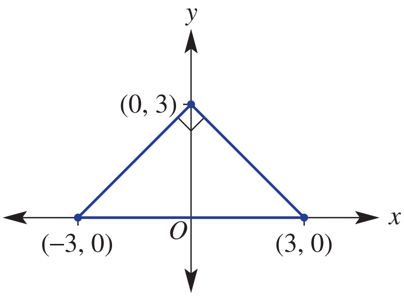
d Write the equation of the line perpendicular to $x = 6$ which passes through these points.
 i $(0, 7)$
 ii $\left(0, -\frac{1}{2}\right)$

PROBLEM-SOLVING	5	5, 6	5(½), 6, 7

5 Find the equation of the line which is:

- **a** parallel to the line with equation $y = -3x - 7$ and passes through $(3, 0)$; remember to substitute the point $(3, 0)$ to find the value of the y-intercept
- **b** parallel to the line with equation $y = \frac{1}{2}x + 2$ and passes through $(1, 3)$
- **c** perpendicular to the line with equation $y = 5x - 4$ and passes through $(1, 6)$
- **d** perpendicular to the line with equation $y = -x - \frac{1}{2}$ and passes through $(-2, 3)$.

6 A right-angled isosceles triangle has vertices at $(0, 3)$, $(3, 0)$ and $(-3, 0)$. Find the equation of each side.



7 A parallelogram has two side lengths of 5 units. Three of its sides have equations $y = 0$, $y = 2$, $y = 2x$. Find the equation of the fourth side.

REASONING	8	8, 9	8, 9

8 **a** Using $m_2 = -\frac{1}{m_1}$, find the gradient of a line perpendicular to the line with the given gradient.

- **i** $m_1 = \frac{2}{3}$
- **ii** $m_1 = \frac{1}{5}$
- **iii** $m_1 = -\frac{1}{7}$
- **iv** $m_1 = -\frac{3}{11}$

b If $m_1 = \frac{a}{b}$, find m_2 given $m_1 \times m_2 = -1$.

9 **a** Find the gradient of a line that is parallel to these lines.

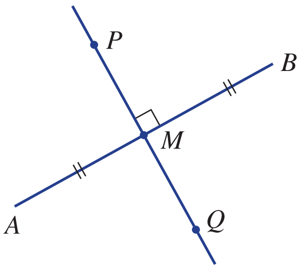
- **i** $2x + 4y = 9$
- **ii** $3x - y = 8$

b Find the gradient of a line that is perpendicular to these lines.

- **i** $5x + 5y = 2$
- **ii** $7x - y = -1$

ENRICHMENT: Perpendicular bisectors	–	–	10(½)

10 If a line segment AB is cut by another line PQ at right angles through the midpoint (M) of AB, then PQ is called the perpendicular bisector.
By first finding the midpoint of AB, find the equation of the perpendicular bisector of the segment connecting these points.



- **a** $A(1, 1)$, $B(3, 5)$
- **b** $A(0, 6)$, $B(4, 0)$
- **c** $A(-2, 3)$, $B(6, -1)$
- **d** $A(-6, -1)$, $B(0, 2)$
- **e** $A(-1, 3)$, $B(2, -4)$
- **f** $A(-6, -5)$, $B(4, 7)$

ISBN 978-1-108-77288-4

Applications and problem-solving

The following problems will investigate practical situations drawing upon knowledge and skills developed throughout the chapter. In attempting to solve these problems, aim to identify the key information, use diagrams, formulate ideas, apply strategies, make calculations and check and communicate your solutions.

Solar car battery

1 The battery power of two electric cars is analysed and compared by data scientists after testing the cars under average city driving conditions. The average distance covered in 1 hour under such conditions is 40 km.

- The Zet car battery is fully charged with 80 kWh reducing at a rate of 20 kWh per hour.
- The Spark car battery is fully charged with 60 kWh reducing at a rate of 12 kWh per hour.

The data scientists wish to investigate the life of each battery charge and compare the performance of each car's battery within the limits of average city driving conditions.

a Find a rule for the battery power P (kWh) over t hours for:
  i the Zet car
  ii the Spark car.

b Find the battery power remaining (the value of P (kWh)) after 3 hours of average city driving for:
  i the Zet car
  ii the Spark car.

c After how long will there be 40 kWh of power remaining for:
  i the Zet car?
  ii the Spark car?

d Which car's battery has the longer travel time? Give reasons.

e If the cars are driven until their batteries are entirely depleted of power, how far could they travel under average city conditions? Recall that 40 km is covered in one hour.

f After how long will the cars have the same remaining battery power?

g A third car, the Watt, has a battery producing the following statistics:
  - 60 kWh after 1 hour of driving
  - 20 kWh after 4 hours of driving.

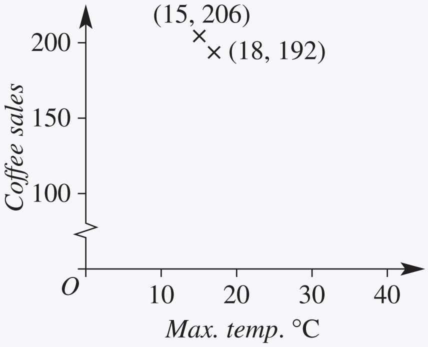
Find a rule for the Watt's remaining battery power P (kWh) after t hours and calculate how far this car can travel on one full charge.

Temperature vs coffee

2 Lydia owns a coffee shop and notices a decline in coffee sales as the maximum daily temperature rises over summer. She collects the following statistics by selecting 5 days within a two-month period.

Data point	1	2	3	4	5
Max. temperature	15	18	27	31	39
Coffee sales	206	192	165	136	110

Lydia wishes to analyse her collected data and see if there is a clear relationship between the maximum temperature and coffee sales. She wants to be better informed, so she can predict income from coffee sales in the future.



a Copy and complete this graph for the 5 data points. Two points have been done for you.

b Do you think there is a clear relationship between coffee sales and maximum temperature? Give a reason.

c Using the coordinates of data points 1 and 5, find a rule linking coffee sales, C, and maximum temperature, T.

d Using your rule from part **c**, predict the number of coffee sales for a day with the following maximum temperatures.

i 25°C **ii** 35°C

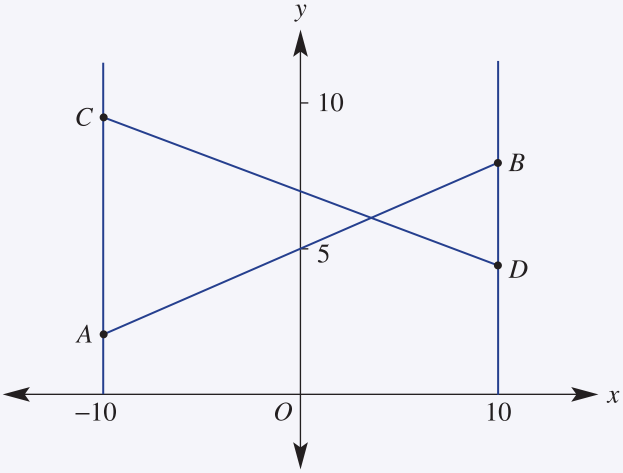
e Using your rule from part **c**, predict the maximum temperature for a day with the following number of coffee sales.

i 150 **ii** 202

f At what maximum temperature does your rule predict that there will be no coffee sales? Is this realistic?

Rope course

3 Douglas is designing a rope obstacle course between two vertical trees which are 20 m apart. To help analyse his design, he draws the two trees represented by $x = -10$ and $x = 10$ on a Cartesian plane as shown.
Two initial ropes AB and CD are used to start with. Douglas sets the heights of the points A, B, C and D as follows:
$A = 3$ m, $B = 7$ m, $C = 9$ m, $D = 4$ m



Douglas wants to be clear about his design so that he has a good idea about the amount of rope required and the positions of various key points on the obstacle course.

a Find the lengths of the following ropes correct to two decimal places.

i AB **ii** CD

b Find the equation of the lines which describes the following ropes.

i AB **ii** CD

c Use an accurate graph to determine the approximate position at which the ropes intersect.

d Douglas decides to place a third rope midway along and perpendicular to rope AB and run it from this position to the left tree.

i Find an equation for this third rope. **ii** How high up the left tree will it reach?

iii What will be the length of the rope correct to one decimal place?

e Douglas decides that he wants a fourth rope running from the point D on the right tree to the left tree, joining somewhere between point A and the midpoint of AC. What are the minimum and maximum lengths of rope that he might need? Round to one decimal place where necessary.

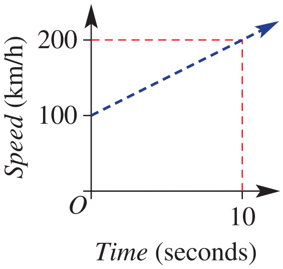
ISBN 978-1-108-77288-4

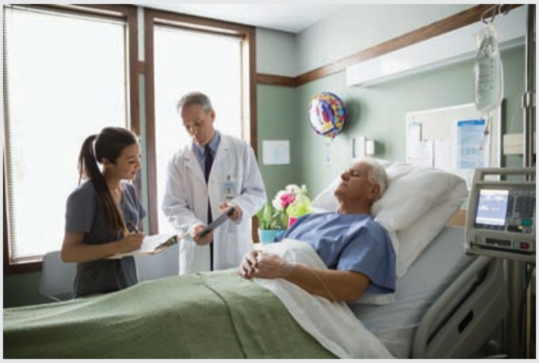
4J Linear modelling

Learning intentions

- To understand whether or not a situation can be modelled by a linear rule
- To be able to form a linear rule to link two variables
- To be able to use a rule to sketch a graph and predict the value of one variable given the other

If a relationship between two variables is linear, the graph will be a straight line and the equation linking the two variables can be written in gradient–intercept form. The process of describing and using such line graphs and rules for the relationship between two variables is called linear modelling. A test car, for example, increasing its speed from 100 km/h to 200 km/h in 10 seconds with constant acceleration could be modelled by the rule $s = 10t + 100$. This rule could then be used to calculate the speed at different times in the test run.





A doctor may prescribe 1400 mL of saline solution to be given intravenously over seven hours. The rate of infusion is 200 mL/h, and the remaining volume, V, after t hours has the linear equation: $V = -200t + 1400$.

LESSON STARTER The test car

The graph shown above describes the speed of a racing car over a 10 second period.

- Explain why the rule is $s = 10t + 100$.
- Why might negative values of t not be considered for the graph?
- How could you accurately calculate the speed after 6.5 seconds?
- If the car continued to accelerate at the same rate, how could you accurately predict the car's speed after 13.2 seconds?

KEY IDEAS

- Many situations can often be **modelled** by using a linear rule or graph. The key elements of linear modelling include:
  - finding the rule linking the two variables
  - sketching a graph
  - using the graph or rule to predict or estimate the value of one variable given the other
  - finding the rate of change of one variable with respect to the other variable – this is equivalent to finding the gradient.

ISBN 978-1-108-77288-4

BUILDING UNDERSTANDING

1 A person gets paid \$50 plus \$20 per hour. Decide which rule describes the relationship between the pay \$$P$ and number of hours worked, n.

A $P = 50 + n$ **B** $P = 50 + 20n$

C $P = 50n + 20$ **D** $P = 20 + 50$

2 The amount of money in a bank account is \$1000 and is increasing by \$100 per month.

a Find the amount of money in the account after:

i 2 months **ii** 5 months **iii** 12 months.

b State a rule for the amount of money A dollars after n months.

3 If $d = 5t - 4$, find:

a d when $t = 10$ **b** d when $t = 1.5$

c t when $d = 6$ **d** t when $d = 11$.

Example 19 Applying linear relations

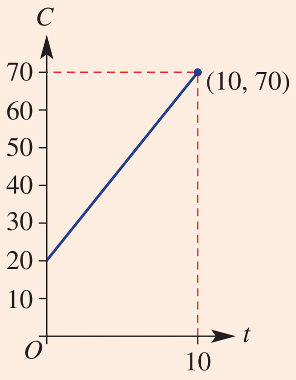
The deal offered by Bikeshare, a bicycle rental company, is a fixed charge of \$20 plus \$5 per hour of use.

a Write a rule for the total cost, \$$C$, of renting a bike for t hours.

b Sketch the graph of C versus t for $0 \leqslant t \leqslant 10$.

c What is the total cost when a bike is rented for 4 hours?

d If the total cost was \$50, for how many hours was a bike rented?

SOLUTION	EXPLANATION
a $C = 20 + 5t$	A fixed amount of \$20 plus \$5 for each hour.
b C-intercept is 20 At $t = 10,\ C = 20 + 5(10) = 70$ ∴ endpoint is (10, 70) 	Let $t = 0$ to find the C-intercept. Letting $t = 10$ gives $C = 70$ and this gives the other endpoint. Sketch the graph using the points (0, 20) and (10, 70).

Continued on next page

c $C = 20 + 5t$ $= 20 + 5(4)$ $= 40$	Substitute $t = 4$ into the rule.
The cost is \$40.	Answer the question using the correct units.
d $C = 20 + 5t$ $50 = 20 + 5t$ $30 = 5t$ $t = 6$	Write the rule and substitute $C = 50$. Solve the resulting equation for t by subtracting 20 from both sides then dividing both sides by 5.
The bike was rented for 6 hours.	Answer the question in words.

Now you try

A salesperson earns \$500 per week plus \$10 for each item sold.

a Write a rule for the total weekly wage, \$$W$, if the salesperson makes x sales.
b Draw a graph of W versus x for $0 \leqslant x \leqslant 100$.
c How much does the salesperson earn in a week if they make 25 sales?
d In a particular week the salesperson earns \$850. How many sales did they make?

Exercise 4J

FLUENCY	1–3	2–4	3, 4

Example 19

1 A sales representative earns \$400 a week plus \$20 for each sale she makes.
a Write a rule which gives the total weekly wage, \$$W$, if she makes x sales.
b Draw a graph of W versus x for $0 \leqslant x \leqslant 40$.
c How much does the sales representative earn in a particular week if she makes 12 sales?
d In a particular week, the sales representative earns \$1000. How many sales did she make?

Example 19

2 A catering company charges \$500 for the hire of a marquee, plus \$25 per guest.
a Write a rule for the cost, \$$C$, of hiring a marquee and catering for n guests.
b Draw a graph of C versus n for $0 \leqslant n \leqslant 100$.
c How much would a party catering for 40 guests cost?
d If a party cost \$2250, how many guests were catered for?

3 A plumber charges a \$40 fee up-front and \$50 for each hour he works.
a Find a linear equation for the total charge, \$$C$, for n hours of work.
b What is the cost if he works 4 hours?
c If the plumber works on a job for two days and averages 6 hours per day, what will the total cost be?

4 The cost, $\$C$, of recording a music CD is \$300, plus \$120 per hour of studio time.

- **a** Write a rule for the cost, $\$C$, of recording a CD requiring t hours of studio time.
- **b** Draw a graph of C versus t for $0 \leqslant t \leqslant 10$.
- **c** How much does a recording cost if it requires 6 hours of studio time?
- **d** If a recording cost \$660 to make, for how long was the studio used?

PROBLEM-SOLVING	5, 6	5, 6	6, 7

5 A petrol tank holds 66 litres of fuel. It contains 12 litres of petrol initially and the petrol pump fills it at 3 litres every 10 seconds.

- **a** Write a linear equation for the amount of fuel (F litres) in the tank after t minutes.
- **b** How long will it take to fill the tank?
- **c** How long will it take to add 45 litres into the petrol tank?

6 A tank is initially full with 4000 litres of water and water is being used at a rate of 20 litres per minute.

- **a** Write a rule for the volume, V litres, of water after t minutes.
- **b** Calculate the volume after 1.5 hours.
- **c** How long will it take for the tank to be emptied?
- **d** How long will it take for the tank to have only 500 litres?

7 A spa pool contains 1500 litres of water. It is draining at the rate of 50 litres per minute.

- **a** Draw a graph of the volume of water, V litres, remaining after t minutes.
- **b** Write a rule for the volume of water at time t minutes.
- **c** What does the gradient represent?
- **d** What is the volume of water remaining after 5 minutes?
- **e** After how many minutes is the pool half empty?

REASONING	8	8	8, 9

8 The rule for distance travelled d km over a given time t hours for a moving vehicle is given by $d = 50 + 80t$.

- **a** What is the speed of the vehicle?
- **b** If the speed was actually 70 km per hour, how would this change the rule? Give the new rule.

9 The altitude, h metres, of a helicopter t seconds after it begins its descent is given by $h = 350 - 20t$.

- **a** At what rate is the helicopter's altitude decreasing?
- **b** At what rate is the helicopter's altitude increasing?
- **c** What is the helicopter's initial altitude?
- **d** How long will it take for the helicopter to reach the ground?
- **e** If instead the rule was $h = 350 + 20t$, describe what the helicopter would be doing.

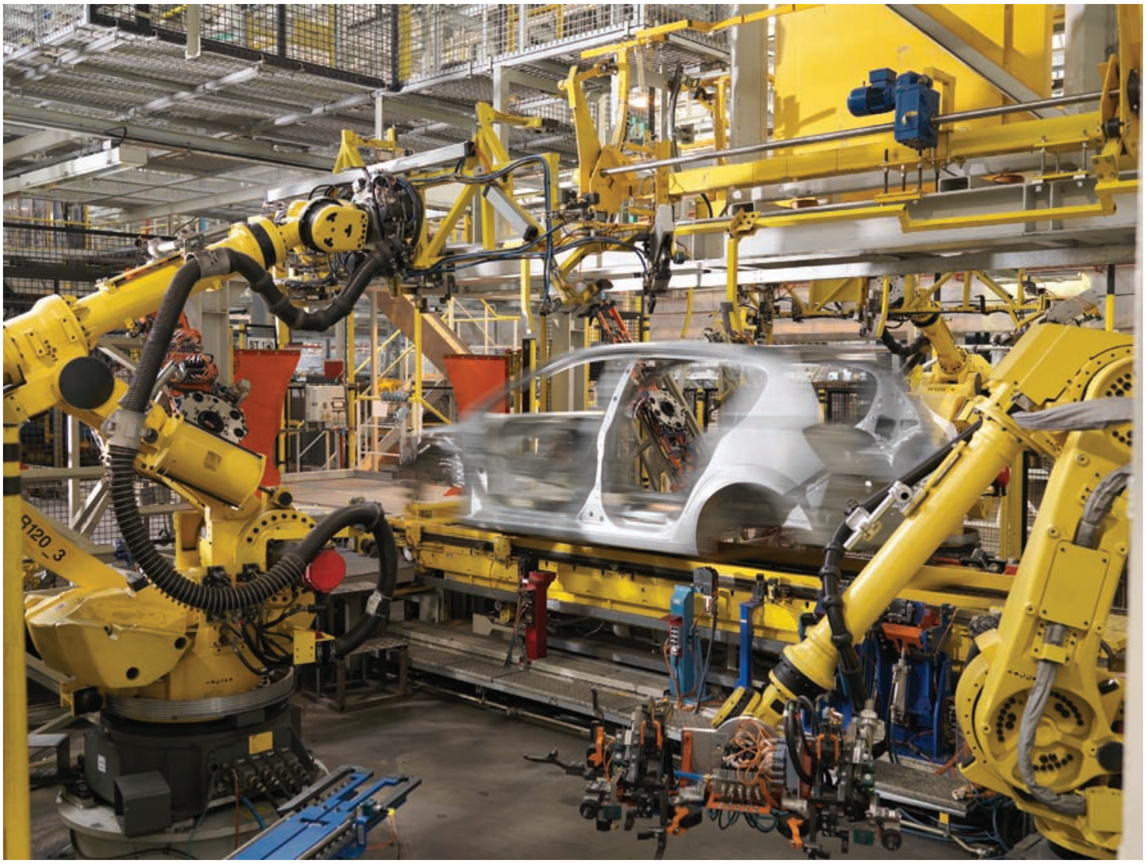
ENRICHMENT: Sausages and cars	–	–	10, 11

10 Joanne organised a sausage sizzle to raise money for her science club. The hire of the barbecue cost Joanne \$20, and the sausages cost 40c each.

- **a** **i** Write a rule for the total cost, $\$C$, if Joanne buys and cooks n sausages.
  - **ii** If the total cost was \$84, how many sausages did Joanne buy?
- **b** **i** If Joanne sells each sausage for \$1.20, write a rule to find her profit, $\$P$, after buying and selling n sausages.
  - **ii** How many sausages must she sell to 'break even'?
  - **iii** If Joanne's profit was \$76, how many sausages did she buy and sell?

11 The directors of a car manufacturing company calculate that the set-up cost for a new component is \$6700 and each component would cost \$10 to make.

- **a** Write a rule for the total cost, $\$C$, of producing x components.
- **b** Find the cost of producing 200 components.
- **c** How many components could be produced for \$13 000?
- **d** Find the cost of producing 500 components.
- **e** If each component is able to be sold for \$20, how many must they sell to 'break even'?
- **f** Write a rule for the profit, $\$P$, in terms of x.
- **g** Write a rule for the profit, $\$T$, per component in terms of x.
- **h** Find x, the number of components, if the profit per component is to be \$5.



4K Graphical solutions to simultaneous equations

Learning intentions

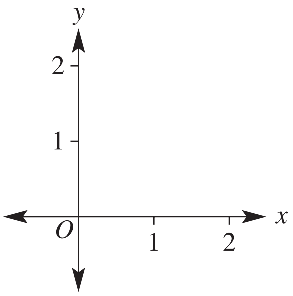
- To understand that two different straight line graphs intersect at exactly one point unless the lines are parallel
- To know that the point of intersection of two lines is the solution of the simultaneous equations
- To know how to use the linear rules to check if a point is at the intersection of the two lines
- To be able to use the graphs of two straight lines to read off the coordinates of the point of intersection

To find a point that satisfies more than one equation involves finding the solution to simultaneous equations. An algebraic approach was considered in Chapter 2. A graphical approach involves locating an intersection point.

Revenue, R, and costs, C, often have linear equations, such as $R = 1000n$ and $C = 400n + 3000$, for producing n surfboards. The break-even point of a business is where these two linear graphs intersect; profit starts as revenue begins to exceed costs.

LESSON STARTER Accuracy counts

Two graphs have the rules $y = x$ and $y = 2 - 4x$.
Accurately sketch the graphs of both rules on a large set of axes like the one shown.

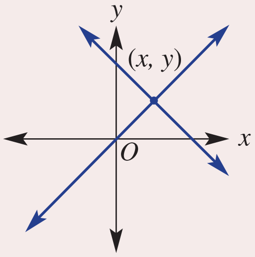


- State the x-value of the point at the intersection of your two graphs.
- State the y-value of the point at the intersection of your two graphs.
- Discuss how you could use the rules to check if your point is correct.
- If your point does not satisfy both rules, check the accuracy of your graphs and try again.

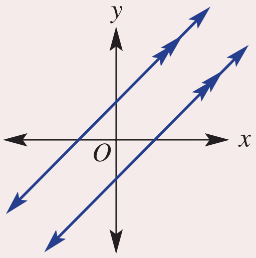
ISBN 978-1-108-77288-4

KEY IDEAS

- When we consider two or more equations at the same time, they are called **simultaneous equations**.
- To determine the **point of intersection** of two lines, we can use an accurate graph and determine its coordinates (x, y).
- Two situations can arise for the intersection of two different lines.
  - The two graphs intersect at one point only and there is one solution (x, y).
    - The point of intersection is simultaneously on both lines and is the **solution** to the simultaneous equations.

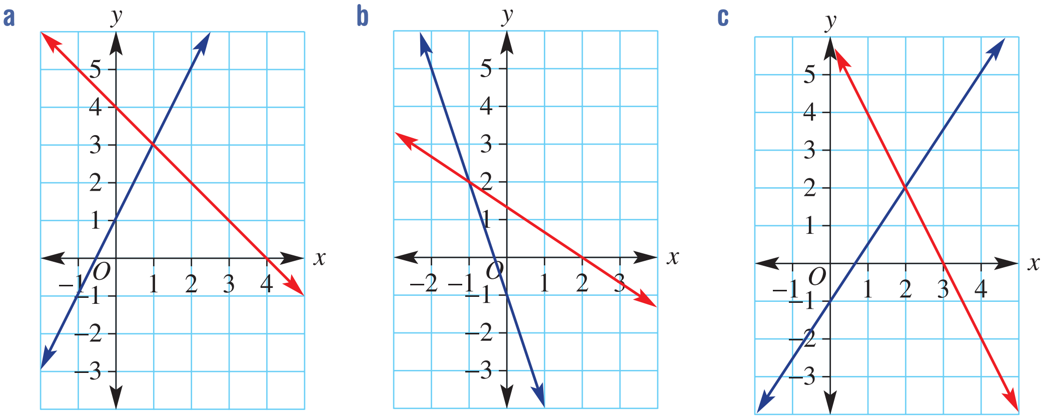


  - The two lines are parallel and there is no intersection.



BUILDING UNDERSTANDING

1 Give the coordinates of the point of intersection for these pairs of lines.



2 Decide if the point (1, 3) satisfies these equations. (*Hint*: Substitute (1, 3) into each equation to see if the equation is true.)

a $y = 2x + 1$ **b** $y = -x + 5$ **c** $y = -2x - 1$ **d** $y = 4x - 1$

3 Consider the two lines with the rules $y = 5x$ and $y = 3x + 2$ and the point (1, 5).

a Substitute (1, 5) into $y = 5x$. Does (1, 5) sit on the line with equation $y = 5x$?

b Substitute (1, 5) into $y = 3x + 2$. Does (1, 5) sit on the line with equation $y = 3x + 2$?

c Is (1, 5) the intersection point of the lines with the given equations?

Example 20 Checking an intersection point

Decide if the given point is at the intersection of the two lines with the given equations.

a $y = 2x + 3$ and $y = -x$ with point $(-1, 1)$

b $y = -2x$ and $3x + 2y = 4$ with point $(2, -4)$

SOLUTION	EXPLANATION
a Substitute $x = -1$. $y = 2x + 3$ $= 2 \times (-1) + 3$ $= -2 + 3$ $= 1$ So $(-1, 1)$ satisfies $y = 2x + 3$.	Substitute $x = -1$ into $y = 2x + 3$ to see if $y = 1$. If so, then $(-1, 1)$ is on the line.
$y = -x$ $= -(-1)$ $= 1$ So $(-1, 1)$ satisfies $y = -x$.	Repeat for $y = -x$.
$\therefore$ $(-1, 1)$ is the intersection point.	If $(-1, 1)$ is on both lines then it must be the intersection point.
b Substitute $x = 2$. $y = -2x$ $= -2 \times (2)$ $= -4$ So $(2, -4)$ satisfies $y = -2x$.	Substitute $x = 2$ into $y = -2x$ to see if $y = -4$. If so, then $(2, -4)$ is on the line.
$3x + 2y = 4$ $3 \times (2) + 2 \times (-4) = 4$ $6 + (-8) = 4$ $-2 = 4$ (false)	Substitute $x = 2$ and $y = -4$ to see if $3x + 2y = 4$ is true. Clearly, the equation is not satisfied.
So $(2, -4)$ is not on the line. $\therefore$ $(2, -4)$ is not the intersection point.	As the point is not on both lines, it cannot be the intersection point.

Now you try

Decide if the given point is at the intersection of the two lines with the given equations.

a $y = 3x - 5$ and $y = -2x$ with point $(1, -2)$

b $y = -4x$ and $3x - y = 2$ with point $(2, -8)$

Example 21 Solving simultaneous equations graphically

Solve the simultaneous equations $y = 2x - 2$ and $x + y = 4$ graphically.

SOLUTION

$y = 2x - 2$

y-intercept (let $x = 0$):

$y = 2 \times (0) - 2$

$y = -2$

x-intercept (let $y = 0$):

$0 = 2x - 2$

$2 = 2x$

$x = 1$

$x + y = 4$

y-intercept (let $x = 0$):

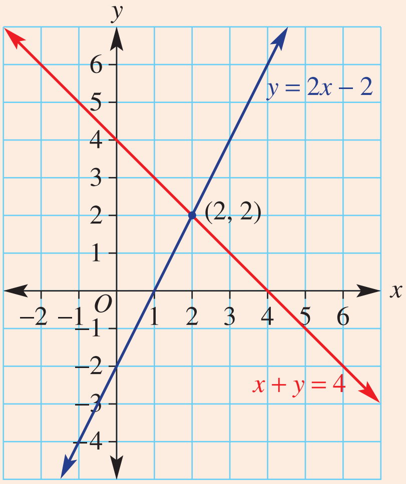
$(0) + y = 4$

$y = 4$

x-intercept (let $y = 0$):

$x + (0) = 4$

$x = 4$



The intersection point is (2, 2).

EXPLANATION

Sketch each linear graph by first finding the y-intercept (substitute $x = 0$) and the x-intercept (substitute $y = 0$) and solve the resulting equation.

Repeat the process for the second equation.

Sketch both graphs on the same set of axes by marking the intercepts and joining with a straight line.

Ensure that the axes are scaled accurately.

Locate the intersection point and read off the coordinates.

The point (2, 2) simultaneously belongs to both lines.

Now you try

Solve the simultaneous equations $y = -x + 1$ and $2x + y = 3$ graphically.

ISBN 978-1-108-77288-4

Updated September 2021

Exercise 4K

FLUENCY	1, 2–3(½)	2–3(½)	2–3(½)

Example 20

1 Decide if the given point is at the intersection of the two lines with the given equations.

a $y = x + 2$ and $y = 3x$ with point $(1, 3)$

b $x + 2y = 6$ and $3x - 4y = -1$ with point $(2, 2)$

Example 20

2 Decide if the given point is at the intersection of the two lines with the given equations.

a $y = 3x - 4$ and $y = 2x - 2$ with point $(2, 2)$

b $y = -x + 3$ and $y = -x$ with point $(2, 1)$

c $y = -4x + 1$ and $y = -x - 1$ with point $(1, -3)$

d $x - y = 10$ and $2x + y = 8$ with point $(6, -4)$

e $2x + y = 0$ and $y = 3x + 4$ with point $(-1, 2)$

f $x - 3y = 13$ and $y = -x - 1$ with point $(4, -3)$

Example 21

3 Solve these pairs of simultaneous equations graphically by finding the coordinates of the intersection point.

a $2x + y = 6$
$x + y = 4$

b $3x - y = 7$
$y = 2x - 4$

c $y = x - 6$
$y = -2x$

d $y = 2x - 4$
$x + y = 5$

e $2x - y = 3$
$3x + y = 7$

f $y = 2x + 1$
$y = 3x - 2$

g $x + y = 3$
$3x + 2y = 7$

h $y = x + 2$
$y = 3x - 2$

i $y = x - 3$
$y = 2x - 7$

j $y = 3$
$x + y = 2$

k $y = 2x - 3$
$x = -1$

l $y = 4x - 1$
$y = 3$

PROBLEM-SOLVING	4, 5	4, 5	5, 6

4 A company manufactures electrical components. The cost, \$$C$ (including rent, materials and labour), is given by the rule $C = n + 3000$, and its revenue, \$$R$, is given by the rule $R = 5n$, where n is the number of components produced.

a Sketch the graphs of C and R on the same set of axes and determine the point of intersection.

b State the number of components, n, for which the costs \$$C$ are equal to the revenue \$$R$.

5 Dvdcom and Associates manufactures DVDs. Its costs, \$$C$, are given by the rule $C = 4n + 2400$, and its revenue, \$$R$, is given by the rule $R = 6n$, where n is the number of DVDs produced. Sketch the graphs of C and R on the same set of axes and determine the number of DVDs produced if the costs equal the revenue.

6 Two asteroids are 1000 km apart and are heading straight for each other. One asteroid is travelling at 59 km per second and the other at 41 km per second. How long will it be before they collide?

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REASONING	7	7, 8	7–9

7 Explain why the graphs of the rules $y = 3x - 7$ and $y = 3x + 4$ have no intersection point.

8 For the following families of graphs, determine their points of intersection (if any).

a $y = x, y = 2x, y = 3x$

b $y = x, y = -2x, y = 3x$

c $y = x + 1, y = x + 2, y = x + 3$

d $y = -x + 1, y = -x + 2, y = -x + 3$

e $y = 2x + 1, y = 3x + 1, y = 4x + 1$

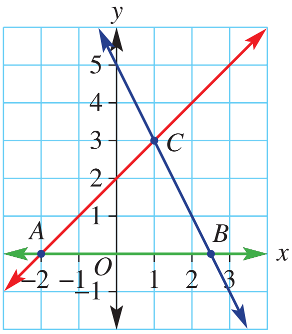
f $y = 2x + 3, y = 3x + 3, y = 4x + 3$

9 **a** If two lines have the equations $y = 3x + 1$ and $y = 2x + c$, find the value of c if the intersection point is at $x = 1$.

b If two lines have the equations $y = mx - 4$ and $y = -2x - 3$, find the value of m if the intersection point is at $x = -1$.

ENRICHMENT: Intersecting to find triangular areas	–	–	10, 11

10 The three lines with equations $y = 0$, $y = x + 2$ and $y = -2x + 5$ are illustrated here.



a State the coordinates of the intersection point of $y = x + 2$ and $y = -2x + 5$.

b Use $A = \frac{1}{2}bh$ to find the area of the enclosed triangle ABC.

11 Use the method outlined in Question **10** to find the area enclosed by these sets of three lines.

a $y = 0$, $y = x + 3$ and $y = -2x + 9$

b $y = 0$, $y = \frac{1}{2}x + 1$ and $y = -x + 10$

c $y = 2$, $x - y = 5$ and $x + y = 1$

d $y = -5$, $2x + y = 3$ and $y = x$

e $x = -3$, $y = -3x$ and $x - 2y = -7$

Using CAS and non-CAS calculators 4K: Finding intersection points

This activity is in the Interactive Textbook in the form of a printable PDF.

Swimming at number 4

At a swim meet, Jess is the fourth and fastest swimmer in her team of 4 which swims the medley relay. Her average speed is 1.5 metres per second. During the race, she often finds herself in second position just before she dives in and needs to catch up so that she wins the race for her team. Each swimmer needs to swim 50 metres.

In the relay Adrianna is the last swimmer of another team which is currently in the lead before she dives in. To win Jess needs to catch up and overtake Adrianna. Adrianna's average speed is 1.4 metres per second.

For this task, d metres is the distance travelled by a swimmer and t seconds is the time in seconds after Adrianna dives into the water.

Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate.

Preliminary task

- **a** Write a rule for Adrianna's distance (d metres) in terms of time (t seconds).
- **b** Find how long it takes for Adrianna to complete the 50 metres to the nearest tenth of a second.
- **c** If Jess dives into the water 2 seconds after Adrianna, then the rule describing Jess's distance after t seconds is given by: $d = 1.5t - 3$.
  - **i** Find how long it takes for Jess to complete the 50 metres, after Adrianna dives into the water, to the nearest tenth of a second.
  - **ii** Sketch a graph of d vs t for both Jess and Adrianna on the same set of axes. Show the coordinates of any endpoints and intercepts.
  - **iii** Does Jess overtake Adrianna? If so, after how many seconds?

Modelling task

Formulate

- **a** The problem is to determine the maximum head start that Adrianna can have so that Jess wins the relay for her team. Write down all the relevant information that will help solve this problem.

Solve

- **b** Jess dives into the water 1 second after Adrianna. i.e. when $t = 1$ then $d = 0$ for Jess.
  - **i** Find a rule for Jess's distance (d metres) after t seconds.
  - **ii** Find the time taken for Jess to complete the 50 metres, after Adrianna dives into the water, to the nearest tenth of a second.
  - **iii** Sketch a graph of d vs t for both Jess and Adrianna on the same set of axes. Show the coordinates of any endpoints and intercepts.
  - **iv** Decide if Jess overtakes Adrianna. If so, after how many seconds?
- **c** Repeat part **b** if Jess dives into the water 3 seconds after Adrianna.

ISBN 978-1-108-77288-4

Evaluate and verify

d If Jess catches up to Adrianna at exactly the time when Adrianna finishes the relay:
  i illustrate this situation with a graph
  ii determine the head start, in seconds to one decimal place, that Adrianna had in this situation.

e Decide under what circumstances will Jess win the relay for her team. Justify your answer.

Communicate

f Summarise your results and describe any key findings.

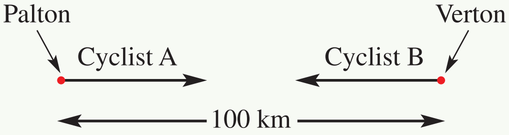
Extension questions

a If Jess's swim speed was able to be increased to 1.6 m/s, investigate the maximum head start that Adrianna could have so that Jess wins the relay.

b If in one particular race Adrianna has a 2.1 second head start, determine Jess's minimum speed if she is to win the relay.

Coming and going

The distance between two towns, Palton and Verton, is 100 km. Two cyclists travel in opposite directions between the towns, starting their journeys at the same time. Cyclist A travels from Palton to Verton at a speed of 20 km/h while cyclist B travels from Verton to Palton at a speed of 25 km/h.



Measuring the distance from Palton

a Using d_A km as the distance cyclist A is from Palton after t hours, explain why the rule connecting d_A and t is $d_A = 20t$.

b Using d_B km as the distance cyclist B is from Palton after t hours, explain why the rule connecting d_B and t is $d_B = 100 - 25t$.

Technology: Spreadsheet (alternatively use a graphics or CAS calculator – see parts* e *and* f *below)

c Instructions:

- Enter the time in hours into column A, starting at 0 hours.
- Enter the formulas for the distances d_A and d_B into columns B and C.
- Use the **Fill down** function to fill in the columns. Fill down until the distances show that both cyclists have completed their journey.

	A	B	C
1	0	= 20* A1	= 100 – 25* A1
2	= A1 + 1		
3			

d **i** Determine how long it takes for cyclist A to reach Verton.

ii Determine how long it takes for cyclist B to reach Palton.

iii After which hour are the cyclists the closest?

Alternative technology: Graphics or CAS calculator

e Instructions:

- Enter or define the formulas for the distances d_A and d_B.
- Go to the table and scroll down to view the distance for each cyclist at hourly intervals. You may need to change the settings so that t increases by 1 each time.

f **i** Determine how long it takes for cyclist A to reach Verton.

ii Determine how long it takes for cyclist B to reach Palton.

iii After which hour are the cyclists the closest?

Investigating the intersection

a Change the time increment to a smaller unit for your chosen technology.

- Spreadsheet: Try 0.5 hours using '= A1 + 0.5', or 0.1 hours using '= A1 + 0.1' in column A.
- Graphics or CAS calculator: Try changing the t increment to 0.5 or 0.1.

b Fill or scroll down to ensure that the distances show that both cyclists have completed their journey.

c Determine the time at which the cyclists are the closest.

d Continue altering the time increment until you are satisfied that you have found the time of intersection of the cyclists correct to one decimal place.

e *Extension:* Complete part **d** above but find an answer correct to three decimal places.

The graph

a Sketch a graph of d_A and d_B on the same set of axes. Scale your axes carefully to ensure that the full journey for both cyclists is represented.

b Determine the intersection point as accurately as possible on your graph and hence estimate the time when the cyclists meet.

c Use technology (graphing calculator) to confirm the point of intersection, and hence determine the time at which the cyclists meet, correct to three decimal places.

Algebra and proof

a At the point of intersection it could be said that $d_A = d_B$.
This means that $20t = 100 - 25t$.
Solve this equation for t.

b Find the exact distance from Palton at the point where the cyclists meet.

Reflection

Write a paragraph describing the journey of the two cyclists. Comment on:

- the speeds of the cyclists
- their meeting point
- the difference in computer and algebraic approaches in finding the time of the intersection point.

Up for a challenge? If you get stuck on a question, check out the 'Working with unfamiliar problems' poster at the end of the book to help you.

1 A tank with 520 L of water begins to leak at a rate of 2 L per day. At the same time, a second tank is being filled at a rate of 1 L per hour starting at 0 L. How long does it take for the tanks to have the same volume?

2 Two cars travel towards each other on a 400 km stretch of road. One car travels at 90 km/h and the other at 70 km/h. How long does it take before they pass each other?

3 The points $(-1, 4)$, $(4, 6)$, $(2, 7)$ and $(-3, 5)$ are the vertices of a parallelogram. Find the midpoints of its diagonals. What do you notice?

4 A trapezium is enclosed by the straight lines $y = 0$, $y = 3$, $y = 7 - x$ and $x = k$, where k is a constant. Find the possible values of k given the trapezium has an area of 24 units^2.

5 Prove that the triangle with vertices at the points $A(-1, 3)$, $B(0, -1)$ and $C(3, 2)$ is isosceles.

6 Find the perimeter (to the nearest whole number) and area of the triangle enclosed by the lines with equations $x = -4$, $y = x$ and $y = -2x - 3$.

7 $ABCD$ is a parallelogram. A, B and C have coordinates $(5, 8)$, $(2, 5)$ and $(3, 4)$ respectively. Find the coordinates of D.

8 A kite is formed by joining the points $A(a, b)$, $B(-1, 3)$, $C(x, y)$ and $D(3, -5)$.

- **a** Determine the equations of the diagonals BD and AC of this kite.
- **b** Given $a = -5$ and $y = 4$, find the values of b and x.
- **c** Find the area of the kite (without the use of a calculator).

9 A trapezium is enclosed by the straight lines $y = 0$, $y = 6$, $y = 8 - 2x$ and $y = x + k$, where k is a constant. Find the possible values of k given the trapezium has an area of 66 units^2.

Chapter summary

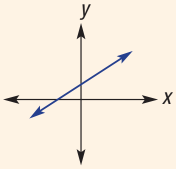
Linear relations

Direct proportion

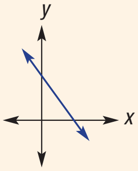
If two variables are directly proportional, their rule is of the form $y = mx$. The gradient of the graph represents the rate of change of y with respect to x; e.g. for a car travelling at 60 km/h for t hours, distance $= 60t$.

Gradient

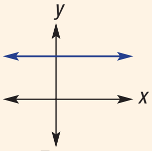
This measures the slope of a line.



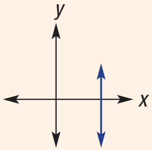
Positive



Negative



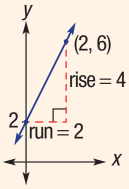
Zero



Undefined

Gradient $m = \frac{\text{rise}}{\text{run}}$

$m = \frac{4}{2} = 2$



Parallel and perpendicular lines (Ext)

Parallel lines have the same gradient e.g. $y = 2x + 3$, $y = 2x - 1$ and $y = 2x$

Perpendicular lines are at right angles. Their gradients m_1 and m_2 are such that $m_1 m_2 = -1$, i.e. $m_2 = -\frac{1}{m_1}$.

A line perpendicular to $y = 2x + 3$ has gradient $-\frac{1}{2}$.

Sketching using intercepts

Sketch with two points. Often we find the x-intercept ($y = 0$) and y-intercept ($x = 0$).

e.g. $y = 2x - 4$

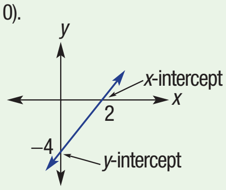
y-int: $x = 0$, $y = 2(0) - 4$

$= -4$

x-int: $y = 0$, $0 = 2x - 4$

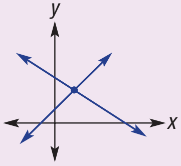
$2x = 4$

$x = 2$

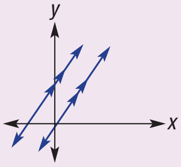


Graphical solutions of simultaneous equations

Graph each line and read off the point of intersection.



Parallel lines have no intersection point.



Straight line

$y = mx + c$

gradient y-intercept

In the form $ax + by = d$, rearrange to $y = mx + c$ form to read off gradient and y-intercept.

A linear relation is made up of points (x, y) that form a straight line when plotted.

Linear modelling

Define variables to represent the problem and write a rule relating the two variables. The rate of change of one variable with respect to the other is the gradient.

Finding the equation of a line

Gradient–intercept form ($y = mx + c$)

Require gradient and y-intercept or any other point to substitute; e.g. line with gradient 3 passes through point (2, 1)

$\therefore\ y = 3x + c$

substitute $x = 2$, $y = 1$

$1 = 3(2) + c$

$1 = 6 + c$

$\therefore\ c = -5$

$y = 3x - 5$

Midpoint and length of a line segment

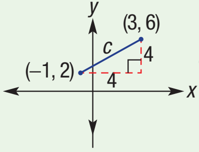
Midpoint is halfway between segment endpoints, e.g. midpoint of segment joining (−1, 2) and (3, 6).

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$x = \frac{-1 + 3}{2} = 1$$

$$y = \frac{2 + 6}{2} = 4\text{, i.e. } (1, 4)$$

Length of segment



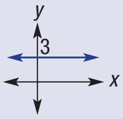
$c^2 = 4^2 + 4^2$

$c = \sqrt{32}$

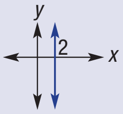
$c = 5.7$ (to 1 d.p.)

Special lines

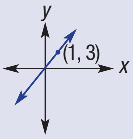
Horizontal line $y = c$
e.g. $y = 3$



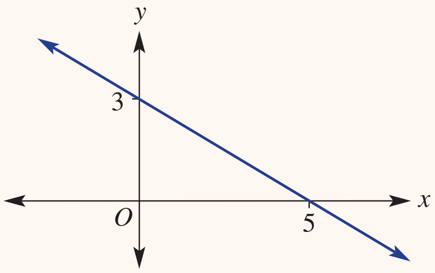
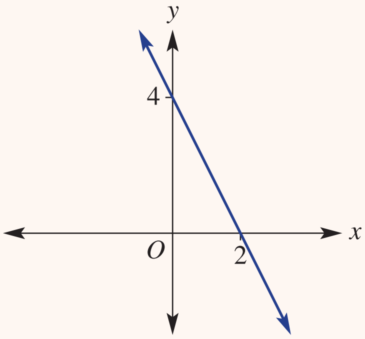
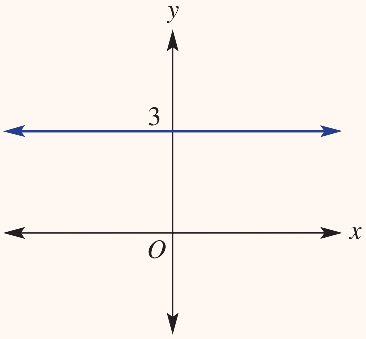
Vertical line $x = b$
e.g. $x = 2$

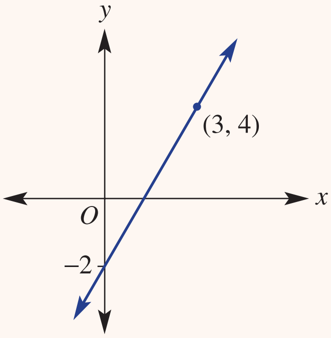


$y = mx$ passes through the origin, substitute $x = 1$ to find another point, e.g. $y = 3x$



Chapter checklist: Success criteria

		✔
4A	**1. I can plot points to graph a straight line.** e.g. Using $-3 \leqslant x \leqslant 3$, construct a table of values and plot a graph of $y = 2x - 4$.	
4A	**2. I can identify the *x*- and *y*-intercept.** e.g. Write down the x- and y-intercepts from this graph. 	
4A	**3. I can determine if a point is on a line.** e.g. Decide if the point $(-2, 9)$ is on the line with rule $y = -2x + 3$.	
4B	**4. I can sketch a graph of a straight line labelling intercepts.** e.g. Sketch the graphs of $y = -2x + 4$ and $3x - 4y = 12$ showing x- and y-intercepts.	
4C	**5. I can graph horizontal and vertical lines.** e.g. Sketch $y = -2$ and $x = 5$.	
4C	**6. I can sketch lines of the form $y = mx$.** e.g. Sketch the graph of $y = 2x$.	
4D	**7. I can find the gradient of a line.** e.g. For the graphs shown, state whether the gradient is positive, negative, zero or undefined and find the gradient.  	
4D	**8. I can find the gradient between two points.** e.g. Find the gradient of the line joining the points $(-1, 3)$ and $(4, 9)$.	
4E	**9. I can work with direct proportion.** e.g. A bathtub is being filled at a constant rate. It takes 10 minutes to fill the bath with 60 litres. Draw a graph of volume (V litres) vs time (t minutes) for $0 \leqslant t \leqslant 10$. Find the gradient of the graph and a rule for V. Use the rule to find the time for the bathtub to fill to 45 litres.	
4F	**10. I can rearrange linear equations.** e.g. Rearrange $2x + 3y = 8$ to the form $y = mx + c$.	

		✔
4F	**11. I can state the gradient and *y*-intercept and use these to sketch a graph.** e.g. State the gradient and *y*-intercept of $y = 3x - 2$ and $2x + 3y = 8$, and sketch their graphs.	
4G	**12. I can determine the equation of a straight line graph given the *y*-intercept.** e.g. Determine the equation of the straight line shown here. 	
45	**13. I can determine the equation of a line given the gradient and a point.** e.g. Find the equation of the line which has a gradient of 2 and passes through the point $(-2, 5)$.	
4H	**14. I can find the midpoint of a line segment.** e.g. Find the midpoint $M(x, y)$ of the line segment joining the points $(2, -1)$ and $(6, 4)$.	
4H	**15. I can find the length of a line segment.** e.g. Find the length of the segment joining $(-2, 1)$ and $(3, 5)$, correct to two decimal places.	
4I	**16. I can find the equation of a parallel line.** e.g. Find the equation of a line that is parallel to the line $y = -2x + 3$ and passes through $(0, 1)$. Ext	
4I	**17. I can find the equation of a perpendicular line.** e.g. Find the equation of a line that is perpendicular to the line $y = 4x + 2$ and passes through $(0, -3)$. Ext	
4J	**18. I can apply linear relations.** e.g. The hire of a jumping castle involves a set-up cost of \$80 and a per hour rate of \$40. Write a rule for the total cost, \$*C*, of hiring a jumping castle for *h* hours and sketch its graph for $0 \leqslant h \leqslant 8$. Determine the total cost for hiring for 5 hours and how many hours it was hired for if the cost was \$200.	
4K	**19. I can check if a point is the intersection point of two lines.** e.g. Determine if the point $(2, -1)$ is at the intersection of the lines with equations $y = -2x + 3$ and $2x - 3y = 7$.	
4K	**20. I can solve simultaneous equations graphically.** e.g. Solve the simultaneous equations $y = 2x - 4$ and $x + y = 5$ graphically.	

Short-answer questions

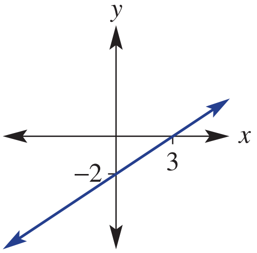
4A

1 Read off the x- and y-intercepts from the table and graph.

a

x	-2	-1	0	1	2
y	0	2	4	6	8

b



4B

2 Sketch the following linear graphs labelling x- and y-intercepts.

a $y = 2x - 4$ **b** $y = 3x + 9$ **c** $y = -2x + 5$ **d** $y = -x + 4$

e $2x + 4y = 8$ **f** $4x - 2y = 10$ **g** $2x - y = 7$ **h** $-3x + 6y = 12$

4B

3 Violet leaves her beach house by car and drives back to her home. Her distance d kilometres from her home after t hours is given by $d = 175 - 70t$.

a How far is her beach house from her home?

b How long does it take to reach her home?

c Sketch a graph of her journey between the beach house and her home.

4C

4 Sketch the following lines.

a $y = 3$ **b** $y = -2$ **c** $x = -4$

d $x = 5$ **e** $y = 3x$ **f** $y = -2x$

4D

5 By first plotting the given points, find the gradient of the line passing through the points.

a $(3, 1)$ and $(5, 5)$ **b** $(2, 5)$ and $(4, 3)$ **c** $(1, 6)$ and $(3, 1)$

d $(-1, 2)$ and $(2, 6)$ **e** $(-3, -2)$ and $(1, 6)$ **f** $(-2, 6)$ and $(1, -4)$

4E

6 An inflatable backyard swimming pool is being filled with water by a hose. It takes 4 hours to fill 8000 L.

a What is the rate at which water is poured into the pool?

b Draw a graph of volume (V litres) versus time (t hours) for $0 \leqslant t \leqslant 4$.

c Find the gradient of your graph and give the rule for V in terms of t.

d Use your rule to find the time to fill 5000 L.

4F

7 For each of the following linear relations, state the value of the gradient and the y-intercept, and then sketch using the gradient–intercept method.

a $y = 2x + 3$

b $y = -3x + 7$

c $2x + 3y = 9$

d $2y - 3x - 8 = 0$

4G

8 Give the equation of the straight line that:

a has gradient 3 and passes through the point $(0, 2)$

b has gradient -2 and passes through the point $(3, 0)$

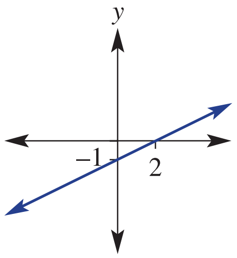
c has gradient $\frac{4}{3}$ and passes through the point $(6, 3)$.

ISBN 978-1-108-77288-4

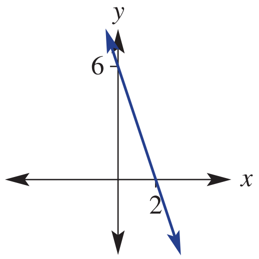
4G

9 Find the equations of the linear graphs below.

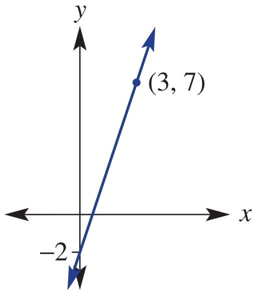
a



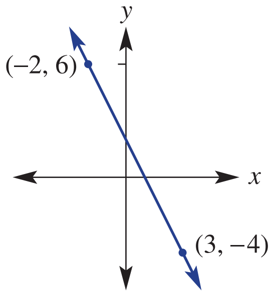
b



c



d



4H

10 For the line segment joining the following pairs of points, find:

i the midpoint **ii** the length (to two decimal places where applicable).

a $(2, 4)$ and $(6, 8)$

b $(5, 2)$ and $(10, 7)$

c $(-2, 1)$ and $(2, 7)$

d $(-5, 7)$ and $(-1, -2)$

4H

11 Find the missing coordinate, n, if:

a the line joining $(-1, 3)$ and $(2, n)$ has gradient 2

b the line segment joining $(-2, 2)$ and $(4, n)$ has length 10, $n > 0$

c the midpoint of the segment joining $(n, 1)$ and $(1, -3)$ is $(3.5, -1)$.

4I

Ext

12 Determine the linear equation that is:

a parallel to the line $y = 2x - 1$ and passes through the point $(0, 4)$

b parallel to the line $y = -x + 4$ and passes through the point $(0, -3)$

c perpendicular to the line $y = 2x + 3$ and passes through the point $(0, -1)$

d perpendicular to the line $y = -\frac{1}{3}x - 2$ and passes through the point $(0, 4)$

e parallel to the line $y = 3x - 2$ and passes through the point $(1, 4)$

f parallel to the line $3x + 2y = 10$ and passes through the point $(-2, 7)$.

4K

13 Determine if the point $(2, 5)$ is the intersection point of the graphs of the following pairs of equations.

a $y = 4x - 3$ and $y = -2x + 6$

b $3x - 2y = -4$ and $2x + y = 9$

4K

14 Find the point of intersection of the following straight lines.

a $y = 2x - 4$ and $y = 6 - 3x$

b $2x + 3y = 8$ and $y = -x + 2$

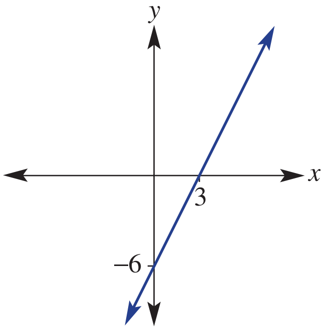
Multiple-choice questions

4A **1** The x- and y-intercepts of the graph of $2x + 4y = 12$ are respectively:

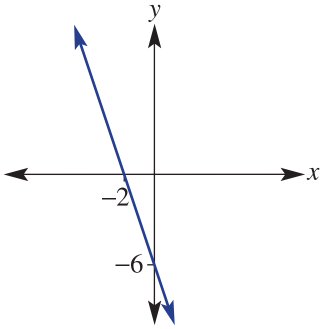
A 4, 2 **B** 3, 2 **C** 6, 3 **D** 2, 12 **E** 8, 6

4B **2** The graph of $y = 3x - 6$ is represented by:

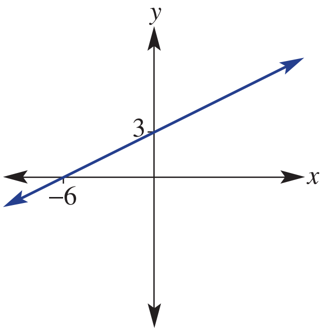
A



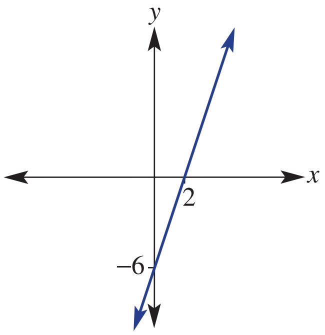
B



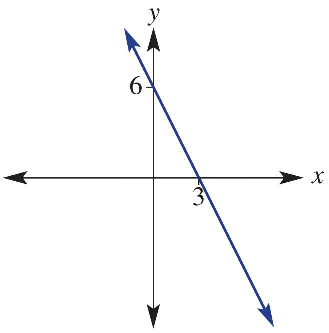
C



D



E



4D **3** The gradient of the line joining the points $(1, -2)$ and $(5, 6)$ is:

A 2 **B** 1 **C** 3 **D** -1 **E** $\frac{1}{3}$

4A

4 The point that is not on the straight line $y = -2x + 3$ is:

A $(1, 1)$
B $(0, 3)$
C $(2, 0)$
D $(-2, 7)$
E $(3, -3)$

4F

5 The linear graph that *does not* have a gradient of 3 is:

A $y = 3x + 7$
B $\frac{y}{3} = x + 2$
C $2y - 6x = 1$
D $y - 3x = 4$
E $3x + y = -1$

4G

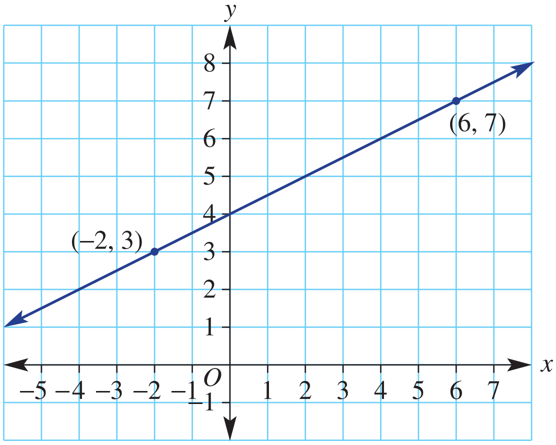
6 A straight line has a gradient of -2 and passes through the point $(0, 5)$. Its equation is:

A $2y = -2x + 10$
B $y = -2x + 5$
C $y = 5x - 2$
D $y - 2x = 5$
E $y = -2(x - 5)$

4G

7 The equation of the graph shown is:

A $y = x + 1$
B $y = \frac{1}{2}x + 4$
C $y = 2x + 1$
D $y = -2x - 1$
E $y = \frac{1}{2}x + 2$



4H

8 The length of the line segment joining the points $A(1, 2)$ and $B(4, 6)$ is:

A 5
B $\frac{4}{3}$
C 7
D $\sqrt{5}$
E $\sqrt{2}$

4I

Ext

9 The gradient of a line perpendicular to $y = 4x - 7$ would be:

A $\frac{1}{4}$
B 4
C -4
D $-\frac{1}{4}$
E -7

4K

10 The point of intersection of $y = 2x$ and $y = 6 - x$ is:

A $(6, 2)$
B $(-1, -2)$
C $(6, 12)$
D $(2, 4)$
E $(3, 6)$

Extended-response questions

1 Joe requires an electrician to come to his house to do some work. He is trying to choose between two electricians he has been recommended.

a The first electrician's cost, \$$C$, is given by $C = 80 + 40n$ where n is the number of hours the job takes.

i State the hourly rate this electrician charges and his initial fee for coming to the house.

ii Sketch a graph of C versus n for $0 \leqslant n \leqslant 8$.

iii What is the cost of a job that takes 2.5 hours?

iv If the job costs \$280, how many hours did it take?

b The second electrician charges a callout fee of \$65 to visit the house and then \$45 per hour.

i Give the equation for the cost, \$$C$, of a job that takes n hours.

ii Sketch the graph of part **b i** for $0 \leqslant n \leqslant 8$ on the same axes as the graph in part **a**.

c Determine the point of intersection of the two graphs.

d After how many hours does the first electrician become the cheaper option?

2 Abby has set up a small business making clay vases. Her production costs include a fixed weekly cost for equipment hire and a cost per vase for clay.

Abby has determined that the total cost of producing 7 vases in a week was \$146 and the total cost of producing 12 vases in a week was \$186.

a Find a linear rule relating the production cost, \$$C$, to the number of vases produced, v.

b Use your rule to state:

i the initial cost of materials each week

ii the ongoing cost of production per vase.

c At a selling price of \$12 per vase Abby determines her weekly profit to be given by $P = 4v - 90$.

How many vases must she sell in order to make a profit?

CHAPTER 5

Measurement

Body surface area

Body surface area (BSA) is measured in square metres and is the total surface area of a person's body. It is used in medicine to calculate correct drug dose and the volume of fluids needing to be administered intravenously.

Calculating the surface area of prisms and pyramids is relatively easy, as we use the area formulas for each face, and then find the total. However, calculating the actual surface area of the irregular solid, such as a person, is a difficult task.

To find the surface area of a person the following formula can be used.

$$\text{BSA } (\text{m}^2) = \sqrt{\frac{\text{body weight (kg)} \times \text{height (cm)}}{3600}}$$

Note that the average BSA for a 12–13-year-old is $1.33\,\text{m}^2$ and $1.7\,\text{m}^2$ for the average adult.

Paediatricians need to know a child's BSA to use Clarke's body surface area formula to calculate

ISBN 978-1-108-77288-4

the correct dose required by each patient. Clarke's formula states:

$$\textit{Child's dose} = \frac{\text{surface area of child (m}^2) \times \text{adult dose}}{\text{average surface area of adult (1.7 m}^2)}$$

Doctors and nurses need to be able to perform these calculations quickly and accurately in order to save lives.

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

In this chapter

5A Length and perimeter (CONSOLIDATING)
5B Circumference and perimeter of a sector
5C Area
5D Composite shapes
5E Surface area of prisms and pyramids
5F Surface area of a cylinder
5G Volume of a prism
5H Volume of a cylinder

Australian Curriculum

MEASUREMENT AND GEOMETRY
Using units of measurement

Calculate the areas of composite shapes (ACMMG216)

Calculate the surface area and volume of cylinders and solve related problems (ACMMG217)

Solve problems involving the surface area and volume of right prisms (ACMMG218)

© ACARA

5A Length and perimeter CONSOLIDATING

Learning intentions

- To review the concept of perimeter
- To review metric units of length and how to convert between these
- To be able to find the perimeter of simple shapes

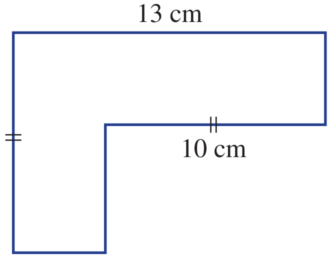
Length is at the foundation of measurement from which the concepts of perimeter, circumference, area and volume are developed. From the use of the royal cubit (distance from tip of middle finger to the elbow) used by the ancient Egyptians to the calculation of pi (π) by modern computers, units of length have helped to create the world in which we live.

To determine the length of scaffolding required for a multistorey building, multiples of its perimeter are calculated.

LESSON STARTER Not enough information?

All the angles at each vertex in this shape are 90° and the two given lengths are 10 cm and 13 cm.

- Do you think there is enough information to find the perimeter of the shape?
- If there is enough information, find the perimeter and discuss your method. If not, then list what information needs to be provided.

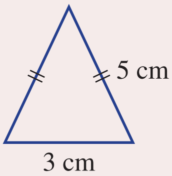


KEY IDEAS

- To convert between **metric units** of length, multiply or divide by the appropriate power of 10.

$$\begin{tikzcd}
\text{km} \arrow[r, bend left, "\times 1000"] & \text{m} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 100"] & \text{cm} \arrow[l, bend left, "\div 100"] \arrow[r, bend left, "\times 10"] & \text{mm} \arrow[l, bend left, "\div 10"]
\end{tikzcd}$$

- **Perimeter** is the distance around the outside of a closed shape.
  - Sides with the same markings are of equal length.



$$\begin{aligned} P &= 2\times 5+3 \\ &= 13 \text{ cm} \end{aligned}$$

BUILDING UNDERSTANDING

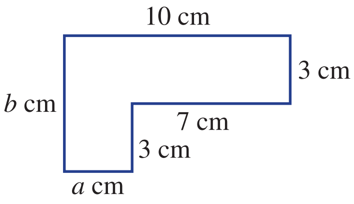
① Convert the following length measurements to the units given in the brackets.

a 5 cm (mm)
b 2.8 m (cm)
c 521 mm (cm)
d 83.7 cm (m)
e 4.6 km (m)
f 2170 m (km)

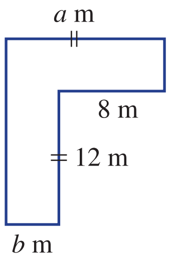
② A steel beam is 8.25 m long and 22.5 mm wide. State the length and the width of the beam in centimetres.

③ Give the values of the pronumerals in these shapes. All angles at vertices are 90°.

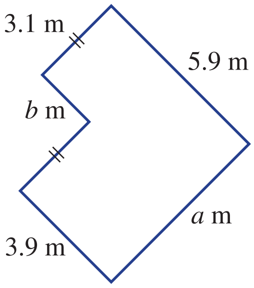
a



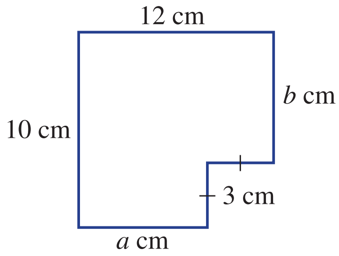
b



c



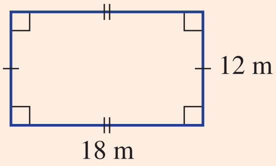
d



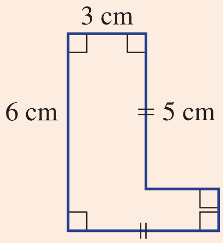
Example 1 Finding perimeters of simple shapes

Find the perimeter of each of the following shapes.

a



b



SOLUTION	EXPLANATION
a Perimeter $= 2 \times 12 + 2 \times 18$ $= 24 + 36$ $= 60$ m	There are two lengths of 12 m and two lengths of 18 m.

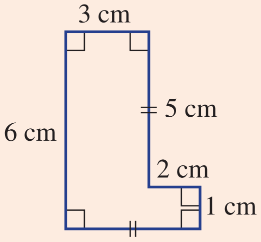
Continued on next page

b Perimeter $= (2 \times 5) + 6 + 3 + 2 + 1$
$= 22$ cm

Alternative method:

Perimeter $= 2 \times 6 + 2 \times 5$
$= 22$ cm

Missing sides are:
5 cm − 3 cm = 2 cm
6 cm − 5 cm = 1 cm

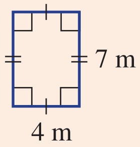


There are two opposite sides each of combined length 6 cm and two opposite sides each of combined length 5 cm.

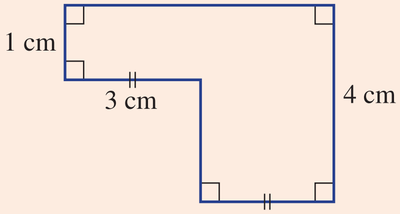
Now you try

Find the perimeter of each of the following shapes.

a

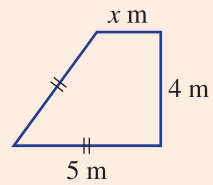


b



Example 2 Finding missing sides given a perimeter

Find the unknown side length in this shape with the given perimeter.



Perimeter = 16 m

SOLUTION

$2 \times 5 + 4 + x = 16$
$14 + x = 16$
$x = 2$

∴ the missing length is 2 m.

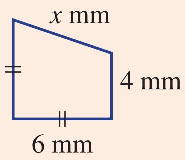
EXPLANATION

Add all the lengths and set equal to the given perimeter.

Solve for the unknown.

Now you try

Find the unknown side length in this shape with the given perimeter.



Perimeter = 23 mm

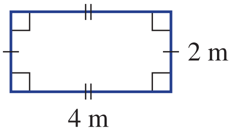
Exercise 5A

FLUENCY	1, 2, 3(½), 4, 5	2, 3(½), 4, 5, 6(½)	3(½), 4, 5–6(½)

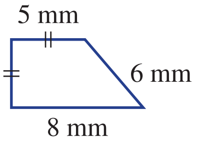
Example 1

1 Find the perimeter of each of the following shapes

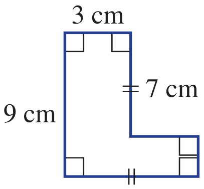
a



b



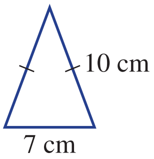
c



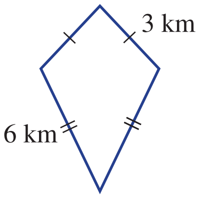
Example 1a

2 Find the perimeter of each of the following shapes.

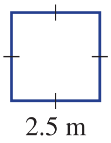
a



b



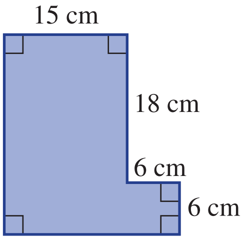
c



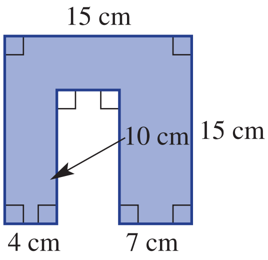
Example 1b

3 Find the perimeter of each of the following composite shapes.

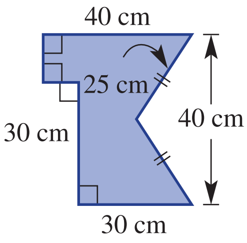
a



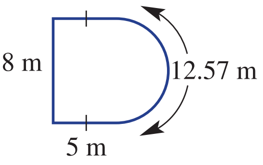
b



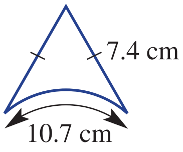
c



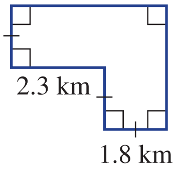
d



e

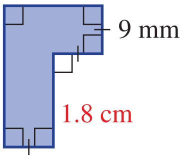


f

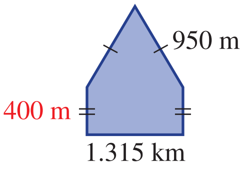


4 Find the perimeter of each of these shapes. You will need to convert the measurements to the same units. Give your answers in the units given in red.

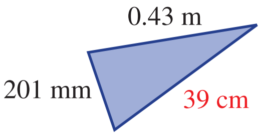
a



b



c



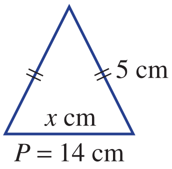
5 Convert the following measurements to the units given in the brackets.

- **a** 8 m (mm)
- **b** 110 000 mm (m)
- **c** 0.00001 km (cm)
- **d** 0.02 m (mm)
- **e** 28 400 cm (km)
- **f** 62 743 000 mm (km)

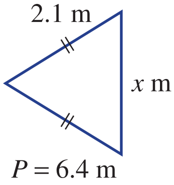
Example 2

6 Find the unknown side length in these shapes with the given perimeters.

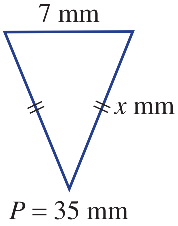
a



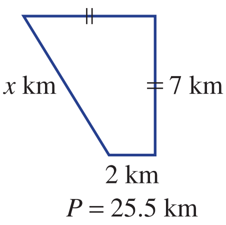
b



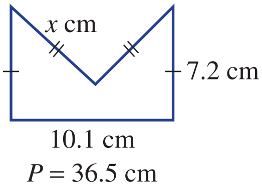
c



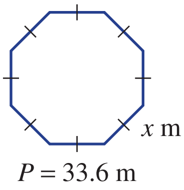
d



e



f

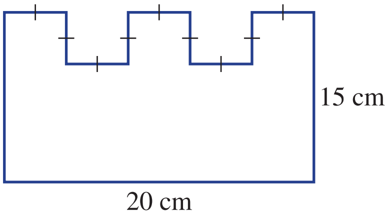


PROBLEM-SOLVING	7, 8	7–9	8–10

7 A lion enclosure is made up of five straight fence sections. Three sections are 20 m in length and the other two sections are 15.5 m and 32.5 m. Find the perimeter of the enclosure.

8 Find the perimeter of these shapes. Assume all angles are right angles.

a



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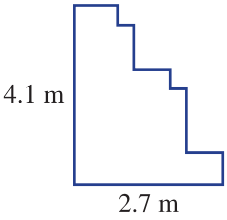
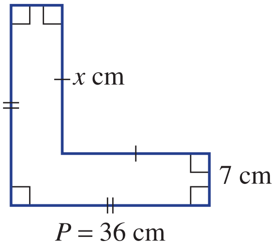


9 Find the value of x in these shapes with the given perimeter.

a



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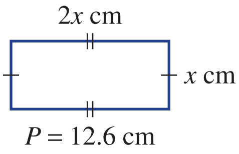

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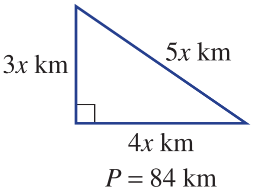
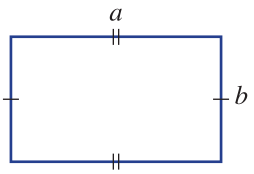


10 A photo 12 cm wide and 20 cm long is surrounded by a picture frame 3 cm wide. Find the outside perimeter of the framed photo.

REASONING	11	11, 12	12, 13

11 Give the rule for the perimeter of these shapes using the given pronumerals, e.g. $P = 3a + 2b$.

a



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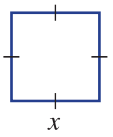
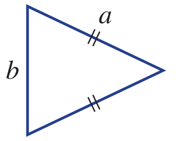
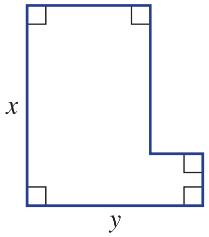


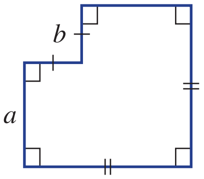
c



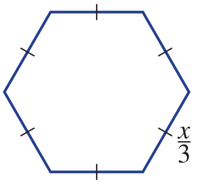
d



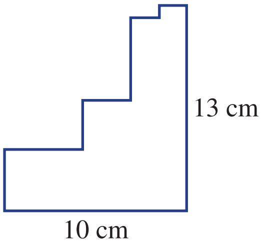
e



f



12 Explain why you do not need any more information to find the perimeter of this shape, assuming all angles are right angles.

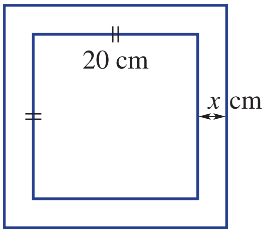


13 A piece of string 1 m long is divided into parts according to the given ratios. Find the length of each part.

a 1 : 3 **b** 2 : 3 **c** 5 : 3 **d** 1 : 2 : 3 : 4

ENRICHMENT: Picture framing	–	–	14

14 A square picture of side length 20 cm is surrounded by a frame of width x cm.



a Find the outside perimeter of the framed picture if:
  i $x = 2$ **ii** $x = 3$ **iii** $x = 5$.

b Write a rule for the perimeter P of the framed picture in terms of x.

c Use your rule to find the perimeter if:
  i $x = 3.7$ **ii** $x = 7.05$.

d Use your rule to find the value of x if:
  i the perimeter is 90 cm
  ii the perimeter is 102 cm.

e Is there a value of x for which the perimeter is 75 cm? Explain.

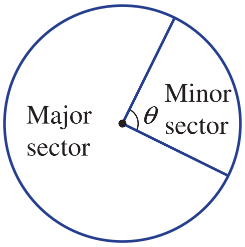
ISBN 978-1-108-77288-4

5B Circumference and perimeter of a sector

Learning intentions

- To know the features of a circle: circumference, radius, diameter, sector
- To know the formula for the circumference of a circle
- To be able to find an exact circumference or use a calculator to find it to a desired number of decimal places
- To understand that the arc length of a sector represents a fraction of the circumference
- To be able to find the perimeter of a sector

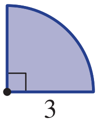
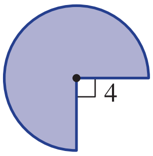
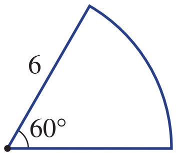
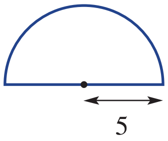
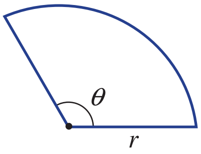
A portion of a circle enclosed by two radii and an arc is called a sector. The perimeter of a sector is made up of three components: two radii and the circular arc. Given an angle θ, it is possible to find the length of the arc using the rule for the circumference of a circle: $C = 2\pi r$ or $C = \pi d$.



LESSON STARTER Perimeter of a sector

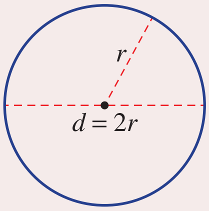
A sector is formed by dividing a circle with two radius cuts. The angle between the two radii determines the size of the sector. The perimeter will therefore depend on both the length of the radii and the angle between them.

- Complete this table to see if you can determine a rule for the perimeter of a sector. Remember that the circumference, C, of a circle is given by $C = 2\pi r$ where r is the radius length.

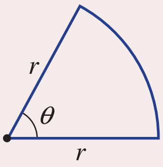
Shape	Fraction of full circle	Working and answer
	$\frac{90}{360} = \frac{1}{4}$	$P = 2 \times 3 + \frac{1}{4} \times 2\pi \times 3 \approx 10.71$
	$\frac{270}{360} =$	$P =$
		$P =$
		$P =$
		$P =$

KEY IDEAS

- **Circumference** of a circle $C = 2\pi r$ or $C = \pi d$.
  - $\frac{22}{7}$ or 3.14 can be used to approximate π or use technology for more precise calculations.

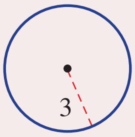


- A **sector** is a portion of a circle enclosed by two radii and an arc. Special sectors include:
  - a half circle, called a **semicircle**
  - a quarter circle, called a **quadrant**.
- The perimeter of a sector is given by $P = 2r + \frac{\theta}{360} \times 2\pi r$.



- The symbol for pi (π) can be used to write an answer exactly.

For example:
$$\begin{aligned} C &= 2\pi r \\ &= 2\pi \times 3 \\ &= 6\pi \end{aligned}$$



BUILDING UNDERSTANDING

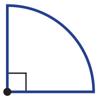
1 **a** What is the radius of a circle if its diameter is 5.6 cm?
b What is the diameter of a circle if its radius is 48 mm?

2 Simplify these numbers to give an exact answer. Do not evaluate with a calculator or round off. The first one is done for you.

a $2 \times 3 \times \pi = 6\pi$
b $6 \times 2\pi$
c $3 + \frac{1}{2} \times 4\pi$
d $2 \times 6 + \frac{1}{4} \times 12\pi$
e $2 \times 4 + \frac{90}{360} \times 2 \times \pi \times 4$
f $3 + \frac{270}{360} \times \pi$

3 Determine the fraction of a circle shown in these sectors. Write the fraction in simplest form.

a



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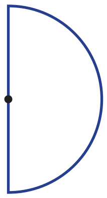
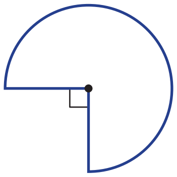
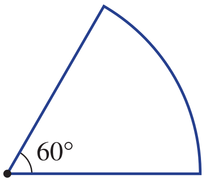


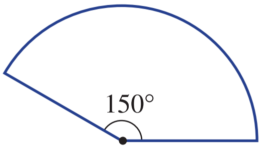
c



d



e



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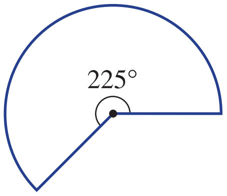
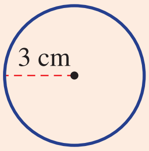


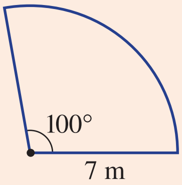
Example 3 Finding the circumference of a circle and the perimeter of a sector

Find the circumference of this circle and the perimeter of this sector correct to two decimal places.

a



b



SOLUTION

a $C = 2\pi r$

$= 2 \times \pi \times 3$

$= 6\pi$

$= 18.85$ cm (to 2 d.p.)

b $P = 2r + \frac{\theta}{360} \times 2\pi r$

$= 2 \times 7 + \frac{100}{360} \times 2 \times \pi \times 7$

$= 14 + \frac{35\pi}{9}$

$= 26.22$ m (to 2 d.p.)

EXPLANATION

Use the formula $C = 2\pi r$ or $C = \pi d$ and substitute $r = 3$ (or $d = 6$).

6π would be the exact answer and 18.85 is the rounded answer. Give units.

Write the formula.

The fraction of the circle is $\frac{100}{360}\left(\text{or } \frac{5}{18}\right)$.

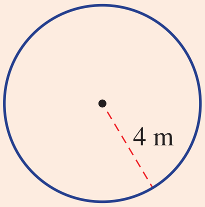
$14 + \frac{35\pi}{9}$ is the exact answer.

Use a calculator and round to two decimal places.

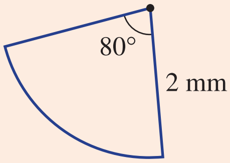
Now you try

Find the circumference of this circle and the perimeter of this sector correct to two decimal places.

a



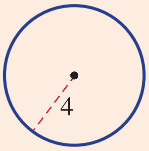
b



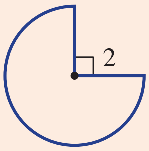
Example 4 Finding the exact perimeter of a shape

Give the exact circumference/perimeter of these shapes.

a



b



SOLUTION

a $C = 2\pi r$

$= 2 \times \pi \times 4$

$= 8\pi$

b $P = 2 \times r + \dfrac{\theta}{360} \times 2\pi r$

$= 2 \times 2 + \dfrac{270}{360} \times 2\pi \times 2$

$= 4 + \dfrac{3}{4} \times 4\pi$

$= 4 + 3\pi$

EXPLANATION

Write the formula with $r = 4$:

$2 \times \pi \times 4 = 2 \times 4 \times \pi$

Write the answer exactly in terms of π.

Use the perimeter of a sector formula.

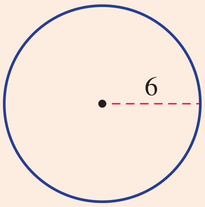
The angle inside the sector is 270° so the fraction is $\dfrac{270}{360} = \dfrac{3}{4}$.

$4 + 3\pi$ cannot be simplified further.

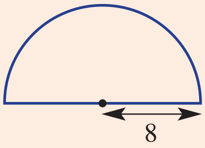
Now you try

Give the exact circumference/perimeter of these shapes.

a



b



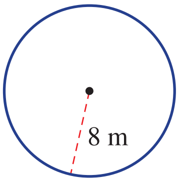
Exercise 5B

FLUENCY	1, 2, 3–5(1/2)	2, 3–6(1/2)	3–6(1/2)

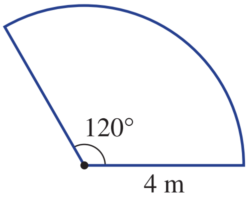
Example 3

1 Find the circumference of this circle and perimeter of this sector correct to two decimal places.

a



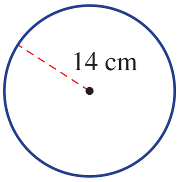
b



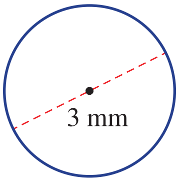
Example 3a

2 Find the circumference of these circles, correct to two decimal places. Use a calculator for the value of π.

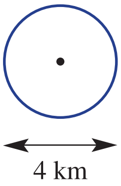
a



b



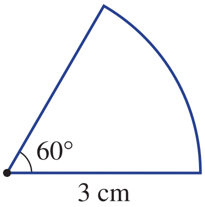
c



Example 3b

3 Find the perimeter of these sectors, correct to two decimal places.

a



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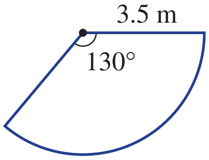

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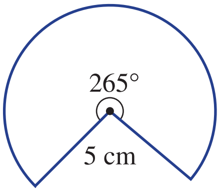
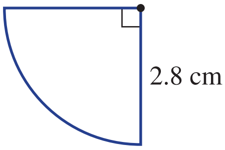
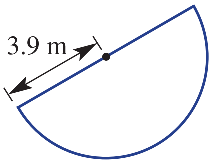


d (Quadrant)



e (Semicircle)



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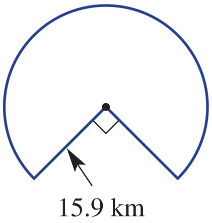
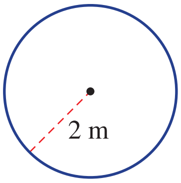


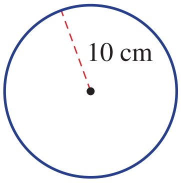
4 Find the circumference of these circles using the given approximation of π. Do not use a calculator.

a



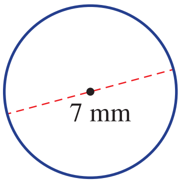
$\pi = 3.14$

b



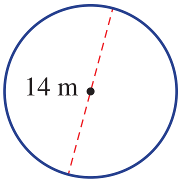
$\pi = 3.14$

c



$\pi = \frac{22}{7}$

d

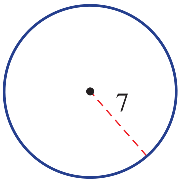


$\pi = \frac{22}{7}$

Example 4a

5 Give the exact circumference of these circles.

a



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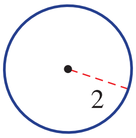

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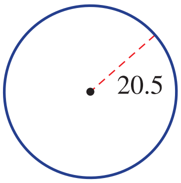

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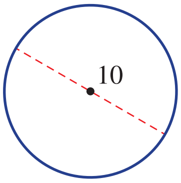

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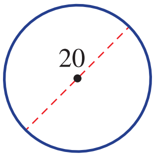

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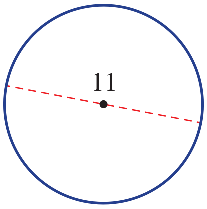

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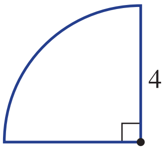
Example 4b

6 Give the exact perimeter of these sectors.

a



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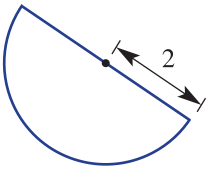

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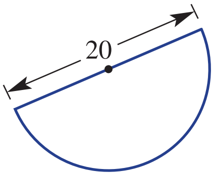

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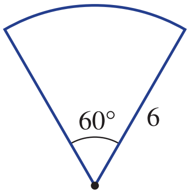

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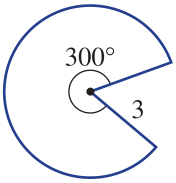

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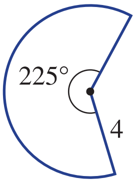


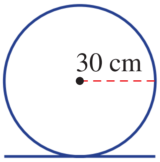
PROBLEM-SOLVING	7, 8	7–9	8–10

7 Find the distance around the outside of a circular pool of radius 4.5 m, correct to two decimal places.

8 Find the length of string required to surround the circular trunk of a tree that has a diameter of 1.3 m, correct to one decimal place.

9 The end of a cylinder has a radius of 5 cm. Find the circumference of the end of the cylinder, correct to two decimal places.

10 A wheel of radius 30 cm is rolled in a straight line.



a Find the circumference of the wheel correct to two decimal places.

b How far, correct to two decimal places, has the wheel rolled after completing:

i 2 rotations? **ii** 10.5 rotations?

c Can you find how many rotations would be required to cover at least 1 km in length? Round to the nearest whole number.

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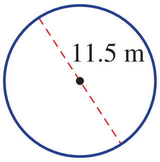
REASONING

11($\frac{1}{2}$) | 11 | 11($\frac{1}{2}$), 12

11 Give exact answers for the perimeter of these shapes.

a



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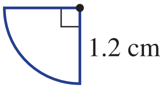

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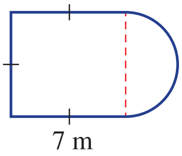

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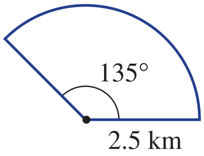

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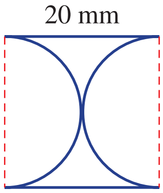

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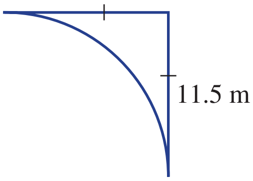


12 We know that the rule for the circumference of a circle is $C = 2\pi r$.

a Find a rule for r in terms of C.

b Find the radius of a circle to one decimal place if its circumference is:

i 10 cm **ii** 25 m.

c Give the rule for the diameter of a circle in terms of its circumference C.

d After 1000 rotations a wheel has travelled 2.12 km. Find its diameter to the nearest centimetre.

ENRICHMENT: The ferris wheel

– | – | 13

13 A large ferris wheel has a radius of 21 m. Round the answers to two decimal places for these questions.

a Find the distance a person will travel on one rotation of the wheel.

b A ride includes six rotations of the wheel. What distance is travelled in one ride?

c How many rotations would be required to ride a distance of:

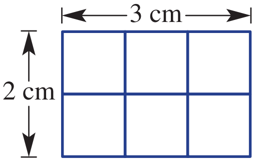
i 500 m? **ii** 2 km?

d Another ferris wheel has a sign which reads, ‘One ride of 10 rotations will cover 2 km’. What must be the diameter of the wheel?

5C Area

Learning intentions

- To review the concept of area
- To understand what a square unit represents
- To know how to convert between metric units of area
- To be able to find the area of two-dimensional shapes using their formulas



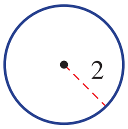
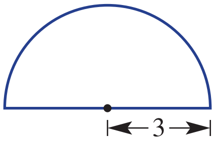
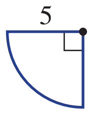
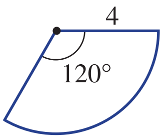
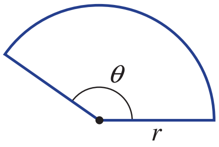
The number of square centimetres in this rectangle is 6; therefore the area is 6 cm^2.

A quicker way to find the number of squares is to note that there are two rows of three squares and hence the area is $2 \times 3 = 6\ \text{cm}^2$. This leads to the formula $A = l \times w$ for the area of a rectangle.

For many common shapes, such as the parallelogram and trapezium, the rules for their area can be developed through consideration of simple rectangles and triangles. Shapes that involve circles or sectors rely on calculations involving pi (π).

LESSON STARTER Formula for the area of a sector

We know that the area of a circle with radius r is given by the rule $A = \pi r^2$. Complete this table of values to develop the rule for the area of a sector.

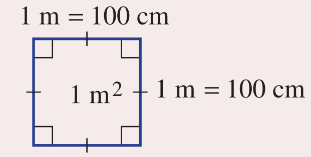
Shape	Fraction of full circle	Working and answer
	1	$A = \pi \times 2^2 \approx 12.57$ units^2
	$\frac{180}{360} =$	$A = \frac{1}{2} \times$
		$A =$
		$A =$
		$A =$

ISBN 978-1-108-77288-4

Updated September 2021

KEY IDEAS

- Conversion of area units



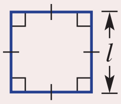
1 m^2 = 100^2 cm^2

$$\begin{tikzcd}
\text{km}^2 \arrow[r, bend left, "\times 1000^2"] & \text{m}^2 \arrow[l, bend left, "\div 1000^2"] \arrow[r, bend left, "\times 100^2"] & \text{cm}^2 \arrow[l, bend left, "\div 100^2"] \arrow[r, bend left, "\times 10^2"] & \text{mm}^2 \arrow[l, bend left, "\div 10^2"]
\end{tikzcd}$$

$10^2 = 10 \times 10 = 100$
$100^2 = 100 \times 100 = 10\,000$
$1000^2 = 1000 \times 1000 = 1\,000\,000$

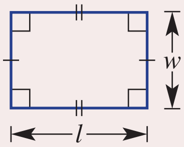
- The area of a two-dimensional shape is a measure of the space enclosed within its boundaries.

Square



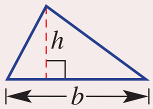
Area = l^2

Rectangle



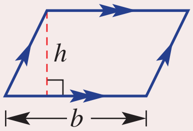
Area = $l \times w$

Triangle



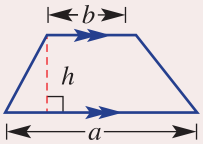
Area = $\frac{1}{2} b \times h$

Parallelogram



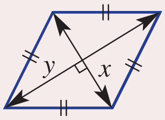
Area = $b \times h$

Trapezium



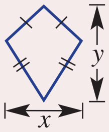
Area = $\frac{1}{2}(a + b)h$

Rhombus



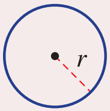
Area = $\frac{1}{2} xy$

Kite



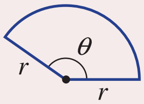
$A = \frac{1}{2} xy$

Circle



$A = \pi r^2$

Sector

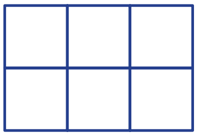


$A = \frac{\theta}{360} \times \pi r^2$

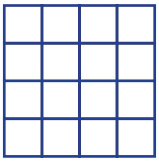
BUILDING UNDERSTANDING

① Count the number of squares to find the area of these shapes. Each square in each shape represents one square unit.

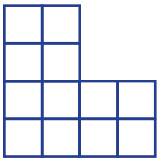
a



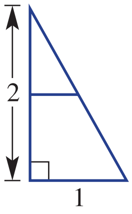
b



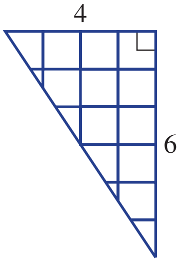
c



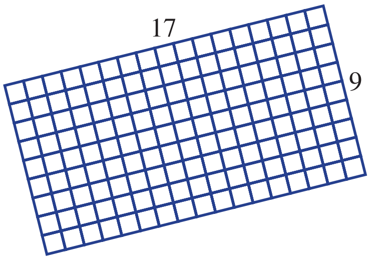
d



e



f



2 Name the shape that has the given area formula.

a $A = lw$

b $A = \pi r^2$

c $A = \frac{1}{2}xy$ (2 shapes)

d $A = \frac{\theta}{360} \times \pi r^2$

e $A = \frac{1}{2}bh$

f $A = \frac{1}{2}(a + b)h$

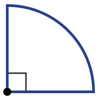
g $A = bh$

h $A = l^2$

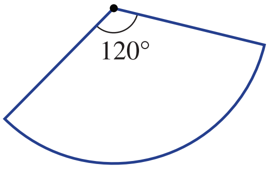
i $A = \frac{1}{2}\pi r^2$

3 What fraction of a full circle is shown by these sectors? Simplify your fraction.

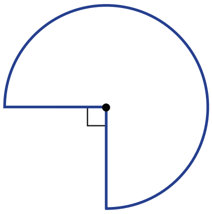
a



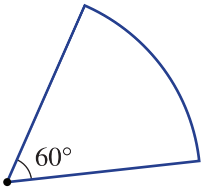
b



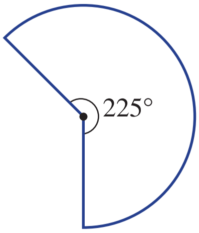
c



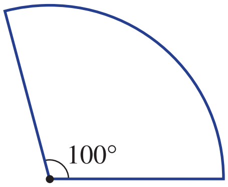
d



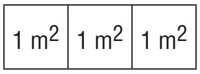
e



f



4 The shape below represents 3 m^2.



a What is the length and width of the shape in cm?

b Hence, give the area of the shape in cm^2.

Example 5 Converting units of area

Convert the following area measurements to the units given in the brackets.

a 859 mm^2 (cm^2)

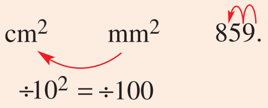
b 2.37 m^2 (cm^2)

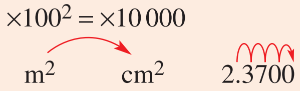
SOLUTION

a $$\begin{aligned} 859\ \text{mm}^2 &= 859 \div 10^2\ \text{cm}^2 \\ &= 8.59\ \text{cm}^2 \end{aligned}$$

b $$\begin{aligned} 2.37\ \text{m}^2 &= 2.37 \times 100^2\ \text{cm}^2 \\ &= 23\,700\ \text{cm}^2 \end{aligned}$$

EXPLANATION





Now you try

Convert the following area measurements to the units given in the brackets.

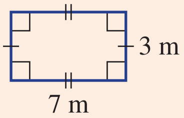
a 14 210 cm^2 (m^2)

b 3.16 cm^2 (mm^2)

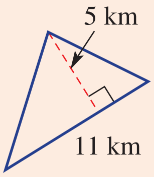
Example 6 Finding areas of rectangles, triangles and parallelograms

Find the area of each of the following plane figures.

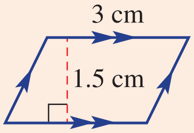
a



b



c



SOLUTION

a $$\begin{aligned} \text{Area} &= l \times w \\ &= 7 \times 3 \\ &= 21\ \text{m}^2 \end{aligned}$$

b $$\begin{aligned} \text{Area} &= \frac{1}{2} \times b \times h \\ &= \frac{1}{2} \times 11 \times 5 \\ &= 27.5\ \text{km}^2 \end{aligned}$$

c $$\begin{aligned} \text{Area} &= b \times h \\ &= 3 \times 1.5 \\ &= 4.5\ \text{cm}^2 \end{aligned}$$

EXPLANATION

a Use the area formula for a rectangle.
Substitute $l = 7$ and $w = 3$.
Include the correct units.

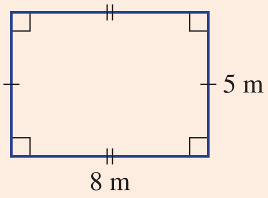
b Use the area formula for a triangle.
Substitute $b = 11$ and $h = 5$.
Include the correct units.

c Use the area formula for a parallelogram.
Multiply the base length by the perpendicular height.

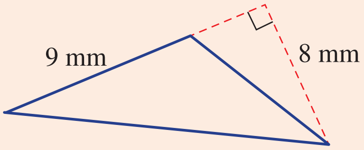
Now you try

Find the area of each of the following plane figures.

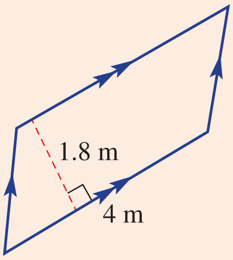
a



b



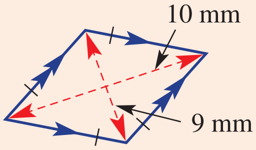
c



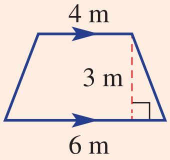
Example 7 Finding areas of rhombuses and trapeziums

Find the area of each of the following plane figures.

a



b

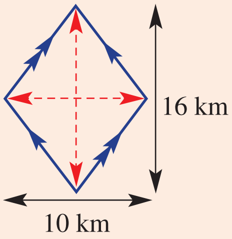


SOLUTION	EXPLANATION
a Area $= \frac{1}{2} \times x \times y$	Use the area formula for a rhombus.
$= \frac{1}{2} \times 10 \times 9$	Substitute $x = 10$ and $y = 9$.
$= 45\text{ mm}^2$	Include the correct units.
b Area $= \frac{1}{2}(a + b)h$	Use the area formula for a trapezium.
$= \frac{1}{2}(4 + 6) \times 3$	Substitute $a = 4$, $b = 6$ and $h = 3$.
$= 15\text{ m}^2$	$\frac{1}{2} \times (4 + 6) \times 3 = 5 \times 3$

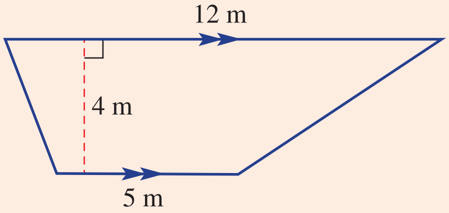
Now you try

Find the area of each of the following plane figures.

a



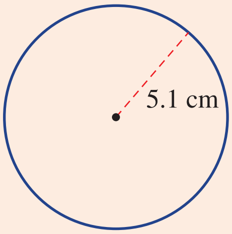
b



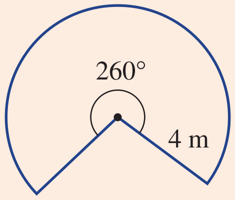
Example 8 Finding areas of circles and sectors

Find the area of this circle and sector correct to two decimal places.

a



b



SOLUTION

a $A = \pi r^2$
$= \pi \times (5.1)^2$
$= 81.71\text{ cm}^2$ (to 2 d.p.)

b $A = \dfrac{\theta}{360} \times \pi r^2$
$= \dfrac{260}{360} \times \pi \times 4^2$
$= \dfrac{13}{18} \times \pi \times 16$
$= 36.30\text{ m}^2$ (to 2 d.p.)

EXPLANATION

Write the rule and substitute $r = 5.1$.

81.7128… rounds to 81.71 as the third decimal place is 2.

Use the sector formula.

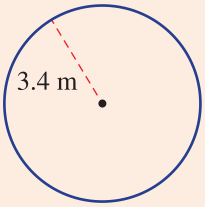
The fraction of the full circle is $\dfrac{260}{360} = \dfrac{13}{18}$, so multiply this by πr^2 to get the sector area.

$\dfrac{104\pi}{9}\text{ m}^2$ would be the exact answer.

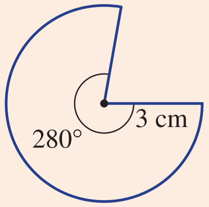
Now you try

Find the area of this circle and sector correct to two decimal places.

a



b



Exercise 5C

FLUENCY	1, 2–5(1/2), 6, 7(1/2)	2–5(1/2), 6, 7(1/2)	2–5(1/3), 6, 7(1/3)

1 Convert the following area measurements to the units given in the brackets.

Example 5a
- **a** **i** 236 mm^2 (cm^2)
  - **ii** $48\,000\text{ cm}^2$ (m^2)

Example 5b
- **b** **i** 4.16 m^2 (cm^2)
  - **ii** 3.5 cm^2 (mm^2)

ISBN 978-1-108-77288-4

Updated September 2021

Example 5

2 Convert the following area measurements to the units given in the brackets.

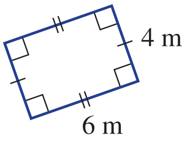
a 2 cm^2 (mm^2) **b** 500 mm^2 (cm^2) **c** 2.1 m^2 (cm^2)

d 210 000 cm^2 (m^2) **e** 0.001 km^2 (m^2) **f** 3 200 000 m^2 (km^2)

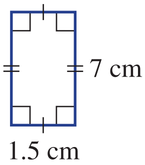
Example 6

3 Find the area of each of the following plane figures.

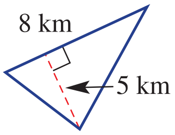
a



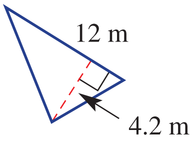
b



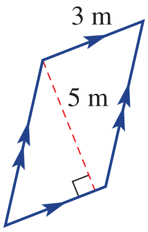
c



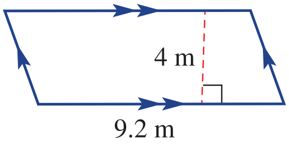
d



e



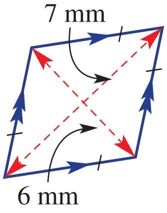
f



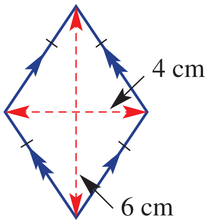
Example 7

4 Find the area of each of the following plane figures.

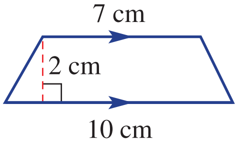
a



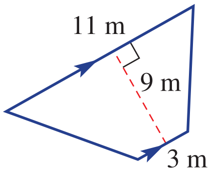
b



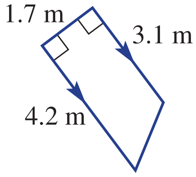
c



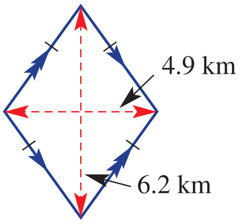
d



e

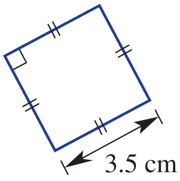


f

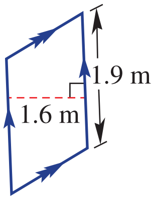


5 Find the area of each of the following mixed plane figures.

a



b



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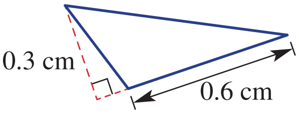

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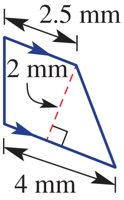

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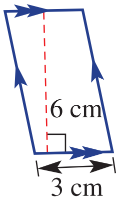

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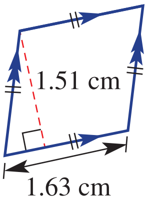
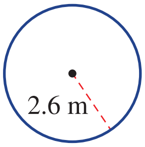


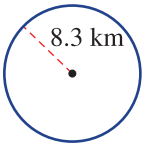
Example 8a

6 Find the area of these circles using the given value for pi (π). Round to two decimal places.

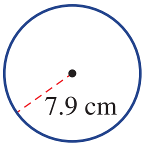
a $\pi = 3.14$



b $\pi = \frac{22}{7}$



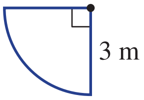
c π (from calculator)



Example 8b

7 Find the area of these sectors rounding to two decimal places.

a



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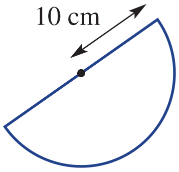

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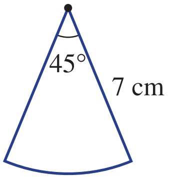

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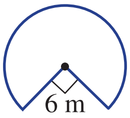

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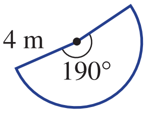

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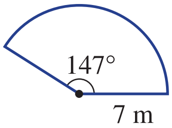


PROBLEM-SOLVING	8–10	8, 9, 11, 12	8, 11–13

8 Convert the following measurements to the units given in the brackets.

a 1.5 km^2 (cm^2)

b 0.000005 m^2 (mm^2)

c 75 000 mm^2 (m^2)

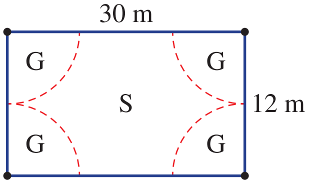
9 A valuer tells you that your piece of land has an area of one-half a square kilometre (0.5 km^2). How many square metres (m^2) do you own?

10 A rectangular park covers an area of 175 000 m^2. Give the area of the park in km^2.

11 An old picture frame that was once square now leans to one side to form a rhombus. If the distances between pairs of opposite corners are 85 cm and 1.2 m, find the area enclosed within the frame in m^2.

12 A pizza shop is considering increasing the diameter of its family pizza tray from 32 cm to 34 cm. Find the percentage increase in area, correct to two decimal places.

13 A tennis court area is illuminated by four corner lights. The illumination of the sector area close to each light is considered to be good (G) while the remaining area is considered to be lit satisfactorily (S).
What percentage of the area is considered ‘good’? Round to the nearest per cent.



REASONING	14	14	14, 15

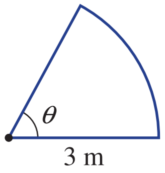
14 The rule for the area of a circle is given by $A = \pi r^2$.

a Rearrange this rule to find a rule for r in terms of A.

b Find the radius of a circle with the given areas. Round to one decimal place.

i 5 cm^2 **ii** 6.9 m^2 **iii** 20 km^2

15 A sector has a radius of 3 m.



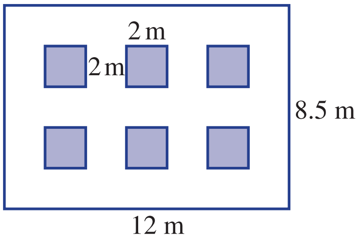
a Find the angle θ, correct to the nearest degree, if its area is:

i 5 m^2 **ii** 25 m^2.

b Explain why the area of the sector could not be 30 m^2.

ENRICHMENT: Windows	–	–	16, 17

16 Six square windows of side length 2 m are to be placed into a 12 m wide by 8.5 m high wall as shown. The windows are to be positioned so that the vertical spacings between the windows and the wall edges are equal. Similarly, the horizontal spacings are also equal.



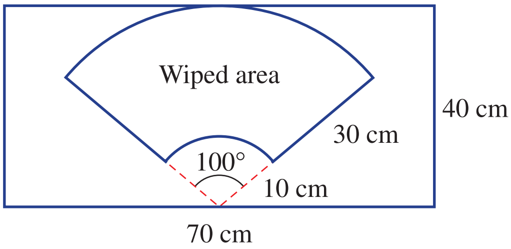
a **i** Find the horizontal distance between the windows.

ii Find the vertical distance between the windows.

b Find the area of the wall not including the window spaces.

c If the wall included 3 rows of 4 windows (instead of 2 rows of 3), investigate if it would be possible to space all the windows so that the horizontal and vertical spacings are uniform (although not necessarily equal to each other).

17 A rectangular window is wiped by a wiper blade forming the given sector shape. What percentage area is cleaned by the wiper blade? Round to one decimal place.



Using CAS and non-CAS calculators 5C: Measurement formulas

This activity is in the Interactive Textbook in the form of a printable PDF.

5D Composite shapes

Learning intentions

- To know that composite shapes are made up of more than one basic shape
- To be able to identify the regular shapes that make up a composite shape
- To be able to use addition or subtraction of known shapes to find the perimeter and area of composite shapes

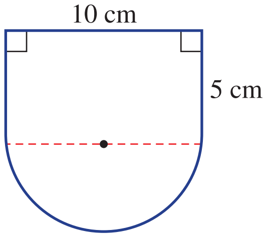
Composite shapes can be thought of as a combination of simpler shapes such as triangles and rectangles. Finding perimeters and areas of such shapes is a matter of identifying the more basic shapes they consist of and combining any calculations in an organised fashion.

For this high-rise office building in Oslo, Norway, the architect has designed windows with triangular, trapezoidal and composite shapes.

LESSON STARTER Incorrect layout

Three students write their solution to finding the area of this shape on the board.

Chris	Matt	Moira
$A = l \times w$ $= 50 + \frac{1}{2}\pi r^2$ $= \frac{1}{2}\pi \times 5^2$ $= 39.27 + 50$ $= 89.27\text{ cm}^2$	$A = \frac{1}{2}\pi \times 5^2$ $= 39.27 + 10 \times 5$ $= 89.27\text{cm}^2$	$A = l \times w + \frac{1}{2}\pi r^2$ $= 10 \times 5 + \frac{1}{2}\pi \times 5^2$ $= 89.27\text{ cm}^2$



- All three students have the correct answer but only one student receives full marks. Who is it?
- Explain what is wrong with the layout of the other two solutions.

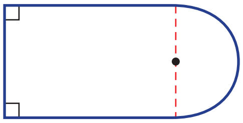
KEY IDEAS

- **Composite shapes** are made up of more than one basic shape.
- Addition and/or subtraction can be used to find areas and perimeters of composite shapes.
- The layout of the relevant mathematical working needs to make sense so that the reader of your work understands each step.

BUILDING UNDERSTANDING

1 Name the two different shapes that make up these composite shapes, e.g. square and semicircle.

a



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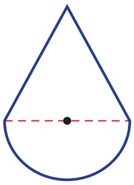

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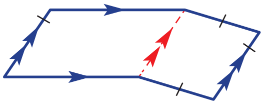
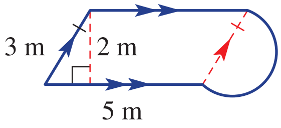


2 State the missing components to complete the working to find the perimeter and area of these composite shapes.

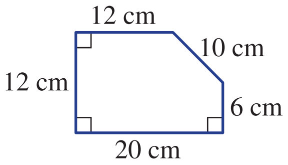
a



$$\begin{aligned} P &= 2 \times \underline{\qquad} + 3 + \frac{1}{2} \times 2\pi r \\ &= \underline{\qquad} + 3 + \underline{\qquad} \\ &= \underline{\qquad}\ \text{m} \end{aligned}$$

$$\begin{aligned} A &= bh + \frac{1}{2}\underline{\qquad} \\ &= 5 \times \underline{\qquad} + \frac{1}{2} \times \underline{\qquad} \\ &= \underline{\qquad} + \underline{\qquad} \\ &= \underline{\qquad}\ \text{m}^2 \end{aligned}$$

b

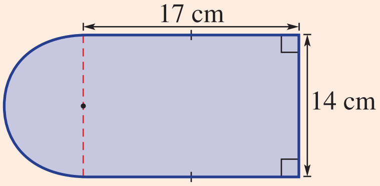


$$\begin{aligned} P &= 20 + 12 + \underline{\qquad} + \underline{\qquad} + \underline{\qquad} \\ &= \underline{\qquad}\ \text{cm} \end{aligned}$$

$$\begin{aligned} A &= lw - \frac{1}{2}bh \\ &= 12 \times \underline{\qquad} - \frac{1}{2} \times 8 \times \underline{\qquad} \\ &= \underline{\qquad} - \underline{\qquad} \\ &= \underline{\qquad}\ \text{cm}^2 \end{aligned}$$

Example 9 Finding perimeters and areas of composite shapes

Find the perimeter and area of this composite shape, rounding answers to two decimal places.



SOLUTION	EXPLANATION
$P = 2 \times l + w + \frac{1}{2} \times 2\pi r$	3 straight sides ⊐ + semicircle arc ⌒
$= 2 \times 17 + 14 + \frac{1}{2} \times 2\pi \times 7$	Substitute $l = 17$, $w = 14$ and $r = 7$.
$= 34 + 14 + \pi \times 7$	Simplify.
$= 69.99$ cm (to 2 d.p.)	Calculate and round to two decimal places.

Continued on next page

$A = l \times w + \frac{1}{2}\pi r^2$ Area of rectangle ▭ + area of semicircle ⌓

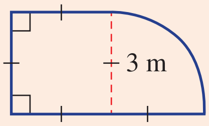
$= 17 \times 14 + \frac{1}{2} \times \pi \times 7^2$ Substitute $l = 17$, $w = 14$ and $r = 7$.

$= 238 + \frac{1}{2} \times \pi \times 49$ Simplify.

$= 314.97\text{ cm}^2$ (to 2 d.p.) Calculate and round to two decimal places.

Now you try

Find the perimeter and area of this composite shape, rounding answers to two decimal places.



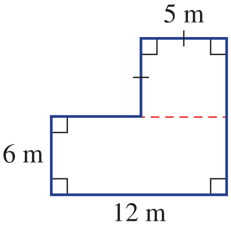
Exercise 5D

FLUENCY	1, 2–4(1/2)	2–5(1/2)	2–5(1/2)

Example 9

1 Find the perimeter and the area of each of these simple composite shapes, rounding answers to two decimal places where necessary.

a



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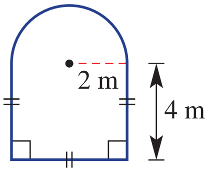
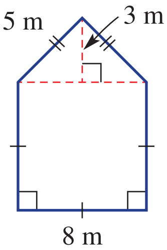


Example 9

2 Find the perimeter and the area of each of these simple composite shapes, rounding answers to two decimal places where necessary.

a



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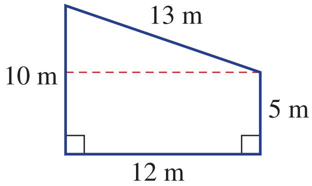

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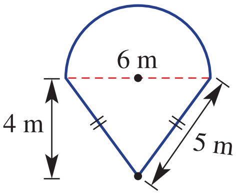

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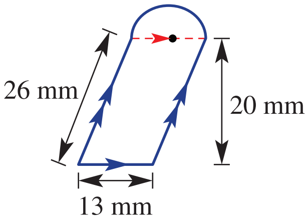

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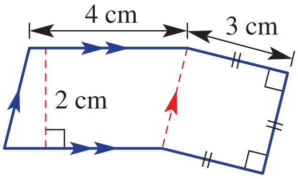
Example 9

3 Find the area of the following composite shapes.

a



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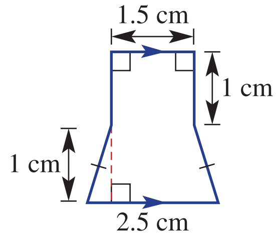

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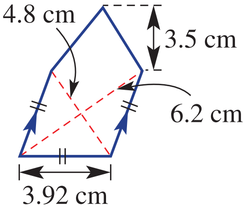

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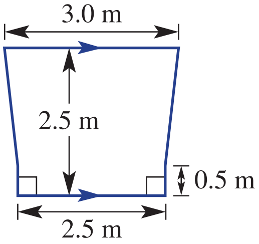

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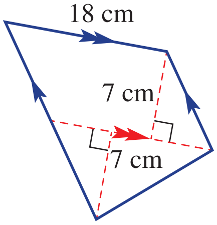

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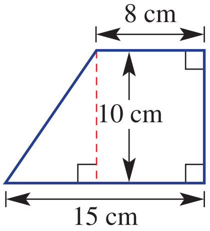
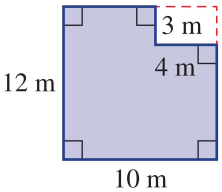


4 Find the shaded area of each of these composite shapes. (*Hint*: Use subtraction.)

a



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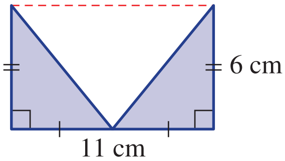

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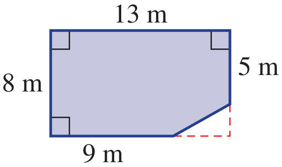

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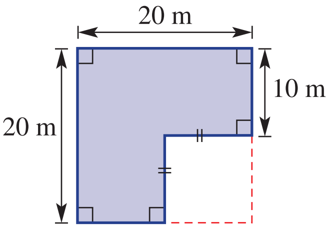

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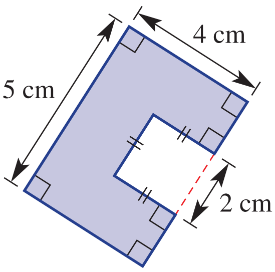

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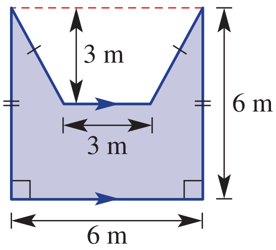
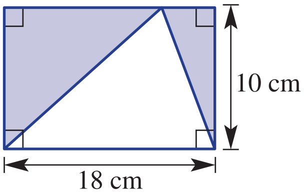


5 Find the area of the shaded region of the following shapes by subtracting the area of the clear shape from the total area. Round to one decimal place where necessary.

a



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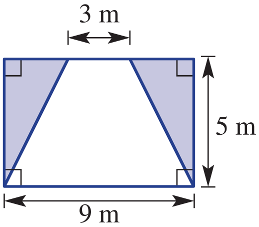

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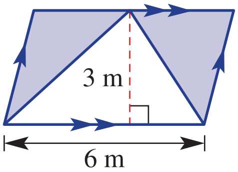

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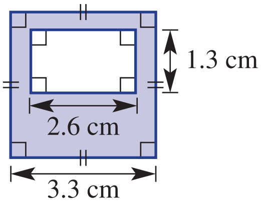

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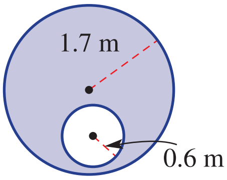

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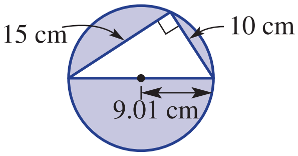

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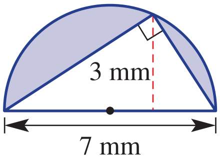

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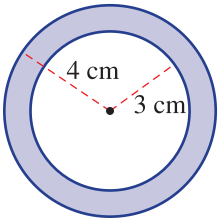

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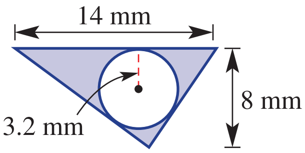


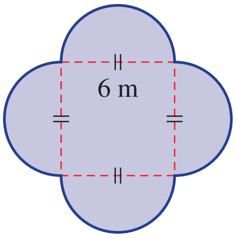
PROBLEM-SOLVING 6, 7 | 7, 8(½), 9 | 7–10

6 An area of lawn is made up of a rectangle measuring 10 m by 15 m and a semicircle of radius 5 m. Find the total area of lawn, correct to two decimal places.

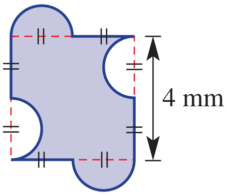
7 Twenty circular pieces of pastry, each of diameter 4 cm, are cut from a rectangular sheet of pastry 20 cm long and 16 cm wide. What is the area, correct to two decimal places, of pastry remaining after the 20 pieces are removed?

8 Find the perimeter and the area of the following composite shapes, correct to two decimal places where necessary.

a



b



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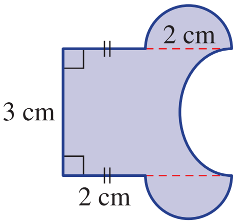

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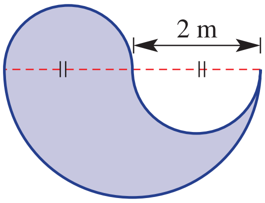

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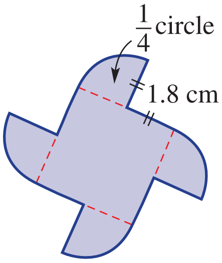

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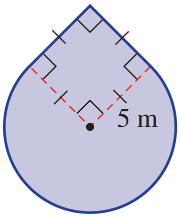
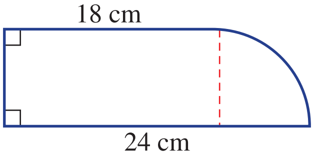


9 These shapes include sectors. Find their area, correct to one decimal place.

a



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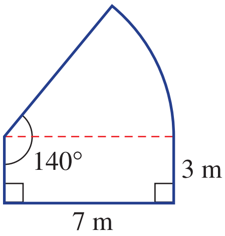

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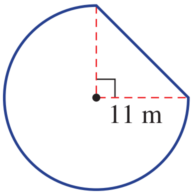
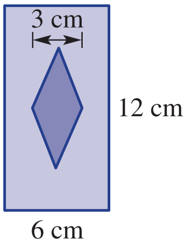


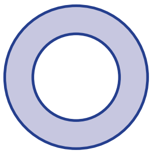
10 A new car manufacturer is designing a logo. It is in the shape of a diamond inside a rectangle. The diamond is to have a horizontal width of 3 cm and an area equal to one-sixth of the area of the rectangle. Find the required height of the diamond.



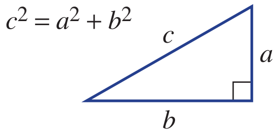
REASONING	11(1/2)	11, 12	11(1/2), 12, 13

11 Using exact values (e.g. $10 + 4\pi$), find the area of the shapes given in Question **8**.

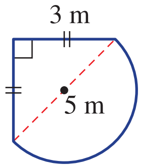
12 A circle of radius 10 cm has a hole cut out of its centre to form a ring. Find the radius of the hole if the remaining area is 50% of the original area. Round to one decimal place.



13 Use Pythagoras' theorem (illustrated in this diagram) to help explain why these composite shapes include incorrect information.



a



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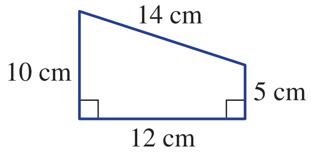

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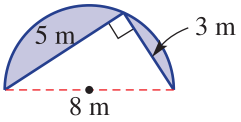
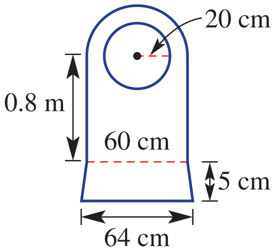
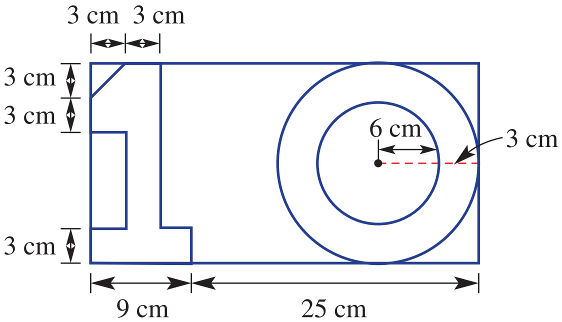


ENRICHMENT: Construction cut-outs	–	–	14, 15

14 The front of a grandfather clock consists of a timber board with dimensions as shown. A circle of radius 20 cm is cut from the board to form the clock face. Find the remaining area of the timber board correct to one decimal place.



15 The number 10 is cut from a rectangular piece of paper. The dimensions of the design are shown below.



a Find the length and width of the rectangular piece of paper.
b Find the sum of the areas of the two cut-out digits, 1 and 0, correct to one decimal place.
c Find the area of paper remaining after the digits have been removed (include the centre of the '0' in your answer) and round to one decimal place.

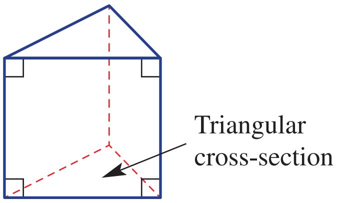
5E Surface area of prisms and pyramids

Learning intentions

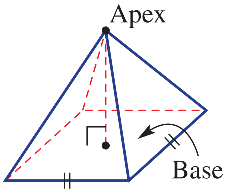
- To understand what is meant by the total surface area of a solid
- To know how to identify the surfaces of a three-dimensional shape using a net or otherwise
- To be able to use area formulas to calculate the surface area of a prism or pyramid

Three-dimensional objects or solids have outside surfaces that together form the total surface area. Nets are very helpful for determining the number and shape of the surfaces of a three-dimensional object.

For this section we will deal with right prisms and pyramids. A right prism has a uniform cross-section with two identical ends and the remaining sides are rectangles. A right pyramid has its apex sitting above the centre of its base.



Right triangular prism

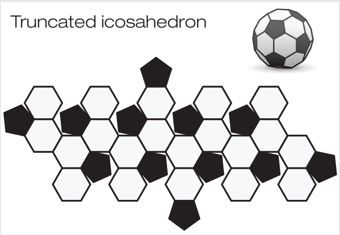


Right square-based pyramid

LESSON STARTER Drawing prisms and pyramids

Prisms are named by the shape of their cross-section and pyramids by the shape of their base.

- Try to draw as many different right prisms and pyramids as you can.
- Describe the different kinds of shapes that make up the surface of your solids.
- Which solids are the most difficult to draw and why?

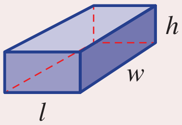
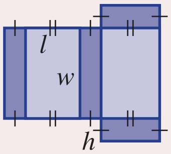
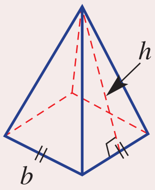
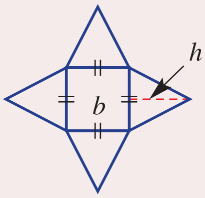


An icosahedron has 20 equilateral triangle faces. If its 12 vertices are truncated, i.e. sliced off, it then has 20 hexagon and 12 pentagon faces. These faces form the surface area net of a soccer ball.

KEY IDEAS

- The **total surface area** (TSA) of a solid is the sum of the areas of all the surfaces.
- A **net** is a two-dimensional illustration of all the surfaces of a solid.
- A right **prism** is a solid with a uniform cross-section with two identical ends and the remaining sides are rectangles.
  - They are named by the shape of their cross-section.

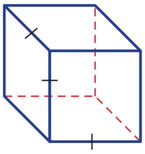
- The nets for a **rectangular prism** (**cuboid**) and square-based **pyramid** are shown here.

Solid	**Net**	**TSA**
Rectangular prism 		TSA = 2 × top + 2 × side + 2 × end $\text{TSA} = 2lw + 2wh + 2lh$
Square-based pyramid 		TSA = base + 4 × side $\text{TSA} = b^2 + 4\left(\frac{1}{2}bh\right)$ $= b^2 + 2bh$

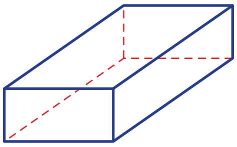
BUILDING UNDERSTANDING

1 Sketch a suitable net for these prisms and pyramids and name each solid.

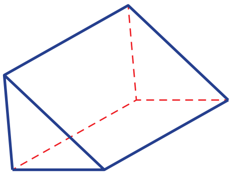
a



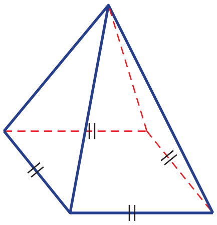
b



c



d



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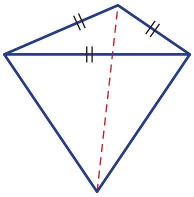

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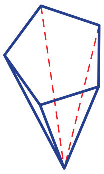
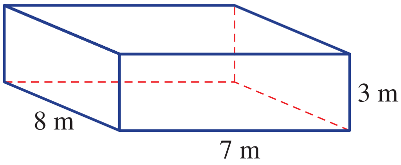


2 State the missing components to complete the working to find the surface area of these solids.

a



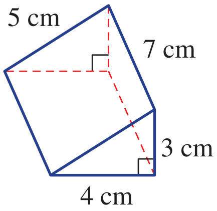
$\text{TSA} = 2 \times 8 \times 7 + 2 \times 8 \times \_\_\_\_\_$

$+ 2 \times \_\_\_\_\_ \times \_\_\_\_\_$

$= \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_$

$= \_\_\_\_\_ \text{ m}^2$

b



$\text{TSA} = 2 \times \frac{1}{2} \times 4 \times \_\_\_\_\_ + 5 \times 7$

$+ 4 \times \_\_\_\_\_ + \_\_\_\_\_ \times \_\_\_\_\_$

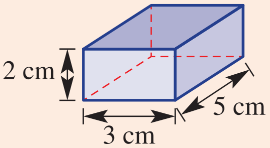
$= \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_$

$= \_\_\_\_\_ \text{ cm}^2$

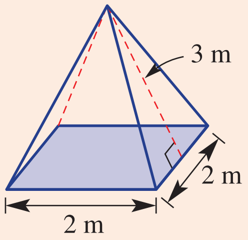
Example 10 Finding a total surface area

Find the total surface area of this right rectangular prism and this right square-based pyramid.

a



b

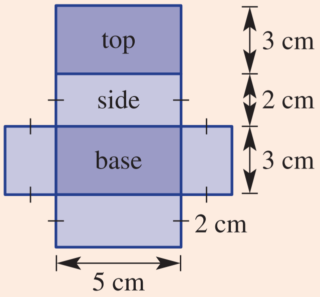


SOLUTION

a TSA = 2 × top + 2 × side + 2 × end

$$\begin{aligned}\text{TSA} &= 2 \times (5 \times 3) + 2 \times (5 \times 2) + 2 \times (2 \times 3)\\ &= 30 + 20 + 12\\ &= 62\text{ cm}^2\end{aligned}$$

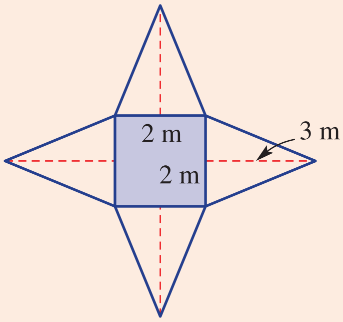
EXPLANATION



b TSA = base + 4 × side

$$\begin{aligned}\text{TSA} &= 2 \times 2 + 4 \times \frac{1}{2} \times 2 \times 3\\ &= 4 + 12\\ &= 16\text{ m}^2\end{aligned}$$

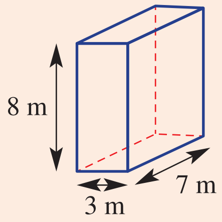
Square base + 4 identical triangles



Now you try

Find the total surface area of this right rectangular prism and this right square-based pyramid.

a



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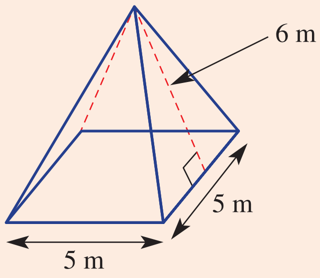


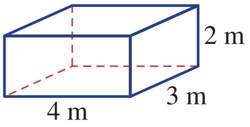
Exercise 5E

FLUENCY	1–3, 4(½), 5	2, 3, 4(½), 5	2(⅓), 3, 4(½), 5

Example 10

1 Find the total surface area of this right rectangular prism and this right square-based pyramid.

a



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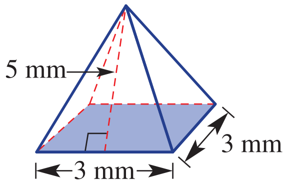
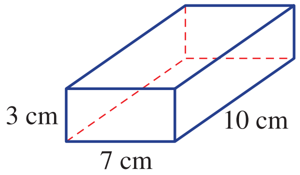


Example 10a

2 Find the surface area of the following right rectangular prisms. Draw a net of the solid to help you.

a



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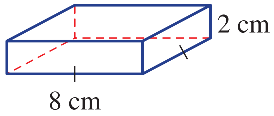

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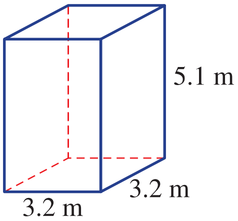
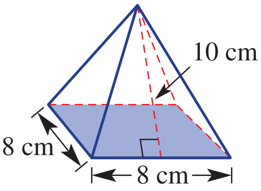


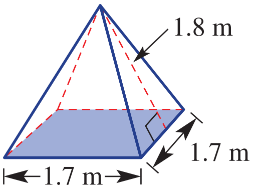
Example 10b

3 Find the surface area of each of these right square-based pyramids. Draw a net to help you.

a

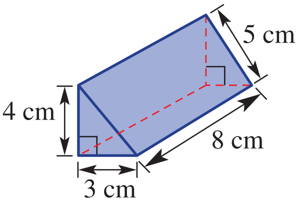


b



4 Find the surface area of each of the following solid objects.

a



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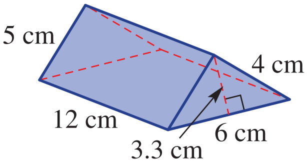

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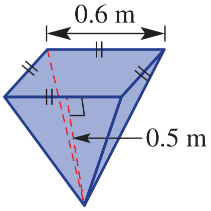

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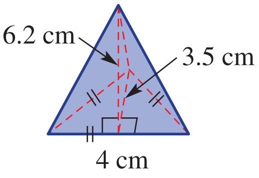

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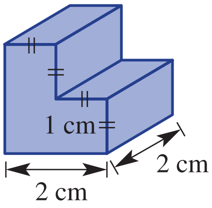

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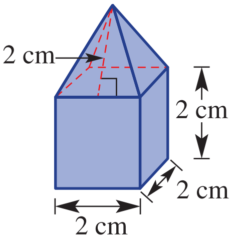


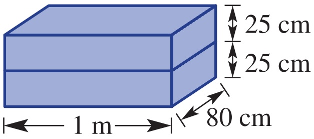
5 Find the total surface area of a cube of side length 1 metre.

ISBN 978-1-108-77288-4

PROBLEM-SOLVING 6, 7 7, 8 8, 9

6 A rectangular box is to be covered in material. How much is required to cover the entire box if it has the dimensions 1.3 m, 1.5 m and 1.9 m?

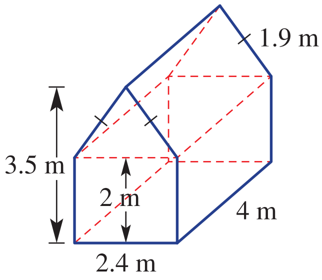
7 Two wooden boxes, both with dimensions 80 cm, 1 m and 25 cm, are placed on the ground, one on top of the other as shown. The entire outside surface is then painted. Find the area of the painted surface.



8 The outside four walls and roof of a barn (shown) are to be painted.

a Find the surface area of the barn, not including the floor.

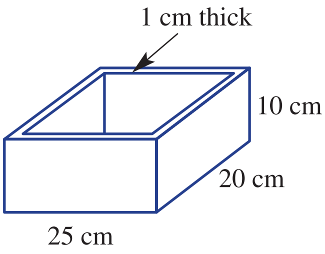
b If 1 litre of paint covers 10 m^2, find how many litres are required to complete the job.



9 An open top rectangular box 20 cm wide, 25 cm long and 10 cm high is made from wood 1 cm thick. Find the surface area:

a outside the box (do not include the top edge)

b inside the box (do not include the top edge).

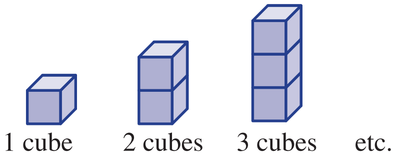


REASONING 10 10 10, 11

10 Draw the stack of 1 cm cube blocks that gives the minimum outside surface area and state this surface area if there are:

a 2 blocks **b** 4 blocks **c** 8 blocks.

11 Cubes of side length one unit are stacked as shown.



a Complete this table.

Number of cubes (n)	1	2	3	4	5	6	7	8	9
Surface area (S)									

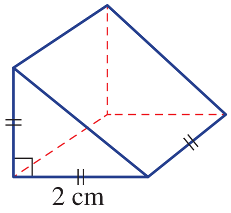
b Can you find the rule for the surface area (S) for n cubes stacked in this way? Write down the rule for S in terms of n.

c Investigate other ways of stacking cubes and look for rules for surface area in terms of n, the number of cubes.

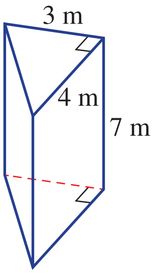
ENRICHMENT: Pythagoras required	–	–	12

12 For prisms and pyramids involving triangles, Pythagoras' theorem ($c^2 = a^2 + b^2$) can be used. Apply the theorem to help find the surface area of these solids. Round to one decimal place.

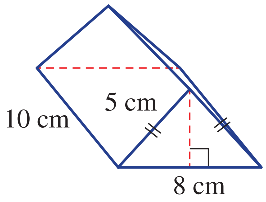
a



b



c



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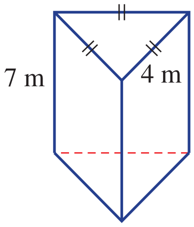

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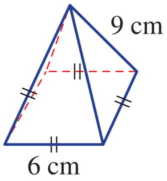

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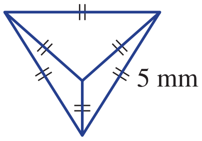
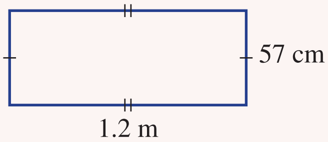


Progress quiz

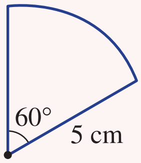
5A/5B

1 Find the perimeter of each of the following shapes. Round to one decimal place in part **b**.

a



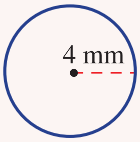
b



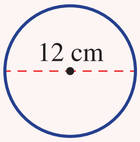
5B/5C

2 Find, correct to one decimal place, the circumference and area of these circles.

a



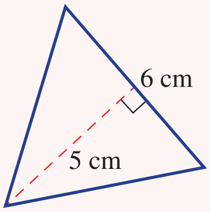
b



5C

3 Find the exact area of the following.

a



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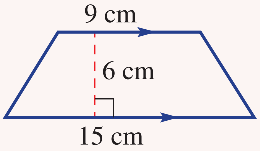

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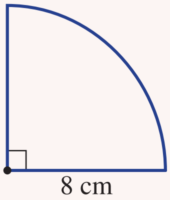


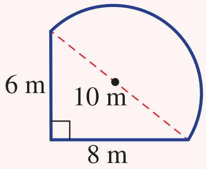
5C

4 Convert 85 000 mm^2 to cm^2.

5D

5 Find the perimeter and area of these composite shapes, rounding answers to two decimal places in part **a**. (*Hint*: Use Pythagoras' theorem in part **b**.)

a



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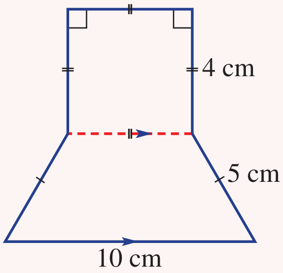
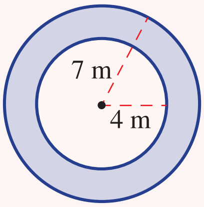


5D

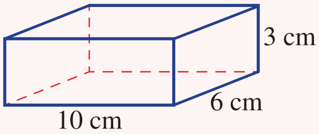
6 Find, correct to two decimal places, the area of the shaded region.



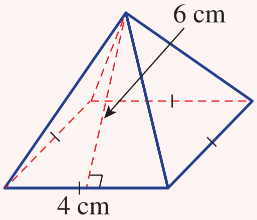
5E

7 Find the total surface area of each of the following solids. Round to one decimal place in part **c** and use Pythagoras' theorem.

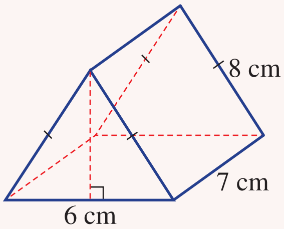
a



b



c

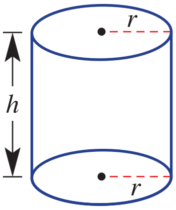


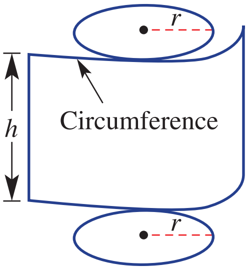
5F Surface area of a cylinder

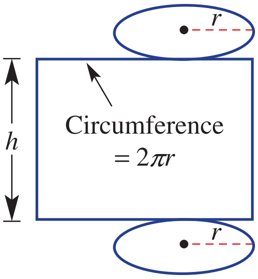
Learning intentions

- To know the shapes that form the net of a cylinder
- To know the formulas for the area of the different surfaces of a cylinder
- To be able to find the surface area of a cylinder and identify which surfaces to include
- To be able to adapt the formulas to find the surface area of cylindrical portions

The net of a cylinder includes two circles and one rectangle with the length of the rectangle equal to the circumference of the circle. The sum of the area of all these shapes will give the total surface area of a cylinder.







LESSON STARTER Curved area

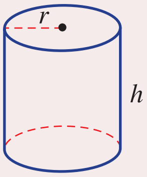
- Roll a piece of paper to form the curved surface of a cylinder.
- Do not stick the ends together because this will allow the paper to return to a flat surface.
- What shape is the paper when lying flat on a table?
- When curved to form the cylinder, what do the sides of the rectangle represent on the cylinder? How does this help to find the surface area of a cylinder?

KEY IDEAS

- Surface area of a **cylinder** = 2 circles + 1 rectangle

$$= 2 \times \pi r^2 + 2\pi r \times h$$

$$\therefore \text{TSA} = \underbrace{2\pi r^2}_{\text{2 circular ends}} + \underbrace{2\pi rh}_{\text{curved area}}$$



  - $\text{TSA} = 2\pi r^2 + 2\pi rh = 2\pi r(r + h)$
- In many problems, you will need to decide which parts of the surface of the cylinder should be included.

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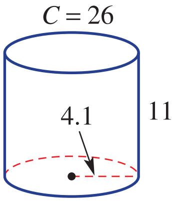
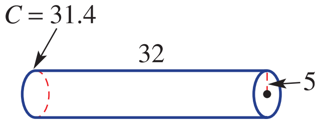
BUILDING UNDERSTANDING

1 Sketch a net suited to these cylinders. Label the sides using the given measurements. C represents the circumference.

a

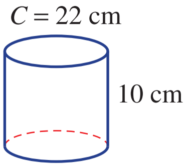


b

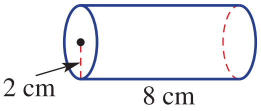


2 The curved surface of these cylinders is flattened out to form a rectangle. What would be the length and width of each rectangle? Round to two decimal places where necessary.

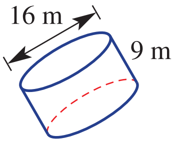
a



b

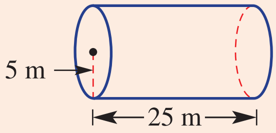


c



Example 11 Finding the surface area of a cylinder

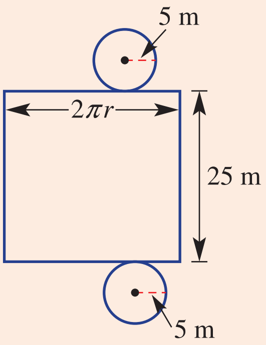
Find the surface area of this cylinder, rounding to two decimal places.



SOLUTION

$$\begin{aligned} \text{TSA} &= 2 \times \text{circular end} + \text{rolled rectangle side} \\ \text{TSA} &= 2\pi r^2 + 2\pi rh \\ &= 2 \times \pi \times 5^2 + 2\pi \times 5 \times 25 \\ &= 50 \times \pi + 250 \times \pi \\ &= 50\pi + 250\pi \\ &= 300\pi \\ &= 942.48 \text{ m}^2 \text{ (to 2 d.p.)} \end{aligned}$$

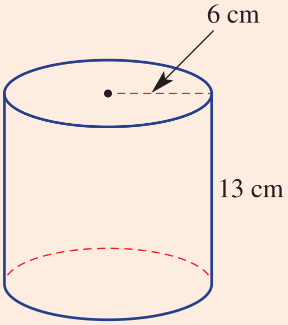
EXPLANATION



Apply the formula with $r = 5$ and $h = 25$.
Round as required.

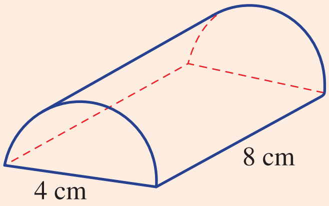
Now you try

Find the surface area of this cylinder, rounding to two decimal places.



Example 12 Finding surface areas of cylindrical portions

Find the total surface area of this half cylinder, rounding to two decimal places.

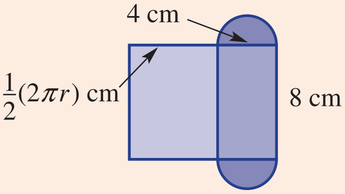


SOLUTION

$$\begin{aligned} \text{TSA} &= 2 \times \text{semicircular end} \\ &\quad + \frac{1}{2} \times \text{rolled rectangle side} + \text{flat side} \\ \text{TSA} &= 2\left(\frac{1}{2}\pi r^2\right) + \frac{1}{2}(2\pi r) \times 8 + 4 \times 8 \\ &= 2 \times \frac{1}{2}\pi \times 2^2 + \frac{1}{2} \times 2 \times \pi \times 2 \times 8 + 32 \\ &= 20\pi + 32 \\ &= 94.83 \text{ cm}^2 \text{ (to 2 d.p.)} \end{aligned}$$

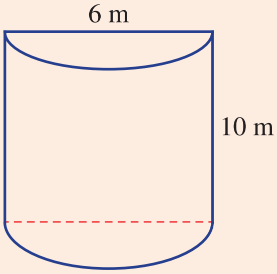
EXPLANATION

As well as half the curved surface of the cylinder, include the rectangular base and the two semicircular ends, with $r = 2$.



Now you try

Find the total surface area of this half cylinder, rounding to two decimal places.



If three tennis balls fit exactly into a cylindrical tube, which is greater, the tube's height or its top circumference?

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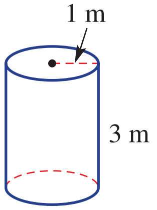
Exercise 5F

FLUENCY	1, 2–4	2–5	3–5

Example 11

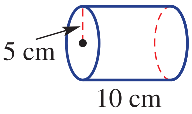
1 Find the surface area of this cylinder, rounding to two decimal places.



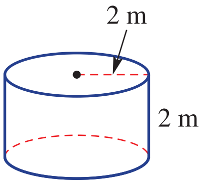
Example 11

2 Find the surface area of these cylinders, rounding to two decimal places. Use a net to help.

a

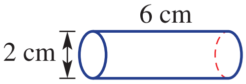


b



3 Find the surface area of these cylinders, rounding to one decimal place. Remember that the radius of a circle is half the diameter.

a



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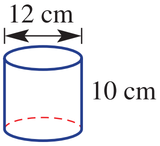
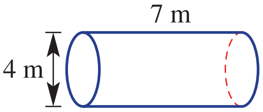


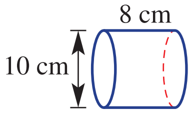
c



4 Find the surface area of a cylindrical plastic container of height 18 cm and with a circle of radius 3 cm at each end. Give the answer correct to two decimal places.

5 Find the area of the curved surface only for these cylinders, correct to two decimal places.

a



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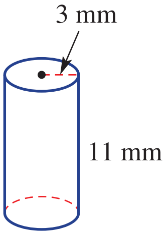

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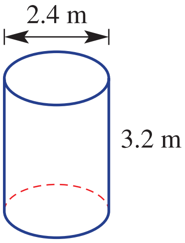
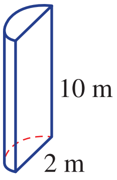


PROBLEM-SOLVING	6(½), 7	6(½), 7, 8	6(½), 8, 9

Example 12

6 Find the total surface area of these solids, rounding to two decimal places.

a



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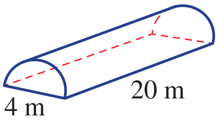

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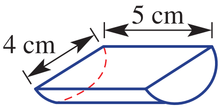

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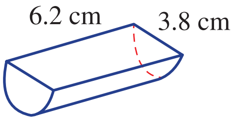

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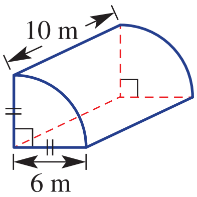

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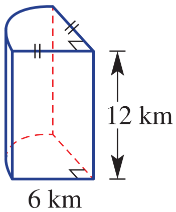

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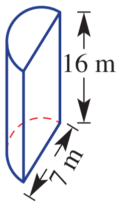

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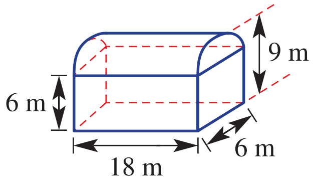
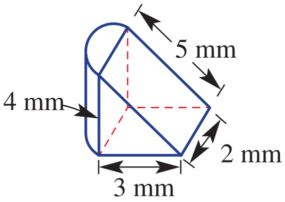
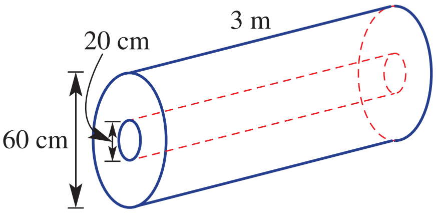


i



7 A water trough is in the shape of a half cylinder. Its semicircular ends have diameter 40 cm and the trough length is 1 m. Find the outside surface area in cm^2 of the curved surface plus the two semicircular ends, correct to two decimal places.

8 A log with diameter 60 cm is 3 m in length. Its hollow centre is 20 cm in diameter. Find the surface area of the log in cm^2, including the ends and the inside, correct to one decimal place.



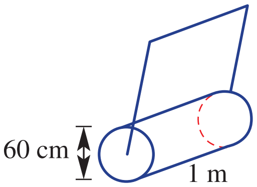
9 A cylindrical roller is used to press crushed rock in preparation for a tennis court. The rectangular tennis court area is 30 m long and 15 m wide. The roller has a width of 1 m and diameter 60 cm.

a Find the surface area of the curved part of the roller in cm^2 correct to three decimal places.

b Find the area, in m^2 to two decimal places, of crushed rock that can be pressed after:

i 1 revolution **ii** 20 revolutions.

c Find the minimum number of complete revolutions required to press the entire tennis court area.



REASONING	10	10, 11	10, 11

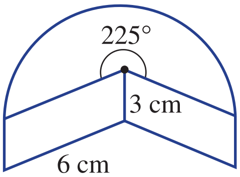
10 It is more precise to give exact values for calculations involving π, e.g. 24π. Give the exact answers for the surface area of the cylinders in Question **2**.

11 A cylinder cut in half gives half the volume but not half the surface area. Explain why.

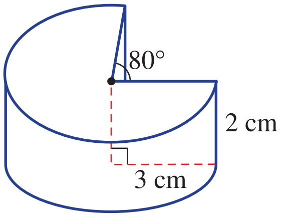
ENRICHMENT: Solid sectors	–	–	12

12 The sector area rule $A = \dfrac{\theta}{360} \times \pi r^2$ can be applied to find the surface areas of solids which have ends that are sectors. Find the exact total surface area of these solids.

a



b



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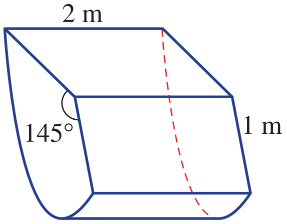


Applications and problem-solving

The following problems will investigate practical situations drawing upon knowledge and skills developed throughout the chapter. In attempting to solve these problems, aim to identify the key information, use diagrams, formulate ideas, apply strategies, make calculations and check and communicate your solutions.

The penny-farthing

1 The penny-farthing was the first machine to be called a bicycle. It was a popular mode of transport in the late 19th century. Edward's penny-farthing has a large wheel diameter of 135 cm and a small wheel diameter of 45 cm.

Edward is interested in the number of wheel rotations required over a certain distance and the relationship between the number of rotations of the small wheel and the large wheel.

- **a** Find the circumference of the following correct to two decimal places.
  - **i** The small wheel
  - **ii** The large wheel
- **b** To cover the same distance, how many times does the small wheel need to turn for each turn of the large wheel?
- **c** Edward rides the penny-farthing and the large wheel rotates 10 times.
  - **i** How far does he travel correct to the nearest cm?
  - **ii** How many times does the small wheel turn?
- **d** Edward wants to ride to school, which is a 1 km journey. How many rotations of the large wheel will occur? Round to the nearest integer.
- **e** On another journey, the small wheel makes 100 more rotations than the large wheel. How far did the penny-farthing travel? Round to the nearest metre.

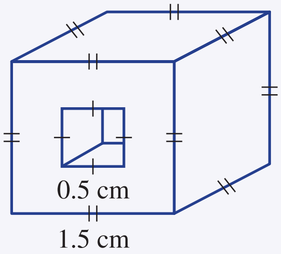
Fishing sinkers

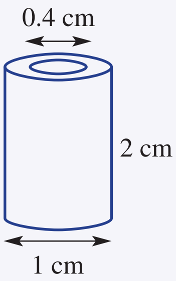
2 Fishing lines often have a metal sinker attached to the line so that the hook and bait sink below the surface of the water. Each sinker must have a hole in it so the line can be properly attached.

The 'Catch Fish' company produces sinkers of various shapes and sizes and coats them with a special anti-rust paint.

The Catch Fish company is interested in the total surface area of each sinker so that it can calculate how much paint is required and the associated cost.

- **a** The most basic sinker is a cube of side length 1.5 cm with a single hole of width 0.5 cm as shown. Find the total surface area of this sinker including the surface inside the hole.





b A cylindrical sinker has dimensions as shown. Find the total surface area of this sinker, including inside the hole, correct to two decimal places.

c The company chooses to make 1000 of the cube sinkers (from part **a** above) and 1000 of the cylindrical sinkers (from part **b** above). The paint costs \$120 per litre and each litre covers 10 m^2 or 100 000 cm^2.

  i What is the total surface area for the 2000 sinkers? Round to the nearest cm^2.

  ii How much will it cost to paint all the sinkers? Assume that paint can be purchased in any quantity and round to the nearest dollar.

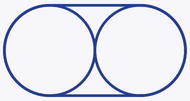
d The Catch Fish company decides to adjust the 2 cm height of the cylindrical sinker so that its surface area is the same as the cube sinker. Investigate to find the new height of the cylindrical sinker.

Circle bands

3 Bridget is playing with an elastic band and a number of one dollar coins, each of diameter 25 mm.

She lays a number of the coins flat on a table to form different shapes and then places an elastic band around them.

Bridget is interested in the length of the elastic band around each shape formed by the coins and how much further the band needs to stretch as she adds more coins.



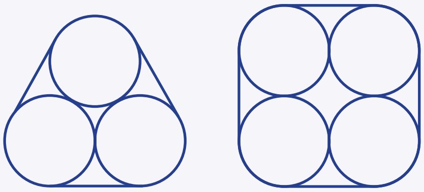
a What is the length of elastic band required to stretch around the following shapes? Round to the nearest mm.

  i A single dollar coin

  ii Two touching coins as shown

b Bridget tries placing a band around three and four coins.

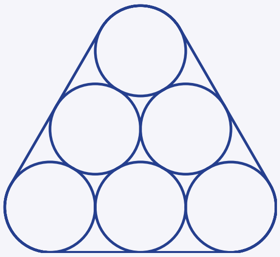
How much further will the band need to stretch for the four-coin shape compared to the three-coin shape?



c Bridget arranges six coins in three rows to form a triangle as shown.

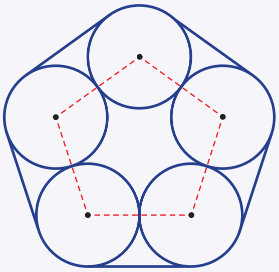
  i How much further does the band need to stretch compared to the three-coin shape above?

  ii If the triangle pattern had n rows, find the rule for the length, L mm, of the elastic band in terms of n.



d Bridget arranges five coins so that the line segments joining the centres form a regular pentagon (with internal angle sum 540°).

How far does the elastic band need to stretch? Round to the nearest mm.



ISBN 978-1-108-77288-4

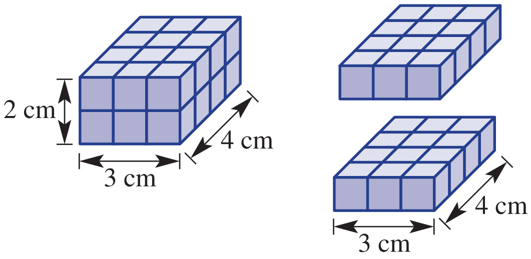
5G Volume of a prism

Learning intentions

- To review the concept of volume
- To know how to convert between metric units of volume
- To understand that volume can also be measured as capacity using litres, millilitres ...
- To know that solids with a uniform cross-section have a formula for volume
- To be able to find the volume of right prisms and solids with a constant cross-section

Volume is the number of cubic units contained within a three-dimensional object.

To find the volume for this solid we can count 24 individual cubic centimetres (24 cm^3) *or* multiply $3 \times 4 \times 2 = 24\ cm^3$.

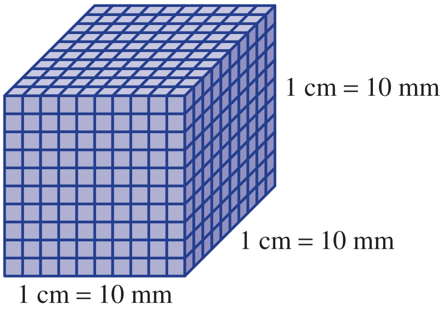


We can see that the area of the base ($3 \times 4 = 12\ cm^2$) also gives the volume of the base layer 1 cm high.
The number of layers equals the height, hence, multiplying the area of the base by the height will give the volume.

This idea can be applied to all right prisms provided a uniform cross-section can be identified. In such solids, the height or length used to calculate the volume is the length of the edge running perpendicular to the base or cross-section, as this measurement counts the number of layers.

LESSON STARTER Cubic units

Consider this 1 cm cube (not to scale) divided into cubic millimetres.



- How many cubic mm sit on one edge of the 1 cm cube?
- How many cubic mm sit on one layer of the 1 cm cube?
- How many layers are there?
- How many cubic mm are there in total in the 1 cm cube?
- Complete this statement $1\ cm^3 = \_\_\_\_\_\ mm^3$
- Explain how you can find how many:
  - **a** cm^3 in 1 m^3
  - **b** m^3 in 1 km^3.

KEY IDEAS

- Common metric units for **volume** include **cubic kilometres** (km^3), **cubic metres** (m^3), **cubic centimetres** (cm^3) and **cubic millimetres** (mm^3).

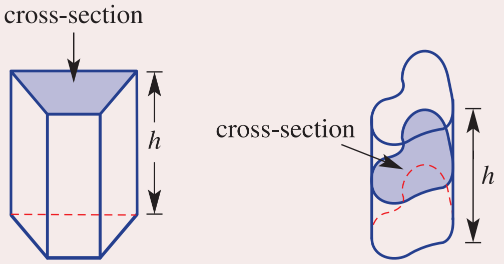
$$\begin{tikzcd}
\text{km}^3 \arrow[r, bend left, "\times 1000^3"] & \text{m}^3 \arrow[l, bend left, "\div 1000^3"] \arrow[r, bend left, "\times 100^3"] & \text{cm}^3 \arrow[l, bend left, "\div 100^3"] \arrow[r, bend left, "\times 10^3"] & \text{mm}^3 \arrow[l, bend left, "\div 10^3"]
\end{tikzcd}$$

$$\begin{aligned} 1000^3 &= 1\,000\,000\,000 \\ 100^3 &= 1\,000\,000 \\ 10^3 &= 1000 \end{aligned}$$

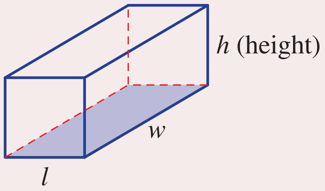
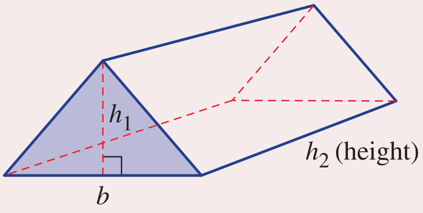
- Common units of **capacity** include:
  - **megalitres** (ML) 1 ML = 1000 kL
  - **kilolitres** (kL) 1 kL = 1000 L
  - **litres** (L) 1 L = 1000 mL
  - **millilitres** (mL)
- Also 1 cm^3 = 1 mL so 1 L = 1000 cm^3 and 1 m^3 = 1000 L
- For solids with a uniform **cross-section**:

  Volume = area of cross-section × perpendicular height

  $V = A \times$ perp. height



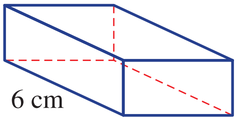
- The perpendicular ‘height’ is the length of the edge that runs perpendicular to the cross-section. This edge can be the vertical height or the length of a prism.
- Some common formulas for volume include:

Rectangular prism (cuboid)	**Triangular prism**
 $V = A \times \text{perp. height}$ $= l \times w \times h$ $= lwh$	 $V = A \times \text{perp. height}$ $= \left(\frac{1}{2} \times b \times h_1\right) \times h_2$

BUILDING UNDERSTANDING

1 Describe the cross-sectional shape for these prisms and state the perpendicular ‘height’ (i.e. the length of the edge perpendicular to the cross-section).

a



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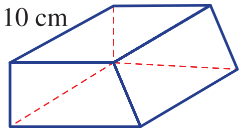

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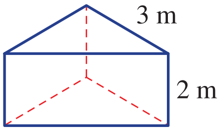

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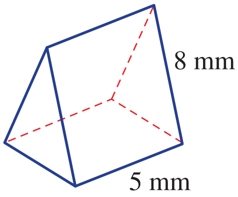
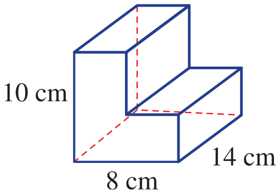


e



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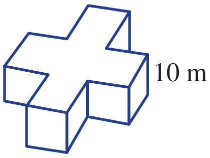
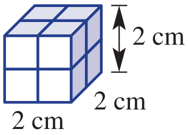


2 Find the volume of these three-dimensional rectangular prisms. Count the number of blocks if you wish.

a



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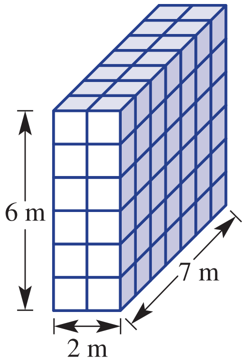

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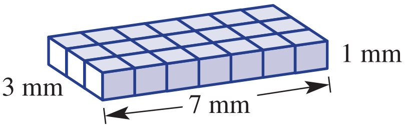

Example 13 Converting units of volume

Convert the following volume measurements to the units given in the brackets.

a 2.5 m^3 (cm^3)

b 458 mm^3 (cm^3)

SOLUTION

a

$$2.5\ \text{m}^3 = 2.5 \times 100^3\ \text{cm}^3$$
$$= 2\,500\,000\ \text{cm}^3$$

EXPLANATION

$$\times 100^3 = \times 1\,000\,000$$
$$\text{m}^3 \longrightarrow \text{cm}^3$$

2.500000

SOLUTION

b

$$458\ \text{mm}^3 = 458 \div 10^3\ \text{cm}^3$$
$$= 0.458\ \text{cm}^3$$

EXPLANATION

$$\text{cm}^3 \longleftarrow \text{mm}^3$$
$$\div 10^3 = \div 1000$$

Now you try

Convert the following volume measurements to the units given in the brackets.

a 4.8 cm^3 (mm^3)

b 612 000 cm^3 (m^3)

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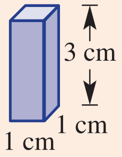
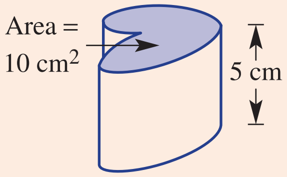
Example 14 Finding volumes of prisms and other solids

Find the volume of each of these three-dimensional objects.

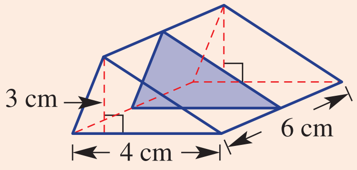
a

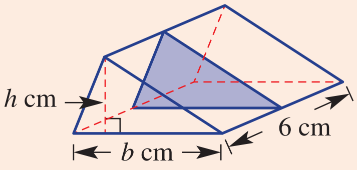


b



c

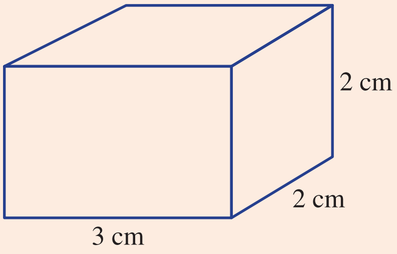


SOLUTION	EXPLANATION
a $V = l \times w \times h$ $= 1 \times 1 \times 3$ $= 3\text{ cm}^3$	The solid is a rectangular prism. Length = 1 cm, width = 1 cm and height = 3 cm
b $V = A \times \text{perp. height}$ $= 10 \times 5$ $= 50\text{ cm}^3$	Volume = area of cross-section × perpendicular height Substitute cross-sectional area = 10 and perpendicular height = 5.
c $V = A \times \text{perp. height}$ $= \left(\frac{1}{2} \times b \times h\right) \times 6$ $= \left(\frac{1}{2} \times 4 \times 3\right) \times 6$ $= 36\text{ cm}^3$	The cross-section is a triangle. The edge perpendicular to the cross-section is the length = 6 cm. 

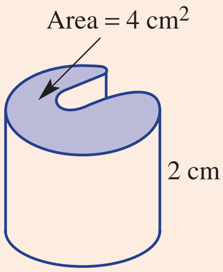
Now you try

Find the volume of each of these three-dimensional objects.

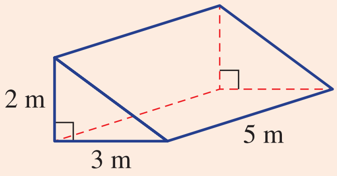
a



b



c



Exercise 5G

FLUENCY 1, 2($1/2$), 3, 4($1/2$), 5, 6 | 2($1/2$), 3, 4($1/2$), 5–7 | 2($1/2$), 4($1/3$), 5, 7

Example 13

1 Convert the following volume measurements to the units given in the brackets.

a 3.5 m^3 (cm^3)
b 216 mm^3 (cm^3)

Example 13

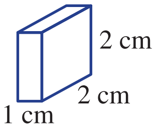
2 Convert the following volume measurements to the units given in the brackets.

a 3 cm^3 (mm^3)
b 2000 mm^3 (cm^3)
c 8.7 m^3 (cm^3)
d 5900 cm^3 (m^3)
e 0.00001 km^3 (m^3)
f 21 700 m^3 (km^3)
g 3 L (mL)
h 0.2 kL (L)
i 3500 mL (L)
j 0.021 L (mL)
k 37 000 L (kL)
l 42 900 kL (ML)

Example 14a

3 Find the volume of these rectangular prisms.

a



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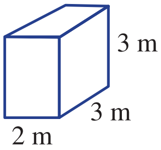

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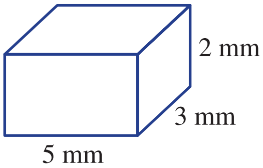
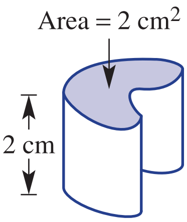


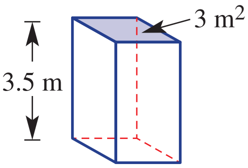
Example 14b

4 Find the volume of each of these three-dimensional objects. The cross-sectional area has been given.

a



b



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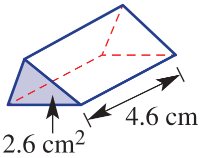

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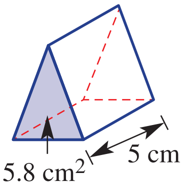

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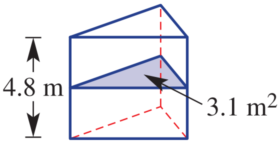

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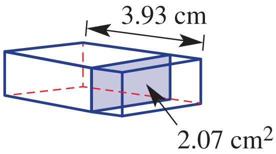

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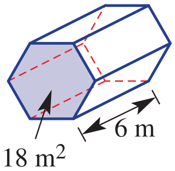

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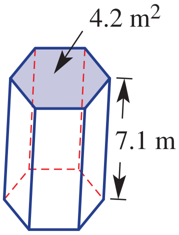
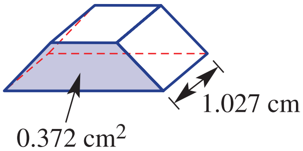


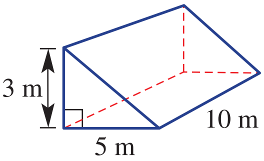
i



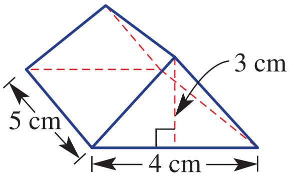
Example 14c

5 Find the volume of these triangular prisms.

a



b



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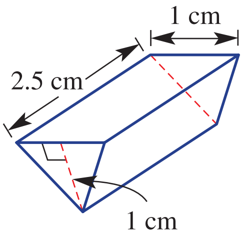
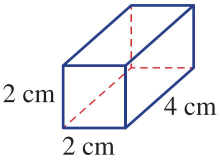
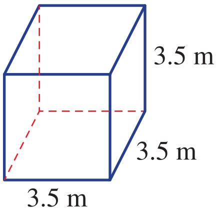


6 Find the volume of each of these rectangular prisms (cuboids).

a



b



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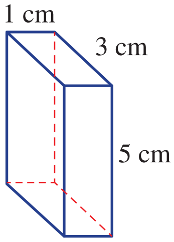
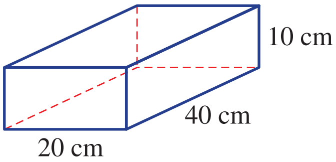


7 Find the volume of these solids. Convert your answer to litres. Recall $1\,\text{cm}^3 = 1\,\text{mL}$ and $1\text{ L} = 1000\text{ cm}^3$.

a



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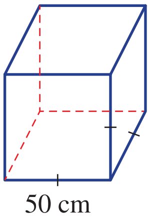

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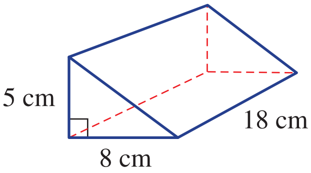

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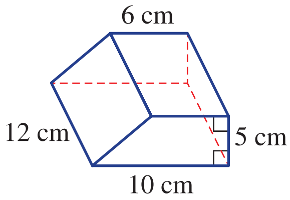


PROBLEM-SOLVING	8, 9	9–11	10–13

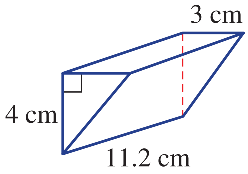
8 A brick is 10 cm wide, 20 cm long and 8 cm high. How much space would five of these bricks occupy?

9 How much air space is contained inside a rectangular cardboard box that has the dimensions 85 cm by 62 cm by 36 cm. Answer using cubic metres (m^3), correct to two decimal places.

10 25 L of water is poured into a rectangular fish tank which is 50 cm long, 20 cm wide and 20 cm high. Will it overflow?

11 Find the volume of each of the following solids, rounding to one decimal place where necessary.

a



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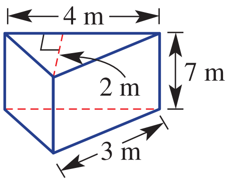

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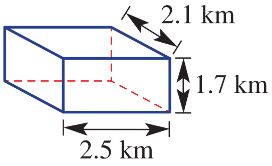

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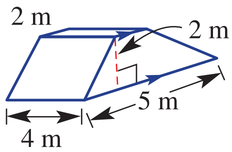

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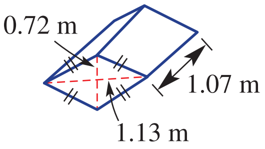

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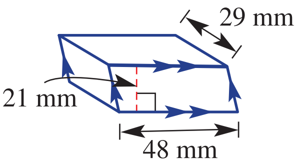
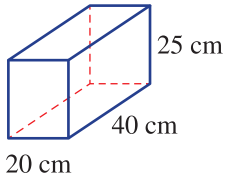


12 Use units for capacity to find the volume of these solids in litres.

a



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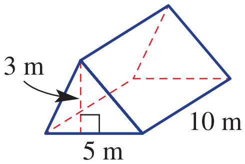

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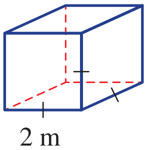
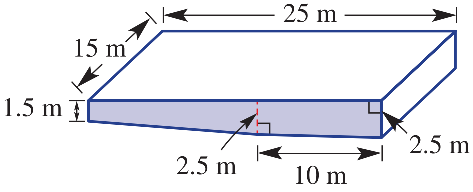


13 The given diagram is a sketch of a new 25 m swimming pool to be installed in a school sports complex.

- **a** Find the area of the shaded side of the pool.
- **b** Find the volume of the pool in litres.



REASONING	14	14	14, 15

14 **a** What single number do you multiply by to convert from:

- **i** L to cm^3? **ii** L to m^3? **iii** mL to mm^3?

b What single number do you divide by to convert from:

- **i** mm^3 to L? **ii** m^3 to ML? **iii** cm^3 to kL?

15 Write rules for the volume of these solids using the given pronumerals.

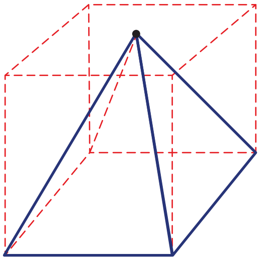
- **a** A rectangular prism with length = width = x and height h
- **b** A cube with side length s
- **c** A rectangular prism with a square base (with side length t) and height six times the side length of the base

ENRICHMENT: Volume of a pyramid	–	–	16

16 Earlier we looked at finding the total surface area of a right pyramid like the one shown here.

Imagine the pyramid sitting inside a prism with the same base.

- **a** Make an educated guess as to what fraction the volume of the pyramid is of the volume of the prism.
- **b** Use the internet to find the actual answer to part **a**.
- **c** Draw some pyramids and find their volume using the results from part **b**.



5H Volume of a cylinder

Learning intentions

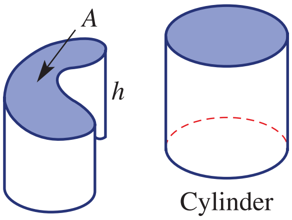
- To know that a cylinder is a solid with a constant circular cross-section
- To know how to find the volume of a cylinder
- To be able to convert the volume of a cylinder to capacity measured in mL, L, …

A right cylinder has its outside surface at right angles to its base and top. It has a uniform cross-section (a circle) and so the volume of a cylinder can be calculated in a similar way to that of a right prism. Cylindrical objects are commonly used to store gases and liquids, and so working out the volume of a cylinder is an important measurement calculation.

An oil refinery converts crude oil into many products including petrol, diesel, jet fuel, motor oils and asphalt base. Cylinder volumes are calculated for its storage tanks and pipeline capacity, i.e. the volume of oil held in a full pipeline.

LESSON STARTER Writing the rule

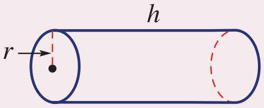
Previously we used the formula $V = A \times h$ to find the volume of solids with a uniform cross-section.



- Discuss any similarities between the two given solids.
- How can the rule $V = A \times h$ be developed further to find the rule for the volume of a cylinder?

KEY IDEAS

- The volume of a cylinder is given by
  $V = \pi r^2 \times h$ or $V = \pi r^2 h$
  - r is the radius of the circular ends.
  - h is the length or distance between the circular ends.



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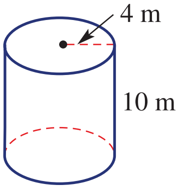
BUILDING UNDERSTANDING

1 Referring to the formula $V = \pi r^2 h$, state the values of r and h for these cylinders.

a



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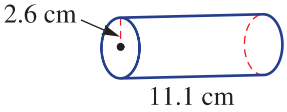

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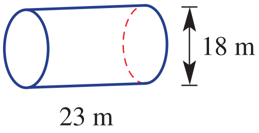


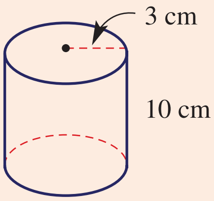
2 Convert the following to the units given in the brackets. Remember, 1 L = 1000 cm^3 and 1 m^3 = 1000 L.

a 2000 cm^3 (L)
b 4.3 cm^3 (mL)
c 3.7 L (cm^3)
d 1 m^3 (L)
e 38 000 L (m^3)
f 0.0002 m^3 (mL)

Example 15 Finding the volume of a cylinder

Find the volume of these cylinders correct to two decimal places.

a



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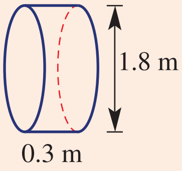
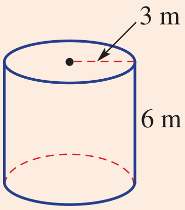


SOLUTION	EXPLANATION
a $V = \pi r^2 h$	Substitute $r = 3$ and $h = 10$ into the rule.
$= \pi \times (3)^2 \times 10$	
$= 90\pi$	90π cm^3 would be the exact answer.
$= 282.74$ cm^3 (to 2 d.p.)	Include volume units.
b $V = \pi r^2 h$	The diameter is 1.8 m so $r = 0.9$.
$= \pi \times (0.9)^2 \times 0.3$	
$= 0.76$ m^3 (to 2 d.p.)	

Now you try

Find the volume of these cylinders correct to two decimal places.

a



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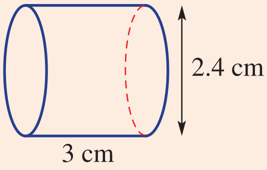


Example 16 Finding the capacity of a cylinder

Find the capacity, in litres, of a cylinder with radius 30 cm and height 90 cm. Round to the nearest litre.

SOLUTION

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \times (30)^2 \times 90 \\ &= 254\,469\ \text{cm}^3 \\ &= 254\ \text{L (to the nearest litre)} \end{aligned}$$

EXPLANATION

Substitute $r = 30$ and $h = 90$.

There are 1000 cm^3 in 1 L so divide by 1000 to convert to litres. 254.469 to nearest litre is 254 L.

Now you try

Find the capacity, in litres, of a cylinder with radius 40 cm and height 100 cm. Round to the nearest litre.

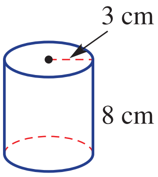
Exercise 5H

FLUENCY	1, 2–3(1/2)	2–3(1/2)	2–3(1/2)

Example 15

1 Find the volume of these cylinders correct to two decimal places.

a



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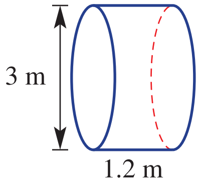
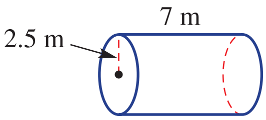


Example 15

2 Find the volume of these cylinders correct to two decimal places.

a



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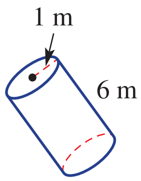

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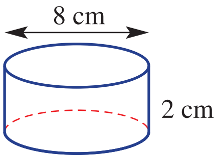

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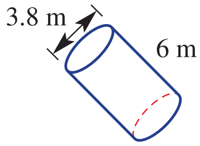

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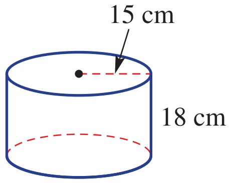
Example 16

3 Find the capacity, in litres, of these cylinders. Round to the nearest litre. Remember $1\text{ L} = 1000\text{ cm}^3$ and $1\text{ m}^3 = 1000\text{ L}$.

a



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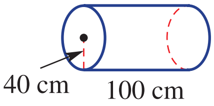

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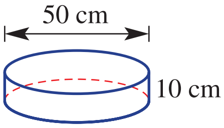

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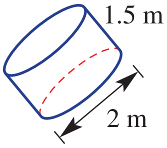

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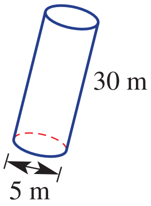

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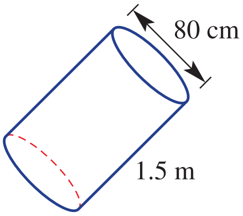


PROBLEM-SOLVING	4, 5	5, 6, 7(1/2)	6, 7

4 A cylindrical water tank has a radius of 2 m and a height of 2 m.

a Find its capacity, in m^3, rounded to three decimal places.

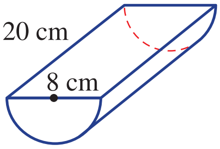
b Find its capacity, in litres, rounded to the nearest litre.

5 How many litres of gas can a tanker carry if it has a cylindrical tank that is 2 m in diameter and 12 m in length? Round to the nearest litre.

6 Of the following, determine which has the bigger volume and calculate the difference in volume to two decimal places: a cube with side length 1 m or a cylinder with radius 1 m and height 0.5 m.

7 Find the volume of these cylindrical portions correct to two decimal places.

a



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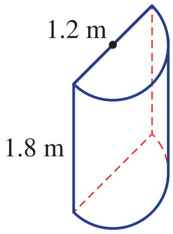

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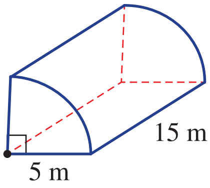

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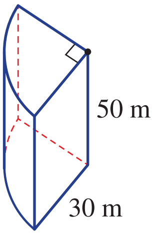

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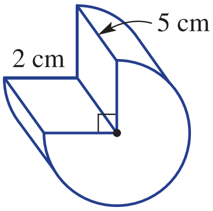

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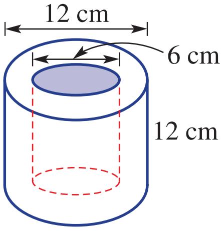

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REASONING	8	8, 9(½)	8, 9(½), 10

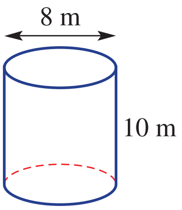
8 The rule for the volume of a cylinder is $V = \pi r^2 h$. Show how you could use this rule to find, correct to three decimal places:

a h when $V = 20$ and $r = 3$

b r when $V = 100$ and $h = 5$.

9 Using exact values (e.g. 20π) find the volume of these solids.

a



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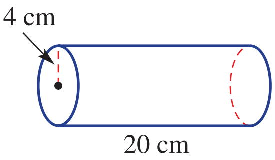

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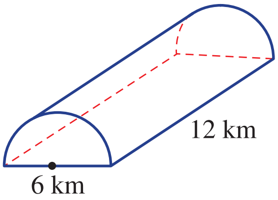

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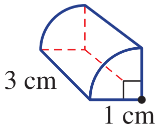

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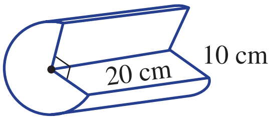

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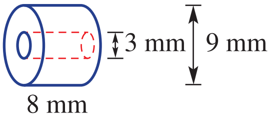
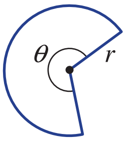


10 Draw a cylinder that has a circumference equal to its height. Try to draw it to scale.

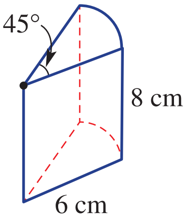
ENRICHMENT: Solid sectors	–	–	11

11 You will recall that the area of a sector is given by $A = \frac{\theta}{360} \times \pi r^2$.



Use this fact to help find the volume of these solids, correct to two decimal places.

a



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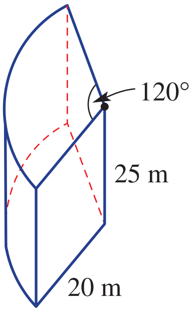

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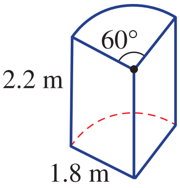

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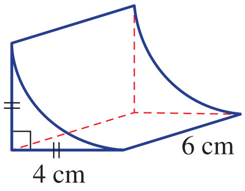

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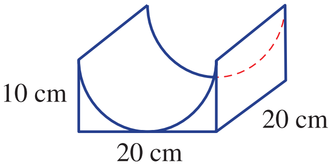

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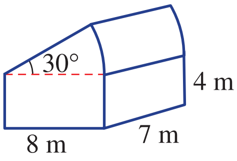
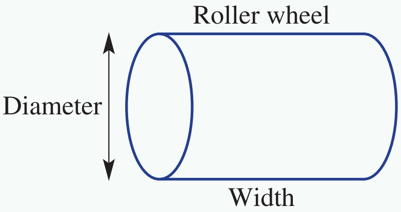


Modelling

Pressing a new road

The construction of a new asphalt road involves pressing the surface with a roller. Two rollers are available, each with main roller wheels of varying diameter and width. The options are:

- Roller 1: Wheel diameter 1 metre and width 2 metres
- Roller 2: Wheel diameter 1.2 metres and width 2.5 metres.



The cost of hiring each roller depends on the number of revolutions in the following way:

- Roller 1: \$2.20 per revolution
- Roller 2: \$3 per revolution.

The rectangular section of road being built is 1 km long and 10 metres wide.

Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate.

Preliminary task

a Find the following for Roller 1 correct to two decimal places:
  i the surface area of the roller wheel (curved surface area only)
  ii the area of ground pressed after 10 revolutions.

b Find the total surface area in square metres of a section of road which is 1 km long and 10 metres wide.

c How many rotations of the wheel would need to occur if Roller 1 was to travel a single-length of the road 1 km long? Round to one decimal place.

Modelling task

Formulate

a The problem is to determine the cost of hiring each roller to press the entire 1 km stretch of road. Write down all the relevant information that will help solve this problem with the aid of diagrams.

Solve

b Using Roller 1 determine the total number of revolutions that the wheel needs to rotate if the entire 10 metre-wide road is to be pressed. Use the following assumptions.
  - It is not necessary to overlap one pressed strip with the next.
  - No revolutions are counted while the roller is manoeuvring into position at the start of each strip.

c Calculate the cost of hiring Roller 1 to press the 1 km stretch of road.

d Repeat parts **b** and **c** for Roller 2.

Evaluate and verify

e Assume that each strip rolled must now overlap the previous pressed strip by 10 cm. Determine the total number of revolutions required for each of the two rollers under these conditions.

f Compare and comment on the cost of hiring the rollers if:
  i no overlap is required between strips
  ii a 10 cm overlap is required between strips.

Communicate

g Summarise your results and describe any key findings.

Extension questions

a If the overlap of each rolled strip is to be 50 cm instead, which roller would be cheaper to use? Give reasons.

b A third roller has wheel diameter 2 metres and width 4 metres and its cost per revolution is \$5. Will this roller be a cheaper option to press the 1 km stretch of road if a 10 cm overlap is required? Justify you answer.

ISBN 978-1-108-77288-4

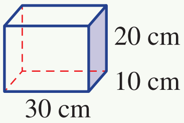
Updated September 2021

Capacity and depth

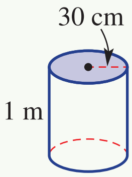
Finding capacity

Find the capacity, in litres, of these containers (i.e. find the total volume of fluid they can hold).

a



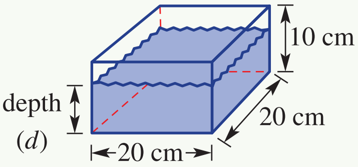
b



> Remember: 1 millilitre (mL) of fluid occupies 1 cm^3 of space, therefore 1 litre (L) occupies 1000 cm^3 as there are 1000 mL in 1 litre.

Finding depth

The depth of water in a prism can be found if the base (cross-sectional) area and volume of water are given.



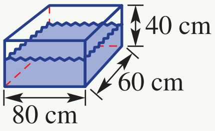
Consider a cuboid, as shown, with 2.4 L of water.

To find the depth of water:

- Convert the volume to cm^3: $2.4\text{ L} = 2.4 \times 1000$
  $= 2400\text{ cm}^3$
- Find the depth: Volume = area of base $\times\, d$
  $2400 = 20 \times 20 \times d$
  $2400 = 400 \times d$
  $\therefore\ d = 6$
  $\therefore$ depth is 6 cm

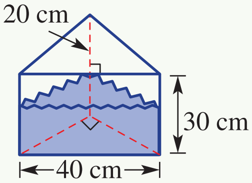
Use the above method to find the depth of water in these prisms.

a



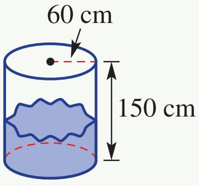
Volume = 96 L

b



Volume = 8 L

c



Volume = 500 L

Volumes of odd-shaped objects

Some solids may be peculiar in shape and their volumes may be difficult to measure.

- **a** A rare piece of rock is placed into a cylindrical jug of water and the water depth rises from 10 cm to 11 cm. The radius of the jug is 5 cm.
  - **i** Find the area of the circular base of the cylinder.
  - **ii** Find the volume of water in the jug before the rock is placed in the jug.
  - **iii** Find the volume of water in the jug including the rock.
  - **iv** Hence find the volume of the rock.
- **b** Use the procedure outlined in part **a** to find the volume of an object of your choice. Explain and show your working and compare your results with those of other students in your class if they are measuring the volume of the same object.

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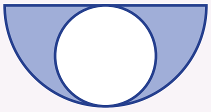
Problems and challenges

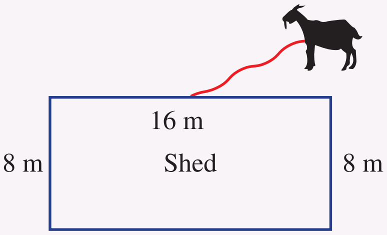
Up for a challenge? If you get stuck on a question, check out the 'Working with unfamiliar problems' poster at the end of the book to help you.

1 The 100 m^2 flat roof of a factory feeds all the water collected to a rainwater tank. If there is 1 mm of rainfall, how many litres of water go into the tank?

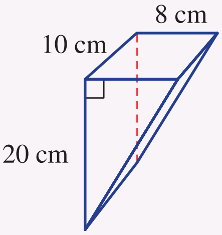
2 What is the relationship between the shaded and non-shaded regions in this circular diagram?



3 A goat is tethered to the centre of one side of a shed with a 10 m length of rope. What area of grass can the goat graze?



4 A rain gauge is in the shape of a triangular prism with dimensions as shown. What is the depth of water when it is half full?



5 A rectangular fish tank has base area 0.3 m^2 and height 30 cm and is filled with 80 L of water. Ten large fish, each with volume 50 cm^3, are placed into the tank. By how much does the water level rise?

6 Find the rule for the volume of a cylinder in terms of r only if the height is equal to its circumference.

7 The surface area of a cylinder is 2π square units. Find a rule for h in terms of r.

8 Give the dimensions of a cylinder that can hold 1 L of milk. What considerations are needed when designing containers for use by consumers?

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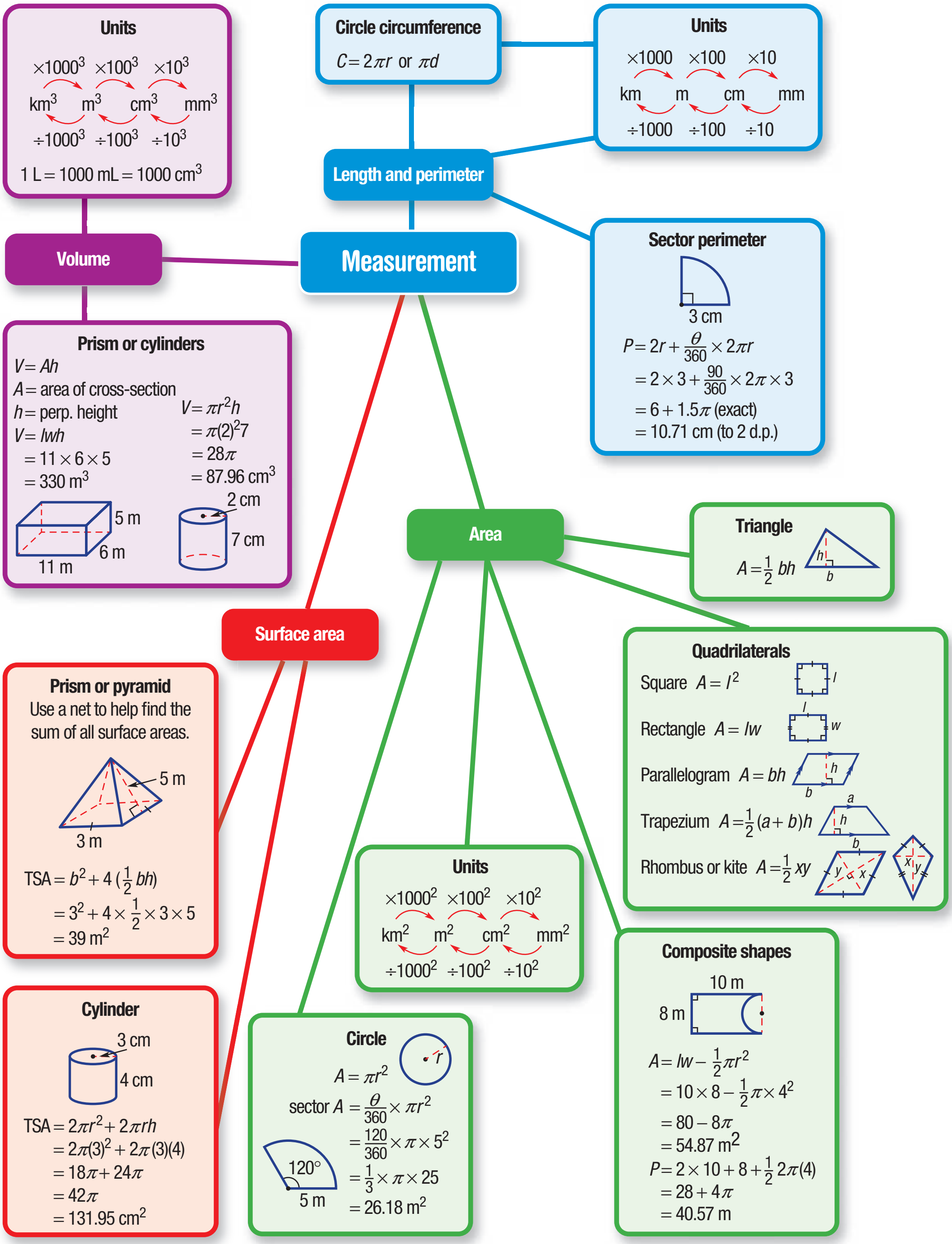


Measurement

Length and perimeter

Circle circumference

$C = 2\pi r$ or πd

Units

×1000 ×100 ×10

km m cm mm

÷1000 ÷100 ÷10

Sector perimeter

$P = 2r + \frac{\theta}{360} \times 2\pi r$

$= 2 \times 3 + \frac{90}{360} \times 2\pi \times 3$

$= 6 + 1.5\pi$ (exact)

$= 10.71$ cm (to 2 d.p.)

Volume

Units

$\times 1000^3$ $\times 100^3$ $\times 10^3$

km^3 m^3 cm^3 mm^3

$\div 1000^3$ $\div 100^3$ $\div 10^3$

1 L = 1000 mL = 1000 cm^3

Prism or cylinders

$V = Ah$

$A =$ area of cross-section

$h =$ perp. height

$V = lwh$

$= 11 \times 6 \times 5$

$= 330 \text{ m}^3$

$V = \pi r^2 h$

$= \pi(2)^2 7$

$= 28\pi$

$= 87.96 \text{ cm}^3$

Surface area

Prism or pyramid

Use a net to help find the sum of all surface areas.

$\text{TSA} = b^2 + 4\left(\frac{1}{2}bh\right)$

$= 3^2 + 4 \times \frac{1}{2} \times 3 \times 5$

$= 39 \text{ m}^2$

Cylinder

$\text{TSA} = 2\pi r^2 + 2\pi rh$

$= 2\pi(3)^2 + 2\pi(3)(4)$

$= 18\pi + 24\pi$

$= 42\pi$

$= 131.95 \text{ cm}^2$

Area

Triangle

$A = \frac{1}{2}bh$

Quadrilaterals

Square $A = l^2$

Rectangle $A = lw$

Parallelogram $A = bh$

Trapezium $A = \frac{1}{2}(a + b)h$

Rhombus or kite $A = \frac{1}{2}xy$

Units

$\times 1000^2$ $\times 100^2$ $\times 10^2$

km^2 m^2 cm^2 mm^2

$\div 1000^2$ $\div 100^2$ $\div 10^2$

Circle

$A = \pi r^2$

sector $A = \frac{\theta}{360} \times \pi r^2$

$= \frac{120}{360} \times \pi \times 5^2$

$= \frac{1}{3} \times \pi \times 25$

$= 26.18 \text{ m}^2$

Composite shapes

$A = lw - \frac{1}{2}\pi r^2$

$= 10 \times 8 - \frac{1}{2}\pi \times 4^2$

$= 80 - 8\pi$

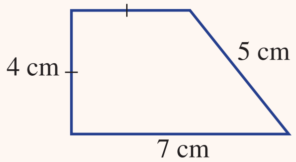
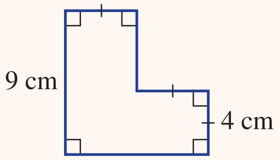
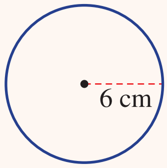
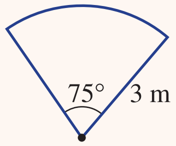
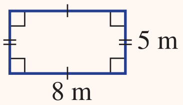
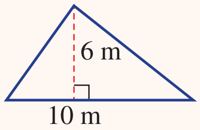
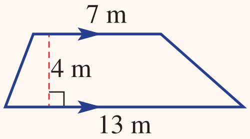
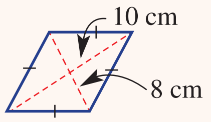
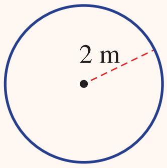
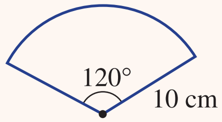
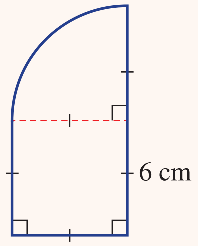
$= 54.87 \text{ m}^2$

$P = 2 \times 10 + 8 + \frac{1}{2} 2\pi(4)$

$= 28 + 4\pi$

$= 40.57$ m

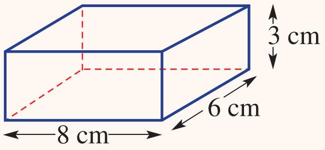
Chapter checklist: Success criteria

		✔
5A	**1. I can find the perimeter of simple shapes.** e.g. Find the perimeter of the following shape. 	
5A	**2. I can find the perimeter of composite shapes.** e.g. Find the perimeter of the following shape. 	
5B	**3. I can find the circumference of a circle.** e.g. Find the circumference of this circle both in exact form and rounded to two decimal places. 	
5B	**4. I can find the perimeter of a sector.** e.g. Find the perimeter of this sector correct to two decimal places. 	
5C	**5. I can convert between units of area.** e.g. Convert 3200 cm^2 to m^2 and 2.4 cm^2 to mm^2.	
5C	**6. I can find the area of simple shapes.** e.g. Find the area of this rectangle and triangle.  	
5C	**7. I can find the area of a trapezium or rhombus.** e.g. Find the area of the following trapezium and rhombus.  	
5C	**8. I can find the area of a circle or sector.** e.g. Find the area of this circle and sector correct to two decimal places.  	
5D	**9. I can find the perimeter and area of composite shapes.** e.g. Find the perimeter and area of this composite shape, rounding answers to two decimal places. 	

✓

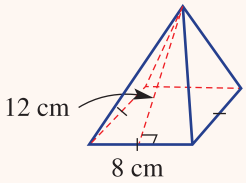
5E

10. I can find the surface area of a prism.
e.g. Find the total surface area of this right rectangular prism.



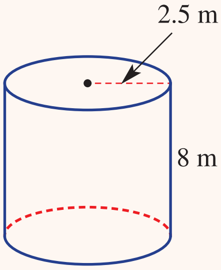
5E

11. I can find the surface area of a pyramid.
e.g. Find the total surface area of this right square-based pyramid.



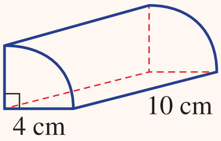
5F

12. I can find the surface area of a cylinder.
e.g. Find the surface area of this cylinder correct to two decimal places.



5F

13. I can find the surface area of cylindrical portions.
e.g. Find the total surface area of this quarter-cylinder, rounding to two decimal places.

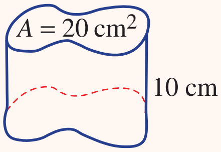


5G

14. I can convert between units of volume.
e.g. Convert 3 m^3 to cm^3 and 1276 mm^3 to cm^3.

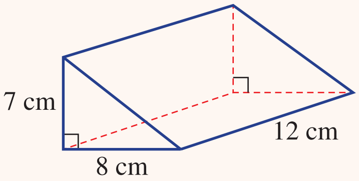
5G

15. I can find the volume of an object with a constant cross-section.
e.g. Find the volume of this solid object.



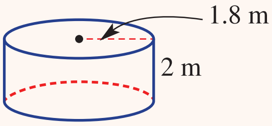
5G

16. I can find the volume of a prism.
e.g. Find the volume of this triangular prism.



5H

17. I can find the volume of a cylinder.
e.g. Find the volume of this cylinder correct to two decimal places.



5H

18. I can find volume in terms of capacity.
e.g. Find the capacity in litres of a cylinder with radius 20 cm and height 80 cm. Round to the nearest litre.

Short-answer questions

5A/C/G

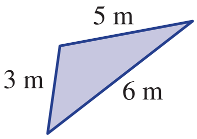
1 Convert the following measurements to the units given in brackets.

a 3.8 m (cm)
b 1.27 km (m)
c 273 mm^2 (cm^2)
d 5.2 m^2 (cm^2)
e 0.01 m^3 (cm^3)
f 53 100 mm^3 (cm^3)
g 3100 mL (L)
h 0.043 L (mL)
i 2.83 kL (L)

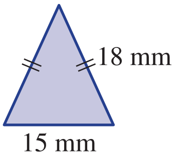
5A

2 Find the perimeter of each of the following shapes.

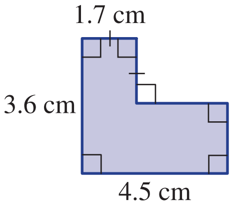
a



b



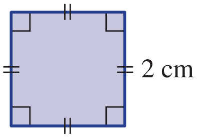
c



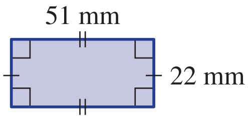
5C

3 Find the area of each of the following plane figures.

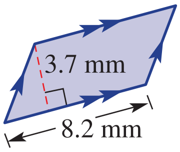
a



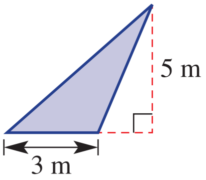
b



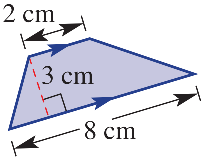
c



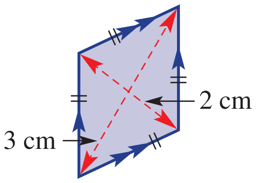
d



e



f



5D

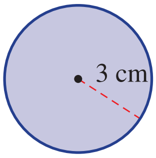
4 A new garden bed is to be constructed inside a rectangular lawn area of length 10.5 m and width 3.8 m. The garden bed is to be the shape of a triangle with base 2 m and height 2.5 m. With the aid of a diagram, find:

a the area of the garden bed
b the area of the lawn remaining around the garden bed.

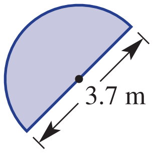
5B/C

5 Find the area and circumference or perimeter of each of the following shapes correct to two decimal places.

a



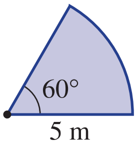
b



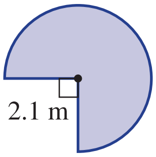
5B/C

6 Find the perimeter and area of these sectors. Round to two decimal places.

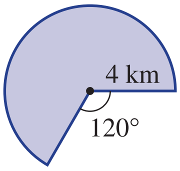
a



b



c



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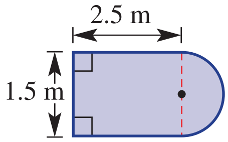
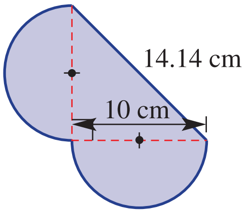
5D

7 Find the perimeter and area of each of the following composite shapes, correct to two decimal places.

a



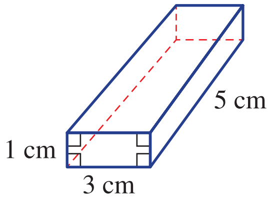
b



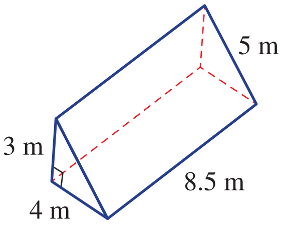
5E

8 Find the total surface area of each of these solid objects.

a



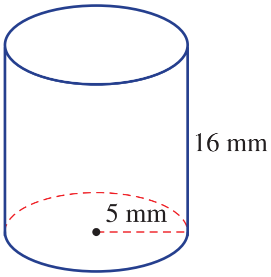
b



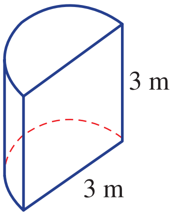
5F

9 Find the total surface area of each of the following solid objects, correct to two decimal places.

a



b

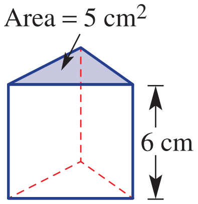


Half-cylinder

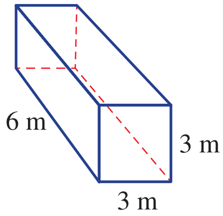
5G/H

10 Find the volume of each of these solid objects, rounding to two decimal places where necessary.

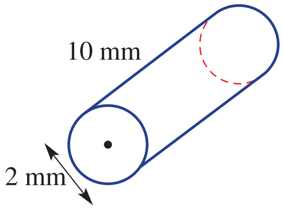
a



b



c



Multiple-choice questions

5A/C

1 If the area of a square field is 25 km^2, its perimeter is:

A 10 km **B** 20 km **C** 5 km **D** 50 km **E** 25 km

5C

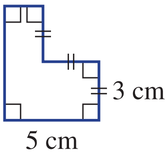
2 2.7 m^2 is the same as:

A 270 cm^2 **B** 0.0027 km^2 **C** 27 000 cm^2 **D** 2700 mm^2 **E** 27 cm^2

5A

3 The perimeter of this shape is:

A 21 cm **B** 14 cm **C** 24 cm
D 20 cm **E** 22 cm



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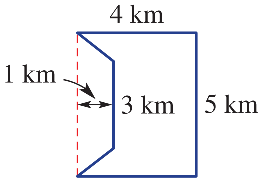
5C **4** A parallelogram has area 10 m^2 and base 5 m. Its perpendicular height is:

A 50 m **B** 2 m **C** 50 m^2 **D** 2 m^2 **E** 0.5 m

5D **5** This composite shape could be considered as a rectangle with an area in the shape of a trapezium removed. The shape's area is:

A 16 km^2 **B** 12 km^2 **C** 20 km^2

D 24 km^2 **E** 6 km^2



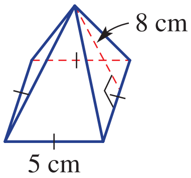
5B **6** A semicircular goal area has a diameter of 20 m. Its perimeter correct to the nearest metre is:

A 41 m **B** 36 m **C** 83 m **D** 51 m **E** 52 m

5E **7** The surface area of the pyramid shown is:

A 185 cm^2 **B** 105 cm^2 **C** 65 cm^2

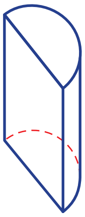
D 100 cm^2 **E** 125 cm^2



5F **8** The area of the curved surface only of this half cylinder with radius 5 mm and height 12 mm is closest to:

A 942.5 mm^2 **B** 94.2 mm^2 **C** 377 mm^2

D 471.2 mm^2 **E** 188.5 mm^2



5G **9** The cross-sectional area of a prism is 100 m^2. If its volume is 6500 m^3, the prism's total height would be:

A 0.65 m **B** 650 000 m **C** 65 m **D** 6.5 m **E** 650 m

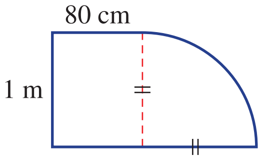
5H **10** The exact volume of a cylinder with radius 3 cm and height 10 cm is:

A 60π cm **B** 80π cm **C** 45π cm^3 **D** 30π cm^3 **E** 90π cm^3

Extended-response questions

1 An office receives five new desks with a bench shape made up of a rectangle and quarter-circle as shown.

The edge of the bench is lined with a rubber strip at a cost of \$2.50 per metre.



a Find the length, in centimetres, of the rubber edging strip for one desk correct to two decimal places.

b By converting your answer in part **a** to metres, find the total cost of the rubber strip for the five desks. Round to the nearest dollar.

The manufacturer claims that the desktop area space is more than 1.5 m^2.

c Find the area of the desktop in cm^2 correct to two decimal places.

d Convert your answer to m^2 and determine whether or not the manufacturer's claim is correct.

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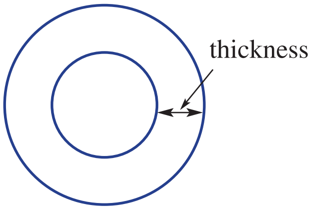
2 Circular steel railing of diameter 6 cm is to be used to fence the side of a bridge. The railing is hollow and the radius of the hollow circular space is 2 cm.



- **a** By adding the given information to this diagram of the cross-section of the railing, determine the thickness of the steel.
- **b** Determine, correct to two decimal places, the area of steel in the cross-section.

Eight lengths of railing each 10 m long are required for the bridge.

- **c** Using your result from part **b**, find the volume of steel required for the bridge in cm^3.
- **d** Convert your answer in part **c** to m^3.

The curved outside surface of the steel railings is to be painted to help protect the steel from the weather.

- **e** Find the outer circumference of the cross-section of each railing correct to two decimal places.
- **f** Find the total surface area in m^2 of the eight lengths of railing that are to be painted. Round to the nearest m^2.

The cost of the railing paint is \$80 per m^2.

- **g** Using your answer from part **f**, find the cost of painting the bridge rails to the nearest dollar.

Reviewing number and financial mathematics

Short-answer questions

1 Evaluate the following.

a $\frac{3}{7} + \frac{1}{4}$ **b** $2\frac{1}{3} - 1\frac{5}{9}$ **c** $\frac{9}{10} \times \frac{5}{12}$ **d** $3\frac{3}{4} \div 2\frac{1}{12}$

2 Convert each of the following to a percentage.

a 0.6 **b** $\frac{5}{16}$ **c** 2 kg out of 20 kg **d** 75c out of $3

3 Write these rates and ratios in simplest form.

a Prize money is shared between two people in the ratio 60 : 36.

b Jodie travels 165 km in 3 hours.

c 3 mL of rain falls in $1\frac{1}{4}$ hours.

4 Calculate the new value.

a Increase $60 by 12% **b** Decrease 70 cm by 8%

5 Jeff earns a weekly retainer of $400 plus 6% of the sales he makes. If he sells $8200 worth of goods, how much will he earn for the week?

Multiple-choice questions

1 $-3 + (4 + (-10)) \times (-2)$ is equal to:

A 21 **B** 9 **C** 18 **D** −15 **E** −6

2 The estimate of $221.7 \div 43.4 - 0.0492$ using one significant figure rounding is:

A 4.9 **B** 5.06 **C** 5.5 **D** 5 **E** 4.95

3 $450 is divided in the ratio 4 : 5. The value of the smaller portion is:

A $210 **B** $250 **C** $90 **D** $200 **E** $220

4 A book that costs $27 is discounted by 15%. The new price is:

A $20.25 **B** $31.05 **C** $4.05 **D** $22.95 **E** $25.20

5 Anna is paid a normal rate of $12.10 per hour. If in a week she works 6 hours at the normal rate, 2 hours at time and a half and 3 hours at double time, how much does she earn?

A $181.50 **B** $145.20 **C** $193.60 **D** $175.45 **E** $163.35

Extended-response question

Jim and Jill are trialling new banking arrangements.

a Jill plans to trial a simple interest plan. Before investing her money she increases the amount in her account by 20% to $21 000.

i What was the original amount in her account?

ii Jill invests the $21 000 for 4 years at an interest rate of 3% p.a. How much does she have in her account at the end of the 4 years?

iii Jill continues with this same plan and after a certain number of years has received $5670 interest. For how many years has she had the money invested?

iv What percentage increase does this interest represent on her initial investment?

b Jim is investing his \$21 000 in an account that compounds annually at 3% p.a. How much does he have after 4 years to the nearest cent?

c **i** Who had the more money in their investment after 4 years and by how much? Round to the nearest dollar.
  ii Who will have the more money after 10 years and by how much? Round to the nearest dollar.

Linear and simultaneous equations

Short-answer questions

1 Solve the following equations and inequalities.

a $3x + 7 = 25$
b $\frac{2x - 1}{4} > 2$
c $4(2m + 3) = 15$
d $-3(2y + 4) - 2y = -4$
e $3(a + 1) \leqslant 4 - 8a$
f $3(2x - 1) = -2(4x + 3)$

2 Noah receives m dollars pocket money per week. His younger brother Jake gets \$6, which is half of \$3 less than Noah's amount.

a Write an equation to represent the problem.
b Solve the equation in part **a** to determine how much Noah receives each week.

3 The formula $S = \frac{n}{2}(a + l)$ gives the sum S of a sequence of n numbers with first term a and last term l.

a Find the sum of the sequence of ten terms 2, 5, 8, …, 29.
b Rearrange the formula to make l the subject.
c If a sequence of eight terms has a sum of 88 and a first term equal to 4, use your answer to part **b** to find the last term of this sequence.

4 Solve the following equations simultaneously.

a $x + 4y = 18$
$x = 2y$

b $7x - 2y = 3$
$y = 2x - 3$

c $2x + 3y = 4$
$x + y = 3$

d $3x + 4y = 7$
$5x + 2y = -7$

Multiple-choice questions

1 The simplified form of $5ab + 6a \div 2 + a \times 2b - a$ is:

A $10ab$ **B** $10ab + 3a$ **C** $13ab - a$ **D** $5ab + 3a + 2b$ **E** $7ab + 2a$

2 The expanded form of $-2(3m - 4)$ is:

A $-6m + 8$ **B** $-6m + 4$ **C** $-6m - 8$ **D** $-5m - 6$ **E** $5m + 8$

3 The solution to $\frac{d}{4} - 7 = 2$ is:

A $d = -20$ **B** $d = 15$ **C** $d = 36$ **D** $d = 1$ **E** $d = 30$

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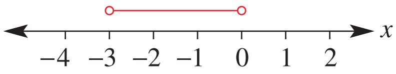
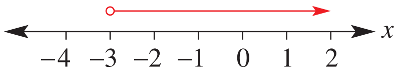
4 The solution to $1 - 3x < 10$ represented on a number line is:

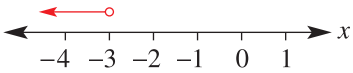
A



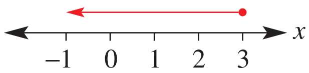
B



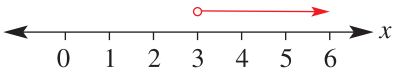
C



D



E



5 The formula $m = \sqrt{\frac{b-1}{a}}$ with b as the subject is:

A $b = a\sqrt{m} + 1$

B $b = \frac{m^2}{a} + 1$

C $b = a^2m^2 + 1$

D $b = am^2 + 1$

E $b = m^2\sqrt{a} + 1$

Extended-response question

a Chris referees junior basketball games on a Sunday. He is paid \$20 plus \$12 per game he referees. He is trying to earn more than \$74 one Sunday. Let x be the number of games he referees.

- **i** Write an inequality to represent the problem.
- **ii** Solve the inequality to find the minimum number of games he must referee.

b Two parents support the game by buying raffle tickets and badges. One buys 5 raffle tickets and 2 badges for \$11.50 while the other buys 4 raffle tickets and 3 badges for \$12. Determine the cost of a raffle ticket and the cost of a badge by:

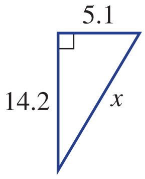
- **i** defining two variables
- **ii** setting up two equations to represent the problem
- **iii** solving your equations simultaneously.

Pythagoras' theorem and trigonometry

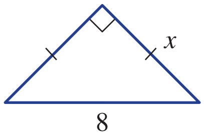
Short-answer questions

1 Find the value of the pronumerals, correct to one decimal place.

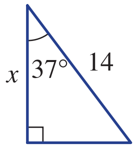
a



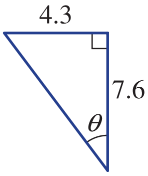
b



c

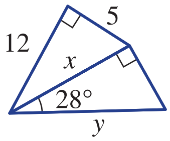


d

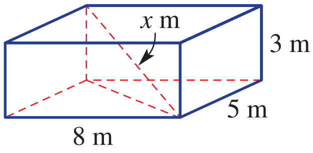


2 Find the value of the pronumerals. Round to one decimal place where necessary.

a



b



3 A wire is to be connected from the edge of the top of a 28 m high building to the edge of the top of a 16 m high building. The buildings are 15 m apart.

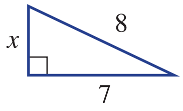
- **a** What length of wire, to the nearest centimetre, is required?
- **b** What is the angle of depression from the top of the taller building to the top of the smaller building? Round to one decimal place.

4 A yacht sails 18 km from its start location on a bearing of 295°T.

- **a** How far east or west is it from its start location? Answer correct to one decimal place.
- **b** On what true bearing would it need to sail to return directly to its start location?

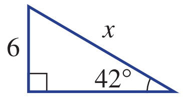
Multiple-choice questions

1 The exact value of x in the triangle shown is:



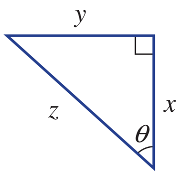
A 3.9 **B** $\sqrt{113}$ **C** $\sqrt{57}$
D $\sqrt{15}$ **E** 2.7

2 The correct expression for the triangle shown is:



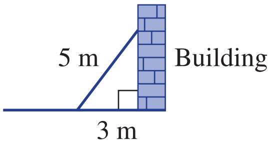
A $x = \dfrac{6}{\sin 42^\circ}$ **B** $x = 6\tan 42^\circ$ **C** $x = 6\sin 42^\circ$
D $x = \dfrac{6}{\cos 42^\circ}$ **E** $x = \dfrac{\sin 42^\circ}{6}$

3 The correct expression for the angle θ is:



A $\theta = \tan^{-1}\left(\dfrac{x}{z}\right)$ **B** $\theta = \sin^{-1}\left(\dfrac{x}{z}\right)$ **C** $\theta = \tan^{-1}\left(\dfrac{y}{x}\right)$
D $\theta = \cos^{-1}\left(\dfrac{z}{x}\right)$ **E** $\theta = \cos^{-1}\left(\dfrac{x}{y}\right)$

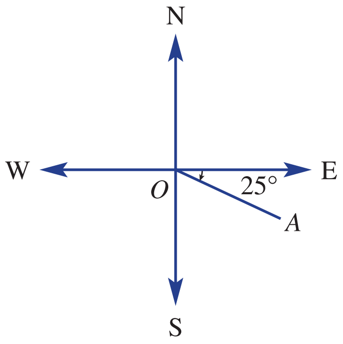
4 A 5 m plank of wood is leaning up against the side of a building as shown. If the plank touches the ground 3 m from the base of the building, the angle the wood makes with the building is closest to:



A 36.9° **B** 59° **C** 53.1° **D** 31° **E** 41.4°

5 The true bearing of A from O is:

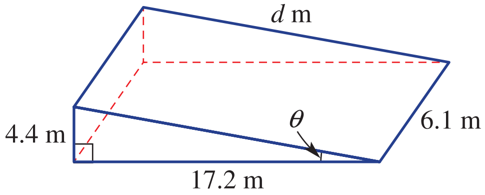
A 025° **B** 125° **C** 155° **D** 065° **E** 115°



Extended-response question

A skateboard ramp is constructed as shown.

a Calculate the distance d metres up the ramp, correct to two decimal places.



b What is the angle of inclination (θ) between the ramp and the ground, correct to one decimal place?

c **i** A skateboarder rides from one corner of the ramp diagonally to the other corner. What distance would be travelled? Round to one decimal place.

ii If the skateboarder travels at an average speed of 10 km/h, how many seconds does it take to ride diagonally across the ramp? Answer correct to one decimal place.

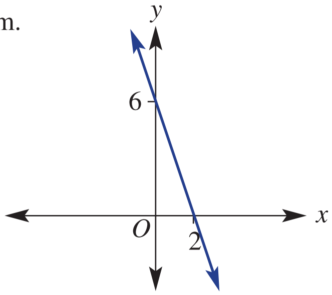
Linear relations

Short-answer questions

1 Sketch the following linear graphs labelling x- and y-intercepts.
- **a** $y = 2x - 6$
- **b** $3x + 4y = 24$
- **c** $y = 4x$

2 Find the gradient of each of the following.
- **a** The line passing through the points $(-1, 2)$ and $(2, 4)$
- **b** The line passing through the points $(-2, 5)$ and $(1, -4)$
- **c** The line with equation $y = -2x + 5$
- **d** The line with equation $-4x + 3y = 9$

3 Give the equation of the following lines in gradient–intercept form.
- **a** The line with the given graph
- **b** The line with gradient 3 and passing through the point $(2, 5)$
- **c** The line parallel to the line with equation $y = 2x - 1$ and passing through the origin
- **d** The line perpendicular to the line with equation $y = 3x + 4$ and passing through the point $(0, 2)$



4 Solve the simultaneous equations $y = 2x - 4$ and $x + y = 5$ graphically by finding the coordinates of the point of intersection.

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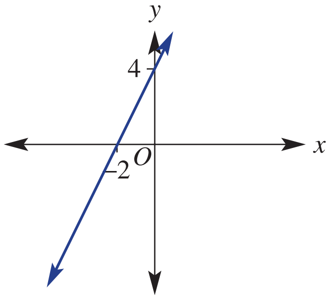
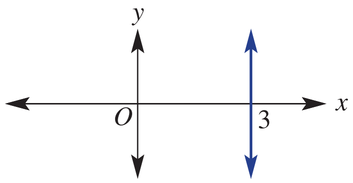
Multiple-choice questions

1 The coordinates of the x- and y-intercepts respectively for the graph shown are:



A $(-2, 4)$ and $(4, -2)$
B $(0, 4)$ and $(-2, 0)$
C $(-2, 0)$ and $(0, 4)$
D $(4, 0)$ and $(0, -2)$
E $(2, 0)$ and $(0, -4)$

2 The graph shown has equation:



A $y = 3x$
B $y = 3$
C $y = x + 3$
D $x = 3$
E $x + y = 3$

3 If the point $(-1, 3)$ is on the line $y = 2x + c$, the value of c is:
A 1 **B** 5 **C** -7 **D** -5 **E** -1

4 The line passing through the points $(-3, -1)$ and $(1, y)$ has gradient 2. The value of y is:
A 3 **B** 5 **C** 7 **D** 1 **E** 4

5 The midpoint and length (to one decimal place) of the line segment joining the points $(-2, 1)$ and $(4, 6)$ are:
A $(1, 3.5)$ and 7.8 **B** $(3, 5)$ and 5.4 **C** $(3, 3.5)$ and 6.1
D $(1, 3.5)$ and 3.3 **E** $(3, 3.5)$ and 3.6

Extended-response question

Doug works as a labourer. He is digging a trench and has 180 kg of soil to remove. He has taken 3 hours to remove 36 kg.

a What is the rate at which he is removing the soil?

b If he maintains this rate, write a rule for the amount of soil, S (kg), remaining after t hours.

c Draw a graph of your rule.

d How long will it take to remove all of the soil?

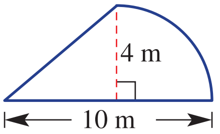
e Doug is paid $40 for the job plus $25 per hour worked.
  i Write a rule for his pay P dollars for working h hours.
  ii How much will he be paid to remove all the soil?

Measurement

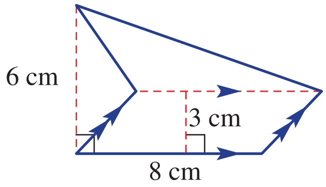
Short-answer questions

1 Find the area of each of the figures below. Round to two decimal places where necessary.

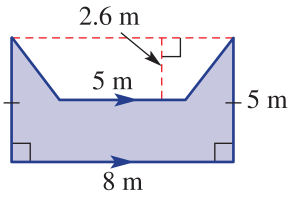
a



b

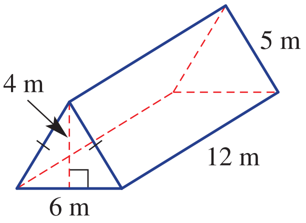


2 A tin of varnish for the timber (shaded) on the deck shown covers 6.2 square metres. How many tins will be required to completely varnish the deck?

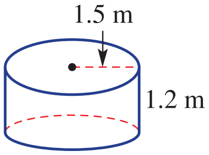


3 Find the total surface area of these solid objects. Round to two decimal places where necessary.

a

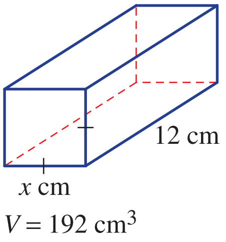


b



4 Find the value of the pronumeral for the given volume.

a



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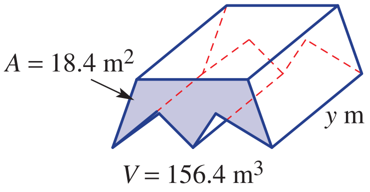
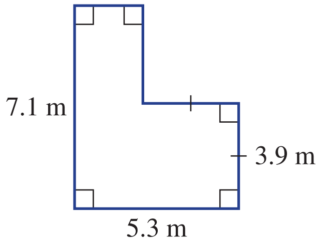


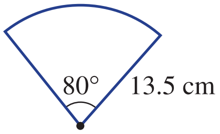
Multiple-choice questions

1 The perimeter and area of the figure shown are:

A 20.2 m, 22.42 m^2
B 24.8 m, 22.42 m^2
C 24.8 m, 25.15 m^2
D 20.2 m, 25.15 m^2
E 21.6 m, 24.63 m^2



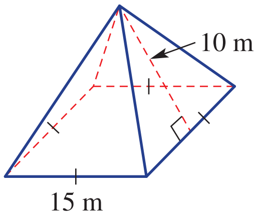
2 The exact perimeter in centimetres of this sector is:



A 127.2
B $\frac{81\pi}{2} + 27$
C $12\pi + 27$
D 45.8
E $6\pi + 27$

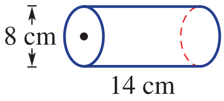
3 420 cm^2 is equivalent to:
A 4.2 m^2
B 0.42 m^2
C 42 000 m^2
D 0.042 m^2
E 0.0042 m^2

4 This square pyramid has a total surface area of:



A 525 m^2
B 300 m^2
C 750 m^2
D 450 m^2
E 825 m^2

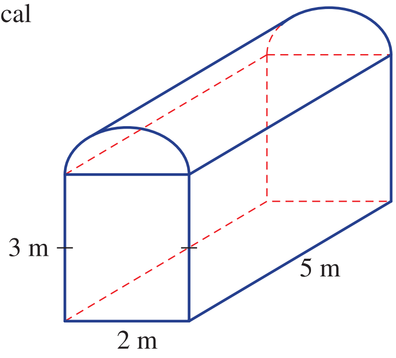
5 The volume of the cylinder shown is closest to:



A 703.7 cm^3
B 351.9 cm^3
C 2814.9 cm^3
D 452.4 cm^3
E 1105.8 cm^3

Extended-response question

A barn in the shape of a rectangular prism with a semicylindrical roof and with the dimensions shown is used to store hay.



a The roof and the two long side walls of the barn are to be painted. Calculate the surface area to be painted correct to two decimal places.

b A paint roller has a width of 20 cm and a radius of 3 cm.
  i Find the area of the curved surface of the paint roller in m^2. Round to four decimal places.
  ii Hence, state the area that the roller will cover in 100 revolutions.

c Find the minimum number of revolutions required to paint the area of the barn in part **a** with one coat.

d Find the volume of the barn correct to two decimal places.

e A rectangular bail of hay has dimensions 1 m by 40 cm by 40 cm. If there are 115 bails of hay in the barn, what volume of air space remains? Answer to two decimal places.

CHAPTER 6 Indices and surds

Musical notes and indices

Each octave on the piano has 12 notes or semitones with the highest note having double the frequency of the lowest note. The frequency of a piano's lowest C, for example, multiplied by 2, 2^2, 2^3, 2^4, 2^5, 2^6 calculates the higher C frequencies, each an octave apart. The highest C frequency multiplied by 2^{-1}, 2^{-2}, 2^{-3}, 2^{-4}, 2^{-5}, 2^{-6} gives the lower C frequencies.

Within an octave, the frequency of each note increases by $\times\, 2^{\frac{1}{12}}$ because $\left(2^{\frac{1}{12}}\right)^{12} = 2^{\frac{1}{12} \times 12} = 2$. Hence the frequency of each note can be found by multiplying the frequency of the lowest note by $2^{\frac{1}{12}}, \left(2^{\frac{1}{12}}\right)^2, \left(2^{\frac{1}{12}}\right)^3, \ldots \left(2^{\frac{1}{12}}\right)^{11}, \left(2^{\frac{1}{12}}\right)^{12}$. For example:

- Middle C frequency is 261.6 Hz (hertz = cycles/s)
- C sharp frequency $= 261.6 \times 2^{\frac{1}{12}} = 277.2$ Hz
- D frequency $= 261.6 \times \left(2^{\frac{1}{12}}\right)^2 = 293.6$ Hz

At the top of this octave:

- C frequency $= 261.6 \times \left(2^{\frac{1}{12}}\right)^{12} = 261.6 \times 2 = 523.2$ Hz

Tuning a piano to concert pitch means that A above

middle C is tuned to exactly 440 Hz. Each note above A is × $2^{\frac{1}{12}}$ the frequency of the note below. Each note below A is × $2^{-\frac{1}{12}}$ the frequency of the note above it. Middle C is 9 notes or semitones below A, giving:

Middle C frequency $= 440 \times \left(2^{-\frac{1}{12}}\right)^9 = 261.6$ Hz.

Piano notes played together that sound pleasing to the ear have close to whole number ratios of their frequencies.

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

In this chapter

6A Index notation (CONSOLIDATING)
6B Index laws 1 and 2
6C Index law 3 and the zero power
6D Index laws 4 and 5
6E Negative indices
6F Scientific notation
6G Scientific notation using significant figures
6H Fractional indices and surds (EXTENDING)
6I Simple operations with surds (EXTENDING)

Australian Curriculum

NUMBER AND ALGEBRA
Real numbers

Apply index laws to numerical expressions with integer indices (ACMNA209)

Express numbers in scientific notation (ACMNA210)

Patterns and algebra

Extend and apply the index laws to variables, using positive integer indices and the zero index (ACMNA212)

MEASUREMENT AND GEOMETRY
Using units of measurement

Investigate very small and very large time scales and intervals (ACMMG219)

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6A Index notation CONSOLIDATING

Learning intentions

- To know that indices are used as a shorthand way of representing repeated multiplication of the same factor
- To know how to interpret the base and index to write a term in expanded form
- To be able to evaluate powers
- To be able to use indices to write a product in index form
- To know how to use a factor tree to write a number as a product of its prime factors

When a product includes the repeated multiplication of the same factor, indices can be used to produce a more concise expression. For example, $5 \times 5 \times 5$ can be written as 5^3 and $x \times x \times x \times x \times x$ can be written as x^5. The expression 5^3 is called a power, and we say '5 to the power of 3'. The 5 is called the base and the 3 is the index, exponent or power. Numbers written with indices are common in mathematics and can be applied to many types of problems. The mass of a 100 kg limestone block, for example, might decrease by 2 per cent per year for 20 years. The mass after 20 years could be calculated by multiplying 100 by 0.98, 20 times. This is written as $100 \times (0.98)^{20}$.

A wide range of people use the index laws in their profession, including scientists, chemists, radiographers, financial planners, economists, actuaries, astronomers, geologists, computer programmers, and medical, electrical, sound and aerospace engineers.

LESSON STARTER Who has the most?

A person offers you one of two prizes.

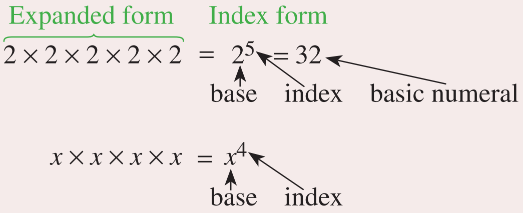
Prize A
$1000 now

Prize B
1 cent then doubled every day for 20 days.

- Which offer would you take?
- Try to calculate the final amount for prize B.
- How might you use indices to help calculate the value of prize B?
- How can a calculator help to find the amount for prize B using the power button $\boxed{\wedge}$?

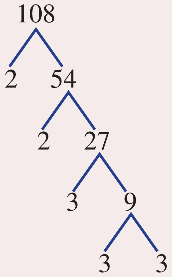
KEY IDEAS

- **Indices** (plural of **index**) can be used to represent a product of the same factor.
- The **base** is the factor in the product.
- The index (**exponent** or **power**) is the number of times the factor (base number) is written.


Expanded form
Index form
$2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$
base
index
basic numeral
$x \times x \times x \times x = x^4$
base
index

- Note that $a^1 = a$.
  For example: $5^1 = 5$
- **Prime factorisation** involves writing a number as a product of its prime factors.

$$108 = 2 \times 2 \times 3 \times 3 \times 3$$
$$= \underbrace{2^2 \times 3^3}_{\text{prime factor form}}$$



BUILDING UNDERSTANDING

① State the missing index.

a $5^{\square} = 25$
b $2^{\square} = 8$
c $(-2)^{\square} = -8$
d $(-4)^{\square} = 16$

② State the number or pronumeral that is the base in these expressions.

a 3^7
b $(-7)^3$
c $\left(\frac{2}{3}\right)^4$
d y^{10}

③ State the number that is the index in these expressions.

a 4^3
b $(-3)^7$
c $\left(\frac{1}{2}\right)^4$
d $(xy)^{13}$

④ State the prime factors of these numbers.

a 6
b 15
c 30
d 77

ISBN 978-1-108-77288-4

Example 1 Writing in expanded form

Write in expanded form.

a a^3 **b** $(xy)^4$ **c** $2a^3b^2$

SOLUTION	EXPLANATION
a $a^3 = a \times a \times a$	Factor *a* is written three times.
b $(xy)^4 = xy \times xy \times xy \times xy$	Factor *xy* is written four times.
c $2a^3b^2 = 2 \times a \times a \times a \times b \times b$	Factor *a* is written three times and factor *b* is written twice. Factor 2 only appears once.

Now you try

Write in expanded form.

a b^4 **b** $(mn)^3$ **c** $5a^2b$

Example 2 Expanding and evaluating

Write each of the following in expanded form and then evaluate.

a 5^3 **b** $(-2)^5$ **c** $\left(\frac{2}{5}\right)^3$

SOLUTION	EXPLANATION
a $\begin{aligned} 5^3 &= 5 \times 5 \times 5 \\ &= 125 \end{aligned}$	Write in expanded form with 5 written three times and evaluate.
b $\begin{aligned} (-2)^5 &= (-2) \times (-2) \times (-2) \times (-2) \times (-2) \\ &= -32 \end{aligned}$	Write in expanded form with −2 written five times and evaluate.
c $\begin{aligned} \left(\frac{2}{5}\right)^3 &= \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \\ &= \frac{8}{125} \end{aligned}$	Write in expanded form. Evaluate by multiplying numerators and denominators.

Now you try

Write each of the following in expanded form and then evaluate.

a 2^4 **b** $(-3)^3$ **c** $\left(\frac{4}{3}\right)^2$

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Example 3 Writing in index form

Write each of the following in index form.

a $6 \times x \times x \times x \times x$ **b** $\frac{3}{7} \times \frac{3}{7} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}$ **c** $8 \times a \times a \times 8 \times b \times b \times a \times b$

SOLUTION

a $6 \times x \times x \times x \times x = 6x^4$

b $\frac{3}{7} \times \frac{3}{7} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = \left(\frac{3}{7}\right)^2 \times \left(\frac{4}{5}\right)^3$

c $8 \times a \times a \times 8 \times b \times b \times a \times b$
$= 8 \times 8 \times a \times a \times a \times b \times b \times b$
$= 8^2a^3b^3$

EXPLANATION

Factor x is written 4 times, 6 only once.

There are two groups of $\left(\frac{3}{7}\right)$ and three groups of $\left(\frac{4}{5}\right)$.

Group the numerals and like pronumerals and write in index form.

Now you try

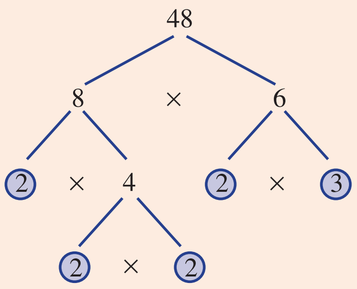
Write each of the following in index form.

a $2 \times y \times y \times y$ **b** $\frac{5}{9} \times \frac{5}{9} \times \frac{5}{9} \times \frac{3}{7} \times \frac{3}{7}$ **c** $3 \times a \times 3 \times 3 \times a$

Example 4 Finding the prime factor form

Express 48 as a product of its prime factors in index form.

SOLUTION



$\therefore\ 48 = 2 \times 2 \times 2 \times 2 \times 3$
$= 2^4 \times 3$

EXPLANATION

Choose a pair of factors of 48, for example 8 and 6.
Choose a pair of factors of 8, i.e. 2 and 4.
Choose a pair of factors of 6, i.e. 2 and 3.
Continue this process until the factors are all prime numbers (circled numbers).

Write the prime factors of 48.
Express in index notation.

Now you try

Express 60 as a product of its prime factors in index form.

Exercise 6A

FLUENCY 1, 2–7(1/2) 2–8(1/2) 2–8(1/3)

Example 1

1 Write in expanded form.

a a^4 **b** $(xp)^6$ **c** $4x^2y^5$ **d** $4(ab)^3$

Example 1

2 Write each of the following in expanded form.

a b^3 **b** x^3 **c** $(5a)^4$ **d** $(3y)^3$

e $(pq)^2$ **f** $-3s^3t^2$ **g** $6x^3y^5$ **h** $5(yz)^6$

Example 2

3 Write each of the following in expanded form and then evaluate.

a 6^2 **b** 2^4 **c** 3^5 **d** 12^1

e $(-2)^3$ **f** $(-1)^7$ **g** $(-3)^4$ **h** $(-5)^2$

i $\left(\frac{2}{3}\right)^3$ **j** $\left(\frac{3}{4}\right)^2$ **k** $\left(\frac{1}{6}\right)^3$ **l** $\left(\frac{5}{2}\right)^2$

m $\left(\frac{2}{-3}\right)^3$ **n** $\left(\frac{-3}{4}\right)^4$ **o** $\left(\frac{-1}{4}\right)^2$ **p** $\left(\frac{5}{-2}\right)^5$

Example 3a

4 Write each of the following in index form.

a $3 \times 3 \times 3$ **b** $8 \times 8 \times 8 \times 8 \times 8 \times 8$

c $y \times y$ **d** $3 \times x \times x \times x$

e $4 \times c \times c \times c \times c \times c$ **f** $5 \times 5 \times 5 \times d \times d$

g $x \times x \times y \times y \times y$ **h** $7 \times b \times 7 \times b \times 7$

Example 3b

5 Write each of the following in index form.

a $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$ **b** $\frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}$

c $\frac{4}{7} \times \frac{4}{7} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}$ **d** $\frac{7x}{9} \times \frac{7x}{9} \times \frac{y}{4} \times \frac{y}{4} \times \frac{y}{4}$

Example 3c

6 Write each of the following in index form.

a $3 \times x \times y \times x \times 3 \times x \times 3 \times y$ **b** $3x \times 2y \times 3x \times 2y$

c $4d \times 2e \times 4d \times 2e$ **d** $6by(6by)(6y)$

e $3pq(3pq)(3pq)(3pq)$ **f** $7mn \times 7mn \times mn \times 7$

Example 4

7 Express each of the following as a product of prime factors in index form.

a 10 **b** 8 **c** 144

d 512 **e** 216 **f** 500

8 If $a = 3$, $b = 2$ and $c = -3$, evaluate these expressions.

a $(ab)^2$ **b** $(bc)^3$ **c** $\left(\frac{a}{c}\right)^4$ **d** $\left(\frac{b}{c}\right)^3$

e $(abc)^1$ **f** $c^2 + ab$ **g** ab^2c **h** c^2ab^3

PROBLEM-SOLVING 9, 10 9–11 10–12

9 Find the missing number.

a $3^{\square} = 81$ **b** $2^{\square} = 256$ **c** $\square^3 = 125$ **d** $\square^5 = 32$

e $\square^3 = -64$ **f** $\square^7 = -128$ **g** $\square^3 = \frac{1}{8}$ **h** $\left(\frac{2}{3}\right)^{\square} = \frac{16}{81}$

10 A bacterium splits in two every 5 minutes. New cells also continue splitting in the same way. Use a table to help.

a How long will it take for 1 cell to divide into:

i 4 cells? **ii** 16 cells? **iii** 64 cells?

b A single cell is set aside to divide for 2 hours. How many cells will there be after this time?

11 A sharebroker says he can triple your money every year, so you invest \$1000 with him.

a How much should your investment be worth in 5 years' time?

b For how many years should you invest if you were hoping for a total of at least \$100 000? Give a whole number of years.

12 The weight of a 12 kg fat cat reduces by 10% each month. How long does it take for the cat to be at least 6 kg lighter than its original weight? Give your answer as a whole number of months.

REASONING	13, 14	13, 14	13, 14, 15(½)

13 **a** Evaluate the following.

i 3^2 **ii** $(-3)^2$ **iii** $-(3)^2$ **iv** $-(-3)^2$

b Explain why the answers to parts **i** and **ii** are positive.

c Explain why the answers to parts **iii** and **iv** are negative.

14 **a** Evaluate the following.

i 2^3 **ii** $(-2)^3$ **iii** $-(2)^3$ **iv** $-(-2)^3$

b Explain why the answers to parts **i** and **iv** are positive.

c Explain why the answers to parts **ii** and **iii** are negative.

15 It is often easier to evaluate a decimal raised to a power by first converting the decimal to a fraction, as shown on the right.

$$\begin{aligned}(0.5)^4 &= \left(\frac{1}{2}\right)^4\\ &= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\\ &= \frac{1}{16}\end{aligned}$$

Use this idea to evaluate these decimals as fractions.

a $(0.5)^3$ **b** $(0.25)^2$ **c** $(0.2)^3$

d $(0.5)^6$ **e** $(0.7)^2$ **f** $(1.5)^4$

g $(2.6)^2$ **h** $(11.3)^2$ **i** $(3.4)^2$

ENRICHMENT: LCM and HCF from prime factorisation	–	–	16

16 Last year you may have used prime factorisation to find the LCM (lowest common multiple) and the HCF (highest common factor) of two numbers. Here are the definitions.

- The LCM of two numbers in their prime factor form is the product of all the different primes raised to their highest power.
- The HCF of two numbers in their prime factor form is the product of all the common primes raised to their smallest power.

For example: $12 = 2^2 \times 3$ and $30 = 2 \times 3 \times 5$

The prime factors 2 and 3 are common.

$$\therefore\ \text{HCF} = 2 \times 3 = 6 \qquad \therefore\ \text{LCM} = 2^2 \times 3 \times 5 = 60$$

Find the LCM and HCF of these pairs of numbers by first writing them in prime factor form.

a 4, 6 **b** 42, 28 **c** 24, 36 **d** 10, 15

e 40, 90 **f** 100, 30 **g** 196, 126 **h** 2178, 1188

ISBN 978-1-108-77288-4

6B Index laws 1 and 2

Learning intentions

- To understand that only terms with the same base can be multiplied or divided
- To know that multiplying terms with the same base involves adding the powers
- To know that dividing terms with the same base involves subtracting the powers
- To be able to combine terms under multiplication and division using index laws

An index law (or identity) is an equation that is true for all possible values of the variables in that equation. When multiplying or dividing numbers with the same base, index laws can be used to simplify the expression.

Consider $a^m \times a^n$.

Using expanded form:

$$a^m \times a^n = \underbrace{\overbrace{a \times a \times a \times \ldots \times a}^{m \text{ factors of } a} \times \overbrace{a \times a \times \ldots \times a}^{n \text{ factors of } a}}_{m + n \text{ factors of } a}$$

$$= a^{m+n}$$

So the total number of factors of a is $m + n$.

Also:

$$a^m \div a^n = \frac{\overbrace{a \times a \times \ldots \times \cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{a}}^{m \text{ factors of } a}}{\underbrace{\cancel{a} \times \ldots \times \cancel{a} \times \cancel{a}}_{n \text{ factors of } a}}$$

$$= a^{m-n}$$

So the total number of factors of a is $m - n$.

Biologists and statisticians use indices when modelling exponential population growth, such as a bacteria population $P = 2^n$ after n generations, and Australia's projected population $P = 25 \times 1.04^n$ in millions, n years from 2018.

LESSON STARTER Discovering laws 1 and 2

Consider the two expressions $2^3 \times 2^5$ and $6^8 \div 6^6$.
Complete this working.

$$2^3 \times 2^5 = 2 \times \square \times \square \times 2 \times \square \times \square \times \square \times \square$$

$$= 2^{\square}$$

$$6^8 \div 6^6 = \frac{6 \times \square \times \square \times \square \times \square \times \square \times \square \times \square}{6 \times \square \times \square \times \square \times \square \times \square}$$

$$= \frac{6 \times 6}{1}$$

$$= 6^{\square}$$

- What do you notice about the given expression and the answer in each case? Can you express this as a rule or law in words?
- Repeat the type of working given above and test your laws on these expressions.

a $3^2 \times 3^7$ **b** $4^{11} \div 4^8$

KEY IDEAS

- **Index law 1:** $a^m \times a^n = a^{m+n}$
  - When multiplying terms with the same base, add the powers.
- **Index law 2:** $a^m \div a^n = \dfrac{a^m}{a^n} = a^{m-n}$
  - When dividing terms with the same base, subtract the powers.

BUILDING UNDERSTANDING

1 State the missing words.

a Index law 1 states that if you ____________ two terms with the same ____________ you ____________ the powers.

b Index law 2 states that if you ____________ two terms with the same ____________ you ____________ the powers.

2 Give the missing numbers to complete the following.

a $3^2 \times 3^4 = 3 \times \square \times 3 \times \square \times \square \times \square$
$= 3^{\square}$

b $5^5 \div 5^3 = \dfrac{5 \times \square \times \square \times \square \times \square}{5 \times \square \times \square}$
$= 5^{\square}$

3 Decide if these statements are true or false.

a $5 \times 5 \times 5 \times 5 = 5^4$

b $2^6 \times 2^2 = 2^{6+2}$

c $7^2 \times 7^4 = 7^{4-2}$

d $8^4 \div 8^2 = 8^{4+2}$

e $a \times a^2 = a^3$

f $x^7 \div x = x^7$

Example 5 Using laws 1 and 2 with numerical bases

Simplify, giving your answer in index form.

a $3^6 \times 3^4$

b $7^9 \div 7^5$

SOLUTION	EXPLANATION
a $3^6 \times 3^4 = 3^{6+4}$ $= 3^{10}$	$a^m \times a^n = a^{m+n}$ (add the powers)
b $7^9 \div 7^5 = 7^{9-5}$ $= 7^4$	$a^m \div a^n = a^{m-n}$ (subtract the powers)

Now you try

Simplify, giving your answer in index form.

a $2^4 \times 2^5$

b $9^7 \div 9^3$

Example 6 Using index law 1

Simplify each of the following using the first index law.

a $x^4 \times x^5$ **b** $x^3y^4 \times x^2y$ **c** $3m^4 \times 2m^5$

SOLUTION	EXPLANATION
a $x^4 \times x^5 = x^{4+5}$ $= x^9$	Use law 1 to add the indices.
b $x^3y^4 \times x^2y = x^{3+2}y^{4+1}$ $= x^5y^5$	Use law 1 to add the indices corresponding to each different base. Recall $y = y^1$.
c $3m^4 \times 2m^5 = 3 \times 2 \times m^{4+5}$ $= 6m^9$	Multiply the numbers and then use law 1 to add the indices of the base m.

Now you try

Simplify each of the following using the first index law.

a $x^2 \times x^9$ **b** $a^2b^4 \times ab^3$ **c** $4m^2 \times 7m^3$

Example 7 Using index law 2

Simplify each of the following using the second index law.

a $x^{10} \div x^2$ **b** $\frac{8a^6b^3}{12a^2b^2}$

SOLUTION	EXPLANATION
a $x^{10} \div x^2 = x^{10-2}$ $= x^8$	Use law 2 to subtract the indices.
b $\frac{8a^6b^3}{12a^2b^2} = \frac{{}^{2}\cancel{8}a^{6-2}b^{3-2}}{{}^{3}\cancel{12}}$ $= \frac{2a^4b}{3}$	Cancel the numbers using the highest common factor (4) and use law 2 to subtract the indices for each different base.

Now you try

Simplify each of the following using the second index law.

a $a^7 \div a^3$ **b** $\frac{4x^6y^4}{10x^3y^3}$

Example 8 Combining index laws 1 and 2

Simplify each of the following using the first two index laws.

a $x^2 \times x^3 \div x^4$

b $\dfrac{2a^3b \times 8a^2b^3}{4a^4b^2}$

SOLUTION

a $x^2 \times x^3 \div x^4 = x^5 \div x^4$
$= x$

b $\dfrac{2a^3b \times 8a^2b^3}{4a^4b^2} = \dfrac{{}^{4}\cancel{16}a^5b^4}{{}^{1}\cancel{4}a^4b^2}$
$= 4ab^2$

EXPLANATION

Use law 1 to add the indices for $x^2 \times x^3$.
Use law 2 to subtract the indices for $x^5 \div x^4$.

Multiply the numbers and use law 1 to add the indices for each different base in the numerator. Use law 2 to subtract the indices of each different base and cancel the 16 with the 4.

Now you try

Simplify each of the following using the first two index laws.

a $a^4 \times a^3 \div a^2$

b $\dfrac{5a^2b^3 \times 4ab^2}{10ab^3}$

Exercise 6B

FLUENCY	1, 2–5(1/2)	2–6(1/3)	2–6(1/4)

1 Simplify, giving your answers in index form.

Example 5a

a **i** $2^4 \times 2^3$
ii $5^6 \times 5^3$

Example 5b

b **i** $3^7 \div 3^4$
ii $6^8 \div 6^3$

Example 5

2 Simplify, giving your answers in index form.

a $7^2 \times 7^4$ **b** $8^9 \times 8$ **c** $3^4 \times 3^4$ **d** $6^5 \times 6^9$
e $5^4 \div 5$ **f** $10^6 \div 10^5$ **g** $9^9 \div 9^6$ **h** $(-2)^5 \div (-2)^3$

Example 6

3 Simplify each of the following using the first index law.

a $x^4 \times x^3$ **b** $a^6 \times a^3$ **c** $t^5 \times t^3$ **d** $y \times y^4$
e $d^2 \times d$ **f** $y^2 \times y \times y^4$ **g** $b \times b^5 \times b^2$ **h** $q^6 \times q^3 \times q^2$
i $x^3y^3 \times x^4y^2$ **j** $x^7y^3 \times x^2y$ **k** $5x^3y^5 \times xy^4$ **l** $xy^4z \times 4xy$
m $3m^3 \times 5m^2$ **n** $4e^4f^2 \times 2e^2f^2$ **o** $5c^4d \times 4c^3d$ **p** $9yz^2 \times 2yz^5$

ISBN 978-1-108-77288-4

Updated September 2021

Example 7

4 Simplify each of the following using the second index law.

a $a^6 \div a^4$
b $x^5 \div x^2$
c $\dfrac{q^{12}}{q^2}$
d $\dfrac{d^7}{d^6}$
e $\dfrac{8b^{10}}{4b^5}$
f $\dfrac{12d^{10}}{36d^5}$
g $\dfrac{4a^{14}}{2a^7}$
h $\dfrac{18y^{15}}{9y^7}$
i $9m^3 \div m^2$
j $14x^4 \div x$
k $5y^4 \div y^2$
l $6a^6 \div a^5$
m $\dfrac{3m^7}{12m^2}$
n $\dfrac{5w^2}{25w}$
o $\dfrac{4a^4}{20a^3}$
p $\dfrac{7x^5}{63x}$
q $\dfrac{16x^8y^6}{12x^2y^3}$
r $\dfrac{6s^6t^3}{14s^5t}$
s $\dfrac{8m^5n^4}{6m^4n^3}$
t $-\dfrac{5x^2y}{xy}$

Example 8

5 Simplify each of the following using the first two index laws.

a $b^5 \times b^2 \div b$
b $y^5 \times y^4 \div y^3$
c $c^4 \div c \times c^4$
d $x^4 \times x^2 \div x^5$
e $\dfrac{t^4 \times t^3}{t^6}$
f $\dfrac{p^2 \times p^7}{p^3}$
g $\dfrac{d^5 \times d^3}{d^2}$
h $\dfrac{x^9 \times x^2}{x}$
i $\dfrac{3x^3y^4 \times 8xy}{6x^2y^2}$
j $\dfrac{9b^4}{2g^3} \times \dfrac{4g^4}{3b^2}$
k $\dfrac{24m^7n^5}{5m^3n} \times \dfrac{5m^2n^4}{8mn^2}$
l $\dfrac{p^4q^3}{p^2q} \times \dfrac{p^6q^4}{p^3q^2}$

6 Simplify each of the following.

a $\dfrac{m^4}{n^2} \times \dfrac{m}{n^3}$
b $\dfrac{x}{y} \times \dfrac{x^3}{y}$
c $\dfrac{a^4}{b^3} \times \dfrac{b^6}{a}$
d $\dfrac{12a}{3c^3} \times \dfrac{6a^4}{4c^4}$
e $\dfrac{3f^2 \times 8f^7}{4f^3}$
f $\dfrac{4x^2b \times 9x^3b^2}{3xb}$
g $\dfrac{8k^4m^5}{5km^3} \times \dfrac{15km}{4k}$
h $\dfrac{12x^7y^3}{5x^4y} \times \dfrac{25x^2y^3}{8xy^4}$
i $\dfrac{9m^5n^2 \times 4mn^3}{12mn \times m^4n^2} \times \dfrac{m^3n^2}{2m^2n}$

PROBLEM-SOLVING

7–8($\frac{1}{2}$)	7–8($\frac{1}{2}$)	8($\frac{1}{2}$), 9

7 Write the missing number.

a $2^7 \times 2^{\square} = 2^{19}$
b $6^{\square} \times 6^3 = 6^{11}$
c $11^6 \div 11^{\square} = 11^3$
d $19^{\square} \div 19^2 = 19$
e $x^6 \times x^{\square} = x^7$
f $a^{\square} \times a^2 = a^{20}$
g $b^{13} \div b^{\square} = b$
h $y^{\square} \div y^9 = y^2$
i $\square \times x^2 \times 3x^4 = 12x^6$
j $15y^4 \div (\square y^3) = y$
k $\square a^9 \div (4a) = \dfrac{a^8}{2}$
l $13b^6 \div (\square b^5) = \dfrac{b}{3}$

8 Evaluate without using a calculator.

a $7^7 \div 7^5$
b $10^6 \div 10^5$
c $13^{11} \div 13^9$
d $2^{20} \div 2^{17}$
e $101^5 \div 101^4$
f $200^{30} \div 200^{28}$
g $7 \times 31^{16} \div 31^{15}$
h $3 \times 50^{200} \div 50^{198}$

9 If m and n are positive integers, how many combinations of m and n satisfy the following?

a $a^m \times a^n = a^8$

b $a^m \times a^n = a^{15}$

REASONING	10	10, 11(½)	11–12(½)

10 The given answers are incorrect. Give the correct answer and explain the error made.

a $a^4 \times a = a^4$ **b** $x^7 \div x = x^7$ **c** $3a^5 \div 6a^3 = 2a^2$

d $5x^7 \div 10x^3 = \dfrac{1}{2x^4}$ **e** $2x^7 \times 3x^4 = 5x^{11}$ **f** $a^5 \div a^2 \times a = a^5 \div a^3 = a^2$

11 Given that $a = 2x$, $b = 4x^2$ and $c = 5x^3$, find expressions for:

a $2a$ **b** $3b$ **c** $2c$ **d** $-2a$

e abc **f** $\dfrac{c}{b}$ **g** $\dfrac{ab}{c}$ **h** $\dfrac{-2bc}{a}$

12 Simplify these expressions using the given pronumerals.

a $2^x \times 2^y$ **b** $5^a \times 5^b$ **c** $t^x \times t^y$

d $3^x \div 3^y$ **e** $10^p \div 10^y$ **f** $2^p \times 2^q \div 2^r$

g $10^p \div 10^q \div 10^r$ **h** $2^a \times 2^{a+b} \times 2^{3a-b}$ **i** $a^{x-2}b^x \times a^{2x}b^3$

j $a^xb^y \times a^yb^x$ **k** $a^xb^y \div (a^yb^x)$ **l** $w^{x+2}b^x \div w^{2x} \times b^3$

m $\dfrac{a^x \times 3a^y}{3a^2}$ **n** $\dfrac{4p^a \times 5q^b}{20q^5}$ **o** $\dfrac{10k^xm^y}{8km^3} \div \dfrac{5k^xm^{2x}}{16k}$

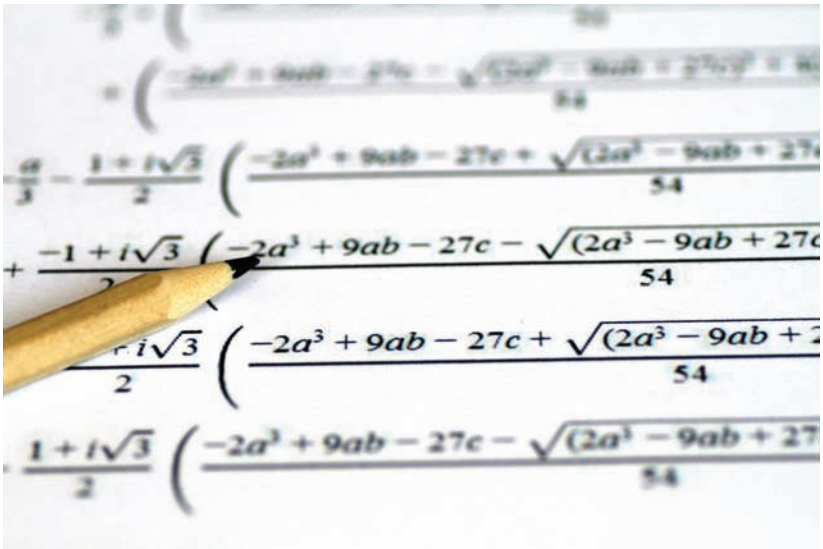
ENRICHMENT: Equal to *ab*	–	–	13, 14

13 Show working to prove that these expressions simplify to ab.

a $\dfrac{5a^2b^7}{9a^3b} \times \dfrac{9a^4b^2}{5a^2b^7}$ **b** $\dfrac{3a^5bc^3}{6a^4c} \times \dfrac{4b^3}{2abc} \times \dfrac{2a^3b^2c}{2a^2b^4c^2}$

c $\dfrac{3a^4b^5}{a^5b^2} \div \dfrac{6b^3}{2a^2b}$ **d** $\dfrac{2a}{3a^2b^3} \times \dfrac{9a^4b^7}{ab^5} \div \dfrac{6a}{b^2}$

14 Make up your own expressions which simplify to ab. Test them on a friend.



ISBN 978-1-108-77288-4

Updated September 2021

6C Index law 3 and the zero power

Learning intentions

- To know how to raise a term in index form to a power
- To know the rule for the zero power
- To be able to simplify expressions involving a power of a power and the zero power

Sometimes we find expressions already written in index form are raised to another power, such as $(2^3)^4$ or $(a^2)^5$

Consider $(a^m)^n$.

$$\text{Using expanded form } (a^m)^n = \overbrace{a^m \times a^m \times \ldots \times a^m}^{n \text{ factors of } a^m}$$

$$= \underbrace{\overbrace{a \times a \times \ldots \times a}^{m \text{ factors of } a} \times \overbrace{a \times a \times \ldots \times a}^{m \text{ factors of } a} \times \ldots \times \overbrace{a \times a \times \ldots \times a}^{m \text{ factors of } a}}_{m \times n \text{ factors of } a}$$

$$= a^{m \times n}$$

So the total number of factors of a is $m \times n$.

$$\text{We also know that } a^m \div a^m = \frac{\cancel{a} \times \cancel{a} \times \cancel{a} \times \cdots \times \cancel{a}}{\cancel{a} \times \cancel{a} \times \cancel{a} \times \cdots \times \cancel{a}}$$

$$= \frac{1}{1}$$

$$= 1$$

$$\text{But using index law number 2:} \quad a^m \div a^m = a^{m-m}$$

$$= a^0$$

This implies that $a^0 = 1$.

LESSON STARTER Discovering law 3 and the zero power

Use the expanded form of 5^3 to simplify $(5^3)^2$ as shown.

$$(5^3)^2 = (5 \times \square \times \square) \times (5 \times \square \times \square)$$

$$= 5^{\square}$$

- Repeat the above steps to simplify $(3^2)^4$ and $(x^4)^2$.
- What do you notice about the given expression and answer in each case? Can you express this as a law or rule in words?

  Now complete this table.

Index form	3^5	3^4	3^3	3^2	3^1	3^0
Basic numeral	243	81				

- What pattern do you notice in the basic numerals?
- What conclusion do you come to regarding 3^0?

KEY IDEAS

- **Index law 3**: $(a^m)^n = a^{mn}$
  - When raising a term in index form to another power, retain the base and multiply the indices. For example: $(x^2)^3 = x^{2\times 3} = x^6$.
- **The zero power**: $a^0 = 1$, where $a \neq 0$
  - Any expression except 0 raised to the power of zero is 1. For example: $(2a)^0 = 1$.

BUILDING UNDERSTANDING

① State the missing words or numbers in these sentences.

a When raising a term or numbers in index form to another power, ____________ the indices.

b Any expression (except 0) raised to the power 0 is equal to ____________.

② State the missing numbers in these tables.

a

Index form	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Basic numeral	64	32					

b

Index form	4^5	4^4	4^3	4^2	4^1	4^0
Basic numeral	1024	256				

③ State the missing numbers in this working.

a $(4^2)^3 = (4 \times \square) \times (4 \times \square) \times (4 \times \square)$
$\quad = 4^{\square}$

b $(a^2)^5 = (a \times \square) \times (a \times \square) \times (a \times \square) \times (a \times \square) \times (a \times \square)$
$\quad = a^{\square}$

Example 9 Using index law 3

Apply index law 3 to simplify each of the following.

a $(x^5)^4$

b $3(y^5)^2$

SOLUTION	EXPLANATION
a $(x^5)^4 = x^{5\times 4}$ $= x^{20}$	Retain x as the base and multiply the indices.
b $3(y^5)^2 = 3y^{5\times 2}$ $= 3y^{10}$	Retain y and multiply the indices. The power only applies to the bracketed term.

Now you try

Apply index law 3 to simplify each of the following.

a $(a^3)^2$

b $5(b^2)^7$

Example 10 Using the zero power

Apply the zero power rule to evaluate each of the following.

a $(-3)^0$ **b** $-(5x)^0$ **c** $2y^0 - (3y)^0$

SOLUTION	EXPLANATION
a $(-3)^0 = 1$	Any number raised to the power of 0 is 1.
b $-(5x)^0 = -1$	Everything in the brackets is to the power of 0 so $(5x)^0$ is 1.
c $2y^0 - (3y)^0 = 2 \times 1 - 1$ $= 2 - 1$ $= 1$	$2y^0$ has no brackets so the power applies to the y only, so $2y^0 = 2 \times y^0 = 2 \times 1$ while $(3y)^0 = 1$.

Now you try

Apply the zero power rule to evaluate each of the following.

a $(-2)^0$ **b** $-(4a)^0$ **c** $6x^0 - (2x)^0$

Example 11 Combining index laws

Simplify each of the following by applying the various index laws.

a $(x^2)^3 \times (x^3)^5$ **b** $\dfrac{(m^3)^4}{m^7}$ **c** $\dfrac{4x^2 \times 3x^3}{6x^5}$

SOLUTION	EXPLANATION
a $(x^2)^3 \times (x^3)^5 = x^6 \times x^{15}$ $= x^{21}$	Use index law 3 to remove brackets first by multiplying indices. Then use index law 1 to add indices.
b $\dfrac{(m^3)^4}{m^7} = \dfrac{m^{12}}{m^7}$ $= m^5$	Remove brackets by multiplying indices and then simplify using index law 2.
c $\dfrac{4x^2 \times 3x^3}{6x^5} = \dfrac{12x^5}{6x^5}$ $= 2x^0$ $= 2 \times 1$ $= 2$	Simplify the numerator first by multiplying numbers and adding indices of base x. Then cancel and subtract indices. The zero power says $x^0 = 1$.

Now you try

Simplify each of the following by applying the various index laws.

a $(y^3)^4 \times (y^2)^3$

b $\dfrac{(t^2)^4}{t^3}$

c $\dfrac{5m^2 \times 2m^4}{10m^5}$

Exercise 6C

FLUENCY	1, 2–5(½)	2–6(½)	2–6(⅓)

1 Apply index law 3 to simplify each of the following. Leave your answers in index form.

Example 9a

a **i** $(y^6)^2$ **ii** $(m^3)^6$

Example 9b

b **i** $5(m^8)^2$ **ii** $2(j^4)^6$

Example 9

2 Apply index law 3 to simplify each of the following. Leave your answers in index form.

a $(x^2)^5$

b $(b^3)^4$

c $(3^2)^3$

d $(4^3)^5$

e $(3^5)^6$

f $(7^5)^2$

g $4(q^7)^4$

h $-3(c^2)^5$

Example 10

3 Evaluate each of the following.

a 5^0

b 9^0

c $(-6)^0$

d $(-3)^0$

e $-(4^0)$

f $\left(\dfrac{3}{4}\right)^0$

g $\left(-\dfrac{1}{7}\right)^0$

h $(4y)^0$

i $5m^0$

j $-3p^0$

k $6x^0 - 2x^0$

l $-5n^0 - (8n)^0$

m $(3x^4)^0$

n $1^0 + 2^0 + 3^0$

o $(1 + 2 + 3)^0$

p $100^0 - a^0$

Example 11a

4 Simplify each of the following by combining various index laws.

a $4 \times (4^3)^2$

b $(3^4)^2 \times 3$

c $x \times (x^0)^5$

d $y^5 \times (y^2)^4$

e $b^5 \times (b^3)^3$

f $(a^2)^3 \times a^4$

g $(d^3)^4 \times (d^2)^6$

h $(y^2)^6 \times (y)^4$

i $z^4 \times (z^3)^2 \times (z^5)^3$

j $a^3f \times (a^4)^2 \times (f^4)^3$

k $x^2y \times (x^3)^4 \times (y^2)^2$

l $(s^2)^3 \times 5(r^0)^3 \times rs^2$

Example 11b

5 Simplify each of the following.

a $7^8 \div (7^3)^2$

b $(4^2)^3 \div 4^5$

c $(3^6)^3 \div (3^5)^2$

d $(m^3)^6 \div (m^2)^9$

e $(y^5)^3 \div (y^6)^2$

f $(h^{11})^2 \div (h^5)^4$

g $\dfrac{(b^2)^5}{b^4}$

h $\dfrac{(x^4)^3}{x^7}$

i $\dfrac{(y^3)^3}{y^3}$

Example 11c

6 Simplify each of the following using various index laws.

a $\dfrac{3x^4 \times 6x^3}{9x^{12}}$

b $\dfrac{5x^5 \times 4x^2}{2x^{10}}$

c $\dfrac{24(x^4)^4}{8(x^4)^2}$

d $\dfrac{4(d^4)^3 \times (e^4)^2}{8(d^2)^5 \times e^7}$

e $\dfrac{6(m^3)^2(n^5)^3}{15(m^5)^0(n^2)^7}$

f $\dfrac{2(a^3)^4(b^2)^6}{16(a)^0(b^6)^2}$

PROBLEM-SOLVING			
	7	7, 8	8, 9

7 There are 100 rabbits on Mt Burrow at the start of the year 2000. The rule for the number of rabbits N after t years (from the start of the year 2000) is $N = 100 \times 2^t$.

a Find the number of rabbits when:
  i $t = 2$ **ii** $t = 6$ **iii** $t = 0$.

b Find the number of rabbits at the beginning of:
  i 2003 **ii** 2007 **iii** 2010.

c How many years will it take for the population to first rise to more than 500 000? Give a whole number of years.

8 If m and n are positive integers, in how many ways can $(a^m)^n = a^{16}$?

9 Evaluate these without using a calculator.

a $(2^4)^8 \div 2^{30}$ **b** $(10^3)^7 \div 10^{18}$ **c** $(x^4)^9 \div x^{36}$

d $((-1)^{11})^2 \times ((-1)^2)^{11}$ **e** $-2((-2)^3)^3 \div (-2)^8$ **f** $\dfrac{(a^2)^3}{(b^4)^7} \times \dfrac{(b^7)^4}{(a^3)^2}$

REASONING			
	10	10, 11	11, 12

10 Explain the error made in the following problems and then give the correct answer.

a $(a^4)^5 = a^9$ **b** $3(x^3)^2 = 9x^6$ **c** $(2x)^0 = 2$

11 **a** Simplify these by first working with the inner brackets. Leave your answer in index form.
  i $((2^3)^4)^2$ **ii** $(((-2)^2)^5)^3$ **iii** $((x^6)^2)^7$ **iv** $(((a^2)^4)^3)^2$

b Simplify these expressions.
  i $((2^a)^b)^c$ **ii** $((a^m)^n)^p$ **iii** $(x^{2y})^{3z}$

12 **a** Show that $\dfrac{5a^2b}{2ab^2} \div \dfrac{10a^4b^7}{4a^3b^8}$ is equal to 1.

b Make up your own expression like the one above for which the answer is equal to 1. Test it on a friend.

ENRICHMENT: Changing the base			
	–	–	13(½)

13 The base of a number in index form can be changed using index law number 3.

For example: $8^2 = (2^3)^2$
$= 2^6$

Change the base numbers and simplify the following using the smallest possible base integer.

a 8^4 **b** 32^3 **c** 9^3

d 81^5 **e** 25^5 **f** 243^{10}

g 256^9 **h** 2401^{20} **i** $100\,000^{10}$

ISBN 978-1-108-77288-4

6D Index laws 4 and 5

Learning intentions

- To understand that all numbers or pronumerals multiplied or divided in brackets are raised to the power outside the brackets
- To be able to expand brackets involving powers
- To be able to combine all index laws to simplify expressions

It is common to find expressions such as $(2x)^3$ and $\left(\frac{x}{3}\right)^4$ in mathematical problems. These differ from most of the expressions in previous sections in that they contain two or more numbers or pronumerals connected by multiplication or division and raised to a power. These expressions can also be simplified using two index laws, which effectively remove the brackets.

Consider $(a \times b)^m$.

Using expanded form:

$$
\begin{aligned}
(a \times b)^m &= \overbrace{ab \times ab \times ab \times \cdots \times ab}^{m \text{ factors of } ab} \\
&= \overbrace{a \times a \times \cdots \times a}^{m \text{ factors of } a} \times \overbrace{b \times b \times \cdots \times b}^{m \text{ factors of } b} \\
&= a^m \times b^m
\end{aligned}
$$

So, this becomes a product of m factors of a and m factors of b.

Also:

$$
\begin{aligned}
\left(\frac{a}{b}\right)^m &= \overbrace{\frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \times \cdots \times \frac{a}{b}}^{m \text{ factors of } \frac{a}{b}} \\
&= \frac{\overbrace{a \times a \times a \times \cdots \times a}^{m \text{ factors of } a}}{\underbrace{b \times b \times b \times \cdots \times b}_{m \text{ factors of } b}} \\
&= \frac{a^m}{b^m}
\end{aligned}
$$

So to remove the brackets, we can raise each of a and b to the power m.

LESSON STARTER Discovering laws 4 and 5

Use the expanded form of $(2x)^3$ and $\left(\frac{x}{3}\right)^4$ to help simplify the expressions.

$$
\begin{aligned}
(2x)^3 &= 2x \times \square \times \square \\
&= 2 \times 2 \times 2 \times \square \times \square \times \square \\
&= 2^{\square} \times \square^{\square}
\end{aligned}
$$

$$
\begin{aligned}
\left(\frac{x}{3}\right)^4 &= \frac{x}{3} \times \square \times \square \times \square \\
&= \frac{x \times \square \times \square \times \square}{3 \times \square \times \square \times \square} \\
&= \frac{\square^{\square}}{\square^{\square}}
\end{aligned}
$$

- Repeat these steps to also simplify the expressions $(3y)^4$ and $\left(\frac{x}{2}\right)^5$.
- What do you notice about the given expressions and the answer in each case? Can you express this as a rule or law in words?

KEY IDEAS

- **Index law 4**: $(a \times b)^m = (ab)^m = a^m b^m$
  - When multiplying two or more numbers raised to the power of m, raise each number in the brackets to the power of m. For example: $(2x)^2 = 2^2x^2 = 4x^2$.
- **Index law 5**: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ and $b \neq 0$
  - When dividing two numbers raised to the power of m, raise each number in the brackets to the power of m. For example: $\left(\frac{y}{3}\right)^3 = \frac{y^3}{3^3} = \frac{y^3}{27}$.

BUILDING UNDERSTANDING

1 State the missing terms to complete index laws 4 and 5.

a $(a \times b)^m = a^m \times \square$

b $\left(\frac{a}{b}\right)^m = \frac{a^m}{\square}$

2 State the missing terms to complete this working.

a
$$\begin{aligned}(5a)^3 &= 5a \times \square \times \square\\ &= 5 \times 5 \times 5 \times a \times \square \times \square\\ &= 5^3 \times \square\end{aligned}$$

b
$$\begin{aligned}\left(\frac{x}{6}\right)^3 &= \frac{x}{6} \times \square \times \square\\ &= \frac{x \times \square \times \square}{6 \times \square \times \square}\\ &= \frac{x^3}{\square}\end{aligned}$$

Example 12 Using index law 4

Expand each of the following using the fourth index law.

a $(5b)^3$

b $(-2x^3y)^4$

c $4(c^2d^3)^5$

SOLUTION	EXPLANATION
a $(5b)^3 = 5^3b^3$ $= 125b^3$	Raise each numeral and pronumeral in the brackets to the power of 3. Evaluate $5^3 = 5 \times 5 \times 5$.
b $(-2x^3y)^4 = (-2)^4(x^3)^4y^4$ $= 16x^{12}y^4$	Raise each value in the brackets to the power of 4. Evaluate $(-2)^4$ and simplify using law 3.
c $4(c^2d^3)^5 = 4(c^2)^5(d^3)^5$ $= 4c^{10}d^{15}$	Raise each value in the brackets to the power of 5. Note that the coefficient (4) is not raised to the power of 5. Simplify using index laws.

Now you try

Expand each of the following using the fourth index law.

a $(3a)^2$

b $(-3x^2y^3)^3$

c $5(a^2b)^4$

Example 13 Using index law 5

Apply the fifth index law to the following.

a $\left(\frac{6}{b}\right)^3$ **b** $\left(\frac{-2a^2}{3bc^3}\right)^4$ **c** $\left(\frac{x^2y^3}{c}\right)^3 \times \left(\frac{xc}{y}\right)^4$

SOLUTION

EXPLANATION

a $$\left(\frac{6}{b}\right)^3 = \frac{6^3}{b^3} = \frac{216}{b^3}$$

Raise each value in the brackets to the power of 3 and evaluate 6^3.

b $$\left(\frac{-2a^2}{3bc^3}\right)^4 = \frac{(-2)^4a^8}{3^4b^4c^{12}} = \frac{16a^8}{81b^4c^{12}}$$

Raise each value in the brackets to the power of 4.

Recall $(a^2)^4 = a^{2\times4}$ and $(c^3)^4 = c^{3\times4}$
Evaluate $(-2)^4$ and 3^4.

c $$\left(\frac{x^2y^3}{c}\right)^3 \times \left(\frac{xc}{y}\right)^4 = \frac{x^6y^9}{c^3} \times \frac{x^4c^4}{y^4} = \frac{x^{10}y^9c^4}{c^3y^4} = x^{10}y^5c$$

Raise each value in the brackets to the power.

Multiply the numerators using law 1 and then divide using law 2.

Now you try

Apply the fifth index law to the following.

a $\left(\frac{2}{x}\right)^5$ **b** $\left(\frac{-3m^3}{2np^2}\right)^2$ **c** $\left(\frac{xy^2}{2}\right)^3 \times \left(\frac{x}{3y}\right)^2$

Exercise 6D

FLUENCY	1, 2–4(1/2)	2–5(1/2)	2–5(1/3)

Example 12

1 Expand each of the following using the fourth index law.

a $(2x)^3$ **b** $(-3w^3y)^3$ **c** $2(x^3y)^2$

Example 12

2 Expand each of the following using the fourth index law.

a $(5y)^2$ **b** $(4a^2)^3$ **c** $(-3r)^2$
d $-(3b)^4$ **e** $-(7r)^3$ **f** $(-2h^2)^4$
g $(5c^2d^3)^4$ **h** $(2x^3y^2)^5$ **i** $9(p^2q^4)^3$
j $(8t^2u^9v^4)^0$ **k** $-4(p^4qr)^2$ **l** $(-5s^7t)^2$

Example 13a,b

3 Apply the fifth index law to expand the following.

a $\left(\frac{p}{q}\right)^3$ **b** $\left(\frac{x}{y}\right)^4$ **c** $\left(\frac{4}{y}\right)^3$ **d** $\left(\frac{2}{r^3}\right)^2$

e $\left(\frac{s^3}{7}\right)^2$ **f** $\left(\frac{2m}{n}\right)^5$ **g** $\left(\frac{3n^3}{2m^4}\right)^3$ **h** $\left(\frac{-2r}{n}\right)^4$

i $\left(\frac{-3f}{2^3g^5}\right)^2$ **j** $\left(\frac{-3x}{2y^3g^5}\right)^2$ **k** $\left(\frac{3km^3}{4n^7}\right)^3$ **l** $-\left(\frac{-5w^4y}{2x^3}\right)^2$

4 Simplify each of the following by applying the various index laws.

a $a(3b)^2$ **b** $a(3b^2)^3$ **c** $-3(2a^3b^4)^2a^2$

d $2(3x^2y^3)^3$ **e** $(-4b^2c^5d)^3$ **f** $a(2a)^3$

g $a(3a^2)^2$ **h** $5a^3(-2a^4b)^3$ **i** $-5(-2m^3pt^2)^5$

j $-(-7d^2f^4g)^2$ **k** $-2(-2^3x^4yz^3)^3$ **l** $-4a^2b^3(-2a^3b^2)^2$

Example 13c

5 Simplify each of the following.

a $((x^2)^3)^4$ **b** $((2x^3)^2)^4$ **c** $(a^3b^2)^3 \times (a^4b)^2$

d $(a^2b)^3 \times (ab^2)^4$ **e** $\frac{(2m^3n)^3}{m^4}$ **f** $\frac{3(2^2c^4d^5)^3}{(2cd^2)^4}$

g $\left(\frac{-3x^2y^0}{5a^5b^3}\right)^3$ **h** $\frac{-3(2^4a^4b^3)^3}{(-2^3a^2b)^4}$ **i** $\frac{-5(3^5m^3n^2)^2}{(-3^3m^2n)^3}$

j $\left(\frac{a^3b}{c}\right)^3 \times \left(\frac{ac^4}{b}\right)^2$ **k** $\left(\frac{x^2z}{y}\right)^4 \times \left(\frac{xy^2}{z}\right)^3$ **l** $\left(\frac{r^3s}{t}\right)^2 \div \left(\frac{s}{rt^4}\right)^3$

PROBLEM-SOLVING	6	6, 7(1/2)	6, 7

6 The rule for the number of seeds germinating in a glasshouse over a two-week period is given by $N = \left(\frac{t}{2}\right)^3$ where N is the number of germinating seeds and t is the number of days.

a Find the number of germinating seeds after:
  i 4 days **ii** 10 days.

b Use index law 5 to rewrite the rule without brackets.

c Use your rule in part **b** to find the number of seeds germinating after:
  i 6 days **ii** 4 days.

d Find the number of days required to germinate:
  i 64 seeds **ii** 1 seed.

7 Find the value of a that makes these equations true, given $a > 0$.

a $\left(\frac{a}{3}\right)^2 = \frac{4}{9}$ **b** $\left(\frac{a}{2}\right)^4 = 16$ **c** $(5a)^3 = 1000$

d $(2a)^4 = 256$ **e** $\left(\frac{2a}{3}\right)^2 = \frac{4}{9}$ **f** $\left(\frac{6a}{7}\right)^3 = 1728$

REASONING	8	8, 9(½)	8, 9

8 Rather than evaluating $\frac{2^4}{4^4}$ as $\frac{16}{256} = \frac{1}{16}$, it is easier to evaluate $\frac{2^4}{4^4}$ in the way shown below.

$$\frac{2^4}{4^4} = \left(\frac{2}{4}\right)^4 = \left(\frac{1}{2}\right)^4 = \frac{1^4}{2^4} = \frac{1}{16}$$

- **a** Explain why this method is helpful.
- **b** Use this idea to evaluate these without the use of a calculator.
  - **i** $\frac{6^3}{3^3}$
  - **ii** $\frac{10^4}{5^4}$
  - **iii** $\frac{4^4}{12^4}$
  - **iv** $\frac{3^3}{30^3}$

9 Decide if the following are true or false. Give reasons.

- **a** $(-2x)^2 = -(2x)^2$
- **b** $(-3x)^3 = -(3x)^3$
- **c** $\left(\frac{-5}{x}\right)^5 = -\left(\frac{5}{x}\right)^5$
- **d** $\left(\frac{-4}{x}\right)^4 = -\left(\frac{4}{x}\right)^4$

ENRICHMENT: False laws	–	–	10

10 Consider the equation $(a + b)^2 = a^2 + b^2$.

- **a** Using $a = 2$ and $b = 3$, evaluate $(a + b)^2$.
- **b** Using $a = 2$ and $b = 3$, evaluate $a^2 + b^2$.
- **c** Would you say that the equation is true for all values of a and b?
- **d** Now decide whether $(a - b)^2 = a^2 - b^2$ for all values of a and b. Give an example to support your answer.
- **e** Decide if these equations are true or false for all values of a and b.
  - **i** $(-ab)^2 = a^2b^2$
  - **ii** $-(ab)^2 = a^2b^2$
  - **iii** $\left(\frac{-a}{b}\right)^3 = \frac{-a^3}{b^3}$ $(b \neq 0)$
  - **iv** $\left(\frac{-a}{b}\right)^4 = \frac{-a^4}{b^4}$ $(b \neq 0)$

Index laws are routinely used in the finance industry, including by corporate and personal financial advisors and investment bankers. For example, $\$P$ invested at $r\%$ p.a. for n years increases to $\$A = P\left(1 + \frac{r}{100}\right)^n$.

6E Negative indices

Learning intentions

- To understand that negative indices relate to division
- To know how to express a term involving a negative index with a positive index
- To be able to write expressions involving negative indices with positive indices
- To be able to evaluate numbers raised to negative indices without a calculator

We know that $2^3 = 8$ and $2^0 = 1$, but what about 2^{-1} or 2^{-6}? Such numbers written in index form using negative indices also have meaning in mathematics.

Audio engineers who set up loud speaker systems for live performances know that as sound travels from its source, its intensity, I, is inversely proportional ($\propto$) to the distance, d, squared, i.e. $I \propto \frac{1}{d^2}$ or $I \propto d^{-2}$.

Consider $a^2 \div a^5$.

Method 1: Using law 2

$$\begin{aligned}\frac{a^2}{a^5} &= a^{2-5}\\ &= a^{-3} \text{ (from index law 2)}\end{aligned}$$

Method 2: By cancelling

$$\begin{aligned}\frac{a^2}{a^5} &= \frac{\cancel{a}^1 \times \cancel{a}^1}{a \times a \times a \times \cancel{a}_1 \times \cancel{a}_1}\\ &= \frac{1}{a^3}\end{aligned}$$

$$\therefore a^{-3} = \frac{1}{a^3}$$

Also, using index law 1 we can write:

$$\begin{aligned}a^m \times a^{-m} &= a^{m+(-m)}\\ &= a^0\\ &= 1\end{aligned}$$

So dividing by a^m we have $a^{-m} = \frac{1}{a^m}$ or dividing by a^{-m} we have $a^m = \frac{1}{a^{-m}}$.

LESSON STARTER Continuing the pattern

Explore the use of negative indices by completing this table.

Index form	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
Whole number or fraction	16	8					$\frac{1}{4} = \frac{1}{2^2}$	

÷2 ÷2 ÷2

- What do you notice about the numbers with negative indices in the top row in comparison to the fractions in the second row?
- Can you describe this connection formally in words?
- What might be another way of writing 2^{-7} or 5^{-4}?

KEY IDEAS

- $a^{-m} = \dfrac{1}{a^m}$ and $a^m = \dfrac{1}{a^{-m}}$

  a raised to the power $-m$ is equal to the reciprocal of a raised to the power m. ($a \neq 0$)

BUILDING UNDERSTANDING

① State the following using positive indices. For example: $\dfrac{1}{8} = \dfrac{1}{2^3}$.

a $\dfrac{1}{4}$ **b** $\dfrac{1}{9}$ **c** $\dfrac{1}{125}$ **d** $\dfrac{1}{27}$

② State the missing numbers in these patterns.

a

Index form	3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}
Whole number or fraction	81	27					$\frac{1}{9} = \frac{1}{3^2}$	

÷3 ÷3 ÷3

b

Index form	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
Whole number or fraction	10 000							$\frac{1}{1000} = \frac{1}{10^3}$

÷10 ÷10

Example 14 Writing expressions using positive indices

Express each of the following with positive indices only.

a x^{-2} **b** $3a^{-2}b^4$

SOLUTION	EXPLANATION
a $x^{-2} = \dfrac{1}{x^2}$	$a^{-m} = \dfrac{1}{a^m}$
b $3a^{-2}b^4 = \dfrac{3}{1} \times \dfrac{1}{a^2} \times \dfrac{b^4}{1}$ $= \dfrac{3b^4}{a^2}$	Rewrite a^{-2} using a positive power and combine numerators and denominators.

Now you try

Express each of the following with positive indices only.

a a^{-3} **b** $2x^{-4}y^3$

Example 15 Using $\frac{1}{a^{-m}} = a^m$

Express each of the following using positive indices only.

a $\frac{1}{c^{-2}}$ **b** $\frac{x^{-3}}{y^{-5}}$ **c** $\frac{5}{x^3y^{-4}}$

SOLUTION	EXPLANATION
a $\frac{1}{c^{-2}} = c^2$	$\frac{1}{a^{-m}} = a^m$
b $\frac{x^{-3}}{y^{-5}} = x^{-3} \times \frac{1}{y^{-5}}$ $= \frac{1}{x^3} \times \frac{y^5}{1}$ $= \frac{y^5}{x^3}$	Express x^{-3} and $\frac{1}{y^{-5}}$ with positive indices using $a^{-m} = \frac{1}{a^m}$ and $\frac{1}{a^{-m}} = a^m$.
c $\frac{5}{x^3y^{-4}} = \frac{5}{x^3} \times \frac{1}{y^{-4}}$ $= \frac{5y^4}{x^3}$	Express $\frac{1}{y^{-4}}$ as a positive power, y^4.

Now you try

Express each of the following using positive indices only.

a $\frac{1}{a^{-3}}$ **b** $\frac{x^{-2}}{y^{-4}}$ **c** $\frac{6}{a^2b^{-5}}$

Example 16 Evaluating without a calculator

Express using positive powers only, and then evaluate without using a calculator.

a 3^{-4} **b** $\frac{-5}{3^{-2}}$ **c** $\left(\frac{2}{3}\right)^{-4}$

SOLUTION	EXPLANATION
a $3^{-4} = \frac{1}{3^4}$ $= \frac{1}{81}$	Express 3^{-4} as a positive power and evaluate 3^4.

b $\dfrac{-5}{3^{-2}} = -5 \times \dfrac{1}{3^{-2}}$ $= -5 \times 3^2$ $= -5 \times 9$ $= -45$	Express $\dfrac{1}{3^{-2}}$ as a positive power and simplify.
c $\left(\dfrac{2}{3}\right)^{-4} = \dfrac{2^{-4}}{3^{-4}}$	Apply the power to each numeral in the brackets using index law 5.
$= 2^{-4} \times \dfrac{1}{3^{-4}}$ $= \dfrac{1}{2^4} \times 3^4$ $= \dfrac{3^4}{2^4}$ $= \dfrac{81}{16}$	Express 2^{-4} and $\dfrac{1}{3^{-4}}$ with positive indices and evaluate.

Now you try

Express using positive powers only, and then evaluate without using a calculator.

a 2^{-5} **b** $\dfrac{-4}{2^{-3}}$ **c** $\left(\dfrac{5}{7}\right)^{-2}$

Exercise 6E

FLUENCY	1, 2–4(½)	2–5(⅓)	2–5(¼)

1 Express each of the following with positive indices only.

Example 14a

a **i** x^{-5}
  ii a^{-7}

Example 14b

b **i** $2a^{-3}b^2$
  ii $-7x^4y^{-5}$

Example 14

2 Express each of the following with positive indices only.

a x^{-1} **b** a^{-4} **c** b^{-6} **d** 5^{-2}
e 4^{-3} **f** 9^{-1} **g** $5x^{-2}$ **h** $4y^{-3}$
i $3m^{-5}$ **j** p^7q^{-2} **k** mn^{-4} **l** x^4y^{-4}
m $2a^{-3}b^{-1}$ **n** $7r^{-2}s^{-3}$ **o** $5^{-1}u^{-8}v^2$ **p** $9^{-1}m^{-3}n^{-5}$

ISBN 978-1-108-77288-4

Updated September 2021

Example 15a

3 Express each of the following using positive indices only.

a $\frac{1}{y^{-1}}$ **b** $\frac{1}{b^{-2}}$ **c** $\frac{1}{m^{-5}}$ **d** $\frac{1}{x^{-4}}$

e $\frac{7}{q^{-1}}$ **f** $\frac{3}{t^{-2}}$ **g** $\frac{5}{h^{-4}}$ **h** $\frac{4}{p^{-4}}$

i $\frac{a}{b^{-2}}$ **j** $\frac{e}{d^{-1}}$ **k** $\frac{2n^2}{m^{-3}}$ **l** $\frac{y^5}{3x^{-2}}$

m $\frac{-3}{7y^{-4}}$ **n** $\frac{-2}{b^{-8}}$ **o** $\frac{-3g}{4h^{-3}}$ **p** $\frac{(-3u)^2}{5t^{-2}}$

Example 15b

4 Express each of the following using positive indices only.

a $\frac{a^{-3}}{b^{-3}}$ **b** $\frac{x^{-2}}{y^{-5}}$ **c** $\frac{g^{-2}}{h^{-3}}$ **d** $\frac{m^{-1}}{n^{-1}}$

e $\frac{5^{-1}}{7^{-3}}$ **f** $\frac{3^{-2}}{4^{-3}}$ **g** $\frac{5^{-2}}{6^{-1}}$ **h** $\frac{4^{-3}}{8^{-2}}$

Example 15c

5 Express each of the following using positive indices only.

a $\frac{7}{x^{-4}y^3}$ **b** $\frac{1}{u^{-3}v^2}$ **c** $\frac{a^{-3}5^{-1}}{y^{-3}}$ **d** $\frac{2a^{-4}}{b^{-5}c^2}$

e $\frac{5a^2c^{-4}}{6b^{-2}d}$ **f** $\frac{5^{-1}h^3k^{-2}}{4^{-1}m^{-2}p}$ **g** $\frac{4t^{-1}u^{-2}}{3^{-1}v^2w^{-6}}$ **h** $\frac{4^{-1}x^2y^{-5}}{4m^{-1}n^{-4}}$

PROBLEM-SOLVING	6(1/2)	6(1/2), 7	6(1/3), 7, 8

Example 16

6 Evaluate without the use of a calculator. (*Hint*: Write expressions using positive indices.)

a 5^{-1} **b** 3^{-2} **c** $(-4)^{-2}$

d -5^{-2} **e** 4×10^{-2} **f** -5×10^{-3}

g -3×2^{-2} **h** $8 \times (2^2)^{-2}$ **i** $6^4 \times 6^{-6}$

j $8^{-7} \times (8^2)^3$ **k** $(5^2)^{-1} \times (2^{-2})^{-1}$ **l** $(3^{-2})^2 \times (7^{-1})^{-1}$

m $\frac{1}{8^{-1}}$ **n** $\frac{1}{10^{-2}}$ **o** $\frac{-2}{5^{-3}}$

p $\frac{2}{2^{-3}}$ **q** $\frac{-5}{2^{-1}}$ **r** $\frac{2^3}{2^{-3}}$

s $\left(\frac{3}{8}\right)^{-2}$ **t** $\left(\frac{-4}{3}\right)^{-3}$ **u** $\frac{(-5)^2}{2^{-2}}$

v $\frac{(3^{-2})^3}{3^{-5}}$ **w** $\frac{(-2^{-3})^{-3}}{(2^{-2})^{-4}}$ **x** $\left(\frac{2^{-4}}{7^{-2}}\right)^{-1} \times \left(\frac{7^{-1}}{2^{-1}}\right)^{-4}$

7 The mass of a small insect is 2^{-9} kg. How many grams is this? Round to two decimal places.

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8 Find the value of x in these equations.

a $2^x = \frac{1}{16}$ **b** $5^x = \frac{1}{625}$ **c** $(-3)^x = \frac{1}{81}$

d $(0.5)^x = 2$ **e** $(0.2)^x = 25$ **f** $3(2^{2x}) = 0.75$

REASONING	9	9	9, 10

9 Describe the error made in these problems and then give the correct answer.

a $2x^{-2} = \frac{1}{2x^2}$ **b** $\frac{5}{a^4} = \frac{a^{-4}}{5}$ **c** $\frac{2}{(3b)^{-2}} = \frac{2b^2}{9}$

10 Consider the number $\left(\frac{2}{3}\right)^{-1}$.

a Complete this working: $\left(\frac{2}{3}\right)^{-1} = \frac{1}{\left(\frac{2}{3}\right)} = 1 \div \boxed{} = 1 \times \boxed{} = \frac{3}{2}$

b Show similar working as in part **a** to simplify these.

i $\left(\frac{5}{4}\right)^{-1}$ **ii** $\left(\frac{2}{7}\right)^{-1}$ **iii** $\left(\frac{x}{3}\right)^{-1}$ **iv** $\left(\frac{a}{b}\right)^{-1}$

c What conclusion can you come to regarding the simplification of fractions raised to the power -1?

d Simplify these fractions.

i $\left(\frac{2}{3}\right)^{-2}$ **ii** $\left(\frac{4}{5}\right)^{-2}$ **iii** $\left(\frac{1}{2}\right)^{-5}$ **iv** $\left(\frac{7}{3}\right)^{-3}$

ENRICHMENT: Exponential equations	–	–	11

11 To find x in $2^x = 32$ you could use trial and error; however, the following approach is more useful.

$2^x = 32$

$2^x = 2^5$ (express 32 using a matching base)

$\therefore x = 5$

Use this idea to solve for x in these equations.

a $2^x = 16$ **b** $3^x = 81$ **c** $\left(\frac{1}{2}\right)^x = \frac{1}{8}$ **d** $\left(\frac{1}{7}\right)^x = \frac{1}{49}$

e $\left(\frac{2}{3}\right)^x = \frac{16}{81}$ **f** $4^{2x} = 64$ **g** $3^{x+1} = 243$ **h** $2^{3x-1} = 64$

Progress quiz

6A **1** Write each of the following in expanded form and then evaluate where possible.

a a^4 **b** $5(hk)^3$

c 2^4 **d** $\left(\frac{-3}{4}\right)^3$

6A **2** Write each of the following in index form.

a $7 \times m \times m \times m \times m$

b $\frac{2}{3} \times \frac{2}{3} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}$

c $8h \times e \times 8h \times e \times 8h \times e \times e$

6B **3** Simplify each of the following using the first or second index laws.

a $2^5 \times 2^3$ **b** $a^3 \times a^5 \times a$ **c** $3k^3m \times 4mk^4$

d $5^{10} \div 5^2$ **e** $\frac{6a^8}{3a^2}$ **f** $\frac{8a^{10}m^6}{24a^5m^2}$

6B **4** Simplify each of the following using the first two index laws.

a $x^4 \times x^5 \div x^3$ **b** $\frac{4x^3y^2 \times 6x^2y}{8x^4y^2}$

6C **5** Apply index law 3 to simplify each of the following. Leave your answers in index form.

a $(x^3)^4$ **b** $-4(q^6)^7$

6C **6** Apply the zero power rule to evaluate each of the following.

a 8^0 **b** $(2x)^0$

c $7m^0$ **d** $-4n^0 - (5n)^0$

6C **7** Simplify each of the following by applying the various index laws.

a $(a^3)^2 \times (a^2)^4$ **b** $\frac{(m^4)^6}{m^2}$

c $\frac{3x^4 \times 12x^2}{9x^6}$ **d** $\frac{8(m^2)^4 \times 5(n^3)^2}{15(m^5)^0 \times 2n^2}$

6D **8** Expand each of the following using the fourth and fifth index laws and simplify.

a $(2b)^3$ **b** $5(h^2j^3k)^3$ **c** $(-3x^4y^2)^3$

d $\left(\frac{4}{c}\right)^3$ **e** $\left(\frac{-2wx^3}{5y^2}\right)^3$ **f** $\left(\frac{a^2b}{c^2}\right)^4 \times \left(\frac{c^5}{b^2}\right)^2$

6E **9** Express each of the following using positive indices only.

a m^{-4} **b** $7x^{-3}y^5$ **c** $\frac{1}{a^{-5}}$

d $\frac{a^{-2}}{c^{-3}}$ **e** $\frac{-13}{m^{-5}}$ **f** $\frac{-5t^{-2}u^3}{3^{-1}v^2w^{-3}}$

6E **10** Express using positive powers only, and then evaluate without using a calculator.

a 4^{-2} **b** $\frac{3}{3^{-2}}$

c $(-6)^{-2}$ **d** $\left(\frac{3}{2}\right)^{-3}$

6F Scientific notation

Learning intentions

- To understand that scientific notation is a shorthand way of writing very large and very small numbers
- To know the general form of scientific notation
- To understand when positive or negative powers of 10 are used in scientific notation
- To be able to convert between scientific notation and decimal form
- To know how to convert between large and small units of time

It is common in the practical world to be working with very large or very small numbers. For example, the number of cubic metres of concrete used to build the Hoover Dam in the United States was 3 400 000 m^3 and the mass of a molecule of water is 0.0000000000000000000000299 grams. Such numbers can be written more efficiently using powers of 10 with positive or negative indices. This is called scientific notation or standard form. The number is written using a number between 1 inclusive and 10 and this is multiplied by a power of 10. Such notation is also used to state very large and very small time intervals.

Nano-robotics is an area of experimental research for disease control, such as targeted cancer drug delivery. Nanobots are micrometres long, e.g. 3×10^{-6} m, with components nanometres long, e.g. 1×10^{-9} m. One nanometre is around $\left(\frac{1}{100\,000}\right)$ of a hair's diameter.

LESSON STARTER Building scientific notation

Use the information given to complete the table.

Decimal form	Working	Scientific notation
2 350 000	$2.35 \times 1\,000\,000$	2.35×10^6
502 170		
314 060 000		
0.000298	$2.98 \div 10\,000 = \frac{2.98}{10^4}$	2.98×10^{-4}
0.000004621		
0.003082		

- Discuss how each number using scientific notation is formed.
- When are positive indices used and when are negative indices used?
- Where does the decimal point appear to be placed when using scientific notation?

ISBN 978-1-108-77288-4

KEY IDEAS

- Numbers written in **scientific notation** are expressed in the form $a \times 10^m$ where $1 \leqslant a < 10$ or $-10 < a \leqslant -1$ and m is an integer.
- Large numbers will use positive powers of 10.

  For example: 38 million years $= 38\,000\,000$ years
  $= 3.8 \times 10^7$ years
- Small numbers will use negative powers of 10.

  For example: 417 nanoseconds $= 0.000000417$ seconds
  $= 4.17 \times 10^{-7}$ seconds
- To write numbers using scientific notation, place the decimal point after the first non-zero digit and then multiply by a power of 10.
- Examples of units for which very large or small numbers may be used:
  - 2178 km $= 2\,178\,000$ m $= 2.178 \times 10^6$ metres
  - 4517 centuries $= 451\,700$ years $= 4.517 \times 10^5$ years
  - 12 million years $= 12\,000\,000$ years or 1.2×10^7 years
  - 2320 tonnes $= 2\,320\,000$ kg or 2.32×10^6 kg
  - 27 microns (millionth of a metre) $= 0.000027$ m or 2.7×10^{-5} metres
  - 109 milliseconds (thousandths of a second) $= 0.109$ seconds or 1.09×10^{-1} seconds
  - 3.8 microseconds (millionth of a second) $= 0.0000038$ or 3.8×10^{-6} seconds
  - 54 nanoseconds (billionth of a second) $= 0.000000054$ or 5.4×10^{-8} seconds

BUILDING UNDERSTANDING

1 Which of the numbers 1000, 10 000 or 100 000 completes each equation?

a $6.2 \times \_\_\_\_\_ = 62\,000$
b $9.41 \times \_\_\_\_\_ = 9410$
c $1.03 \times \_\_\_\_\_ = 103\,000$
d $3.2 \div \_\_\_\_\_ = 0.0032$
e $5.16 \div \_\_\_\_\_ = 0.0000516$
f $1.09 \div \_\_\_\_\_ = 0.000109$

2 State the following as powers of 10.

a 100 000
b 100
c 1 000 000 000

3 If these numbers were written using scientific notation, would positive or negative indices be used?

a 2000
b 0.0004
c 19 300
d 0.00101431

Example 17 Writing numbers using scientific notation

Write the following using scientific notation.

a $4\,500\,000$ **b** 0.0000004

SOLUTION

a $4\,500\,000 = 4.5 \times 10^6$

EXPLANATION

Place the decimal point after the first non-zero digit (4) and then multiply by 10^6 since the decimal place has been moved 6 places to the left.

SOLUTION

b $0.0000004 = 4 \times 10^{-7}$

EXPLANATION

The first non-zero digit is 4. Multiply by 10^{-7} since the decimal place has been moved 7 places to the right.

Now you try

Write the following numbers using scientific notation.

a $720\,000$ **b** 0.000003

Example 18 Writing numbers in decimal form

Express each of the following in decimal form.

a 9.34×10^5 **b** 4.71×10^{-5}

SOLUTION

a $9.34 \times 10^5 = 934\,000$

EXPLANATION

Move the decimal point 5 places to the right.

SOLUTION

b $4.71 \times 10^{-5} = 0.0000471$

EXPLANATION

Move the decimal point 5 places to the left and insert zeros where necessary.

Now you try

Express each of the following in decimal form.

a 6.17×10^3 **b** 4.02×10^{-6}

Exercise 6F

FLUENCY	1, 2–6(1/2)	2–6(1/2)	2–6(1/3)

1 Write the following numbers using scientific notation.

Example 17a **a** **i** 43 000 **ii** 1 490 000 000

Example 17b **b** **i** 0.004 **ii** 0.0000061

Example 17a **2** Write the following using scientific notation.

a 40 000 **b** 2 300 000 000 000 **c** 16 000 000 000
d –7 200 000 **e** –3500 **f** –8 800 000
g 52 hundreds **h** 3 million **i** 21 thousands

Example 17b **3** Write the following using scientific notation.

a 0.000003 **b** 0.0004 **c** –0.00876
d 0.00000000073 **e** –0.00003 **f** 0.000000000125
g –0.00000000809 **h** 0.000000024 **i** 0.0000345

4 Write each of the following numbers using scientific notation.

a 6000 **b** 720 000 **c** 324.5
d 7869.03 **e** 8459.12 **f** 0.2
g 0.000328 **h** 0.00987 **i** –0.00001
j –460 100 000 **k** 17 467 **l** –128

Example 18a **5** Express each of the following in decimal form.

a 5.7×10^4 **b** 3.6×10^6 **c** 4.3×10^8
d 3.21×10^7 **e** 4.23×10^5 **f** 9.04×10^{10}
g 1.97×10^8 **h** 7.09×10^2 **i** 6.357×10^5

Example 18b **6** Express each of the following in decimal form.

a 1.2×10^{-4} **b** 4.6×10^{-6} **c** 8×10^{-10}
d 3.52×10^{-5} **e** 3.678×10^{-1} **f** 1.23×10^{-7}
g 9×10^{-5} **h** 5×10^{-2} **i** 4×10^{-1}

PROBLEM-SOLVING	7–9(1/2)	7–9(1/2), 10, 11	7–9(1/2), 11, 12

7 Express each of the following approximate numbers using scientific notation.

a The mass of Earth is 6 000 000 000 000 000 000 000 000 kg.
b The diameter of Earth is 40 000 000 m.
c The diameter of a gold atom is 0.0000000001 m.
d The radius of Earth's orbit around the Sun is 150 000 000 km.
e The universal constant of gravitation is 0.0000000000667 Nm^2/kg^2.
f The half-life of polonium-214 is 0.00015 seconds.
g Uranium-238 has a half-life of 4 500 000 000 years.

8 Express each of the following in decimal form.

- **a** Neptune is approximately 4.6×10^9 km from Earth.
- **b** A population of bacteria contained 8×10^{12} organisms.
- **c** The Moon is approximately 3.84×10^5 km from Earth.
- **d** A fifty-cent coin is approximately 3.8×10^{-3} m thick.
- **e** The diameter of the nucleus of an atom is approximately 1×10^{-14} m.
- **f** The population of a city is 7.2×10^5.

9 Write the following using scientific notation in the units given in the brackets.
Recall:

$$\begin{aligned} 1 \text{ second} &= 1000 \text{ milliseconds} \\ 1 \text{ millisecond} &= 1000 \text{ microseconds} \\ 1 \text{ microsecond} &= 1000 \text{ nanoseconds} \end{aligned}$$

- **a** 3 million years (months)
- **b** 0.03 million years (months)
- **c** 492 milliseconds (seconds)
- **d** 0.38 milliseconds (seconds)
- **e** 2.1 microseconds (seconds)
- **f** 0.052 microseconds (seconds)
- **g** 4 nanoseconds (seconds)
- **h** 139.2 nanoseconds (seconds)
- **i** 39.5 centuries (years)
- **j** 438 decades (years)
- **k** 2.3 hours (milliseconds)
- **l** 5 minutes (nanoseconds)

10 When Sydney was planning for the 2000 Olympic Games, the Olympic Organising Committee made the following predictions.

- The cost of staging the games would be A\$1.7 billion ($\$1.7 \times 10^9$) (excluding infrastructure). In fact, \$140 million extra was spent on staging the games.
- The cost of constructing or upgrading infrastructure would be \$807 million.

Give each of the following answers in scientific notation.

- **a** The actual total cost of staging the Olympic Games.
- **b** The total cost of staging the games and constructing or upgrading the infrastructure.

11 Two planets are 2.8×10^8 km and 1.9×10^9 km from their closest sun. What is the difference between these two distances in scientific notation?

12 Two particles weigh 2.43×10^{-2} g and 3.04×10^{-3} g. Find the difference in their weight in scientific notation.

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REASONING

13(1/2) 13(1/2) 13–14(1/3)

13 The number 47×10^4 is not written using scientific notation since 47 is not a number between 1 and 10. The following shows how to convert to scientific notation.

$$\begin{aligned} 47 \times 10^4 &= 4.7 \times 10 \times 10^4 \\ &= 4.7 \times 10^5 \end{aligned}$$

Write these numbers using scientific notation.

a 32×10^3 **b** 41×10^5 **c** 317×10^2 **d** 5714×10^2
e 0.13×10^5 **f** 0.092×10^3 **g** 0.003×10^8 **h** 0.00046×10^9
i 61×10^{-3} **j** 424×10^{-2} **k** 1013×10^{-6} **l** 490000×10^{-1}
m 0.02×10^{-3} **n** 0.0004×10^{-2} **o** 0.00372×10^{-1} **p** 0.04001×10^{-6}

14 Use index law 3: $(a^m)^n = a^{m \times n}$ and index law 5: $(a \times b)^m = a^m \times b^m$ to simplify these numbers. Give your answer using scientific notation.

a $(2 \times 10^2)^3$ **b** $(3 \times 10^4)^2$ **c** $(2.5 \times 10^{-2})^2$ **d** $(1.5 \times 10^{-3})^3$
e $(2 \times 10^{-3})^{-3}$ **f** $(5 \times 10^{-4})^{-2}$ **g** $\left(\frac{1}{3} \times 10^2\right)^{-2}$ **h** $\left(\frac{2}{5} \times 10^4\right)^{-1}$

ENRICHMENT: Scientific notation with index laws

– – 15(1/2), 16, 17

15 Use index laws to simplify these and write using scientific notation.

a $(3 \times 10^2) \times (2 \times 10^4)$ **b** $(4 \times 10^4) \times (2 \times 10^7)$
c $(8 \times 10^6) \div (4 \times 10^2)$ **d** $(9 \times 10^{20}) \div (3 \times 10^{11})$
e $(7 \times 10^2) \times (8 \times 10^2)$ **f** $(1.5 \times 10^3) \times (8 \times 10^4)$
g $(6 \times 10^4) \div (0.5 \times 10^2)$ **h** $(1.8 \times 10^6) \div (0.2 \times 10^3)$
i $(3 \times 10^{-4}) \times (3 \times 10^{-5})$ **j** $(15 \times 10^{-2}) \div (2 \times 10^6)$
k $(4.5 \times 10^{-3}) \div (3 \times 10^2)$ **l** $(8.8 \times 10^{-1}) \div (8.8 \times 10^{-1})$

16 Determine, using index laws, how many seconds it takes for light to travel from the Sun to Earth given that Earth is 1.5×10^8 km from the Sun and the speed of light is 3×10^5 km/s.

17 Using index laws and the fact that the speed of light is equal to 3×10^5 km/s, determine:

a how far light travels in one nanosecond (1×10^{-9} seconds). Answer using scientific notation in kilometres and then convert your answer to centimetres.

b how long light takes to travel 300 kilometres. Answer in seconds.

6G Scientific notation using significant figures

Learning intentions

- To know the rules for counting significant figures
- To be able to determine the number of significant figures in a number
- To be able to write numbers in scientific notation rounded to a set number of significant figures
- To be able to use scientific notation on a calculator

The number of digits used to record measurements depends on how accurately the measurements can be recorded. The volume of Earth, for example, has been calculated as 1 083 210 000 000 km^3. This appears to show six significant figures and could be written using scientific notation as 1.08321×10^{12}. A more accurate calculation may include more non-zero digits in the last seven places.

The mass of a single oxygen molecule is known to be 0.000000000000000000000000053 g. This shows two significant figures and is written using scientific notation as 5.3×10^{-26}. On many calculators you will notice that very large or very small numbers are automatically converted to scientific notation using a certain number of significant figures. Numbers can also be entered into a calculator using scientific notation.

Palaeontologists, who study fossils, use 1 or 2 significant figures for the estimated time period when Tyrannosaurus Rex dinosaurs lived on Earth, i.e. from 7×10^7 years ago to 6.5×10^7 years ago.

LESSON STARTER Significant discussions

Begin a discussion regarding significant figures by referring to these questions.

- Why is the volume of Earth given as 1 083 210 000 000 km^3 written using seven zeros at the end of the number? Wouldn't the exact mass of Earth include some other digits in these places?
- Why is the mass of an oxygen molecule given as 5.3×10^{-26} written using only two significant digits? Wouldn't the exact mass of a water molecule include more decimal places?

KEY IDEAS

- **Significant figures** are counted from left to right starting at the first non-zero digit. Zeros with no non-zero digit on their right are not counted. For example:
  - 38 041 has five significant figures
  - 0.0016 has two significant figures
  - 3.21×10^4 has three significant figures.
- When using scientific notation, the first significant figure sits to the left of the decimal point.
- Calculators can be used to work with scientific notation.
  - $\boxed{\text{E}}$ or $\boxed{\text{EE}}$ or $\boxed{\text{EXP}}$ or $\boxed{\times 10^x}$ are common key names on calculators.
  - Pressing 2.37 $\boxed{\text{EE}}$ 5 gives 2.37×10^5 and 2.37E5 means 2.37×10^5.

ISBN 978-1-108-77288-4

BUILDING UNDERSTANDING

1 State the missing numbers in these tables, rounding each number to the given number of significant figures.

a 57 263

Significant figures	Rounded number
4	
3	57 300
2	
1	

b 0.0036612

Significant figures	Rounded number
4	
3	
2	
1	0.004

2 Decide if the following numbers are written using scientific notation with three significant figures.

a 4.21×10^4
b 32×10^{-3}
c 1803×10^6
d 0.04×10^2
e 1.89×10^{-10}
f 9.04×10^{-6}
g 5.56×10^{-14}
h 0.213×10^2
i 26.1×10^{-2}

Example 19 Stating the number of significant figures

State the number of significant figures given in these numbers.

a 451
b 0.005012
c 3.2×10^7

SOLUTION	EXPLANATION
a 3 significant figures	Starting at the '4' and moving right we count three digits.
b 4 significant figures	Start counting at the first non-zero digit.
c 2 significant figures	With scientific notation the first significant figure is to the left of the decimal point.

Now you try

State the number of significant figures given in these numbers.

a 4174
b 0.0103
c 5.81×10^4

Example 20 Writing numbers in scientific notation using significant figures

Write these numbers using scientific notation and three significant figures.

a 2 183 000
b 0.0019482

SOLUTION	EXPLANATION
a $2\,183\,000 = 2.18 \times 10^6$ (to 3 sig. fig.)	Put the decimal point after the first non-zero digit. The decimal point has moved 6 places to the left, so multiply by 10^6. Round the third significant figure down since the following digit is less than 5.
b $0.0019482 = 1.95 \times 10^{-3}$ (to 3 sig. fig.)	Move the decimal point 3 places to the right and multiply by 10^{-3}. Round the third significant figure up to 5 since the following digit is greater than 4.

Now you try

Write these numbers using scientific notation and two significant figures.

a 487 130 **b** 0.06531

Example 21 Using a calculator with scientific notation

Use a calculator to evaluate each of the following, leaving your answers in scientific notation correct to four significant figures.

a $3.67 \times 10^5 \times 23.6 \times 10^4$ **b** $7.6 \times 10^{-3} + \sqrt{2.4 \times 10^{-2}}$

SOLUTION	EXPLANATION
a $3.67 \times 10^5 \times 23.6 \times 10^4$	Use a calculator and locate the button used to enter scientific notation.
$= 8.661 \times 10^{10}$ (to 4 sig. fig.)	Write using scientific notation with four significant figures.
b $7.6 \times 10^{-3} + \sqrt{2.4 \times 10^{-2}}$ $= 0.1625$	Use a calculator and locate the button used to enter scientific notation.
$= 1.625 \times 10^{-1}$ (to 4 sig. fig.)	Write using scientific notation with a number between 1 and 10.

Now you try

Use a calculator to evaluate each of the following, leaving your answers in scientific notation correct to three significant figures.

a $2.1 \times 10^3 \times 36.7 \times 10^5$ **b** $1.04 \times 10^{-4} + \sqrt{4.8 \times 10^{-3}}$

Exercise 6G

FLUENCY	1, 2–5(1/2)	2–6(1/3)	2–6(1/4)

1 State the number of significant figures in these numbers.

Example 19a
a **i** 4361 **ii** 72

Example 19b
b **i** 0.016 **ii** 0.000749

Example 19c
c **i** 1.8×10^4 **ii** 1.402×10^{-3}

Example 19
2 State the number of significant figures in these numbers.

a 202 **b** 1007 **c** 30101 **d** 19
e 0.0183 **f** 0.2 **g** 0.706 **h** 0.00109
i 4.21×10^3 **j** 2.905×10^{-2} **k** 1.07×10^{-6} **l** 5.9×10^5

Example 20
3 Write these numbers using scientific notation and three significant figures.

a 242 300 **b** 171 325 **c** 2829 **d** 3 247 000
e 0.00034276 **f** 0.006859 **g** 0.01463 **h** 0.001031
i 23.41 **j** 326.042 **k** 19.618 **l** 0.172046

4 Write each number using scientific notation rounding to the number of significant figures given in the brackets.

a 47 760 (3) **b** 21 610 (2) **c** 4 833 160 (4)
d 37.16 (2) **e** 99.502 (3) **f** 0.014427 (4)
g 0.00201 (1) **h** 0.08516 (1) **i** 0.0001010 (1)

Example 21a
5 Use a calculator to evaluate each of the following, leaving your answers in scientific notation correct to four significant figures.

a 4^{-6} **b** 78^{-3}

c $(-7.3 \times 10^{-4})^{-5}$ **d** $\dfrac{3.185}{7 \times 10^4}$

e $2.13 \times 10^4 \times 9 \times 10^7$ **f** $5.671 \times 10^2 \times 3.518 \times 10^5$

g $9.419 \times 10^5 \times 4.08 \times 10^{-4}$ **h** $2.85 \times 10^{-9} \times 6 \times 10^{-3}$

i $12\,345^2$ **j** 87.14^8

k $\dfrac{1.8 \times 10^{26}}{4.5 \times 10^{22}}$ **l** $\dfrac{-4.7 \times 10^2 \times 6.1 \times 10^7}{3.2 \times 10^6}$

Example 21b
6 Use a calculator to evaluate each of the following, leaving your answers in scientific notation correct to five significant figures.

a $\sqrt{8756}$ **b** $\sqrt{634 \times 7.56 \times 10^7}$

c $8.6 \times 10^5 + \sqrt{2.8 \times 10^{-2}}$ **d** $-8.9 \times 10^{-4} + \sqrt{7.6 \times 10^{-3}}$

e $\dfrac{5.12 \times 10^{21} - 5.23 \times 10^{20}}{2 \times 10^6}$ **f** $\dfrac{8.942 \times 10^{47} - 6.713 \times 10^{44}}{2.5 \times 10^{19}}$

g $\dfrac{2 \times 10^7 + 3 \times 10^8}{5}$ **h** $\dfrac{4 \times 10^8 + 7 \times 10^9}{6}$

i $\dfrac{6.8 \times 10^{-8} + 7.5 \times 10^{27}}{4.1 \times 10^{27}}$ **j** $\dfrac{2.84 \times 10^{-6} - 2.71 \times 10^{-9}}{5.14 \times 10^{-6} + 7 \times 10^{-8}}$

PROBLEM-SOLVING	7, 8	8–10	9, 10

7 The mass of Earth is approximately 6 000 000 000 000 000 000 000 000 kg. Given that the mass of the Sun is 330 000 times the mass of Earth, find the mass of the Sun. Express your answer using scientific notation correct to three significant figures.

8 The diameter of Earth is approximately 12 756 000 m. If the Sun's diameter is 109 times that of Earth, calculate its diameter in kilometres. Express your answer using scientific notation correct to three significant figures.

9 Using the formula for the volume of a sphere, $V = \frac{4\pi r^3}{3}$, and, assuming Earth to be spherical, calculate the volume of Earth in km^3. Use the data given in Question **8** and express your answer using scientific notation correct to three significant figures.

10 Write these numbers from largest to smallest.
2.41×10^6, 24.2×10^5, 0.239×10^7, 2421×10^3, 0.02×10^8

REASONING	11	11, 12	12, 13

11 The following output is common on a number of different calculators and computers. Write down the number that you think they represent.

a 4.26E6
b 9.1E–3
c 5.04EXP11
d 1.931EXP–1
e 2.1^{06}
f 6.14^{-11}

12 Anton writes down $352\,000 \times 250\,000 = 8.8^{10}$. Explain his error.

13 **a** Round these numbers to three significant figures. Retain the use of scientific notation.
i 2.302×10^2
ii 4.9045×10^{-2}
iii 3.996×10^6

b What do you notice about the digit that is the third significant figure?

c Why do you think that it might be important to a scientist to show a significant figure that is a zero at the end of a number?

ENRICHMENT: Combining bacteria	–	–	14

14 A flask of type A bacteria contains 5.4×10^{12} cells and a flask of type B bacteria contains 4.6×10^8 cells. The two types of bacteria are combined in the same flask.

a How many bacterial cells are there in the flask?

b If type A bacterial cells double every 8 hours and type B bacterial cells triple every 8 hours how many cells are in the flask after:
i one day?
ii a week?
iii 30 days?

Express your answers using scientific notation correct to three significant figures.

Applications and problem-solving

The following problems will investigate practical situations drawing upon knowledge and skills developed throughout the chapter. In attempting to solve these problems, aim to identify the key information, use diagrams, formulate ideas, apply strategies, make calculations and check and communicate your solutions.

Double the success

1 Through active investment, Angela's portfolio doubles in value every decade (10 years) and started with an initial investment of \$100 000.

Angela is interested in the value of her investment over time and the time that it takes for it to reach certain investment goals.

a Complete this table showing the value of Angela's investment over 4 decades.

Decade (n)	0	1	2	3	4
Value (\$$V$)	100 000				

b Which rule links the value of Angela's investment, \$$V$, over n decades?

A $V = 100\,000n$ **B** $V = 100\,000 \times 2^n$ **C** $V = 100\,000 \times n^2$ **D** $V = 2^n$

c Use the correct rule from part **b** to calculate the value of Angela's investment after:

i 6 decades **ii** 1.5 decades (to the nearest cent).

d Use trial and error to find how long it takes for Angela's investment to grow in value to the following amounts. Round to the nearest year.

i \$150 000 **ii** \$1 000 000

e A friend of Angela is claiming that their investment of \$50 000 is tripling in value every decade.

i Find a rule for the value, \$$V$, of the friend's investment after n decades.

ii Assuming that the friend's investment started at the same time as Angela's, find how long it would take for the investments to be of the same value. Round to the nearest year. A trial and error approach is appropriate.

Rupert's Moon facts

2 Rupert is studying the Moon and finds these facts:

- Diameter 3475 km
- Mass: 7.35×10^{22} kg
- Average distance to Earth: 384 400 km
- Formation: 4.6 billion years ago
- NASA program cost: \$25.4 billion (about \$120 billion today)
- Dust particle diameter: 50 micrometres

Rupert is interested in the facts about the Moon and tries to make sense of all the numbers. He wants to use scientific notation to deal with the size of the numbers involved.

a Consider the average distance to Earth.

i Write this number using scientific notation.

ii If a rocket travels at 10 km per second, how long will it take to get to the Moon? Answer using hours and minutes.

ISBN 978-1-108-77288-4

b Consider the Moon's diameter.

  i Write this number using scientific notation.

  ii Find the difference between the Moon's diameter and Earth's diameter (12 742 km). Express your answer using scientific notation and two significant figures.

c Consider the time of formation.

  i Write this number using scientific notation.

  ii Write in decimal form.

d Consider the program cost at the time. If an average car at the time cost \$2000, how many cars could have been purchased for the same amount of money?

e Consider the Moon's mass. How many times heavier is Earth if Earth's mass is 5.972×10^{24}? Round to the nearest integer.

f Consider the diameter of a dust particle.

  i Write this number in metres using scientific notation.

  ii How many dust particles sitting in a single row would you need to make a line 1 m long?

Computer depreciation

3 Reynah buys a new computer and at the end of each year, notices that the computer's value has dropped by 50% from the previous year's value.

The rule for linking the computer's value, $\$V$, n years after purchase is given by:

$$V = 3000 \times 0.5^n$$

Reynah is interested in the rate at which the computer's value is decreasing and the time that it takes for it to be valued at a certain price.

a Find the initial value of the computer (i.e. find the value of V when $n = 0$).

b Find the value of the computer after:

  i 2 years **ii** 5 years.

c Use trial and error to find how long it takes for the computer to decrease in value to the following amounts. Answer in years correct to one decimal place.

  i \$1000 **ii** \$200

d The rule linking V with n can also be written as $V = 3000 \times 2^{-n}$.

  i Explain why the rule can also be written this way.

  ii Use this rule to verify your answers to part **b**.

e Another computer costs \$5000 and reduces in value so that each year its value is one third of the value from the previous year. Write a rule connecting the computer's value, $\$V$, after n years using:

  i a positive index

  ii a negative index.

f After how long will the two computers above have the same value, assuming they were purchased at the same time? Answer in years correct to one decimal place.

ISBN 978-1-108-77288-4

6H Fractional indices and surds EXTENDING

Learning intentions

- To know how to use a root sign to rewrite numbers with a fractional index
- To understand what type of number is a surd
- To be able to evaluate numbers involving a fractional index
- To be able to use index laws with fractional indices

So far we have considered indices including positive and negative integers and zero. Numbers can also be expressed using fractional indices. Two examples are $9^{\frac{1}{2}}$ and $5^{\frac{1}{3}}$.

Using index law 1: $9^{\frac{1}{2}} \times 9^{\frac{1}{2}} = 9^{\frac{1}{2}+\frac{1}{2}} = 9^1 = 9$

Since $\sqrt{9} \times \sqrt{9} = 9$ and $9^{\frac{1}{2}} \times 9^{\frac{1}{2}} = 9$ then $9^{\frac{1}{2}} = \sqrt{9}$.

Using index law 1: $5^{\frac{1}{3}} \times 5^{\frac{1}{3}} \times 5^{\frac{1}{3}} = 5^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = 5^1 = 5$

Since $\sqrt[3]{5} \times \sqrt[3]{5} \times \sqrt[3]{5} = 5$ and $5^{\frac{1}{3}} \times 5^{\frac{1}{3}} \times 5^{\frac{1}{3}} = 5$ then $5^{\frac{1}{3}} = \sqrt[3]{5}$.

This shows that numbers with fractional powers can be written using root signs. In the example above, $9^{\frac{1}{2}}$ is the square root of 9 ($\sqrt{9}$) and $5^{\frac{1}{3}}$ is the cube root of 5 ($\sqrt[3]{5}$).

You will have noticed that $9^{\frac{1}{2}} = \sqrt{9} = 3 = \frac{3}{1}$ and so $9^{\frac{1}{2}}$ is a rational number (a fraction), but $5^{\frac{1}{3}} = \sqrt[3]{5}$ does not appear to be able to be expressed as a fraction. In fact, $\sqrt[3]{5}$ is irrational and cannot be expressed as a fraction and is called a surd. As a decimal $\sqrt[3]{5} = 1.70997594668\ldots$, which is an infinite, non-recurring decimal with no repeated pattern. This is a characteristic of all surds.

LESSON STARTER A surd or not?

Surds are numbers with a root sign that cannot be expressed as a fraction. As a decimal they are infinite and non-recurring (with no pattern).

Use a calculator to help complete the table on the right and then decide if you think the numbers are surds.

Index form	With root sign	Decimal	Surd (Yes or No)
$2^{\frac{1}{2}}$	$\sqrt{2}$		
$4^{\frac{1}{2}}$	$\sqrt{4}$		
$11^{\frac{1}{2}}$			
$36^{\frac{1}{2}}$			
$\left(\frac{1}{9}\right)^{\frac{1}{2}}$			
$(0.1)^{\frac{1}{2}}$			
$3^{\frac{1}{3}}$	$\sqrt[3]{3}$		
$3^{\frac{1}{3}}$	$\sqrt[3]{8}$		
$15^{\frac{1}{3}}$			
$\left(\frac{1}{27}\right)^{\frac{1}{3}}$			
$5^{\frac{1}{4}}$			
$64^{\frac{1}{6}}$			

ISBN 978-1-108-77288-4

KEY IDEAS

- Numbers written with **fractional indices** can also be written using a root sign.
  - $a^{\frac{1}{m}} = \sqrt[m]{a}$
  - $\sqrt[2]{a}$ is written $\sqrt{a}$

  For example: $3^{\frac{1}{2}} = \sqrt{3}, 7^{\frac{1}{3}} = \sqrt[3]{7}, 2^{\frac{1}{5}} = \sqrt[5]{2}$

- **Surds** are irrational numbers written with a root sign.
  - Irrational numbers cannot be expressed as a fraction.
  - The decimal expansion is infinite and non-recurring with no pattern.

$\sqrt{2} = 1.41421356237\ldots$

$\sqrt[3]{10} = 2.15443469003\ldots$

$3^{\frac{1}{2}} = 1.73205080757\ldots$

BUILDING UNDERSTANDING

1 Evaluate these numbers. What do you notice?

a 2^2 and $\sqrt{4}$ **b** 2^3 and $\sqrt[3]{8}$ **c** 3^2 and $\sqrt{9}$

d 3^3 and $\sqrt[3]{27}$ **e** 4^2 and $\sqrt{16}$ **f** 4^3 and $\sqrt[3]{64}$

2 State whether the following are true or false.

a Rational numbers can be written as fractions. **b** A surd is a rational number.

c A surd in decimal form will be infinite and non-recurring (with no pattern).

d $\sqrt{3} = 3^{\frac{1}{2}}$ **e** $\sqrt{8} = 8^{\frac{1}{3}}$ **f** $5^{\frac{1}{3}} = \sqrt{5}$ **g** $10^{\frac{1}{6}} = \sqrt{6}$

3 Use a calculator to evaluate these surds and round to four decimal places.

a $7^{\frac{1}{2}}$ (or $\sqrt{7}$) **b** $13^{\frac{1}{2}}$ (or $\sqrt{13}$) **c** $83^{\frac{1}{2}}$ (or $\sqrt{83}$)

A classical guitar, of scale length L mm, has each fret x mm from the bridge, where $x = L \times 2^{-\frac{n}{12}}$ for the nth fret from the nut. The 12th fret is half the scale length, doubling the pitch to one octave higher.

Example 22 Writing numbers using a root sign

Write these numbers using a root sign.

a $6^{\frac{1}{2}}$

b $2^{\frac{1}{5}}$

SOLUTION

a $6^{\frac{1}{2}} = \sqrt{6}$

b $2^{\frac{1}{5}} = \sqrt[5]{2}$

EXPLANATION

$a^{\frac{1}{m}} = \sqrt[m]{a}$ so $6^{\frac{1}{2}} = \sqrt[2]{6}$ (or $\sqrt{6}$) the square root of 6.

$\sqrt[5]{2}$ is called the 5th root of 2.

Now you try

Write these numbers using a root sign.

a $7^{\frac{1}{2}}$

b $3^{\frac{1}{6}}$

Example 23 Evaluating numbers with fractional indices

Evaluate:

a $144^{\frac{1}{2}}$

b $27^{\frac{1}{3}}$

SOLUTION

a $$\begin{aligned} 144^{\frac{1}{2}} &= \sqrt{144} \\ &= 12 \end{aligned}$$

b $$\begin{aligned} 27^{\frac{1}{3}} &= \sqrt[3]{27} \\ &= 3 \end{aligned}$$

EXPLANATION

$a^{\frac{1}{m}} = \sqrt[m]{a}$ where $m = 2$ and the square root of $144 = 12$ since $12^2 = 144$.

The cube root of 27 is 3 since $3^3 = 3 \times 3 \times 3 = 27$.

Now you try

Evaluate:

a $64^{\frac{1}{2}}$

b $216^{\frac{1}{3}}$

Example 24 Using index laws

Use index laws to simplify these expressions.

a $a^{\frac{1}{2}} \times a^{\frac{3}{2}}$ **b** $\dfrac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}}$ **c** $(y^2)^{\frac{1}{4}}$

SOLUTION	EXPLANATION
a $a^{\frac{1}{2}} \times a^{\frac{3}{2}} = a^{\frac{1}{2}+\frac{3}{2}}$ $= a^{\frac{4}{2}}$ $= a^2$	When multiplying indices with the same base add the powers.
b $\dfrac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}} = x^{\frac{1}{2}-\frac{1}{3}}$ $= x^{\frac{1}{6}}$	When dividing indices with the same base, subtract the powers. $\frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$
c $(y^2)^{\frac{1}{4}} = y^{2 \times \frac{1}{4}}$ $= y^{\frac{1}{2}}$	When raising a power to a power, multiply the indices: $2 \times \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$.

Now you try

Use index laws to simplify these expressions.

a $x^{\frac{1}{2}} \times x^{\frac{5}{2}}$ **b** $\dfrac{a^{\frac{2}{3}}}{a^{\frac{1}{2}}}$ **c** $(b^{\frac{1}{3}})^6$

Exercise 6H

FLUENCY	1, 2–5(1/2)	2–6(1/2)	2–6(1/3)

1 Write these numbers using a root sign.

Example 22a

a **i** $8^{\frac{1}{2}}$ **ii** $15^{\frac{1}{2}}$

Example 22b

b **i** $3^{\frac{1}{4}}$ **ii** $7^{\frac{1}{7}}$

Example 22

2 Write these numbers using a root sign.

a $3^{\frac{1}{2}}$ **b** $7^{\frac{1}{2}}$ **c** $5^{\frac{1}{3}}$ **d** $12^{\frac{1}{3}}$

e $31^{\frac{1}{5}}$ **f** $18^{\frac{1}{7}}$ **g** $9^{\frac{1}{9}}$ **h** $3^{\frac{1}{8}}$

3 Write these numbers in index form.

a $\sqrt{8}$ **b** $\sqrt{19}$ **c** $\sqrt[3]{10}$ **d** $\sqrt[3]{31}$

e $\sqrt[4]{5}$ **f** $\sqrt[5]{9}$ **g** $\sqrt[8]{11}$ **h** $\sqrt[11]{20}$

Example 23

4 Without using a calculator, evaluate these numbers with fractional indices.

a $25^{\frac{1}{2}}$ **b** $49^{\frac{1}{2}}$ **c** $81^{\frac{1}{2}}$ **d** $169^{\frac{1}{2}}$

e $8^{\frac{1}{3}}$ **f** $64^{\frac{1}{3}}$ **g** $125^{\frac{1}{3}}$ **h** $1000^{\frac{1}{3}}$

i $16^{\frac{1}{4}}$ **j** $81^{\frac{1}{4}}$ **k** $625^{\frac{1}{4}}$ **l** $32^{\frac{1}{5}}$

Example 24

5 Use index laws to simplify these expressions. Leave your answer in index form.

a $a^{\frac{1}{2}} \times a^{\frac{1}{2}}$ **b** $a^{\frac{1}{3}} \times a^{\frac{1}{3}}$ **c** $a^{\frac{2}{3}} \times a^{\frac{4}{3}}$ **d** $a^{2} \times a^{\frac{1}{2}}$

e $\dfrac{x^{\frac{2}{3}}}{x^{\frac{1}{3}}}$ **f** $\dfrac{x^{\frac{3}{2}}}{x^{\frac{1}{2}}}$ **g** $\dfrac{x^{\frac{7}{6}}}{x^{\frac{2}{6}}}$ **h** $\dfrac{x^{\frac{4}{3}}}{x^{\frac{1}{3}}}$

i $(y^{2})^{\frac{1}{2}}$ **j** $(y^{3})^{\frac{2}{3}}$ **k** $(y^{\frac{1}{2}})^{3}$ **l** $(x^{\frac{1}{2}})^{\frac{1}{2}}$

m $(x^{\frac{2}{3}})^{4}$ **n** $(a^{\frac{2}{5}})^{\frac{1}{3}}$ **o** $(a^{\frac{3}{4}})^{\frac{1}{2}}$ **p** $(n^{\frac{2}{5}})^{\frac{10}{3}}$

6 Use index laws to simplify these expressions.

a $a \times a^{\frac{1}{3}}$ **b** $a^{\frac{1}{2}} \times a^{\frac{1}{5}}$ **c** $a^{\frac{2}{3}} \times a^{\frac{3}{7}}$

d $a^{5} \div a^{\frac{7}{3}}$ **e** $b^{\frac{2}{3}} \div b^{\frac{1}{2}}$ **f** $x^{\frac{4}{5}} \div x^{\frac{2}{3}}$

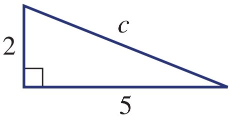
PROBLEM-SOLVING 7–8(1/2) 7–9(1/2) 7–9(1/2)

7 Evaluate the following without a calculator. (*Hint*: First rewrite each question using positive indices using $a^{-m} = \frac{1}{a^m}$.)

a $4^{-\frac{1}{2}}$ **b** $8^{-\frac{1}{3}}$ **c** $32^{-\frac{1}{5}}$ **d** $81^{-\frac{1}{4}}$

e $25^{-\frac{1}{2}}$ **f** $27^{-\frac{1}{3}}$ **g** $1000^{-\frac{1}{3}}$ **h** $256^{-\frac{1}{4}}$

8 Find the length of the hypotenuse (c) in these right-angled triangles. Use Pythagoras' theorem ($c^2 = a^2 + b^2$) and write your answers as surds.

a


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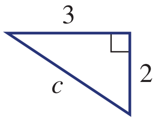
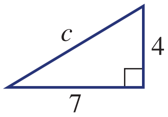
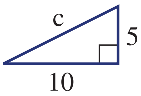
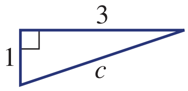
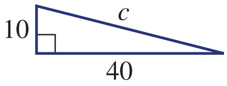


c


d


e


f


ISBN 978-1-108-77288-4

9 Note that
$$\begin{aligned} 8^{\frac{2}{3}} &= (8^{\frac{1}{3}})^2 && \text{using index law 3} \\ &= (\sqrt[3]{8})^2 && \text{since } 8^{\frac{1}{3}} = \sqrt[3]{8} \\ &= 2^2 && \sqrt[3]{8} = 2 \text{ since } 2^3 = 8 \\ &= 4 \end{aligned}$$

Use the approach shown in the example above to evaluate these numbers.

a $27^{\frac{2}{3}}$ **b** $64^{\frac{2}{3}}$ **c** $9^{\frac{3}{2}}$ **d** $25^{\frac{3}{2}}$

e $16^{\frac{5}{4}}$ **f** $4^{\frac{5}{2}}$ **g** $81^{\frac{3}{2}}$ **h** $125^{\frac{5}{3}}$

REASONING	10	10, 11	11, 12

10 Show working to prove that the answers to all these questions simplify to 1. Remember $a^0 = 1$.

a $a^{\frac{1}{2}} \times a^{-\frac{1}{2}}$ **b** $a^{\frac{2}{3}} \times a^{-\frac{2}{3}}$ **c** $a^{\frac{4}{7}} \times a^{-\frac{4}{7}}$

d $a^{\frac{5}{6}} \div a^{\frac{5}{6}}$ **e** $(a^{\frac{1}{2}})^{\frac{1}{2}} \div a^{\frac{1}{4}}$ **f** $a^2 \div (a^3)^{\frac{2}{3}}$

11 A student tries to evaluate $9^{\frac{1}{2}}$ on a calculator and types 9^1/2 and gets 4.5. You know that $9^{\frac{1}{2}} = \sqrt{9} = 3$. What has the student done wrong? (Note: ^ on some calculators is x^y.)

12 a Evaluate the following.

i $\sqrt{3^2}$ **ii** $\sqrt{5^2}$ **iii** $\sqrt{10^2}$

b Simplify $\sqrt{a^2}$ for $a \geqslant 0$.

c Use fractional indices to show that $\sqrt{a^2} = a$ if $a \geqslant 0$.

d Evaluate the following.

i $(\sqrt{4})^2$ **ii** $(\sqrt{9})^2$ **iii** $(\sqrt{36})^2$

e Simplify $(\sqrt{a})^2$.

f Use fractional indices to show that $(\sqrt{a})^2 = a$. Assume $a \geqslant 0$.

g Simplify:

i $\sqrt[3]{a^3}$ **ii** $\sqrt[5]{a^5}$ **iii** $(\sqrt[3]{a})^3$ **iv** $(\sqrt[6]{a})^6$

ENRICHMENT: Fractions raised to fractions	–	–	13(½), 14

13 Note, for example, that $\left(\frac{4}{9}\right)^{\frac{1}{2}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$ since $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$. Now evaluate the following.

a $\left(\frac{16}{25}\right)^{\frac{1}{2}}$ **b** $\left(\frac{9}{49}\right)^{\frac{1}{2}}$ **c** $\left(\frac{4}{81}\right)^{\frac{1}{2}}$ **d** $\left(\frac{8}{27}\right)^{\frac{1}{3}}$

e $\left(\frac{64}{125}\right)^{\frac{1}{3}}$ **f** $\left(\frac{16}{81}\right)^{\frac{1}{4}}$ **g** $\left(\frac{256}{625}\right)^{\frac{1}{4}}$ **h** $\left(\frac{1000}{343}\right)^{\frac{1}{3}}$

14 Note that $\left(\frac{4}{9}\right)^{-\frac{1}{2}} = \dfrac{1}{\left(\frac{4}{9}\right)^{\frac{1}{2}}} = \dfrac{1}{\sqrt{\frac{4}{9}}} = \dfrac{1}{\left(\frac{2}{3}\right)} = \frac{3}{2}$. Now evaluate the following.

a $\left(\frac{9}{4}\right)^{-\frac{1}{2}}$ **b** $\left(\frac{49}{144}\right)^{-\frac{1}{2}}$ **c** $\left(\frac{8}{125}\right)^{-\frac{1}{3}}$ **d** $\left(\frac{1296}{625}\right)^{-\frac{1}{4}}$

6I Simple operations with surds EXTENDING

Learning intentions

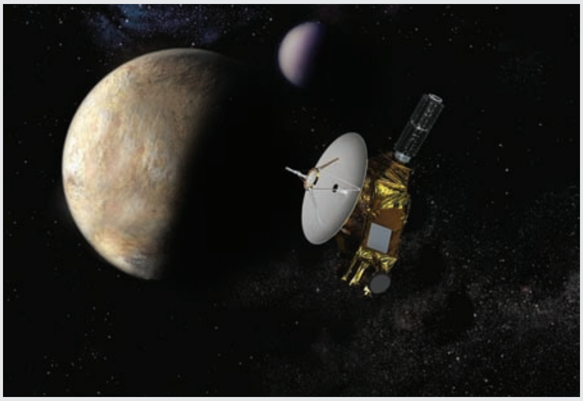
- To understand that only like surds can be added and subtracted
- To know and be able to apply the rules for adding, subtracting, multiplying and dividing simple surds

Since surds, such as $\sqrt{2}$ and $\sqrt{7}$, are numbers, they can be added, subtracted, multiplied or divided. Expressions with surds can also be simplified, but this depends on the surds themselves and the types of operations that sit between them.

$\sqrt{2} + \sqrt{3}$ cannot be simplified since $\sqrt{2}$ and $\sqrt{3}$ are not 'like' surds. This is like trying to simplify $x + y$. However, $\sqrt{2} + 5\sqrt{2}$ simplifies to $6\sqrt{2}$; this is like simplifying $x + 5x = 6x$. Subtraction of surds is treated in the same manner.

Products and quotients involving surds can also be simplified as in these examples:

$$\sqrt{11} \times \sqrt{2} = \sqrt{22} \quad \text{and} \quad \sqrt{30} \div \sqrt{3} = \sqrt{10}$$



Engineers, architects, surveyors, navigators, physicists, geologists, astronomers and space scientists use surds, as they are exact values. For example, the formula for a spacecraft's escape velocity (from gravity) includes the number $\sqrt{2}$: $v = \dfrac{\sqrt{2}\sqrt{GM}}{\sqrt{R}}$.

LESSON STARTER Rules for multiplication and division

Use a calculator to find a decimal approximation for each of the expressions in these pairs.

- $\sqrt{2} \times \sqrt{3}$ and $\sqrt{6}$
- $\sqrt{10} \times \sqrt{5}$ and $\sqrt{50}$

What does this suggest about the simplification of $\sqrt{a} \times \sqrt{b}$?

Repeat the above exploration for the following:

- $\sqrt{6} \div \sqrt{2}$ and $\sqrt{\dfrac{6}{2}}$
- $\sqrt{80} \div \sqrt{8}$ and $\sqrt{\dfrac{80}{8}}$

What does this suggest about the simplification of $\sqrt{a} \div \sqrt{b}$?

KEY IDEAS

- **Surds** can be simplified using addition or subtraction if they are **'like' surds**.
  - $2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$
  - $11\sqrt{7} - 2\sqrt{7} = 9\sqrt{7}$
  - $\sqrt{3} + \sqrt{5}$ cannot be simplified.
- $(\sqrt{a})^2 = a$

  For example: $(\sqrt{5})^2 = 5$
- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

  For example: $\sqrt{5} \times \sqrt{3} = \sqrt{5 \times 3} = \sqrt{15}$
- $\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$

  For example: $\sqrt{10} \div \sqrt{5} = \sqrt{\frac{10}{5}} = \sqrt{2}$

BUILDING UNDERSTANDING

① Decide if the following pairs of numbers contain 'like' surds.

a $3\sqrt{2}, 4\sqrt{2}$ **b** $5\sqrt{3}, 2\sqrt{3}$ **c** $4\sqrt{2}, 5\sqrt{7}$ **d** $\sqrt{3}, 2\sqrt{5}$
e $6\sqrt{6}, 3\sqrt{3}$ **f** $\sqrt{8}, 3\sqrt{8}$ **g** $19\sqrt{2}, -\sqrt{2}$ **h** $-3\sqrt{6}, 3\sqrt{5}$

② **a** Use a calculator to find a decimal approximation for both $\sqrt{5} \times \sqrt{2}$ and $\sqrt{10}$. What do you notice?

b Use a calculator to find a decimal approximation for both $\sqrt{15} \div \sqrt{5}$ and $\sqrt{3}$. What do you notice?

Example 25 Adding and subtracting surds

Simplify:

a $2\sqrt{5} + 6\sqrt{5}$

b $\sqrt{3} - 5\sqrt{3}$

SOLUTION	EXPLANATION
a $2\sqrt{5} + 6\sqrt{5} = 8\sqrt{5}$	This is like simplifying $2x + 6x = 8x$. $2\sqrt{5}$ and $6\sqrt{5}$ contain like surds.
b $\sqrt{3} - 5\sqrt{3} = -4\sqrt{3}$	This is similar to $x - 5x = -4x$ in algebra.

Now you try

Simplify:

a $6\sqrt{3} + 2\sqrt{3}$

b $2\sqrt{7} - 5\sqrt{7}$

Example 26 Multiplying and dividing surds

Simplify:

a $\sqrt{3} \times \sqrt{10}$ **b** $\sqrt{24} \div \sqrt{8}$

SOLUTION	EXPLANATION
a $\sqrt{3} \times \sqrt{10} = \sqrt{3 \times 10}$ $= \sqrt{30}$	Use $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
b $\sqrt{24} \div \sqrt{8} = \sqrt{\frac{24}{8}}$ $= \sqrt{3}$	Use $\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$

Now you try

Simplify:

a $\sqrt{5} \times \sqrt{3}$ **b** $\sqrt{60} \div \sqrt{10}$

Exercise 6I

FLUENCY	1, 2–3(1/2)	2–3(1/2)	2–3(1/3)

1 Simplify:

Example 25a

a **i** $2\sqrt{11} + 3\sqrt{11}$ **ii** $8\sqrt{7} + 3\sqrt{7}$

Example 25b

b **i** $2\sqrt{2} - 5\sqrt{2}$ **ii** $11\sqrt{11} - 9\sqrt{11}$

Example 25

2 Simplify by collecting like surds.

a $3\sqrt{7} + 5\sqrt{7}$ **b** $2\sqrt{11} + 6\sqrt{11}$ **c** $\sqrt{5} + 8\sqrt{5}$

d $3\sqrt{6} + \sqrt{6}$ **e** $3\sqrt{3} + 2\sqrt{5} + 4\sqrt{3}$ **f** $5\sqrt{7} + 3\sqrt{5} + 4\sqrt{7}$

g $3\sqrt{5} - 8\sqrt{5}$ **h** $6\sqrt{7} - 10\sqrt{7}$ **i** $3\sqrt{7} - 2\sqrt{7} + 4\sqrt{7}$

j $5\sqrt{14} + \sqrt{14} - 7\sqrt{14}$ **k** $3\sqrt{2} - \sqrt{5} + 4\sqrt{2}$ **l** $6\sqrt{3} + 2\sqrt{7} - 3\sqrt{3}$

Example 26

3 Simplify:

a $\sqrt{5} \times \sqrt{6}$ **b** $\sqrt{3} \times \sqrt{7}$ **c** $\sqrt{10} \times \sqrt{7}$

d $\sqrt{8} \times \sqrt{2}$ **e** $\sqrt{12} \times \sqrt{3}$ **f** $\sqrt{2} \times \sqrt{11}$

g $\sqrt{3} \times \sqrt{3}$ **h** $\sqrt{12} \times \sqrt{12}$ **i** $\sqrt{36} \div \sqrt{12}$

j $\sqrt{20} \div \sqrt{2}$ **k** $\sqrt{42} \div \sqrt{6}$ **l** $\sqrt{60} \div \sqrt{20}$

m $\sqrt{45} \div \sqrt{5}$ **n** $\sqrt{32} \div \sqrt{2}$ **o** $\sqrt{49} \div \sqrt{7}$

PROBLEM-SOLVING	4–5(½)	4–5(½)	4–6(1/3)

4 Simplify:

a $2 - \sqrt{3} + 6 - 2\sqrt{3}$ **b** $\sqrt{2} - \sqrt{3} + 5\sqrt{2}$ **c** $7\sqrt{5} - \sqrt{2} + 1 + \sqrt{2}$

d $\frac{\sqrt{2}}{3} + \frac{\sqrt{2}}{2}$ **e** $\frac{\sqrt{7}}{2} + \frac{\sqrt{7}}{5}$ **f** $\frac{2\sqrt{6}}{7} - \frac{\sqrt{6}}{2}$

g $\sqrt{10} - \frac{\sqrt{10}}{3}$ **h** $5 - \frac{2\sqrt{3}}{3} + \sqrt{3}$ **i** $\frac{2\sqrt{8}}{7} - \frac{5\sqrt{8}}{8}$

5 Note, for example, that $2\sqrt{3} \times 5\sqrt{2} = 2 \times 5 \times \sqrt{3} \times \sqrt{2}$

$$= 10\sqrt{6}$$

Now simplify the following.

a $5\sqrt{2} \times 3\sqrt{3}$

b $3\sqrt{7} \times 2\sqrt{3}$

c $4\sqrt{5} \times 2\sqrt{6}$

d $2\sqrt{6} \times 5\sqrt{3}$

e $10\sqrt{6} \div 5\sqrt{2}$

f $18\sqrt{12} \div 6\sqrt{2}$

g $20\sqrt{28} \div 5\sqrt{2}$

h $6\sqrt{14} \div 12\sqrt{7}$

6 Expand and simplify. Recall that $a(b + c) = a \times b + a \times c$.

a $2\sqrt{3}(3\sqrt{5} + 1)$

b $\sqrt{5}(\sqrt{2} + \sqrt{3})$

c $5\sqrt{6}(\sqrt{2} + 3\sqrt{5})$

d $7\sqrt{10}(2\sqrt{3} - \sqrt{10})$

e $\sqrt{13}(\sqrt{13} - 2\sqrt{3})$

f $\sqrt{5}(\sqrt{7} - 2\sqrt{5})$

REASONING

7(1/2) 7(1/2) 7–8(1/2)

7 Using $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$, the surd $\sqrt{18}$ can be simplified as shown.

$$\begin{aligned}\sqrt{18} &= \sqrt{9 \times 2}\\ &= \sqrt{9} \times \sqrt{2}\\ &= 3\sqrt{2}\end{aligned}$$

This simplification is possible because 18 has a factor that is a square number (9). Use this technique to simplify these surds.

a $\sqrt{8}$

b $\sqrt{12}$

c $\sqrt{27}$

d $\sqrt{45}$

e $\sqrt{75}$

f $\sqrt{200}$

g $\sqrt{60}$

h $\sqrt{72}$

8 Building on the idea discussed in Question **7**, expressions such as $\sqrt{8} - \sqrt{2}$ can be simplified as shown:

$$\begin{aligned}\sqrt{8} - \sqrt{2} &= \sqrt{4 \times 2} - \sqrt{2}\\ &= 2\sqrt{2} - \sqrt{2}\\ &= \sqrt{2}\end{aligned}$$

Now simplify these expressions.

a $\sqrt{8} + 3\sqrt{2}$

b $3\sqrt{2} - \sqrt{8}$

c $\sqrt{18} + \sqrt{2}$

d $5\sqrt{3} - 2\sqrt{12}$

e $4\sqrt{8} - 2\sqrt{2}$

f $\sqrt{27} + 2\sqrt{3}$

g $3\sqrt{45} - 7\sqrt{5}$

h $6\sqrt{12} - 8\sqrt{3}$

ENRICHMENT: Binomial products

– – 9(1/2)

9 Simplify the following by using the rule $(a + b)(c + d) = ac + ad + bc + bd$.
Note, $(a + b)^2 = (a + b)(a + b)$.

a $(\sqrt{2} + \sqrt{3})(\sqrt{2} + \sqrt{5})$

b $(\sqrt{3} - \sqrt{5})(\sqrt{3} + \sqrt{2})$

c $(2\sqrt{5} - 1)(3\sqrt{2} + 4)$

d $(1 - 3\sqrt{7})(2 + 3\sqrt{2})$

e $(2 - \sqrt{3})(2 + \sqrt{3})$

f $(\sqrt{5} - 1)(\sqrt{5} + 1)$

g $(3\sqrt{2} + \sqrt{3})(3\sqrt{2} - \sqrt{3})$

h $(8\sqrt{2} + \sqrt{5})(8\sqrt{2} - \sqrt{5})$

i $(1 + \sqrt{2})^2$

j $(\sqrt{6} - 3)^2$

k $(2\sqrt{3} - 1)^2$

l $(\sqrt{2} + 2\sqrt{5})^2$

Rabbits and hares

Month	Jan	Feb	Mar	Apr	May	Jun
Rabbit population	500	1000	2000	4000	8000	16 000
Hare population	1	3	9	27	81	343

The population of rabbits and hares in a particular part of the countryside are booming because fox numbers are low. The population numbers in the table on the right were recorded at the beginning of January through to June.

Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate.

Preliminary task

a A population of a different colony of hares is modelled by the rule $P = 4^t$ where P is the population and t is in months. Find:

 i the population after 6 months

 ii the number of whole months for the population to reach at least 1000.

b A population of a different colony of rabbits is modelled by the rule $P = 200 \times 2^t$ where P is the population and t is in months. Find:

 i the population after 6 months

 ii the number of whole months for the population to reach at least 2000.

Modelling task

Formulate

a The problem is to determine the number of months required for the hare population to overtake the rabbit population. Write down all the relevant information that will help solve this problem.

b Copy and extend the given table showing the population of rabbits and hares in the first 12 months by continuing the pattern.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Rabbit population	500	1000	2000	4000	8000	16 000						
Hare population	1	3	9	27	81	343						

c The rule for the population of rabbits is of the form $P = 500 \times (?)^t$. State the missing number.

d The rule for the population of hares is of the form $P = (?)^t$. State the missing number.

Solve

e Use the rules outlined in parts **c** and **d** to confirm the population of:

 i rabbits initially (at $t = 0$) **ii** rabbits after 12 months **iii** hares after 12 months.

f Use the rules outlined in parts **c** and **d** to estimate the population of:

 i rabbits after 2 years **ii** hares after 3 years.

g Use your rules with a trial and error approach to determine the month in which the hare population overtakes the rabbit population.

Evaluate and verity

h Illustrate the increase in the population of rabbits and hares over two years by plotting a graph of the population of rabbits and hares. Use $0 \leqslant t \leqslant 12$ on the horizontal axis.

i If the rule for the population of hares is instead modelled by $P = 100 \times 2^t$, would there be a time when the hare population would overtake that of the rabbit population? Justify your response.

Communicate

j Summarise your results and describe any key findings.

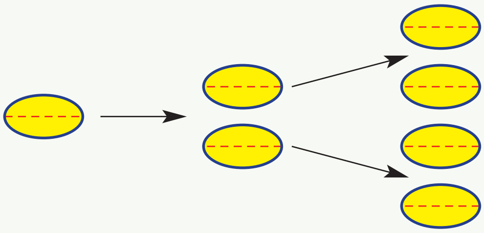
Extension question

a Assume that the initial population of rabbits could vary where the population of rabbits is modelled by the rule $P = k \times 2^t$. Determine possible whole number values of k for which the population of hares overtakes that of rabbits after:

 i 12 months **ii** 6 months **iii** 5 years.

Cell growth

Many cellular organisms reproduce by a process of subdivision called mitosis. A single cell, for example, may divide into two every hour as shown below. After 2 hours the single starting cell has become four:



Dividing into two

A single cell divides into two every hour.

a How many cells will there be after the following number of hours? Explain how you obtained your answers.

i 1 **ii** 2 **iii** 5

b Complete the table showing the number of cells after n hours.

n hours	0	1	2	3	4	5	6
Number of cells, N	1	2	4				
N in index form	2^0	2^1	2^2				

c Write a rule for the number of cells N after n hours.

d Use your rule from part **c** to find the number of cells after:

i 8 hours **ii** 12 hours **iii** 2 days.

e Find how long it takes for a single cell to divide into a total of:

i 128 cells **ii** 1024 cells **iii** 65 536 cells.

Dividing into three or more

a Complete a table similar to the table in the previous section for a cell that divides into 3 every hour.

b Write a rule for N in terms of n if a cell divides into 3 every hour, and then use the rule to find the number of cells after:

i 2 hours **ii** 4 hours **iii** 8 hours.

c Write a rule for N in terms of n if a single cell divides into the following number of cells every hour.

i 4 **ii** 5 **iii** 10

Cell cycle times

a If a single cell divides into two every 20 minutes, investigate how many cells there will be after 4 hours.

b If a single cell divides into three every 10 minutes, investigate how many cells there will be after 2 hours.

c Use the internet to research the cell cycle time and the types of division for at least two different types of cells. Describe the cells and explain the reproductive process.

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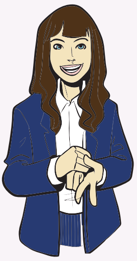
Problems and challenges

Up for a challenge? If you get stuck on a question, check out the 'Working with unfamiliar problems' poster at the end of the book to help you.



1 Determine the last digit of each of the following without using a calculator.

a 2^{222}

b 3^{300}

c 6^{87}

2 Determine the smallest value of n such that:

a $24n$ is a square number

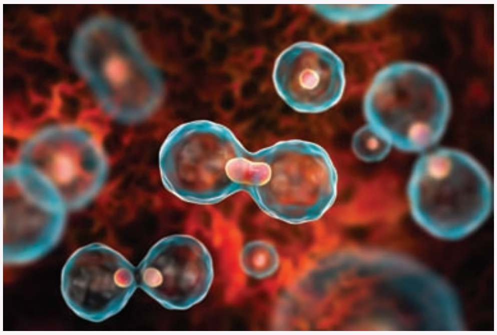
b $750n$ is a square number.

3 Simplify $\left(\dfrac{32}{243}\right)^{-\frac{2}{5}}$ without using a calculator.

4 If $2^x = t$, express the following in terms of t.

a 2^{2x+1} **b** 2^{1-x}

5 A single cell divides in two every 5 minutes and each new cell continues to divide every 5 minutes. How long does it take for the cell population to reach at least 1 million?



6 Find the value of x if $3^{3x-1} = \dfrac{1}{27}$.

7 a Write the following in index form.

i $\sqrt{2\sqrt{2}}$ **ii** $\sqrt{2\sqrt{2\sqrt{2}}}$ **iii** $\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}$

b What value do your answers to part **a** appear to be approaching?

8 Determine the highest power of 2 that divides exactly into 2000000.

9 Simplify these surds.

a $5\sqrt{8} - \sqrt{18}$ **b** $\dfrac{1}{\sqrt{2}} + \sqrt{2}$ **c** $(\sqrt{2} + 3\sqrt{5})^2 - (\sqrt{2} - 3\sqrt{5})^2$

10 Prove that:

a $\dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$ **b** $\dfrac{3}{\sqrt{3}} = \sqrt{3}$ **c** $\dfrac{1}{\sqrt{2} - 1} = \sqrt{2} + 1$

11 Solve for x. There are two solutions for each. (*Hint*: Let $a = 2^x$ in part **a**.)

a $2^{2x} - 3 \times 2^x + 2 = 0$ **b** $3^{2x} - 12 \times 3^x + 27 = 0$

12 Given that $2^4 \times 3^2 \times 5 = 720$, find the smallest whole number x so that $720x$ is a perfect cube.

13 Given that $4^y \times 9^x \times 27 = 4 \times 2^x \times 3^{2y}$, find the values of x and y.

Indices and surds

Index form

a^m ← index

base ↗

e.g. $2^3 = 2 \times 2 \times 2$

$3^4 = 3 \times 3 \times 3 \times 3$

$a \times b \times a \times b \times b = a^2b^3$

Index laws 1 and 2

Law 1: $a^m \times a^n = a^{m+n}$

When multiplying terms with the same base, add indices.

e.g. $x^3 \times x^5 = x^{3+5}$

$= x^8$

Law 2: $a^m \div a^n = a^{m-n}$

When dividing terms with the same base, subtract indices.

e.g. $\frac{x^7}{x^4} = x^{7-4}$

$= x^3$

Index law 3

Law 3: $(a^m)^n = a^{mn}$

To expand brackets, multiply indices.

e.g. $(x^2)^5 = x^{2\times 5}$

$= x^{10}$

Index laws 4 and 5

Law 4: $(ab)^m = a^mb^m$

e.g. $(3x^2)^3 = 3^3(x^2)^3$

$= 27x^6$

Law 5: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

e.g. $\left(\frac{x}{2}\right)^4 = \frac{x^4}{2^4}$

$= \frac{x^4}{16}$

Zero power

$a^0 = 1$

Any expression (except 0) to the power 0 equals 1.

e.g. $3x^0 = 3 \times 1$

$= 3$

$(3x)^0 = 1$

Negative indices

$a^{-m} = \frac{1}{a^m}$

e.g. $2x^{-3} = \frac{2}{1} \times \frac{1}{x^3}$

$= \frac{2}{x^3}$

$\frac{1}{a^{-m}} = a^m$

e.g. $\frac{1}{(2x)^{-2}} = (2x)^2$

$= 4x^2$

Fractional indices (Ext)

$a^{\frac{1}{n}} = \sqrt[n]{a}$

e.g. $a^{\frac{1}{2}} = \sqrt{a}$

$a^{\frac{1}{3}} = \sqrt[3]{a}$

e.g. $25^{\frac{1}{2}} = \sqrt{25} = 5$

since $5^2 = 25$

$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

since $2^3 = 8$

Scientific notation

In scientific notation, very large or very small numbers are written in the form $a \times 10^m$ where $1 \le a < 10$ or $-10 < a \le -1$. Large numbers will use positive powers of 10.

e.g. 2 350 000 kg = 2.35×10^6 kg

Small numbers will use negative powers of 10.

e.g. 16 nano seconds

= 0.000000016 s

= 1.6×10^{-8} seconds

Operations with surds (Ext)

'Like' surds can be added or subtracted.

$3\sqrt{7}$ and $5\sqrt{7}$ are 'like surds'

e.g. $3\sqrt{7} + 5\sqrt{7} = 8\sqrt{7}$

Multiply: $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

e.g. $\sqrt{3} \times \sqrt{10} = \sqrt{3 \times 10} = \sqrt{30}$

Divide: $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

e.g. $\frac{\sqrt{20}}{\sqrt{5}} = \sqrt{\frac{20}{5}}$

$= \sqrt{4} = 2$

Surds (Ext)

A surd is a number written with a root sign which has a decimal representation that is infinite and non-recurring with no pattern. e.g. $\sqrt{2} = 1.41421356\ldots$

ISBN 978-1-108-77288-4

Chapter checklist

Chapter checklist: Success criteria

		✔
6A	**1. I can write a power in expanded form.** e.g. Write $(ab)^3$ in expanded form.	
6A	**2. I can evaluate a power.** e.g. Write $(-4)^3$ in expanded form and then evaluate.	
6A	**3. I can write an expression in index form.** e.g. Write $6 \times x \times x \times y \times x \times y$ in index form.	
6A	**4. I can express a number in prime factor form.** e.g. Express 108 as a product of prime factors in index form.	
6B	**5. I can use index laws 1 and 2 with numerical bases.** e.g. Simplify $5^4 \times 5^3$ and $3^7 \div 3^4$, answering in index form.	
6B	**6. I can use index law 1.** e.g. Simplify $3a^2b \times 4a^4b^3$.	
6B	**7. I can use index law 2.** e.g. Simplify $\dfrac{18x^6y^4}{12xy^3}$.	
6B	**8. I can combine index laws 1 and 2.** e.g. Simplify $\dfrac{2a^5b^4 \times 6ab^2}{3a^4b^5}$.	
6C	**9. I can use index law 3.** e.g. Simplify $(m^6)^5$.	
6C	**10. I can use the zero power.** e.g. Evaluate 7^0 and $4x^0 - (5x)^0$.	
6D	**11. I can use index law 4.** e.g. Expand $4(m^3n)^5$ using the fourth index law.	
6D	**12. I can use index law 5.** e.g. Apply the fifth index law to $\left(\dfrac{-3}{b^5}\right)^4$.	
6D	**13. I can combine index laws.** e.g. Use index laws to simplify $\left(\dfrac{2}{a^0b^3}\right)^2 \times \dfrac{a^4(b^4)^3}{6ab^2}$.	
6E	**14. I can write expressions using only positive indices.** e.g. Express $2m^3n^{-2}$ and $\dfrac{5}{2x^{-3}}$ with positive indices only.	
6E	**15. I can evaluate negative powers without a calculator.** e.g. Express with positive powers and then evaluate 4^{-3} and $\dfrac{4}{3^{-2}}$.	
6F	**16. I can write numbers in scientific notation.** e.g. Write 123 000 and 0.00023 using scientific notation.	

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			✔
6F	**17. I can convert from scientific notation to decimal form.** e.g. Express 4.2×10^5 and 3.5×10^{-3} in decimal form.		
6G	**18. I can state the number of significant figures.** e.g. State the number of significant figures in 1027 and 0.024.		
6G	**19. I can write numbers in scientific notation using significant figures.** e.g. Write 1 738 212 in scientific notation using three significant figures.		
6G	**20. I can use a calculator with scientific notation.** e.g. Evaluate $7.2 \times 10^{-6} \times \sqrt{5.2 \times 10^3}$ using a calculator. Answer in scientific notation correct to three significant figures.		
6H	**21. I can express fractional indices using a root sign.** e.g. Write $10^{\frac{1}{3}}$ using a root sign.	Ext	
6H	**22. I can evaluate numbers with fractional indices.** e.g. Evaluate $81^{\frac{1}{2}}$.	Ext	
6H	**23. I can use index laws with fractional indices.** e.g. Simplify $\left(x^{\frac{1}{2}}\right)^3 \times x^{\frac{5}{2}}$.	Ext	
6I	**24. I can add and subtract like surds.** e.g. Simplify $4\sqrt{5} + 3\sqrt{5} - \sqrt{5}$.	Ext	
6I	**25. I can multiply and divide simple surds.** e.g. Simplify $\sqrt{5} \times \sqrt{6}$ and $\sqrt{14} \div \sqrt{7}$.	Ext	

Short-answer questions

6A **1** Express each of the following in index form.

a $3 \times 3 \times 3 \times 3$ **b** $2 \times x \times x \times x \times y \times y$

c $3 \times a \times a \times a \times \frac{b}{a} \times b$ **d** $\frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{1}{7} \times \frac{1}{7}$

6A **2** Write the following as a product of prime factors in index form.

a 45 **b** 300

6B **3** Simplify using index laws 1 and 2.

a $x^3 \times x^7$ **b** $2a^3b \times 6a^2b^5c$ **c** $3m^2n \times 8m^5n^3 \times \frac{1}{2}m^{-3}$

d $a^{12} \div a^3$ **e** $x^5y^3 \div (x^2y)$ **f** $\frac{5a^6b^3}{10a^8b}$

6C/D **4** Simplify:

a $(m^2)^3$ **b** $(3a^4)^2$ **c** $(-2a^2b)^5$

d $3a^0b$ **e** $2(3m)^0$ **f** $\left(\frac{a^2}{3}\right)^3$

6E **5** Express each of the following with positive indices.

a x^{-3} **b** $4t^{-3}$ **c** $(3t)^{-2}$

d $\frac{2}{3}x^2y^{-3}$ **e** $5\left(\frac{x^2}{y^{-1}}\right)^{-3}$ **f** $\frac{5}{m^{-3}}$

6E **6** Fully simplify each of the following.

a $\frac{5x^8y^{-12}}{x^{10}} \times \frac{(x^2y^5)^2}{10}$ **b** $\left(\frac{(3x)^0}{3x^0y^2}\right)^4 \times \frac{9y^{10}}{x^{-3}}$ **c** $\frac{(4m^2n^3)^2}{2m^5n^4} \div \frac{mn^5}{(m^3n^2)^3}$

6F **7** Arrange the following numbers in ascending order.

$2.35,\ 0.007 \times 10^2,\ 0.0012,\ 3.22 \times 10^{-1},\ 0.4,\ 35.4 \times 10^{-3}$

6F **8** Write the following numbers expressed using scientific notation in decimal form.

a 3.24×10^2 **b** 1.725×10^5 **c** 2.753×10^{-1} **d** 1.49×10^{-3}

6G **9** Write each of the following values using scientific notation with three significant figures.

a The population of the world for 2020 is projected to reach 7 758 156 792.

b The area of the USA is 9 629 091 km^2.

c The time taken for light to travel 1 metre (in a vacuum) is 0.00000000333564 seconds.

d The wavelength of ultraviolet light from a fluorescent lamp is 0.000000294 m.

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6F

10 Write each of the following values using scientific notation in the units given in brackets.

- **a** 25 years (hours). Assume 1 year = 365 days.
- **b** 12 milliseconds (seconds)
- **c** 432 nanoseconds (seconds)

6G

11 Use a calculator to evaluate the following, giving your answer using scientific notation correct to two significant figures.

- **a** $m_s \times m_e$ where m_s (mass of Sun) $= 1.989 \times 10^{30}$ kg and m_e (mass of Earth) $= 5.98 \times 10^{24}$ kg
- **b** The speed, v, in m/s of an object of mass $m = 2 \times 10^{-3}$ kg and kinetic energy $E = 1.88 \times 10^{-12}$ joules, where $v = \sqrt{\dfrac{2E}{m}}$

6H Ext

12 Evaluate without using a calculator.

- **a** $\sqrt[4]{16}$
- **b** $\sqrt[3]{125}$
- **c** $49^{\frac{1}{2}}$
- **d** $81^{\frac{1}{4}}$
- **e** $27^{-\frac{1}{3}}$
- **f** $121^{-\frac{1}{2}}$

6H Ext

13 Simplify the following expressing all answers in positive index form.

- **a** $(s^6)^{\frac{1}{3}}$
- **b** $3x^{\frac{1}{2}} \times 5x^2$
- **c** $(3m^{\frac{1}{2}}n^2)^2 \times m^{-\frac{1}{4}}$
- **d** $\dfrac{4}{a^{\frac{1}{3}}} \times \dfrac{(a^{\frac{1}{2}})^4}{a}$

6I Ext

14 Simplify the following operations with surds.

- **a** $8\sqrt{7} - \sqrt{7} + 2$
- **b** $2\sqrt{3} + 5\sqrt{2} - \sqrt{3} + 4\sqrt{2}$
- **c** $\sqrt{8} \times \sqrt{8}$
- **d** $\sqrt{5} \times \sqrt{3}$
- **e** $2\sqrt{7} \times \sqrt{2}$
- **f** $3\sqrt{2} \times 5\sqrt{11}$
- **g** $\sqrt{42} \div \sqrt{7}$
- **h** $2\sqrt{75} \div \sqrt{3}$
- **i** $\sqrt{50} \div (2\sqrt{10})$

Multiple-choice questions

6B

1 $3x^7 \times 4x^4$ is equivalent to:

- **A** $12x^7$
- **B** $12x^{28}$
- **C** $7x^{11}$
- **D** $12x^{11}$
- **E** $7x^3$

6C

2 $3(2y^2)^0$ simplifies to:

- **A** 6
- **B** 3
- **C** $6y^2$
- **D** $3y$
- **E** 12

ISBN 978-1-108-77288-4

6D

3 $(2x^2)^3$ expands to:

A $2x^5$ **B** $2x^6$ **C** $6x^6$ **D** $8x^5$ **E** $8x^6$

6B

4 $2x^3y \times \dfrac{x^5y^2}{4x^2y}$ simplifies to:

A $\dfrac{x^6y^2}{2}$ **B** $2x^8y$ **C** $2x^6y^2$ **D** $\dfrac{x^4y^2}{2}$ **E** $8x^6y$

6D

5 $\left(\dfrac{-x^2y}{3z^4}\right)^3$ is equal to:

A $\dfrac{x^6y^3}{3z^{12}}$ **B** $\dfrac{-x^5y^4}{9z^7}$ **C** $\dfrac{-x^6y^3}{27z^{12}}$ **D** $\dfrac{-x^2y^3}{3z^{12}}$ **E** $\dfrac{x^6y^3}{9z^{12}}$

6E

6 $2x^{-3}y^4$ expressed with positive indices is:

A $\dfrac{y^4}{2x^3}$ **B** $\dfrac{2y^4}{x^3}$ **C** $-2x^3y^4$ **D** $\dfrac{2}{x^3y^4}$ **E** $\dfrac{y^4}{8x^3}$

6E

7 $\dfrac{3}{(2x)^{-2}}$ is equivalent to:

A $\dfrac{-3}{(2x)^2}$ **B** $6x^2$ **C** $\dfrac{3x^2}{2}$ **D** $12x^2$ **E** $\dfrac{-3x^2}{4}$

6F

8 The weight of a cargo crate is 2.32×10^4 kg. In expanded form this weight in kilograms is:

A 2 320 000 **B** 232 **C** 23 200 **D** 0.000232 **E** 2320

6G

9 0.00032761 using scientific notation rounded to three significant figures is:

A 328×10^{-5} **B** 3.27×10^{-4} **C** 3.28×10^4 **D** 3.30×10^4 **E** 3.28×10^{-4}

6H Ext

10 $36^{\frac{1}{2}}$ is equal to:

A 18 **B** 6 **C** 1296 **D** 9 **E** 81

6I Ext

11 The simplified form of $2\sqrt{7} - 3 + 4\sqrt{7}$ is:

A $-2\sqrt{7} - 3$ **B** $3\sqrt{7}$ **C** $6\sqrt{7} - 3$ **D** $\sqrt{7}$ **E** $8\sqrt{7} - 3$

6I Ext

12 $\sqrt{3} \times \sqrt{7}$ is equivalent to:

A $\sqrt{21}$ **B** $\sqrt{10}$ **C** $2\sqrt{10}$ **D** $10\sqrt{21}$ **E** $21\sqrt{10}$

Extended-response questions

1 Simplify each of the following, expressing answers with positive indices, using a combination of index laws.

a $\dfrac{(4x^2y)^3 \times x^2y}{12(xy^2)^2}$

b $\dfrac{2a^3b^4}{(5a^3)^2} \times \dfrac{20a}{3b^{-4}}$

c $\dfrac{(5m^4n^{-3})^2}{m^{-1}n^2} \div \dfrac{5(m^{-1}n)^{-2}}{mn^{-4}}$

Ext **d** $\dfrac{(8x^4)^{\frac{1}{3}}}{2(y^3)^0} \times \dfrac{(3x^{\frac{1}{3}})^2}{3(x^2)^{\frac{1}{2}}}$

2 The law of gravitational force is given by $F = \frac{Gm_1m_2}{d^2}$ where F is the magnitude of the gravitational force (in newtons, N) between two objects of mass m_1 and m_2 (in kilograms) a distance d (metres) apart. G is the universal gravitational constant, which is approximately 6.67×10^{-11} N m^2 kg^{-2}.

a If two objects of masses 2 kg and 4 kg are 3 m apart, calculate the gravitational force F between them. Answer using scientific notation correct to three significant figures.

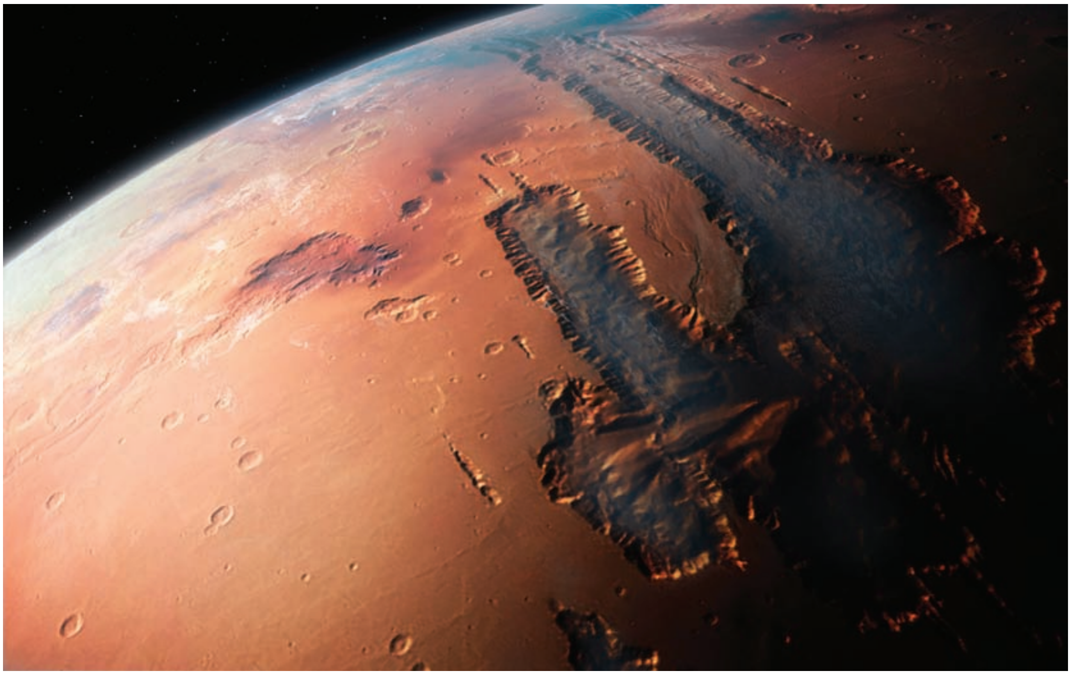
b The average distance between Earth and the Sun is approximately 149 597 870 700 m.

i Write this distance using scientific notation with three significant figures.

ii Hence, if the mass of Earth is approximately 5.98×10^{24} kg and the mass of the Sun is approximately 1.99×10^{30} kg, calculate the gravitational force between them using scientific notation to two significant figures.

c The universal gravitational constant, G, is constant throughout the universe. However, acceleration due to gravity (a, units m/s^2) varies according to where you are in the solar system. Using the formula $a = \frac{Gm}{r^2}$ and the following table, work out and compare the acceleration due to gravity on Earth and on Mars. Answer to three significant figures.

Planet	Mass, m	Radius, r
Earth	5.98×10^{24} kg	6.375×10 m
Mars	6.42×10^{23} kg	3.37×10^6 m



CHAPTER 7 Geometry

Fortifications of Vauban

The Fortifications of Sebastian Le Prestre de Vauban (1633–1707) is a collection of 12 fortified buildings in France. It is listed on the UNESCO (United Nations Educational, Scientific and Cultural Organisation) world heritage list.

Vauban was a military engineer of King Louis XIV. As an engineer and cadet, he had first-hand experience of the needs of a fortress and of a fortified city. The picture shown here is of the Citadel of Neuf-Brisach, in France. Its octagonal layout, which extends out to form the shape of a star, can be seen from this aerial view. The city is divided into 48 blocks, with the most important buildings located in the centre. Vauban understood the symmetry of octagons and squares and how they could be related to each other.

ISBN 978-1-108-77288-4

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

In this chapter

7A Angles and triangles (CONSOLIDATING)
7B Parallel lines (CONSOLIDATING)
7C Quadrilaterals and other polygons
7D Congruent triangles
7E Using congruence in proof (EXTENDING)
7F Enlargement and similar figures
7G Similar triangles
7H Proving and applying similar triangles

Australian Curriculum

MEASUREMENT AND GEOMETRY
Geometric reasoning

Use the enlargement transformation to explain similarity and develop the conditions for triangles to be similar (ACMMG220)

Solve problems using ratio and scale factors in similar figures (ACMMG221)

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The designs of Vauban, and the star fortress design, have been used by architects since to improve the working of forts. Vauban's designs show an understanding not just of the workings of geometry but how geometry could be used to ensure his fortifications could withstand attack.

7A Angles and triangles CONSOLIDATING

Learning intentions
- To review types of angles and triangles
- To be able to work with complementary and supplementary angles, vertically opposite angles and angles in a revolution
- To be able to use the angle sum and other triangle properties to find unknown angles
- To know and be able to apply the exterior angle theorem

Geometry is all around us. The properties of the shape of a window, doorway or roofline depend on their geometry. When lines meet, angles are formed and it is these angles which help define the shape of an object. Fundamental to geometry are the angles formed at a point and the three angles in a triangle. Lines meeting at a point and triangles have special properties and will be revised in this section.

Geometry is an essential tool for architects, builders, navigators, astronomers, engineers and surveyors. The geometry of triangles has been applied by the architect of this 50 m high chimney at Greenwich, London.

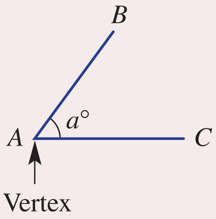
LESSON STARTER Impossible triangles

Triangles are classified either by their side lengths or by their angles.

- First, write the list of three triangles which are classified by their side lengths and the three triangles that are classified by their angles.
- Now try to draw a triangle for the following descriptions. Decide which are possible and which are impossible.
  - Acute scalene triangle
  - Right equilateral triangle
  - Right isosceles triangle
  - Obtuse isosceles triangle
  - Obtuse scalene triangle
  - Acute equilateral triangle

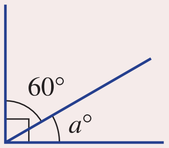
KEY IDEAS

- When two **rays**, **lines** or **line segments** meet at a point, an **angle** is formed.
  - This angle is named $\angle A$ or $\angle BAC$ or $\angle CAB$.
  - The size of this angle in degrees is a°.



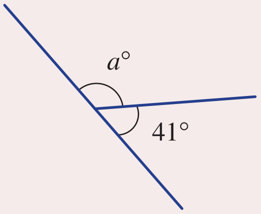
- Angle types
  - **Acute** between 0° and 90°
  - **Obtuse** between 90° and 180°
  - **Reflex** between 180° and 360°
  - **Right** 90°
  - **Straight** 180°
  - **Revolution** 360°

- Angles at a point
  - **Complementary**
    (sum to 90°)



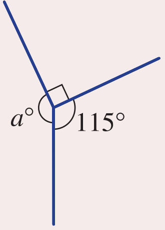
$a + 60 = 90$

  - **Supplementary**
    (sum to 180°)



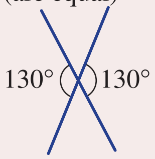
$a + 41 = 180$

  - **Revolution**
    (sum to 360°)

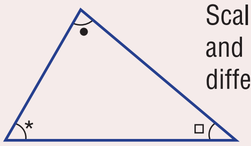
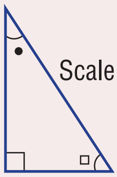
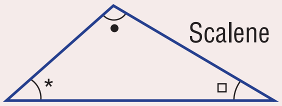
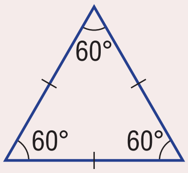
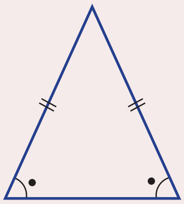
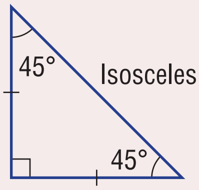
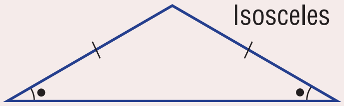


$a + 90 + 115 = 360$

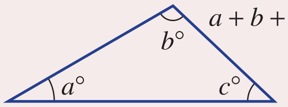
  - **Vertically opposite**
    (are equal)



- Types of triangles

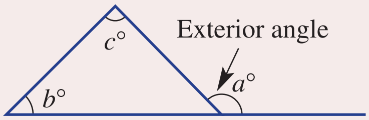
Acute angled (all angles < 90°)	Right-angled (includes a 90° angle)	Obtuse angled (one angle > 90°)
 Scalene (all sides and angles are different sizes)	 Scalene	 Scalene
 Equilateral (all angles 60° and all sides equal)  Isosceles (two angles equal and two sides equal)	 Isosceles	 Isosceles

- The sum of the angles in a triangle is 180°.



$a + b + c = 180$

- An **exterior angle** is formed by extending one side of a shape.
  - **Exterior angle theorem of a triangle**: The exterior angle of a triangle is equal to the sum of the two opposite interior angles.



$a = b + c$

BUILDING UNDERSTANDING

1 Choose a word or number to complete each sentence.

a A 90° angle is called a ______ angle.

b A ______ angle is called a straight angle.

c A 360° angle is called a ______.

d ______ angles are between 90° and 180°.

e ______ angles are between 0° and 90°.

f Reflex angles are between ______ and 360°.

g Complementary angles sum to ______.

h ____________ angles sum to 180°.

i The three angles in a triangle sum to _____.

j Vertically opposite angles are ______.

2 Name the type of triangle that has:

a a pair of equal length sides

b one obtuse angle

c all angles 60°

d one pair of equal angles

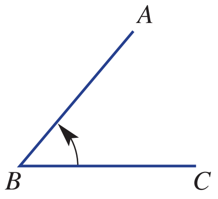
e all angles acute

f all sides of different length

g one right angle.

3 Name the type of each angle, and then estimate the size of the angle and use your protractor to determine an accurate measurement.

a



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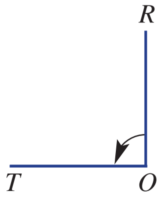
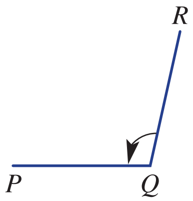


c



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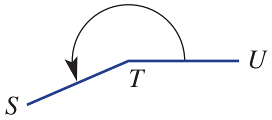

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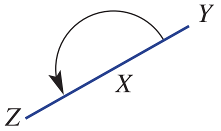

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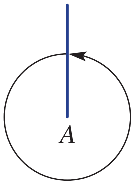


4 State whether each of the following pairs of angles is supplementary (S), complementary (C) or neither (N).

a 30°, 60°

b 45°, 135°

c 100°, 90°

d 50°, 40°

Example 1 Finding supplementary and complementary angles

For an angle of size 47° determine the size of its:

a supplementary angle

b complementary angle.

SOLUTION	EXPLANATION
a 180° − 47° = 133°	Supplementary angles sum to 180°.
b 90° − 47° = 43°	Complementary angles sum to 90°.

Now you try

For an angle of size 72° determine the size of its:

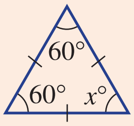
a supplementary angle

b complementary angle.

Example 2 Finding unknown angles in triangles

Name the types of triangles shown here and determine the values of the pronumerals.

a



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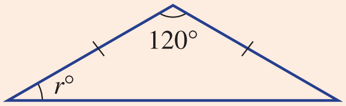

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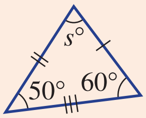
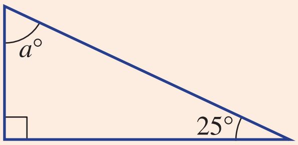


SOLUTION	EXPLANATION
a Equilateral triangle $x = 60$	All sides are equal, therefore all angles are equal.
b Obtuse isosceles triangle $2r + 120 = 180$ $2r = 60$ $r = 30$	One angle is more than 90° and two sides are equal. Angles in a triangle add to 180°. Since the triangle is isosceles, both base angles are $r°$. Subtract 120° from both sides and then divide both sides by 2.
c Acute scalene triangle $s + 50 + 60 = 180$ $s + 110 = 180$ $s = 70$	All angles are less than 90° and all sides are of different length. Angles in a triangle add to 180°. Simplify and solve for s.

Now you try

Name the types of triangles shown here and determine the values of the pronumerals.

a



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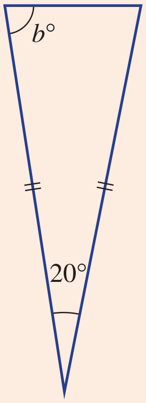

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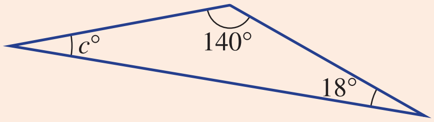
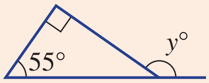


Example 3 Finding exterior and other angles

Find the value of each pronumeral. Give reasons for your answers.

a



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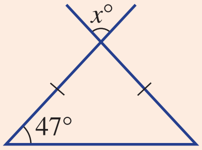


SOLUTION

a $y = 90 + 55$
$= 145$

Use the exterior angle theorem, which states that the exterior angle is equal to the sum of the two opposite interior angles.

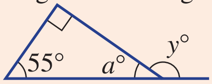
Alternative method:

Let a be the unknown angle.

$$\begin{aligned} a + 90 + 55 &= 180 \text{ (angle sum)} \\ a &= 35 \\ y + 35 &= 180 \\ y &= 145 \end{aligned}$$

Alternatively:

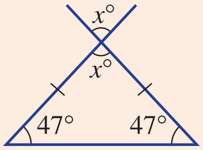
Angles in a triangle sum to 180°.



Angles in a straight line are supplementary (sum to 180°).

b

$$\begin{aligned} x + 47 + 47 &= 180 \text{ (angle sum)} \\ x + 94 &= 180 \\ x &= 86 \end{aligned}$$



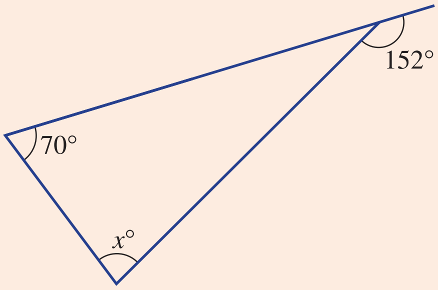
Note the isosceles triangle and vertically opposite angles.

Angles in a triangle add to 180° and vertically opposite angles are equal. Simplify and solve for x.

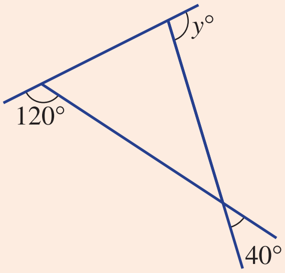
Now you try

Find the value of each pronumeral. Give reasons for your answers.

a



b



Exercise 7A

FLUENCY	1, 2–4(1/2)	2–5(1/2)	2–5(1/3)

Example 1

1 For an angle of size 29° determine the size of its:

a supplementary angle **b** complementary angle.

Example 1

2 For each of the following angle sizes determine the size of:

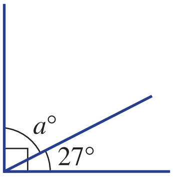
i the supplementary angle

ii the complementary angle.

a 55° **b** 74° **c** 89° **d** 22° **e** 38° **f** 47°

3 Find the value of a in these diagrams. Each diagram contains complementary or supplementary angles, or angles in a revolution.

a



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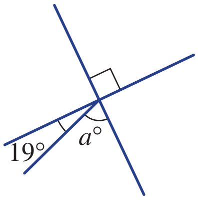
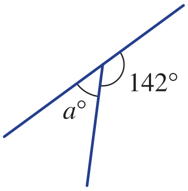
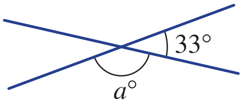


c



d



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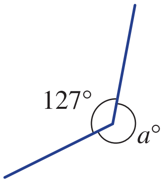

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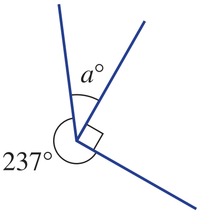
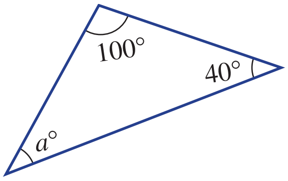


Example 2

4 Determine the values of the pronumerals and name the types of triangles shown here.

a



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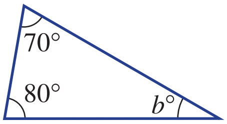

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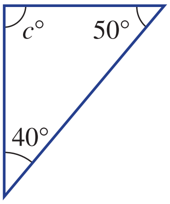

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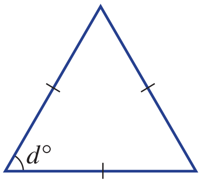

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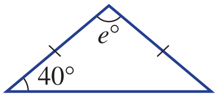

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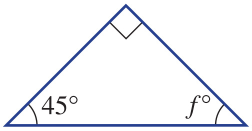
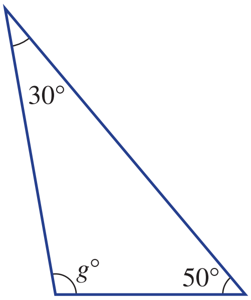
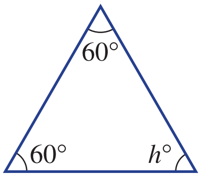


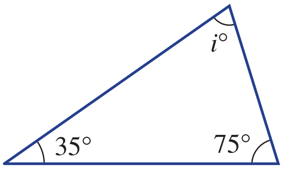
g



h



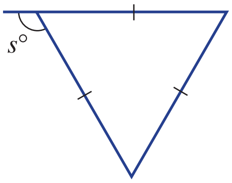
i



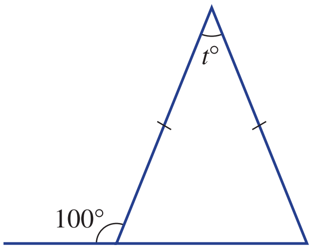
Example 3

5 Find the value of each pronumeral.

a



b



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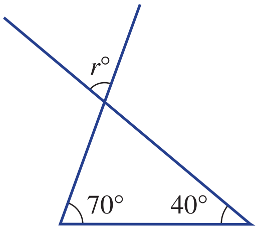

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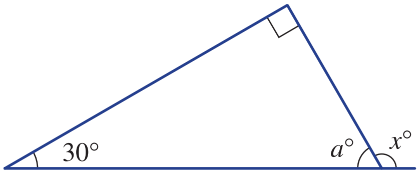

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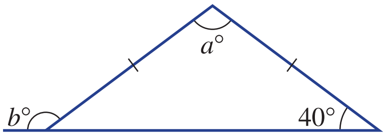

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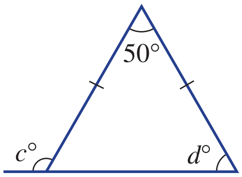


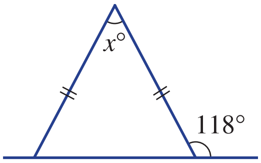
PROBLEM-SOLVING	6, 7	6(½), 7, 8(1/2)	7, 8(1/2)

6 Calculate how many degrees the minute hand of a clock rotates in:

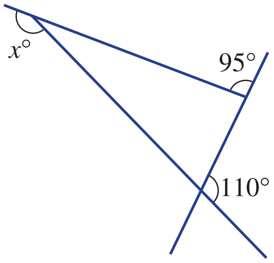
a 1 hour
b $\frac{1}{4}$ of an hour
c 10 minutes
d 15 minutes
e 72 minutes
f 1 minute
g 2 hours
h 1 day.

7 Find the value of x in these diagrams.

a



b



8 Find the acute or obtuse angle between the hour and minute hands at these times. Remember to consider how the hour hand moves between each whole number.

a 3:00 p.m.
b 5:00 a.m.
c 6:30 p.m.
d 11:30 p.m.
e 3:45 a.m.
f 1:20 a.m.
g 4:55 a.m.
h 2:42 a.m.
i 9:27 a.m.

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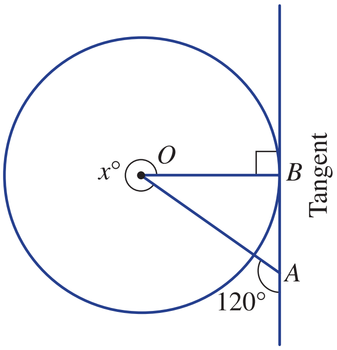
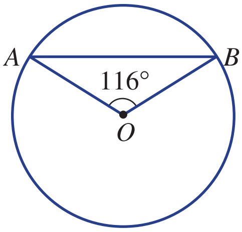
REASONING	9, 10	9, 10	10–12

9 A tangent to a circle is 90° to its radius. Explain why $x = 330$ in this diagram.



10 Explain why $\angle OAB$ is 32° in this circle. O is the centre of the circle.

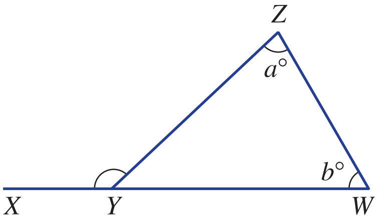


11 In this diagram, $\angle XYZ$ is an exterior angle. *Do not* use the exterior angle theorem in the following.

a If $a = 85$ and $b = 75$, find $\angle XYZ$.

b If $a = 105$ and $b = 60$, find $\angle XYZ$.

c Now using the pronumerals a and b, prove that $\angle XYZ = a° + b°$.

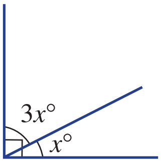


12 Prove that the three exterior angles of a triangle sum to 360°. Use the fact that the three interior angles sum to 180°.

ENRICHMENT: Algebra in geometry	–	–	13

13 Write an equation for each diagram and solve it to find x.

a



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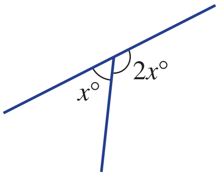

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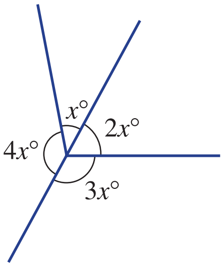

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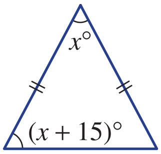
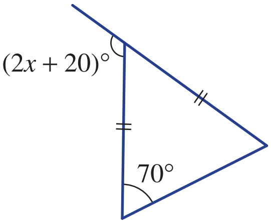


e



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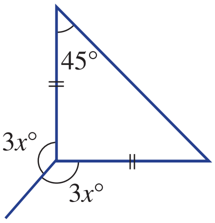

ISBN 978-1-108-77288-4

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7B Parallel lines CONSOLIDATING

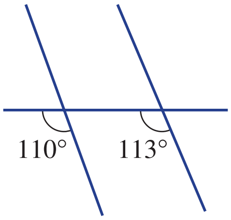
Learning intentions
- To review the types of angles formed by parallel lines and a transversal
- To be able to decide if two lines are parallel based on angle properties
- To be able to find unknown angles in parallel lines, including by adding an extra parallel line

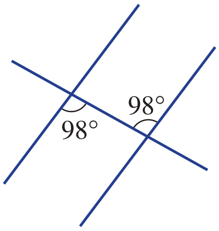
A line crossing two or more other lines (called a transversal) creates a number of special pairs of angles. If the transversal cuts two parallel lines, then these special pairs of angles will be either equal or supplementary.

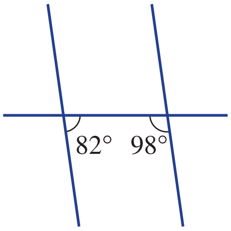
A piano has parallel keys, hammers and strings; however, a violin's strings are not parallel. Musicians write music using two sets of parallel lines: the treble clef (higher notes) and bass clef (lower notes).

LESSON STARTER Are they parallel?

Here are three diagrams in which a transversal crosses two other lines.

1


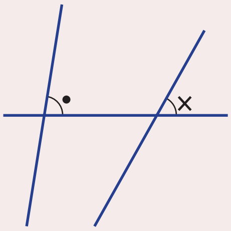
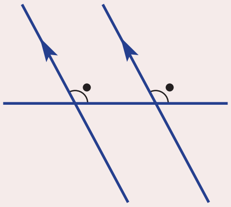
2


3


- Decide if each diagram contains a pair of parallel lines. Give reasons for your answer.
- What words do you remember regarding the name given to each pair of angles shown in the diagrams?

KEY IDEAS

- A **transversal** is a line crossing two or more other lines.

Pair of angles	Non-parallel lines	Parallel lines
Corresponding angles • If lines are parallel corresponding angles are equal.		

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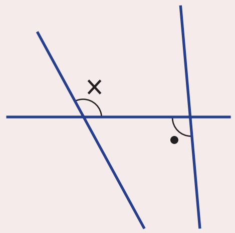
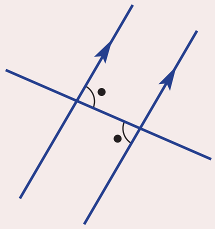
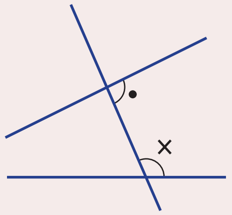
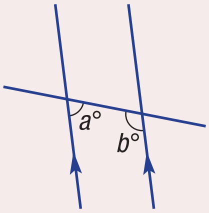
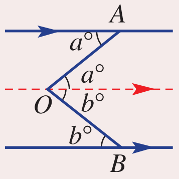
Pair of angles	Non-parallel lines	Parallel lines
Alternate angles • If lines are parallel alternate angles are equal.		
Cointerior angles • If lines are parallel cointerior angles are supplementary.		 $a + b = 180$

- If a line AB is parallel to a line CD we write $AB \parallel CD$.
- A parallel line can be added to a diagram to help find other angles.



$\angle AOB = a^\circ + b^\circ$

BUILDING UNDERSTANDING

① Choose the word *equal* or *supplementary* to complete these sentences.

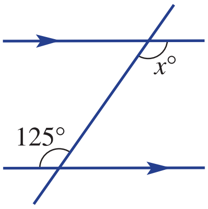
a If two lines are parallel corresponding angles are _________.

b If two lines are parallel alternate angles are _________.

c If two lines are parallel cointerior angles are _________.

② State the values of the pronumerals in these parallel lines. Give reasons for your answers.

a



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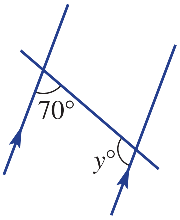

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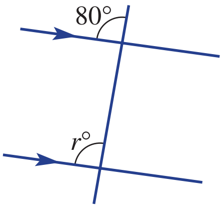

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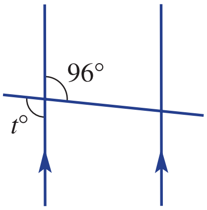

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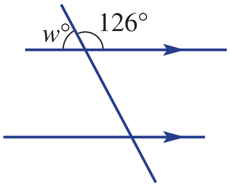

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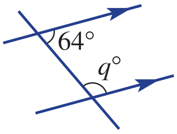
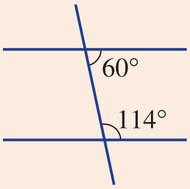


Example 4 Deciding if lines are parallel

Decide if each diagram contains a pair of parallel lines. Give a reason.

a



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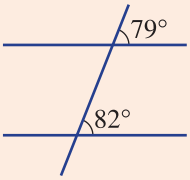

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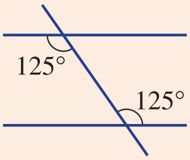
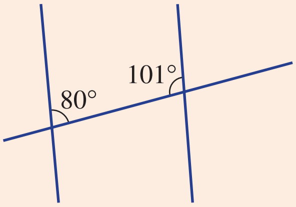


SOLUTION	EXPLANATION
a No. The two cointerior angles are not supplementary.	$60^\circ + 114^\circ = 174^\circ \neq 180^\circ$
b No. The two corresponding angles are not equal.	$79^\circ \neq 82^\circ$
c Yes. The two alternate angles are equal.	If alternate angles are equal then the lines are parallel.

Now you try

Decide if each diagram contains a pair of parallel lines. Give a reason.

a



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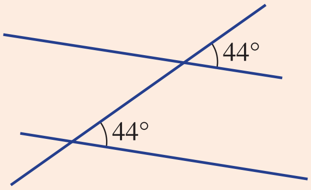

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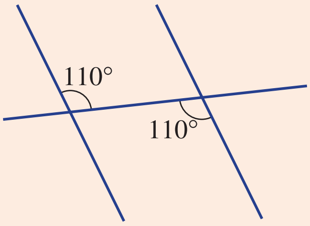
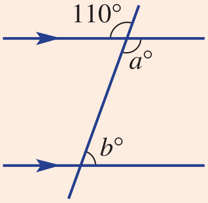


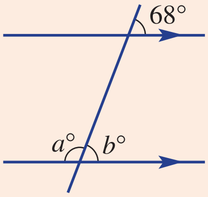
Example 5 Finding angles in parallel lines

Find the value of each of the pronumerals. Give reasons for your answers.

a



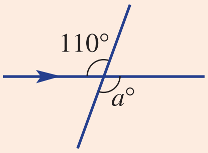
b



SOLUTION

a $a = 110$ (vertically opposite angles)

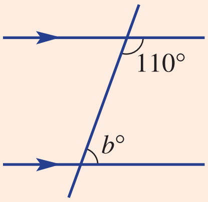
EXPLANATION



Vertically opposite angles are equal.

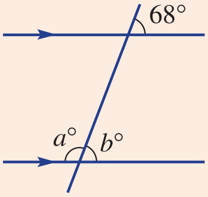
$b + 110 = 180$ (cointerior angles in parallel lines)

$b = 70$



Cointerior angles add to 180° in parallel lines.

b $b = 68$ (corresponding angles in parallel lines)



Corresponding angles are equal in parallel lines.

$a + 68 = 180$ (supplementary angles)

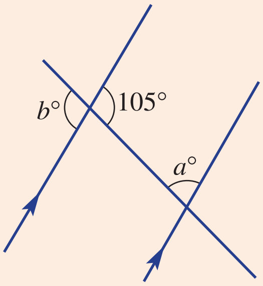
$a = 112$

Supplementary angles add to 180° so $a + b = 180$.

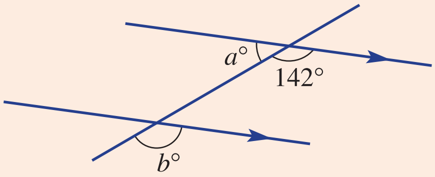
Now you try

Find the value of each of the pronumerals. Give reasons for your answers.

a

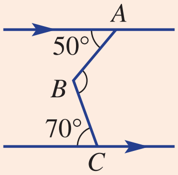


b

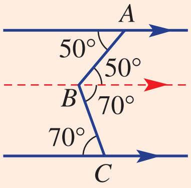


Example 6 Adding a third parallel line

Add a third parallel line to help find ∠ABC in this diagram.



SOLUTION



$$\angle ABC = 50^\circ + 70^\circ$$
$$= 120^\circ$$

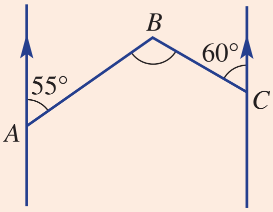
EXPLANATION

Add a third parallel line through B to create two pairs of equal alternate angles.

Add 50° and 70° to give the size of ∠ABC.

Now you try

Add a third parallel line to help find ∠ABC in this diagram.



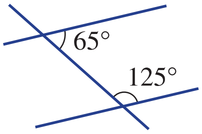
Exercise 7B

FLUENCY	1, 2–3(1/2)	2–3(1/2)	2–3(1/2)

Example 4

1 Decide if each diagram contains a pair of parallel lines. Give a reason.

a



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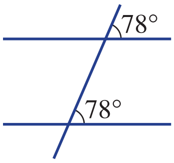
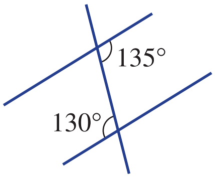


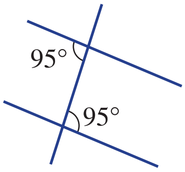
c



Example 4

2 Decide if each diagram contains a pair of parallel lines. Give a reason.

a



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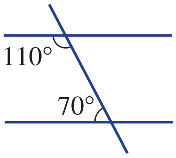

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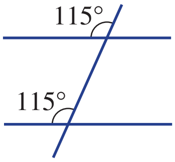

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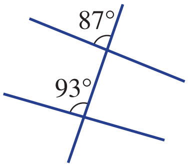

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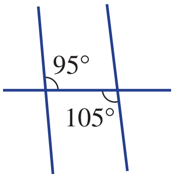

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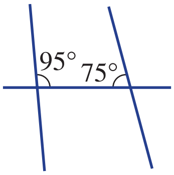

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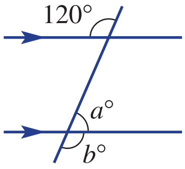
Example 5

3 Find the value of each pronumeral. Give reasons for your answers.

a



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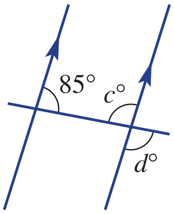

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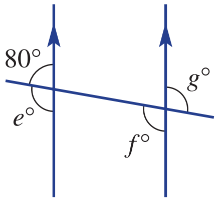
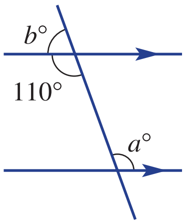
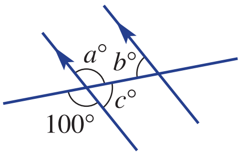


d



e



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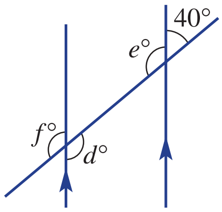
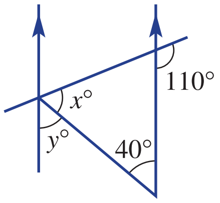


PROBLEM-SOLVING

4	4–5(½)	4–5(½), 6

4 Find the value of each pronumeral.

a



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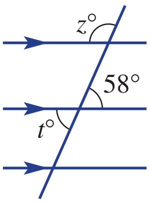

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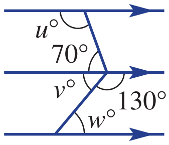

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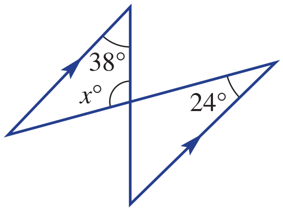

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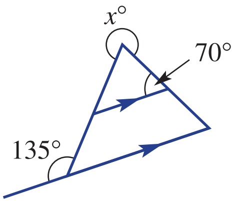

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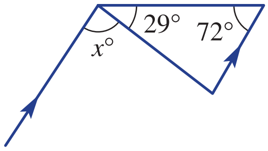
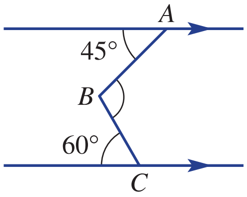


Example 6

5 Add a third parallel line to help find ∠ABC in these diagrams.

a



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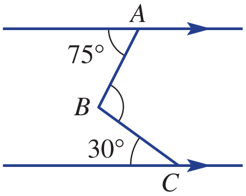
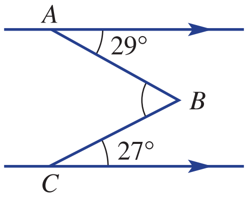


c



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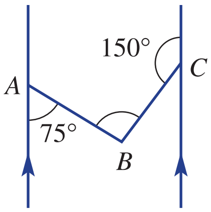

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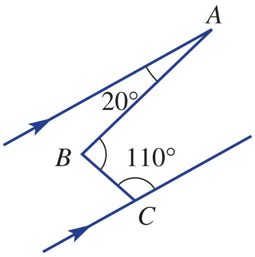

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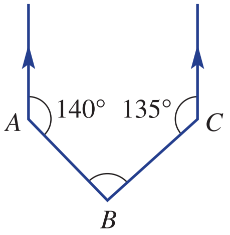

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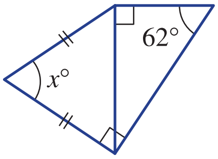
6 Find the value of x.

a



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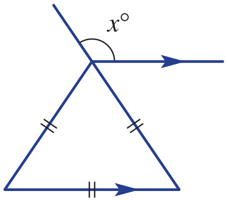

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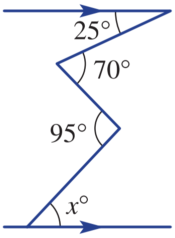
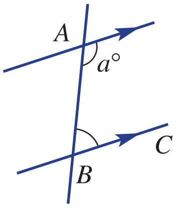


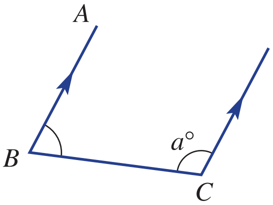
REASONING	7	7(½), 8	7(½), 8, 9

7 Write an expression (for example, $180° - a°$) for $\angle ABC$ in these diagrams.

a



b



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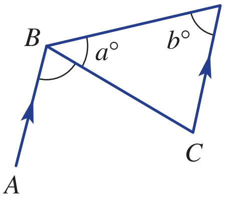

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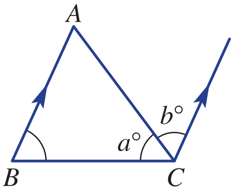
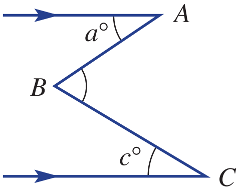


e



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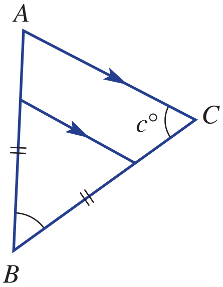
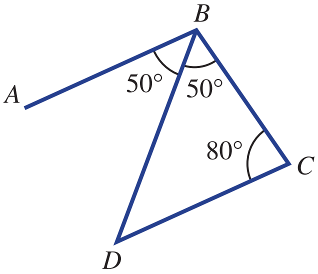


8 Give reasons why $AB \parallel DC$ (AB is parallel to DC) in this diagram.

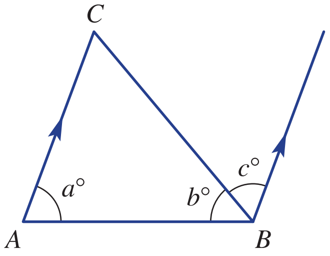


9 The diagram below includes a triangle and a pair of parallel lines.

a Using the parallel lines, explain why $a + b + c = 180$.

b Explain why $\angle ACB = c°$.

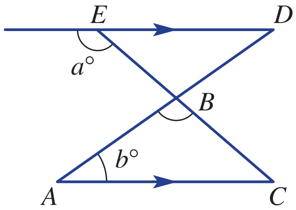
c Explain why this diagram helps to prove that the angle sum of a triangle is 180°.



ENRICHMENT: Proof in geometry

–	–	10

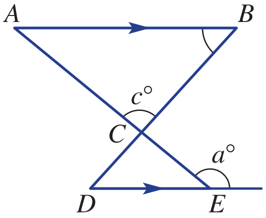
10 Here is a written proof showing that $\angle ABC = a° - b°$.



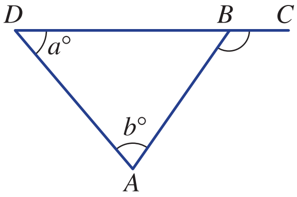
$\angle BED = 180° - a°$ (Supplementary angles)
$\angle BCA = 180° - a°$ (Alternate angles and $ED \parallel AC$)
$\angle ABC = 180° - b° - (180 - a)°$ (Angle sum of a triangle)
$= 180° - b° - 180° + a°$
$= -b° + a°$
$= a° - b°$

Write a similar proof for each of the following angles.

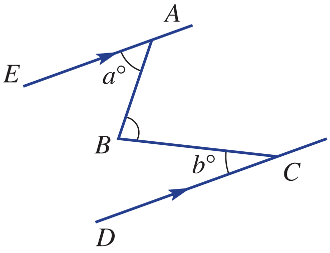
a $\angle ABC = a° - c°$



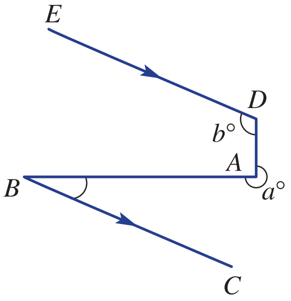
b $\angle ABC = a° + b°$



c $\angle ABC = a° + b°$



d $\angle ABC = 180° + b° - a°$

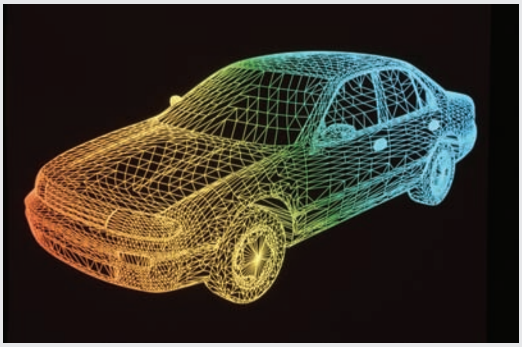


7C Quadrilaterals and other polygons

Learning intentions

- To know how to find the interior angle sum of a polygon
- To know terms related to polygons such as regular, convex and non-convex
- To know the properties of different types of quadrilaterals
- To be able to use the angle sum and polygon properties to find the value of unknown angles

Closed two-dimensional shapes with straight sides are called polygons and are classified by their number of sides. Quadrilaterals have four sides and are classified further by their special properties.

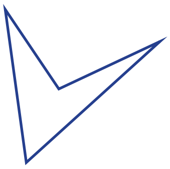


Computer programmers code 3D objects using a polygon mesh of triangles and quadrilaterals. Working with computer-aided design and machine learning, engineers can test and adjust the aerodynamic properties of a new car model before it is built.

LESSON STARTER Draw that shape

Use your knowledge of polygons to draw each of the following shapes. Mark any features, including parallel sides and sides of equal length.

- convex quadrilateral
- non-convex pentagon
- regular hexagon
- square, rectangle, rhombus and parallelogram
- kite and trapezium

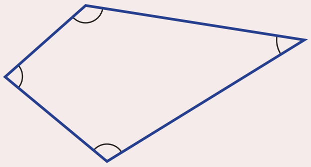


This is a non-convex quadrilateral.

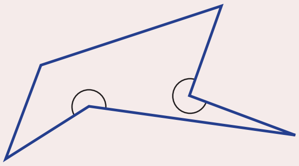
Compare the properties of each shape to ensure you have indicated each property on your drawings.

KEY IDEAS

- **Convex polygons** have all interior angles less than 180°. A **non-convex polygon** has at least one interior angle greater than 180°.



Convex quadrilateral



Non-convex hexagon

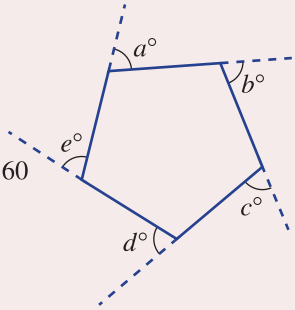
ISBN 978-1-108-77288-4

Updated September 2021

- **The sum of the interior angles**, S°, in a polygon with n sides is given by $S = 180(n - 2)$.

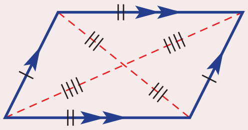
Polygon	Number of sides (n)	Angle sum (S)
Triangle	3	180°
Quadrilateral	4	360°
Pentagon	5	540°
Hexagon	6	720°
Heptagon	7	900°
Octagon	8	1080°
Nonagon	9	1260°
Decagon	10	1440°
Undecagon	11	1620°
Dodecagon	12	1800°
n-gon	n	$180(n - 2)^\circ$

- **Regular polygons** have sides of equal length and equal interior angles.
- The sum of all the exterior angles of every polygon is 360°.
  - This example shows a pentagon but the 360° exterior sum is true for all polygons.

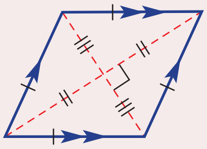


$a + b + c + d + e = 360$

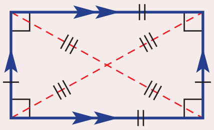
- **Parallelograms** are **quadrilaterals** with two pairs of parallel sides. They include:
  - Parallelogram: a quadrilateral with two pairs of parallel sides



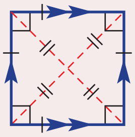
  - Rhombus: a parallelogram with all sides equal



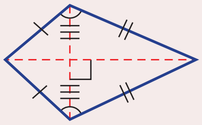
  - Rectangle: a parallelogram with all angles 90°



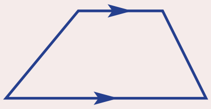
  - Square: a rhombus with all angles 90°.



- The kite and trapezium are also special quadrilaterals.
  - Kite



  - Trapezium



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BUILDING UNDERSTANDING

1 How many sides do these polygons have?

a Pentagon **b** Heptagon **c** Quadrilateral
d Undecagon **e** Nonagon **f** Dodecagon

2 Use $S = 180(n - 2)$ to find the angle sum, S, of these polygons, where n is the number of sides.

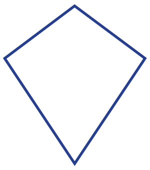
a Hexagon **b** Octagon **c** Undecagon

3 Choose the word to complete these sentences.

a A parallelogram has two pairs of ________ sides.
b The diagonals in a rhombus intersect at ________ angles.
c A ________ has one pair of parallel sides.
d The diagonals in a rectangle are ________ in length.

4 Name each of these shapes and describe them as convex or non convex.

a



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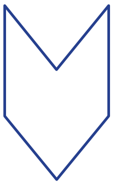

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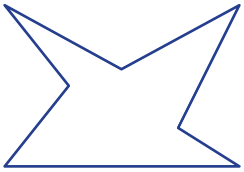
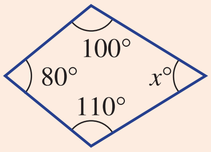


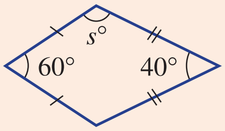
Example 7 Finding angles in quadrilaterals

Find the value of the pronumeral in each of these quadrilaterals.

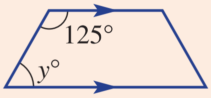
a

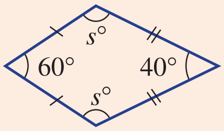


b



c

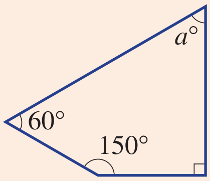


SOLUTION	EXPLANATION
a $x + 80 + 100 + 110 = 360$ $x + 290 = 360$ $x = 70$	The angles in a quadrilateral add to 360°. Simplify and solve for x.
b $2s + 60 + 40 = 360$ $2s + 100 = 360$ $2s = 260$ $s = 130$	 The angles in a quadrilateral add to 360° and the opposite angles ($s°$) are equal in a kite. Simplify and solve for s.
c $y + 125 = 180$ $y = 55$	Cointerior angles inside parallel lines are supplementary.

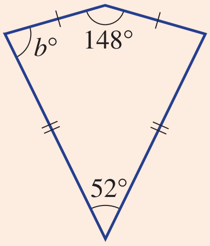
Now you try

Find the value of the pronumeral in each of these quadrilaterals.

a



b



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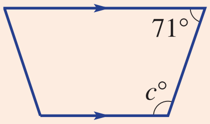
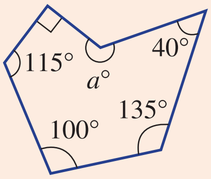


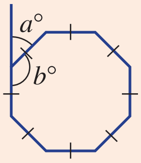
Example 8 Finding angles in polygons

For each polygon, find the angle sum using $S = 180(n - 2)$ and then find the value of any pronumerals. The polygon in part **b** is regular.

a



b



SOLUTION

a

$$\begin{aligned} n = 6 \text{ and } S &= 180(n - 2) \\ &= 180(6 - 2) \\ &= 720 \end{aligned}$$

$$\begin{aligned} a + 90 + 115 + 100 + 135 + 40 &= 720 \\ a + 480 &= 720 \\ a &= 240 \end{aligned}$$

b

$$\begin{aligned} n = 8 \text{ and } S &= 180(n - 2) \\ &= 180(8 - 2) \\ &= 1080 \end{aligned}$$

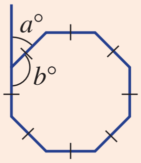
$$\begin{aligned} 8b &= 1080 \\ b &= 135 \\ a + 135 &= 180 \\ a &= 45 \end{aligned}$$

EXPLANATION

The shape is a hexagon with 6 sides so $n = 6$.

The sum of all angles is 720°. Simplify and solve for a.

The regular octagon has 8 sides, so use $n = 8$.



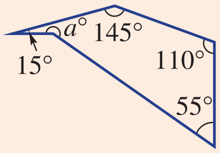
Each interior angle is equal to $b°$, so $8b°$ is the angle sum.
$a°$ is an exterior angle and $a°$ and $b°$ are supplementary.

Continued on next page

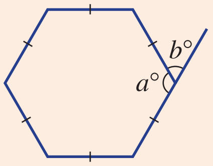
Now you try

For each polygon, find the angle sum using $S = 180\,(n - 2)$ and then find the value of any pronumerals. The polygon in part **b** is regular.

a



b



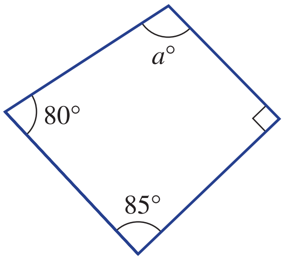
Exercise 7C

FLUENCY	1, 2–4(1/2)	2–4(1/2)	2–4(1/3)

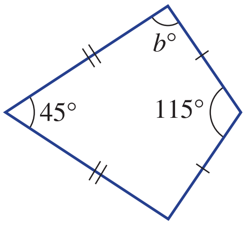
Example 7

1 Find the value of the pronumeral in each of these quadrilaterals.

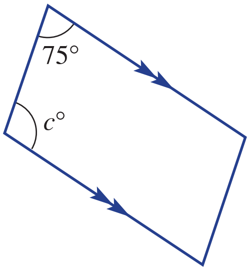
a



b



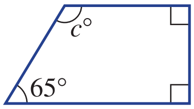
c



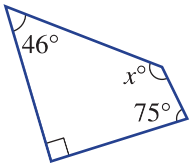
Example 7a

2 Find the values of the pronumerals.

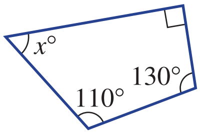
a



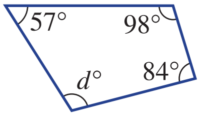
b



c



d



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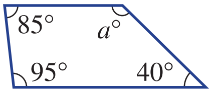
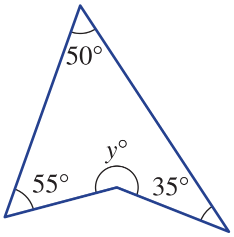


f



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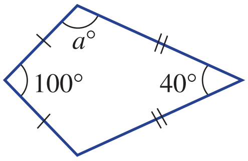
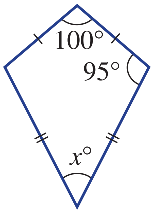
Example 7b,c

3 Find the value of the pronumerals.

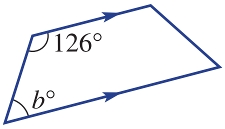
a



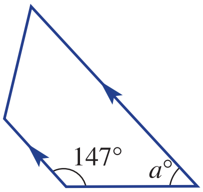
b



c



d



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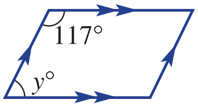

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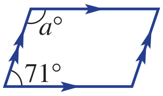
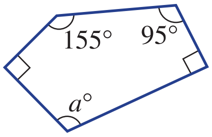


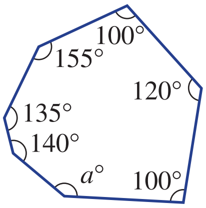
Example 8

4 For each polygon, find the angle sum using $S = 180(n - 2)$ and then find the value of any pronumerals. The polygons in parts **e**–**i** are regular.

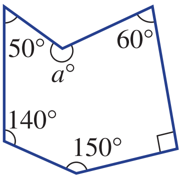
a



b



c



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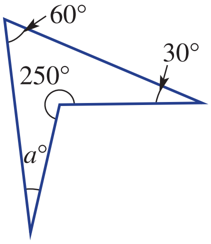

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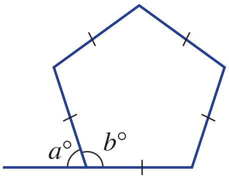

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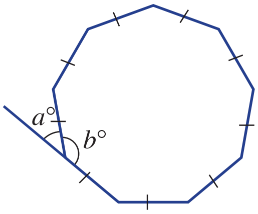

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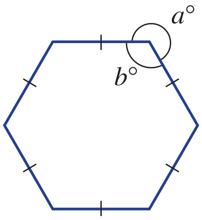

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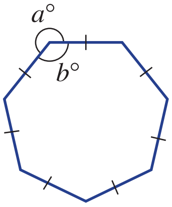

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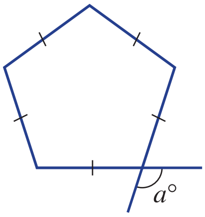


PROBLEM-SOLVING	5	5, 6, 7(1/2)	6, 7

5 List all the quadrilaterals which have these properties.

a 2 pairs of equal length sides

b All interior angles 90°

c Diagonals of equal length

d Diagonals intersecting at right angles

6 Calculate the number of sides of a polygon that has the given angle sum. (*Hint*: Use the rule $S = 180(n - 2)$.)

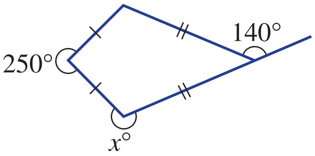
a 2520°

b 4140°

c 18 000°

7 Find the value of x in these diagrams.

a



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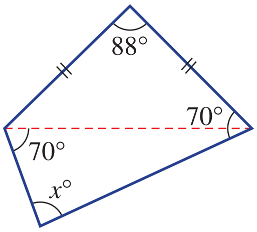

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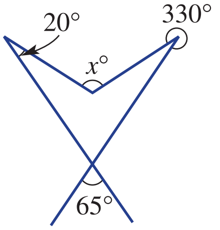

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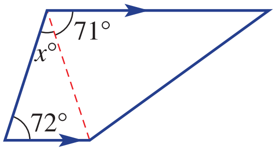

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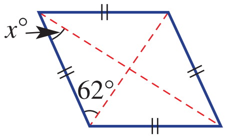

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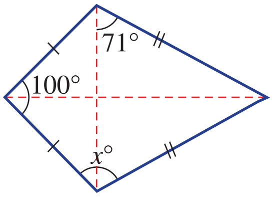


REASONING	8	8, 9	9, 10

8 Explain why a rectangle, a square and a rhombus are all parallelograms.

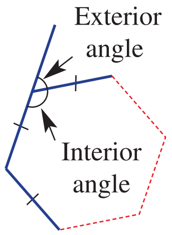
9 Consider a regular polygon with n sides.

a Write the rule for the sum of the interior angles ($S°$).

b Write the rule for the size of each interior angle ($I°$).

c Write the rule for the size of each exterior angle ($E°$).

d Use your rule from part **c** to find the size of the exterior angle of a regular decagon.



10 Recall that a non-convex polygon has at least one reflex interior angle.

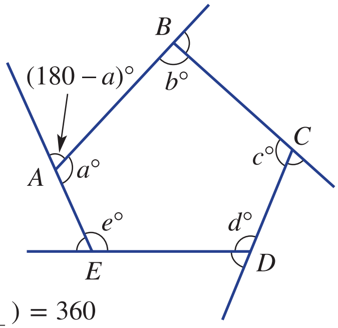
a What is the maximum number of interior reflex angles possible for these polygons?

i quadrilateral **ii** pentagon **iii** octagon

b Write an expression for the maximum number of interior reflex angles for a polygon with n sides.

ENRICHMENT: Angle sum proof	–	–	11

11 Note that if you follow the path around this pentagon starting and finishing at point A (provided you finish by pointing in the same direction as you started) you will have turned a total of 360°.



Complete this proof of the angle sum of a pentagon (540°).

$(180 - a) + (180 - b) + (\qquad) + (\qquad) + (\qquad) = \_\_\_\_$ (sum of exterior angles is $\_\_\_\_$)

$180 + 180 + \_\_\_\_ + \_\_\_\_ + \_\_\_\_ - (a + b + \_\_\_\_ + \_\_\_\_ + \_\_\_\_) = 360$

$\_\_\_\_ - (\qquad\qquad) = 360$

$(\qquad\qquad) = \_\_\_\_$

Now complete a similar proof for the angle sum of these polygons.

a Hexagon **b** Heptagon

(For an additional challenge, try to complete a similar proof for a polygon with n sides.)

7D Congruent triangles

Learning intentions

- To understand the meaning of congruence
- To know the minimal conditions required to prove triangles are congruent
- To be able to identify corresponding pairs of sides or angles in triangles
- To be able to choose which test shows that a pair of triangles are congruent
- To be able to use congruence of triangles to find unknown angles or side lengths

When two objects have the same shape and size we say they are congruent. Matching sides will be the same length and matching angles will be the same size. The areas of congruent shapes will also be equal. However, not every property of a pair of shapes needs to be known in order to determine their congruence. This is highlighted in the study of congruent triangles in which four tests can be used to establish congruence.

Engineers apply congruent triangle geometry in the design and construction of truss bridges, as shown on this railway truss bridge near Mt Fuji, Japan.

LESSON STARTER Constructing congruent triangles

To complete this task you will need a ruler, pencil and protractor. (For accurate constructions you may wish to use compasses.) Divide these constructions up equally among the members of the class. Each group is to construct their triangle with the given properties.

1 Triangle ABC with $AB = 8$ cm, $AC = 5$ cm and $BC = 4$ cm

2 Triangle DEF with $DE = 7$ cm, $DF = 6$ cm and $\angle EDF = 40^\circ$

3 Triangle GHI with $GH = 6$ cm, $\angle IGH = 50^\circ$ and $\angle IHG = 50^\circ$

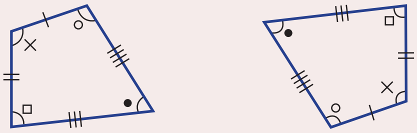
4 Triangle JKL with $\angle JKL = 90^\circ$, $JL = 5$ cm and $KL = 4$ cm

- Now compare all triangles with the vertices ABC. What do you notice? What does this say about two triangles that have three pairs of equal side lengths?
- Compare all triangles with the vertices DEF. What do you notice? What does this say about two triangles that have two pairs of equal side lengths and the included angles equal?
- Compare all triangles with the vertices GHI. What do you notice? What does this say about two triangles that have two equal corresponding angles and one corresponding equal length side?
- Compare all triangles with the vertices JKL. What do you notice? What does this say about two triangles that have one right angle, the hypotenuse and one other pair of corresponding sides equal?

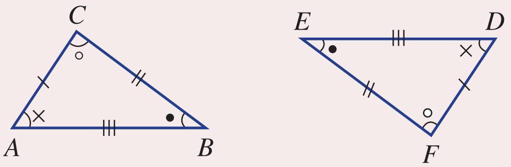
ISBN 978-1-108-77288-4

KEY IDEAS

- **Congruent figures** have the same shape and size.
  - If two figures are congruent, one of them can be transformed by using rotation, reflection and/or translation to match the other figure exactly.

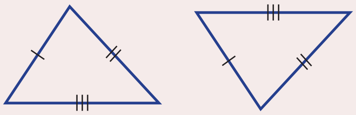


- If triangle *ABC* (ΔABC) is congruent to triangle *DEF* (ΔDEF), we write $\Delta ABC \equiv \Delta DEF$. This is called a **congruence statement**. Letters are usually written in matching order.

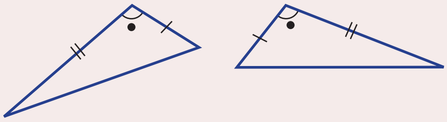


Corresponding sides	Corresponding angles
$AB = DE$	$\angle A = \angle D$
$BC = EF$	$\angle B = \angle E$
$AC = DF$	$\angle C = \angle F$

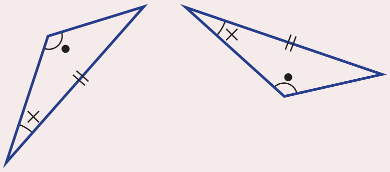
- **Corresponding** sides are opposite equal corresponding angles.
- **Tests for triangle congruence**.
  - Side, Side, Side (SSS)
    Three pairs of corresponding sides are equal.



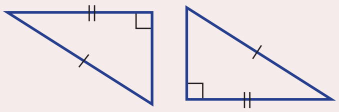
  - Side, Angle, Side (SAS)
    Two pairs of corresponding sides and the included angle are equal.



  - Angle, Angle, Side (AAS)
    Two angles and any pair of corresponding sides are equal.



  - Right angle, Hypotenuse, Side (RHS)
    A right angle, the hypotenuse and one other pair of corresponding sides are equal.

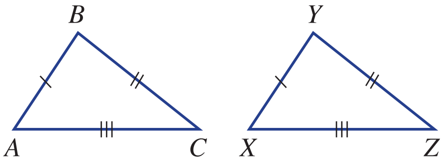


BUILDING UNDERSTANDING

1 State the missing terms to complete the sentences below.

- **a** Congruent figures are exactly the same shape and ________.
- **b** If triangle ABC is congruent to triangle STU then we write $\Delta ABC \equiv$ ________.
- **c** The abbreviated names of the four congruence tests for triangles are SSS, ___, ___ and ___.

2 These two triangles are congruent.



- **a** Name the side on ΔXYZ which corresponds to (matches):
  - **i** AB **ii** AC **iii** BC.
- **b** Name the angle in ΔABC which corresponds to (matches):
  - **i** $\angle X$ **ii** $\angle Y$ **iii** $\angle Z$.

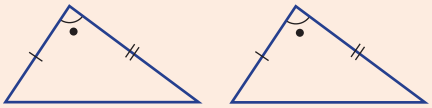
3 State a congruence statement (e.g. $\Delta ABC \equiv \Delta DEF$) given that:

- **a** triangle ABC is congruent to triangle FGH
- **b** triangle DEF is congruent to triangle STU.

Example 9 Choosing a congruence test

Which congruence test (SSS, SAS, AAS or RHS) would be used to show that these pairs of triangles are congruent?

a



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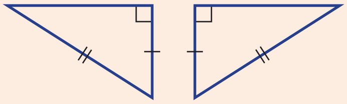

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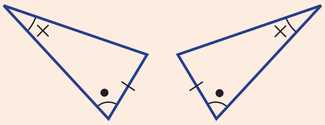

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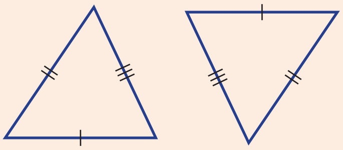


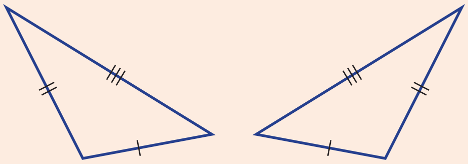
SOLUTION	EXPLANATION
a SAS	Two pairs of corresponding sides and the included angle are equal.
b RHS	A right angle, hypotenuse and one pair of corresponding sides are equal.
c AAS	Two pairs of angles and a pair of corresponding sides are equal.
d SSS	Three pairs of corresponding sides are equal.

Continued on next page

Now you try

Which congruence test (SSS, SAS, AAS or RHS) would be used to show that these pairs of triangles are congruent?

a



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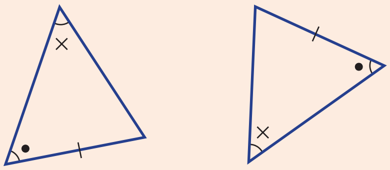

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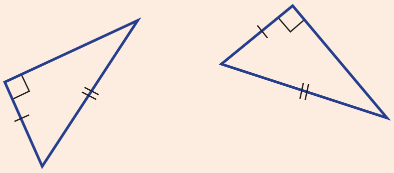

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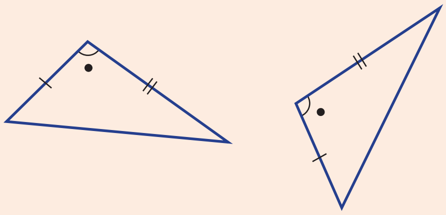
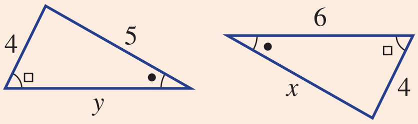


Example 10 Finding missing side lengths and angles using congruence

Find the values of the pronumerals in these pairs of congruent triangles.

a



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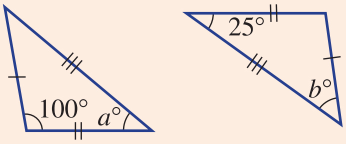
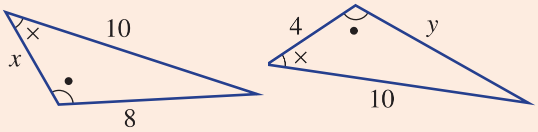


SOLUTION	EXPLANATION
a $x = 5$	The side of length x and the side of length 5 are in corresponding positions (opposite the ☐).
$y = 6$	The longest sides on the two triangles must be equal. The side of length y and the side of length 6 are corresponding sides.
b $a = 25$	The angle marked $a°$ corresponds to the 25° angle in the other triangle.
$b = 180 - 100 - 25$ $= 55$	The angle marked $b°$ corresponds to the missing angle in the first triangle. The sum of three angles in a triangle is 180°.

Now you try

Find the values of the pronumerals in these pairs of congruent triangles.

a



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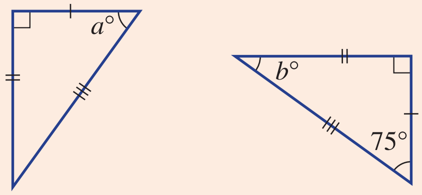


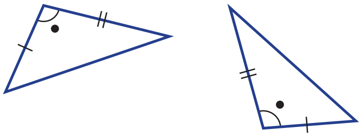
Exercise 7D

FLUENCY	1, 2, 3(½)	2, 3(½)	2, 3(½)

Example 9

1 Which congruence test (SSS, SAS, AAS or RHS) would be used to show that these pairs of triangles are congruent?

a



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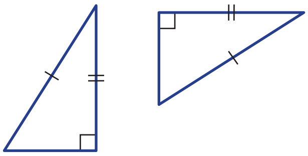

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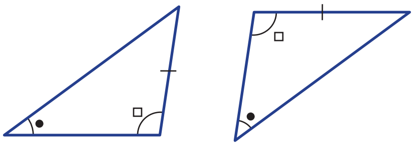

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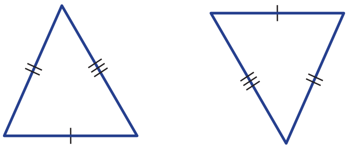
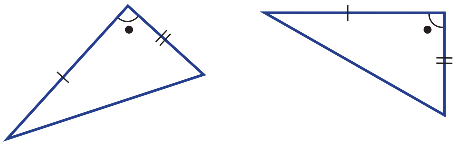


Example 9

2 Which congruence test (SSS, SAS, AAS or RHS) would be used to show that these pairs of triangles are congruent?

a



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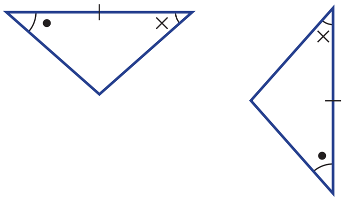

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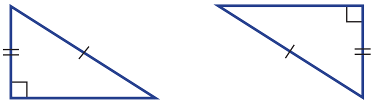

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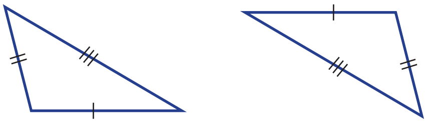
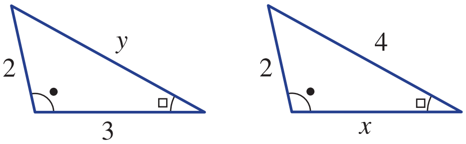


Example 10

3 Find the values of the pronumerals in these pairs of congruent triangles.

a



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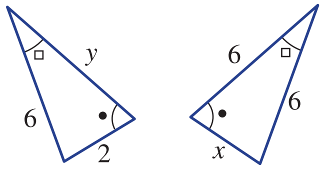

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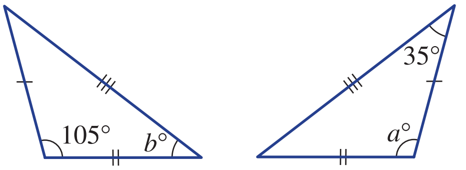

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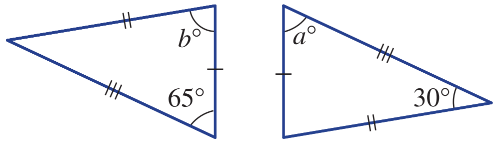

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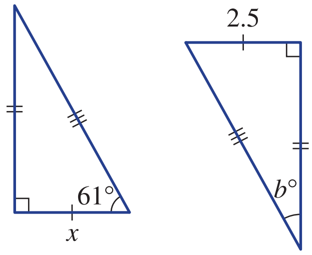

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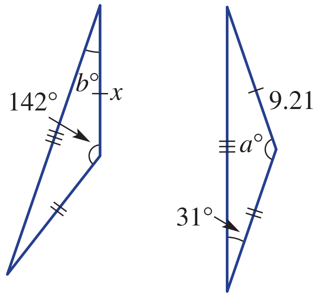

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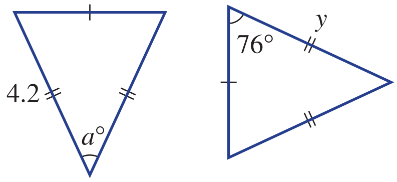

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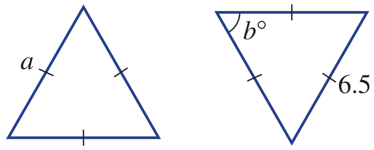
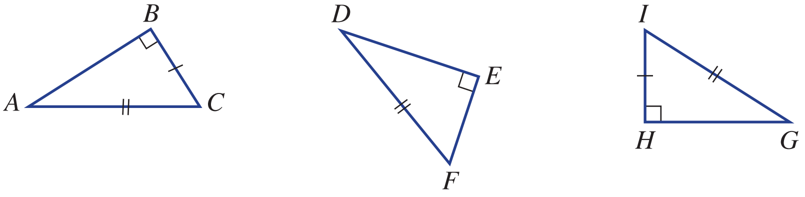


PROBLEM-SOLVING	4, 5	4, 5	4–6

4 For each set of three triangles choose the two which are congruent. Give a reason (SSS, SAS, AAS or RHS) and write a congruence statement (e.g. $\Delta ABC \equiv \Delta FGH$).

a



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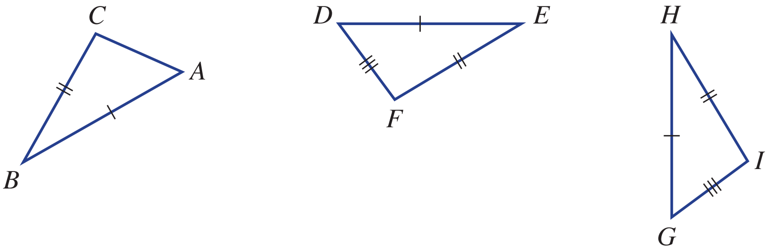

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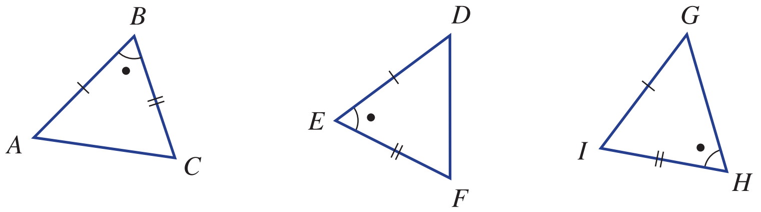

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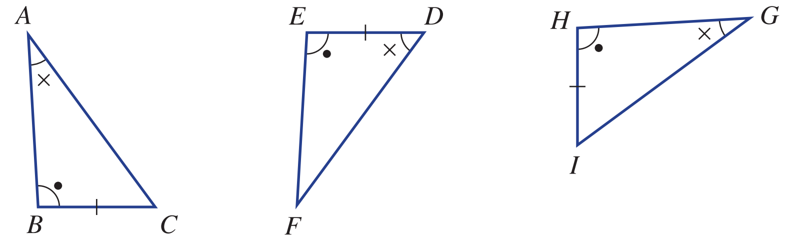

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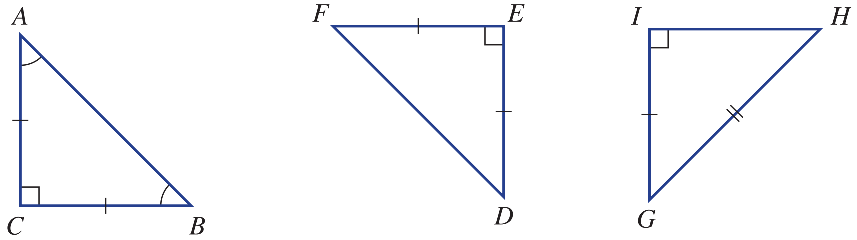
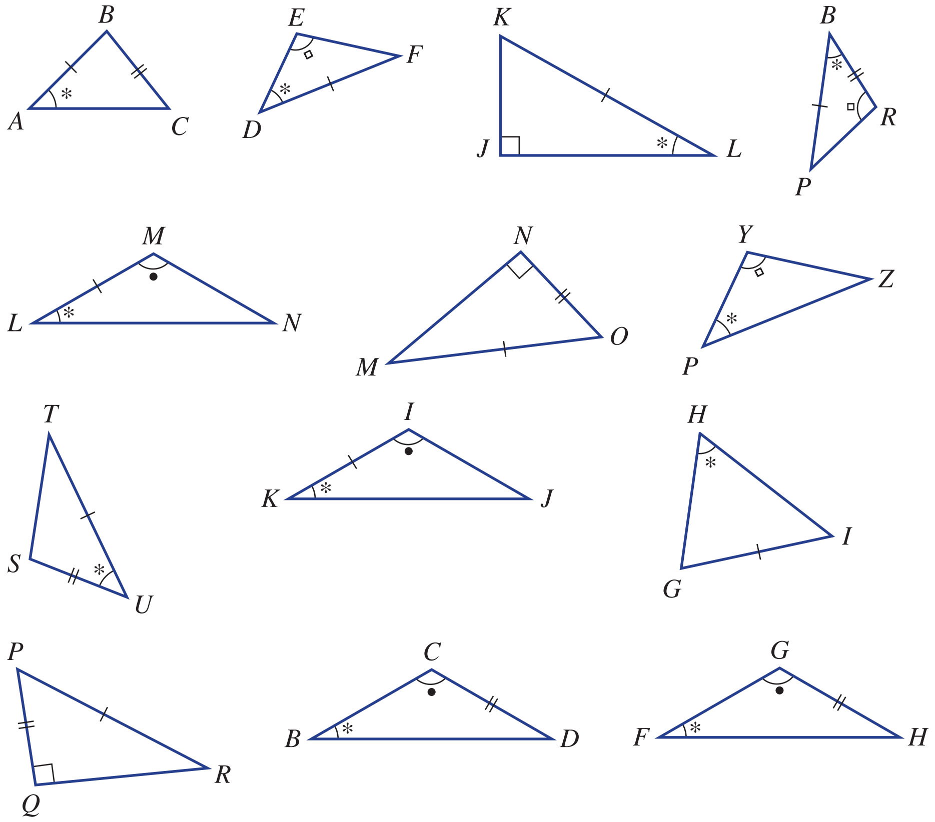


5 Identify all pairs of congruent triangles from those below. Angles with the same mark are equal.

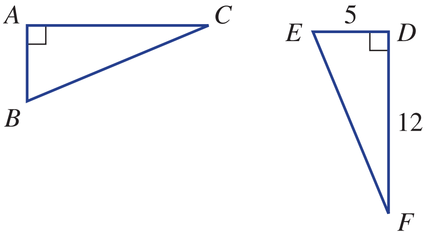


ISBN 978-1-108-77288-4

Updated September 2021

6 Use Pythagoras' theorem to help find the length BC in these pairs of congruent triangles.

a



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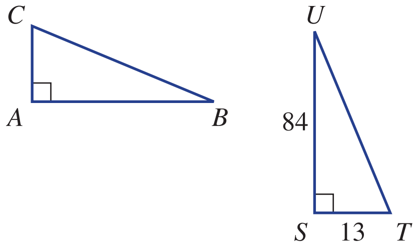
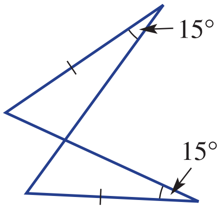


REASONING	7, 8	7–9	8–10

7 Are all triangles with three pairs of equal corresponding angles congruent? Explain why or why not.

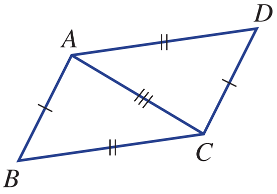
8 Consider this diagram of two triangles.

- **a** Explain why there are two pairs of equal matching angles.
- **b** Give the reason (SSS, SAS, AAS or RHS) why there are two congruent triangles.



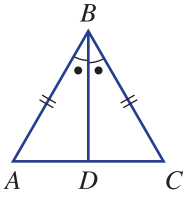
9 $ABCD$ is a parallelogram.

- **a** Give the reason why $\Delta ABC \equiv \Delta CDA$.
- **b** What does this say about $\angle B$ and $\angle D$?



10 Consider the diagram shown at right.

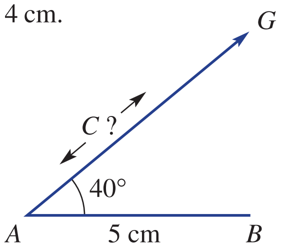
- **a** Explain why there are two pairs of corresponding sides of equal length for the two triangles.
- **b** Give the reason (SSS, SAS, AAS or RHS) why there are two congruent triangles.
- **c** Write a congruence statement.
- **d** Explain why AC is perpendicular (90°) to DB.

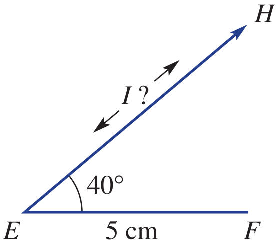


ENRICHMENT: Why not angle, side, side?	–	–	11

11 Angle, Side, Side (ASS) is not a test for congruence of triangles. Complete these tasks to see why.

- **a** Draw two line segments AB and EF both 5 cm long.
- **b** Draw two rays AG and EH so that both $\angle A$ and $\angle E$ are 40°.
- **c** Now place a point C on ray AG so that $BC = 4$ cm.
- **d** Place a point I on ray EH so that FI is 4 cm but place it in a different position so that ΔABC is not congruent to ΔEFI.
- **e** Show how you could use compasses to find the two different places you could put the points C or I so that BC and FI are 4 cm.





7E Using congruence in proof EXTENDING

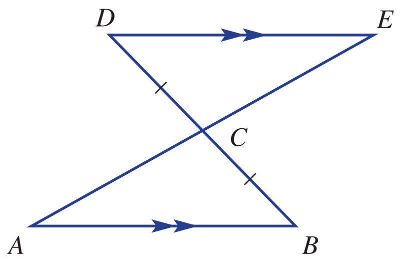
Learning intentions

- To know how to layout the steps of a congruence proof with reasons and correct notation
- To be able to use established congruence to prove other geometrical properties

A mathematical proof is a sequence of correct statements that leads to a result. It should not contain any big 'leaps' and should provide reasons at each step. The proof that two triangles are congruent should list all the corresponding pairs of sides and angles. Showing that two triangles are congruent in more complex problems can then lead to the proof of other geometrical results.

LESSON STARTER Complete the proof

Help complete the proof that $\Delta ABC \equiv \Delta EDC$ for this diagram. Give the missing reasons and congruent triangle in the final statement.



In ΔABC and ΔEDC:
$\angle DCE = \angle BCA$ (_______________)
$\angle ABC = \angle EDC$ (_______________)
$BC = DC$ (given equal sides)
$\therefore\ \Delta ABC \equiv$ ________ (AAS)

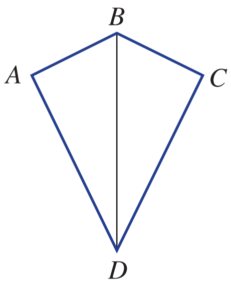
KEY IDEAS

- Prove that two triangles are congruent by listing all the known corresponding equal angles and sides.
  - Give reasons at each step.
  - Conclude by writing a congruence statement and the abbreviated reason (SSS, SAS, AAS or RHS).
  - Vertex labels are usually written in matching order.
- Other geometrical results can be proved by using the properties of congruent triangles.

BUILDING UNDERSTANDING

1 Name the line segment which is common to both triangles in each diagram.

a



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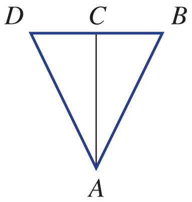
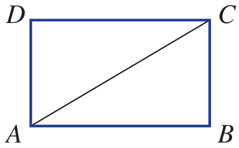
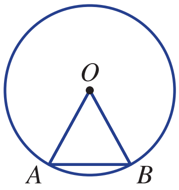


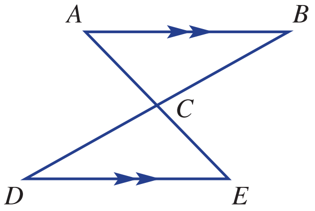
c



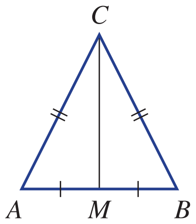
2 Why is ΔOAB an isosceles triangle in this circle? O is the centre of the circle.



3 For this diagram name the angle which is:
- a vertically opposite to $\angle ACB$
- b alternate to $\angle CDE$
- c alternate to $\angle BAC$.

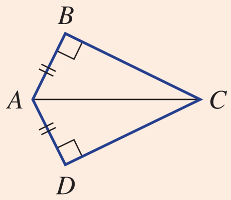


4 For this isosceles triangle, CM is common to ΔAMC and ΔBMC.
- a Which congruence test would be used to prove that $\Delta AMC \equiv \Delta BMC$?
- b Which angle in ΔBMC corresponds to $\angle AMC$ in ΔAMC?



Example 11 Proving that two triangles are congruent

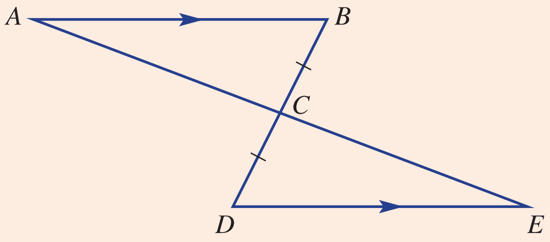
Prove that $\Delta ABC \equiv \Delta ADC$.



SOLUTION	EXPLANATION
In ΔABC and ΔADC:	
$\angle ABC = \angle ADC = 90^\circ$ (given equal angles) (**R**)	Both triangles have a right angle.
AC is common (**H**)	AC is common to both triangles (hypotenuse).
$AB = AD$ (given equal sides) (**S**)	AB and AD are marked as equal.
$\therefore\ \Delta ABC \equiv \Delta ADC$ (RHS)	Write the congruence statement and the abbreviated reason. Write the vertex labels in matching order.

Now you try

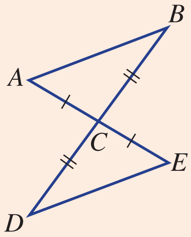
Prove that $\Delta ABC \equiv \Delta EDC$.



Example 12 Proving geometrical results using congruence

a Prove that $\Delta ABC \equiv \Delta EDC$.

b Hence prove that $AB \parallel DE$ (AB is parallel to DE).



SOLUTION

a In ΔABC and ΔEDC:

$AC = EC$ (given equal sides) (**S**)

$BC = DC$ (given equal sides) (**S**)

$\angle ACB = \angle ECD$ (vertically opposite angles) (**A**)

$\Delta ABC \equiv \Delta EDC$ (SAS)

b $\angle BAC = \angle DEC$ (matching angles in congruent triangles)

$\therefore AB \parallel DE$ (alternate angles are equal)

EXPLANATION

List the given pairs of equal length sides and the vertically opposite angles. The included angle is between the given sides, hence SAS.

All matching angles are equal.

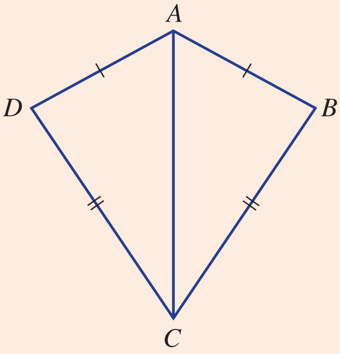
If alternate angles are equal then AB and DE must be parallel.

Now you try

a Prove that $\Delta ABC \equiv \Delta ADC$.

b Hence prove that $\angle ABC = \angle ADE$

(i.e. that a kite has one pair of opposite equal angles).

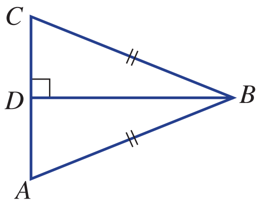


Exercise 7E

FLUENCY	1, 2($\frac{1}{2}$)	2($\frac{1}{2}$)	2($\frac{1}{2}$)

Example 11

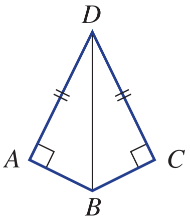
1 Prove that $\Delta ABD \equiv \Delta CBD$.



Example 11

2 Prove that each pair of triangles is congruent. List your reasons and give the abbreviated congruence test.

a



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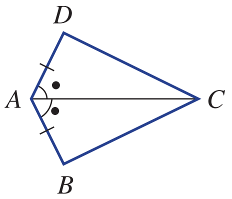

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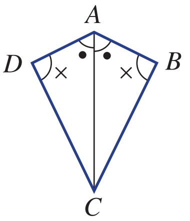

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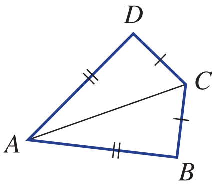

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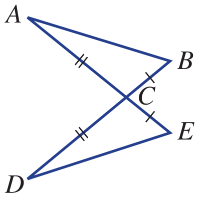

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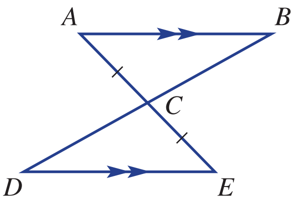

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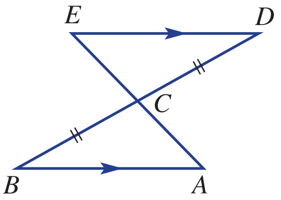

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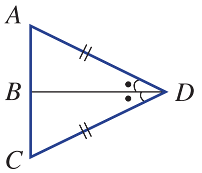

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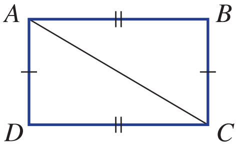

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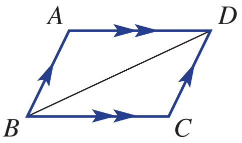

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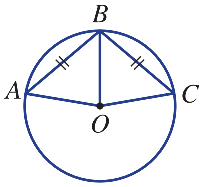

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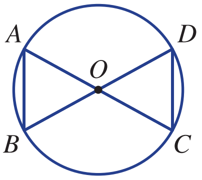
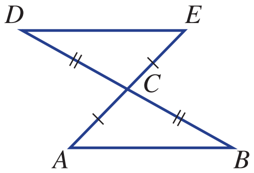


PROBLEM-SOLVING	3, 4	3–6	5–8

Example 12

3 **a** Prove $\Delta ABC \equiv \Delta EDC$.

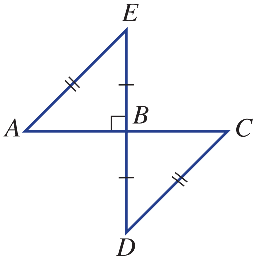
b Hence, prove $AB \parallel DE$.



Example 12

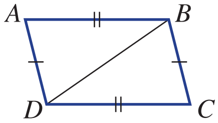
4 **a** Prove $\Delta ABE \equiv \Delta CBD$.

b Hence, prove $AE \parallel CD$.



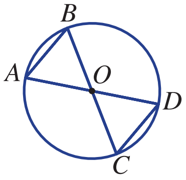
5 **a** Prove $\Delta ABD \equiv \Delta CDB$.

b Hence, prove $AD \parallel BC$.



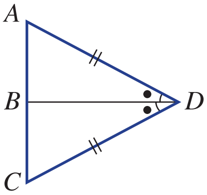
6 **a** Prove that $\Delta AOB \equiv \Delta DOC$. ($O$ is the centre of the circle.)

b Hence, prove that $AB \parallel CD$.



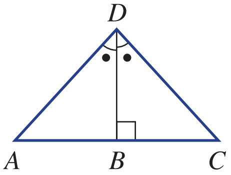
7 **a** Prove that $\Delta ABD \equiv \Delta CBD$.

b Hence, prove that AC is perpendicular to BD. ($AC \perp BD$)



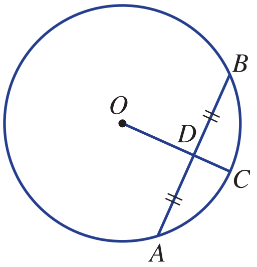
8 **a** Prove that $\Delta ABD \equiv \Delta CBD$.

b Hence, prove that ΔACD is isosceles.

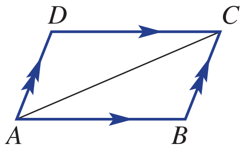


REASONING	9	9, 10	10–12

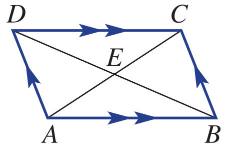
9 Use congruence to explain why OC is perpendicular to AB in this diagram. (*Hint*: Form triangles.)



10 Use congruence to explain why $AD = BC$ and $AB = DC$ in this parallelogram.



11 Use ΔABE and ΔCDE to explain why $AE = CE$ and $BE = DE$ in this parallelogram (i.e. the diagonals bisect each other).



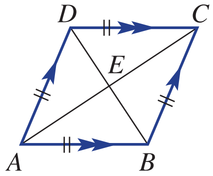
12 Use congruence to show that the diagonals of a rectangle are equal in length.

ENRICHMENT: Extended proofs	–	–	13, 14

13 $ABCD$ is a rhombus. To prove that AC bisects BD at 90°, follow these steps.

a Prove that $\Delta ABE \equiv \Delta CDE$.

b Hence prove that AC bisects BD at 90°.



14 Use congruence to prove that the three angles in an equilateral triangle (given three equal side lengths) are all 60°.

Surveyors must take enough measurements to enable the calculation of every required length and angle. As polygons can be divided into triangles, surveyors use triangle congruence proofs to prove polygons are congruent.

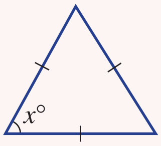
7A **1** For an angle of size 53° determine the size of the:

a complementary angle

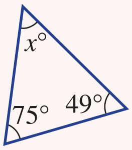
b supplementary angle.

7A **2** Find the value of x in each of these triangles.

a



b



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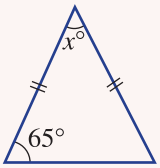

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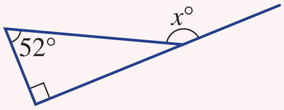

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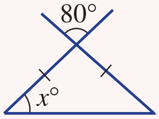

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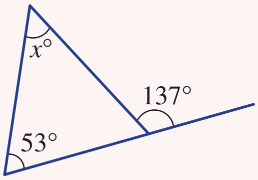
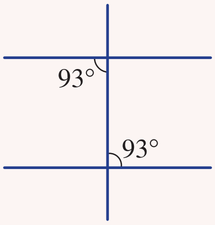


7B **3** Decide if each diagram contains a pair of parallel lines. Give a reason.

a



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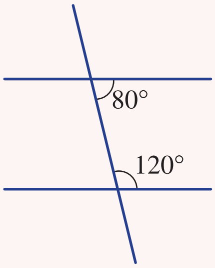
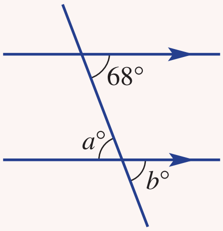


7B/C **4** Find the value of each pronumeral. Give reasons for your answer. Part **d** is regular.

a



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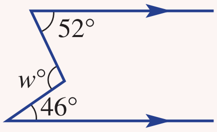

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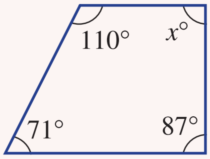
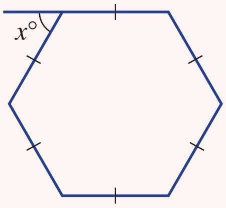
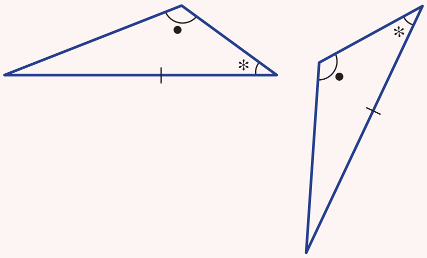


d

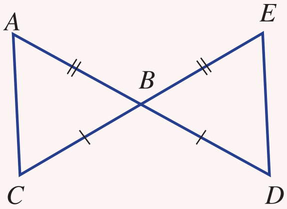


7D **5** Which congruency test could be used to prove these triangles are congruent?

a

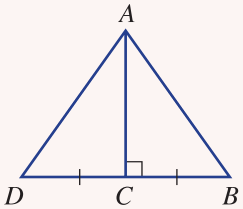


b



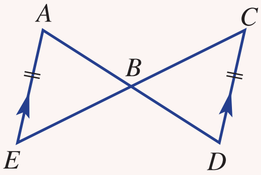
7E Ext

6 Prove that triangle *ABC* is congruent to triangle *ADC*, hence prove that triangle *ABD* is isosceles.



7E Ext

7 Consider the diagram below.



a Prove that $\Delta ABE \equiv \Delta DBC$.

b If $\angle EAB$ is 62°, what is the size of angle *BDC*?

c If $AE = 7$ cm, find the length of the side *CD*.

d If the area of triangle *ABE* is 21 cm^2, what is the area of triangle *DBC*?

7F Enlargement and similar figures

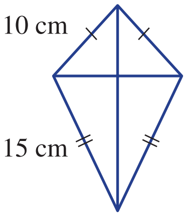
Learning intentions

- To know how to enlarge a figure using a scale factor and a centre of enlargement
- To understand the effect of applying a scale factor to enlarge a shape
- To know the meaning of the term similar figures
- To be able to find the scale factor between two similar figures
- To be able to use the scale factor to find the value of corresponding side lengths in similar figures

Similar figures have the same shape but not necessarily the same size. If two figures are similar then one of them can be enlarged or reduced so that it is identical (congruent) to the other. If a figure is enlarged by a scale factor greater than 1, the image will be larger than the original. If the scale factor is between 0 and 1, the image will be smaller.

A photo forms a similar image to the original, with all matching angles equal and matching lengths in the same ratio. Such as this photo of the Senanque Abbey, in fields of lavender, in southern France.

LESSON STARTER Enlarging a kite

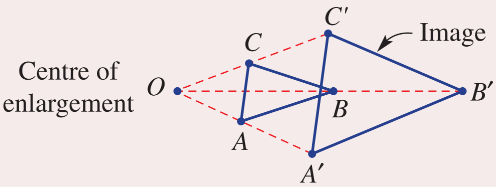


Mandy draws a kite design, and then cuts out a larger shape to make the actual kite. The actual kite shape is to be similar to the design drawing. The 10 cm length on the drawing matches a 25 cm length on the kite.

- How should the interior angles compare between the drawing and the actual kite?
- By how much has the drawing been enlarged; that is, what is the scale factor? Explain your method to calculate the scale factor.
- What length on the kite matches the 15 cm length on the drawing?

KEY IDEAS

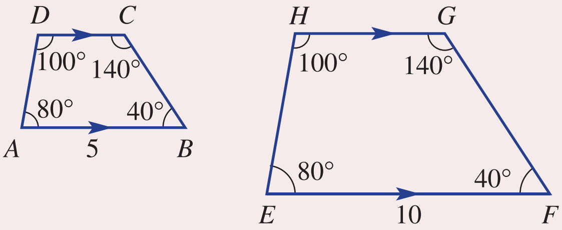
- **Enlargement** is a transformation which involves the increase or decrease in size of an object.
  - The 'shape' of the object is unchanged.
  - Enlargement uses a **centre of enlargement** and an **enlargement factor** or **scale factor**.



Scale factor

$$\frac{OA'}{OA} = \frac{OB'}{OB} = \frac{OC'}{OC}$$

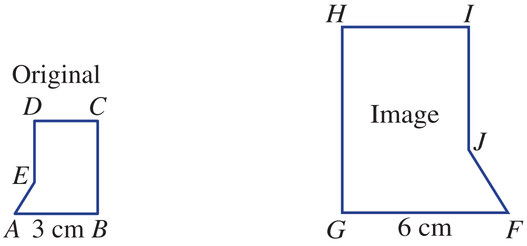
- Two figures are **similar** if one can be enlarged to be congruent to the other.
  - Corresponding angles are equal.
  - Pairs of corresponding sides are in the same proportion or ratio.
- The **scale factor** $= \dfrac{\text{image length}}{\text{original length}}$.
  - If the scale factor is between 0 and 1, the image will be smaller than the original.
  - If the scale factor is greater than 1, the image will be larger than the original.
  - If the scale factor is equal to 1, the image will be congruent to the original.
- The symbols $\mid\mid\mid$ and $\sim$ are used to describe similarity.
  - We can write $ABCD \mid\mid\mid EFGH$ or $ABCD \sim EFGH$.
  - The letters are usually written in matching order.



  - Scale factor $= \dfrac{EF}{AB} = \dfrac{10}{5} = 2$.

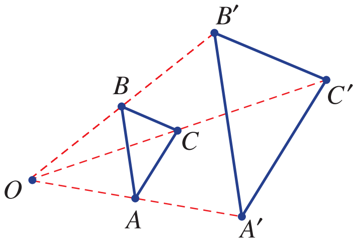
BUILDING UNDERSTANDING

1. The two figures below are similar.



   a Name the angle in the larger figure which corresponds to $\angle A$.
   b Name the angle in the smaller figure which corresponds to $\angle I$.
   c Name the side in the larger figure which corresponds to BC.
   d Name the side in the smaller figure which corresponds to FJ.
   e Use FG and AB to find the scale factor.

2. This diagram shows ΔABC enlarged to give the image $\Delta A'B'C'$.
   a Measure the lengths OA and OA'. What do you notice?
   b Measure the lengths OB and OB'. What do you notice?
   c Measure the lengths OC and OC'. What do you notice?
   d What is the scale factor?
   e Is $A'B'$ twice the length of AB? Measure to check.



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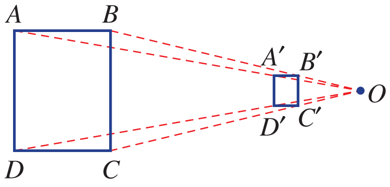
3 This diagram shows rectangle $ABCD$ enlarged (in this case reduced) to rectangle $A'B'C'D'$.

a Measure the lengths OA and OA'. What do you notice?

b Measure the lengths OD and OD'. What do you notice?

c What is the scale factor?

d Compare the lengths AD and $A'D'$. Is $A'D'$ one quarter of the length of AD?

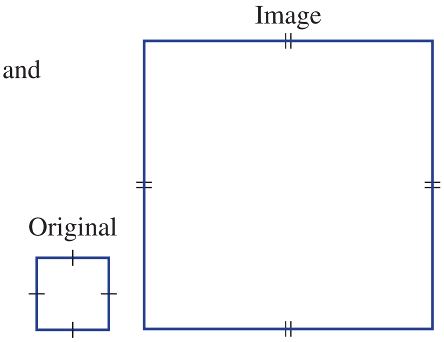


4 A square is enlarged by a scale factor of 4.

a Are the internal angles the same for both the original and the image?

b If the side length of the original square was 2 cm, what would be the side length of the image square?

c If the side length of the image square was 100 m, what would be the side length of the original square?

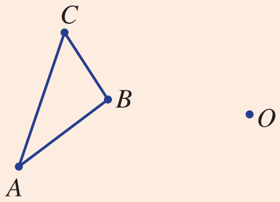


Example 13 Enlarging figures

Copy the given diagram using plenty of space. Use the given centre of enlargement (O) and these scale factors to enlarge ΔABC.

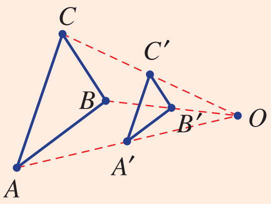
a Scale factor $\frac{1}{2}$

b Scale factor 3



SOLUTION

a



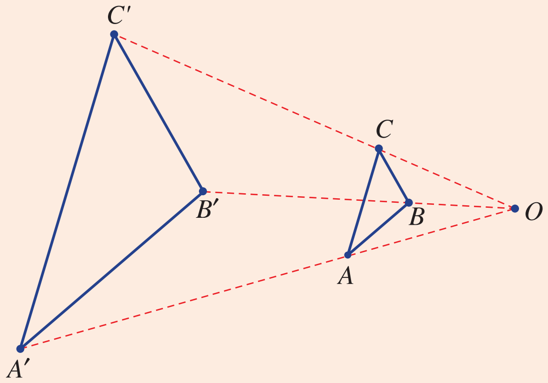
EXPLANATION

Connect dashed lines between O and the vertices A, B and C.

Since the scale factor is $\frac{1}{2}$, place A' so that OA' is half of OA.

Repeat for B' and C'. Join vertices A', B' and C'.

b



Draw dashed lines from O through A, B and C. Place A' so that $OA' = 3OA$. Repeat for B' and C' and form $\Delta A'B'C'$.

Continued on next page

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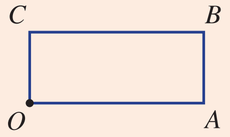
Now you try

Copy the given diagram using plenty of space. Use the given centre of enlargement (O) and these scale factors to enlarge the shape.

a Scale factor 2

b Scale factor $\frac{2}{3}$



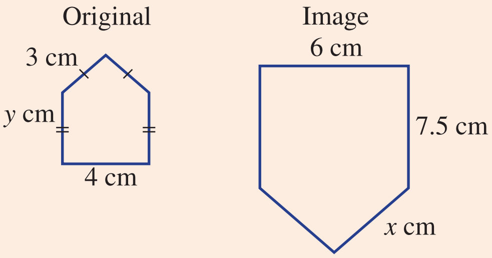
Example 14 Using the scale factor

These figures are similar.

a Find a scale factor.

b Find the value of x.

c Find the value of y.



SOLUTION

EXPLANATION

a Scale factor $= \frac{6}{4} = 1.5$

Choose two corresponding sides and use $\text{scale factor} = \frac{\text{image length}}{\text{original length}}$.

b $x = 3 \times 1.5$
$= 4.5$

Multiply the side lengths on the original by the scale factor to get the length of the corresponding side on the image.

c $y \times 1.5 = 7.5$
$y = 7.5 \div 1.5$
$= 5$

The corresponding side lengths are y cm and 7.5 cm. Divide the side lengths on the image by the scale factor to get the length of the corresponding side on the original.

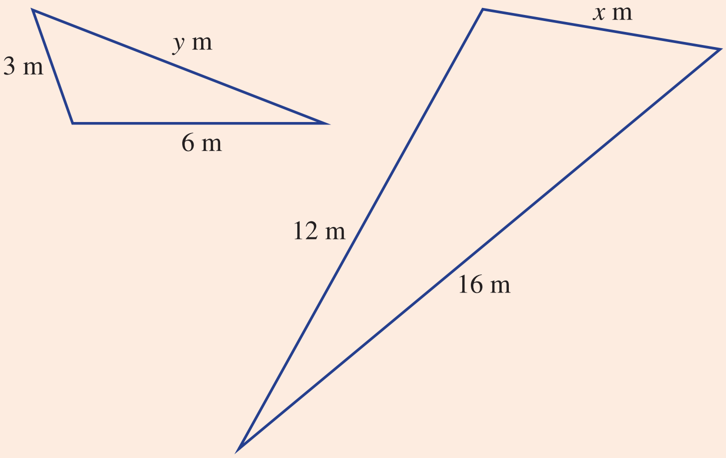
Now you try

These figures are similar.

a Find a scale factor.

b Find the value of x.

c Find the value of y.



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Exercise 7F

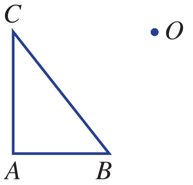
FLUENCY	1, 2, 3(1/2)	2, 3(1/2)	2, 3(1/2)

Example 13

1 Copy the given diagram leaving plenty of space around it, and then use the centre of enlargement (O) and the given scale factors to enlarge ΔABC.

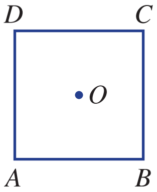
a Scale factor $\frac{1}{3}$

b Scale factor 2



Example 13

2 This diagram includes a square with centre O and vertices $ABCD$. Copy the diagram leaving plenty of space around it.



Enlarge square $ABCD$ by these scale factors and draw the image. Use O as the centre of enlargement.

a $\frac{1}{2}$

b 1.5

Example 14

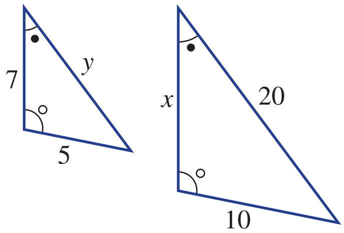
3 Each of the pairs of figures shown here are similar. For each pair, find:

i a scale factor

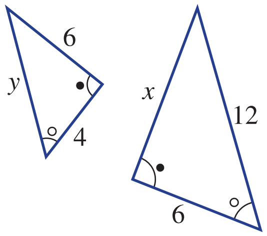
ii the value of x

iii the value of y.

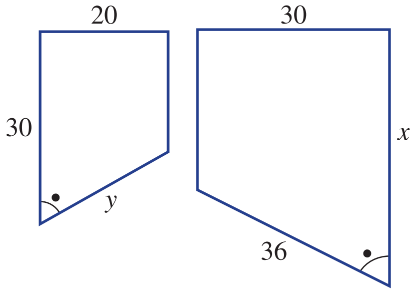
a



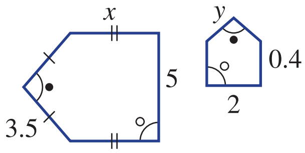
b



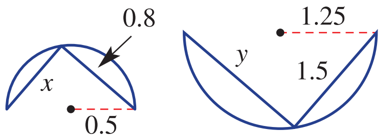
c



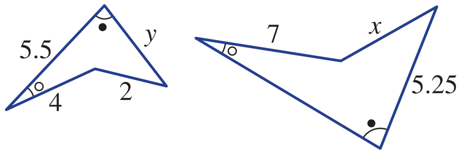
d



e



f



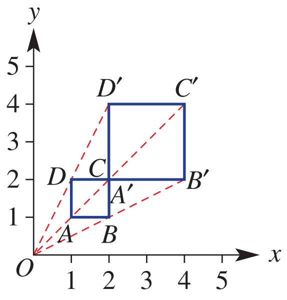
PROBLEM-SOLVING 4, 5 4(½), 5 5, 6

4 These diagrams show a shape and its image after enlargement. For each part, find:

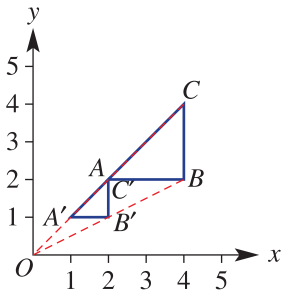
i the scale factor

ii the coordinates (x, y) of the centre of enlargement.

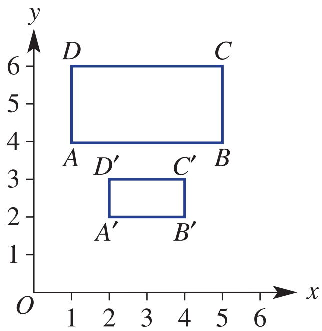
a



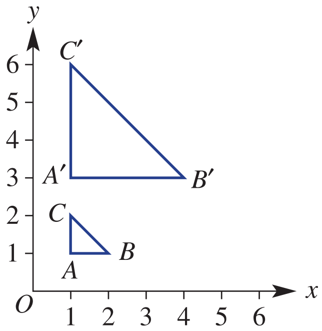
b



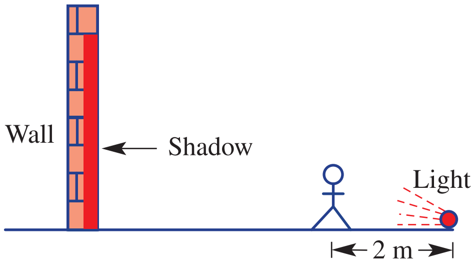
c



d



5 A person 1.8 m tall stands in front of a light that sits on the floor, and casts a shadow on the wall behind them. Form triangles to assist with the following questions.



a How tall will the shadow be if the distance between the wall and the light is:

i 4 m? **ii** 10 m? **iii** 3 m?

b How tall will the shadow be if the distance between the wall and the person is:

i 4 m? **ii** 5 m?

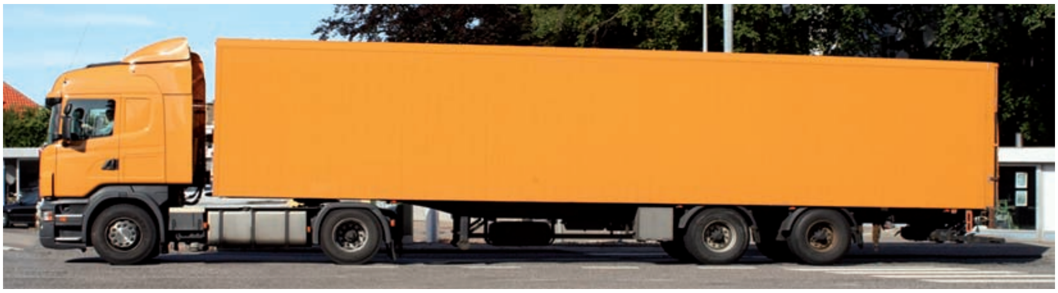
c Find the distance from the wall to the person if the height of the shadow is:

i 7.2 m **ii** 4.5 m.

6 This truck is 12.7 m long.

a Measure the length of the truck in the photo. **b** Measure the height of the truck in the photo.

c Estimate the actual height of the truck.



REASONING	7, 8	7–9	8–10

7 A figure is enlarged by a scale factor of a where $a > 0$.
- **a** For what values of a will the image be larger than the original figure?
- **b** For what values of a will the image be smaller than the original figure?
- **c** For what value of a will the image be congruent to the original figure?

8 Explain why:
- **a** any two squares are similar
- **b** any two equilateral triangles are similar
- **c** any two rectangles are not necessarily similar
- **d** any two isosceles triangles are not necessarily similar.

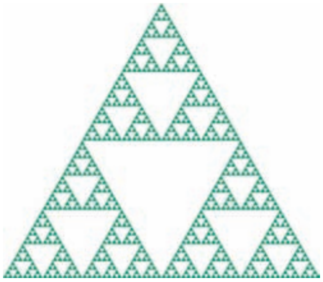
9 An object is enlarged by a factor of k. What scale factor should be used to reverse this enlargement?

10 A map has a scale ratio of 1 : 50 000.
- **a** What length on the ground is represented by 2 cm on the map?
- **b** What length on the map is represented by 12 km on the ground?

ENRICHMENT: The Sierpinski triangle	–	–	11

11 The Sierpinski triangle shown is a mathematically generated pattern.

It is created by repeatedly enlarging triangles by a factor of $\frac{1}{2}$. The steps are listed below.



1. Start with an equilateral triangle as in Figure 1.
2. Enlarge the triangle by a factor $\frac{1}{2}$.
3. Arrange three copies of the image as in Figure 2.
4. Continue repeating steps 2 and 3 with each triangle.

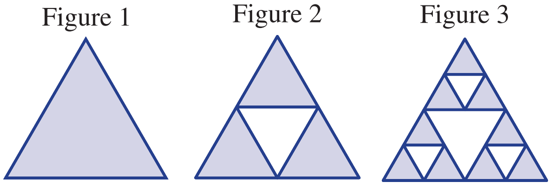


Figure 1 Figure 2 Figure 3

- **a** Make a large copy of Figures 1 to 3 and then draw the next two figures in the pattern.
- **b** If the original triangle (Figure 1) had side length l, find the side length of the smallest triangle in:
  - **i** Figure 2
  - **ii** Figure 3
  - **iii** Figure 8 (assuming Figure 8 is the 8th diagram in the pattern).
- **c** What fraction of the area is shaded in:
  - **i** Figure 2?
  - **ii** Figure 3?
  - **iii** Figure 6 (assuming Figure 6 is the 6th diagram in the pattern)?
- **d** The Sierpinski triangle is one in which the process of enlargement and copying is continued forever. What is the area of a Sierpinski triangle?

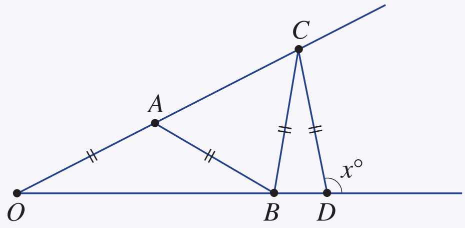
Applications and problem-solving

The following problems will investigate practical situations drawing upon knowledge and skills developed throughout the chapter. In attempting to solve these problems, aim to identify the key information, use diagrams, formulate ideas, apply strategies, make calculations and check and communicate your solutions.

Beam support

1 An angled beam is supported by three equal length struts (AB, BC and CD) as shown. The left strut is connected at point A such that $OA = AB$.

An engineer is interested in finding the possible position of angled beams, that can be supported by struts positioned in such a way.



a Giving reasons, find the value of x if $\angle AOB$ is equal to:

i 10° **ii** 20°.

b Find an expression for x if $\angle AOB = a°$.

c Try drawing a diagram using $\angle AOB = 40°$, labelling all angles. Describe the problem with the diagram if this angle is used.

d Find the maximum angle of the beam ($\angle AOB$) so that all the struts AB, BC and CD can be positioned in the way illustrated in the original diagram.

e What does your result from part **d** mean for the angle x?

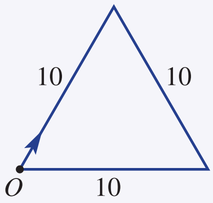
Track navigation

2 A beginner computer programmer is using a simple platform to create a computer game involving different race tracks.

After writing step-by-step commands for polygon-shaped tracks, the programmer wants to be able to write as few commands as possible by using loops to account for commands that are repeated.

The tracks will involve regular polygons in which a car placed on the track has to be programmed to do laps of the track.

a Consider the circuit shown, which is a regular triangle of side length 10 units. The car is placed at O facing in the direction of the track.



The programmed command that can be used to travel along the track is: 'Move x, y', where x is the angle the car needs to first rotate in a clockwise direction so that it is facing in the direction of the track and y represents the distance to travel.

The first command is: Move 0, 10

Complete the final three instructions in the form 'Move __, __', so that the car completes one full circuit and finishes facing in the direction which it started.

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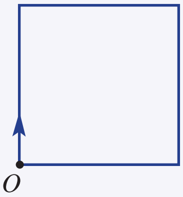
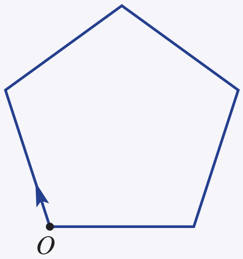
b For the two regular tracks shown, give the set of commands needed to get a car to complete one full lap. Use a side length of 10 units.





The programmer notices that the set of instructions can be simplified by using a loop of the form ‘Move x, y, z’ for the repeating instructions, where z represents the number of times the command is repeated.

c Use the ‘Move x, y, z’ command to rewrite your commands to draw the square and pentagon above. Your first command needs to be ‘Move 0, 10, 1’.

d Consider the sum of all the rotations used for each track. What do you notice?

e Using your result for part **d**, complete the following set of commands to complete one lap of an n-sided regular polygon of side length 10 units. Check that your commands are correct for $n = 8$.

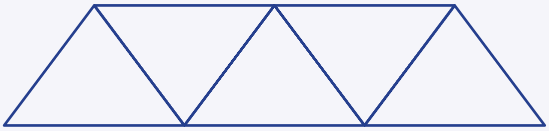
Move 0, 10, 1
Move ___, 10, ___
Move ___, 10, 1

Truss bridges

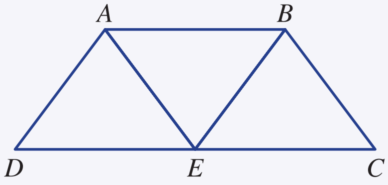
3 Truss bridges use triangles as supports above or below the deck to increase the amount of weight a bridge can hold. Triangles are used for their strength as, unlike a parallelogram, when compressed they maintain their shape and spread the force evenly.

An engineering student is looking at the geometrical structure of different truss designs before simulating experiments to see how the different arrangements affect the weight the bridge can handle.

The following design is a Warren Truss, with repeating triangular sections across the bridge.



Consider the portion of the design shown at right, in which the four diagonals are of equal length.

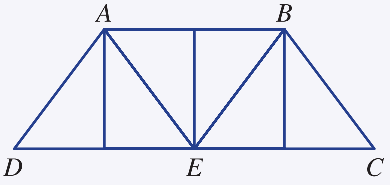


a Classify the type of triangles that have been formed.

b Which test proves that triangles DAE, BEA and EBC are congruent?

c Use the congruence of the triangles to prove that $AB \parallel DC$ and $AD \parallel BE$.

A Pratt Truss bridge involves further strengthening of the design by adding vertical supports from the vertex perpendicular to the base. The drawback of this design is the extra cost in materials.



d How many congruent triangles have now been formed in this diagram? Give reasons why the smaller triangles formed in this bridge frame are all congruent?

7G Similar triangles

Learning intentions

- To know the tests for similarity of triangles
- To be able to choose the appropriate test that proves two triangles are similar
- To be able to use the scale factor from similar triangles to find an unknown length

Many geometric problems can be solved by using similar triangles. Shadows, for example, can be used to determine the height of a tall mast when the shadows form the base of two similar triangles. Solving such problems first involves the identification of two triangles and an explanation as to why they are similar. As with congruence of triangles, there is a set of minimum conditions to establish similarity in triangles.

Similar triangle geometry is the basis of trigonometry and both methods are used by astronomers, navigators, builders, architects, engineers and surveyors. Calculating the height of K2 in Pakistan, Earth's second highest mountain, would involve trigonometry.

LESSON STARTER Are they similar?

Each point below describes two triangles. Accurately draw each pair and decide if they are similar (same shape but of different size).

- ΔABC with $AB = 2$ cm, $AC = 3$ cm and $BC = 4$ cm
  ΔDEF with $DE = 4$ cm, $DF = 6$ cm and $EF = 8$ cm
- ΔABC with $AB = 3$ cm, $AC = 4$ cm and $\angle A = 40^\circ$
  ΔDEF with $DE = 6$ cm, $DF = 8$ cm and $\angle D = 50^\circ$
- ΔABC with $\angle A = 30^\circ$ and $\angle B = 70^\circ$
  ΔDEF with $\angle D = 30^\circ$ and $\angle F = 80^\circ$
- ΔABC with $\angle A = 90^\circ$, $AB = 3$ cm and $BC = 5$ cm
  ΔDEF with $\angle D = 90^\circ$, $DE = 6$ cm and $EF = 9$ cm

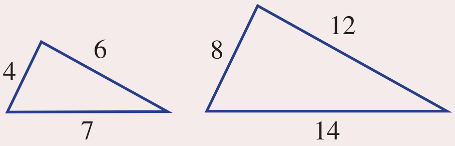
Which pairs are similar and why? For the pairs that are not similar, what measurements could be changed so that they are similar?

KEY IDEAS

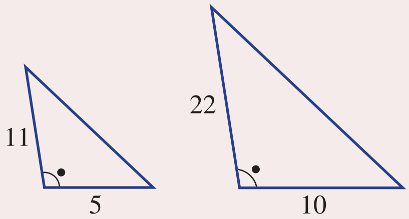
- Two triangles are **similar** if:
  - corresponding angles are equal
  - corresponding sides are in proportion (the same ratio).
- The **similarity statement** for two similar triangles ΔABC and ΔDEF is:
  - $\Delta ABC \mathbin{|||} \Delta DEF$ or
  - $\Delta ABC \sim \Delta DEF$

  Letters are usually written in matching order so AB corresponds to DE etc.

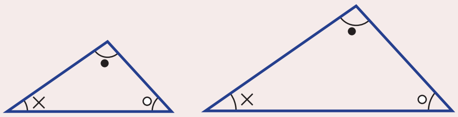
- **Tests for similar triangles**. (Not to be confused with the congruence tests for triangles).
  - Side, Side, Side (SSS)
    All three pairs of corresponding sides are in the same ratio.
    $$\frac{12}{6} = \frac{8}{4} = \frac{14}{7}$$



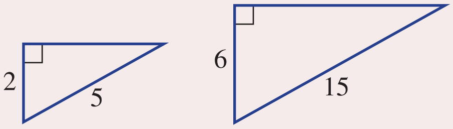
  - Side, Angle, Side (SAS)
    Two pairs of corresponding sides are in the same ratio and the included angle is equal.
    $$\frac{22}{11} = \frac{10}{5}$$



  - Angle, Angle, Angle (AAA or AA)
    All three corresponding angles are equal. (If there are two equal pairs then the third pair must be equal by the angle sum of a triangle.)

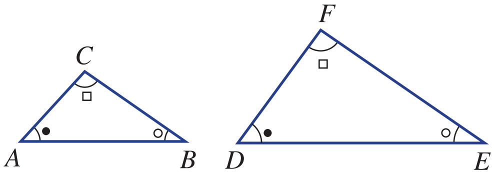


  - Right angle, Hypotenuse, Side (RHS)
    The hypotenuses of right-angled triangles and another corresponding pair of sides are in the same ratio.
    $$\frac{15}{5} = \frac{6}{2}$$



BUILDING UNDERSTANDING

1 These two triangles are similar.



a Which vertex on ΔDEF corresponds to (matches) vertex B on ΔABC?

b Which vertex on ΔABC corresponds to (matches) vertex F on ΔDEF?

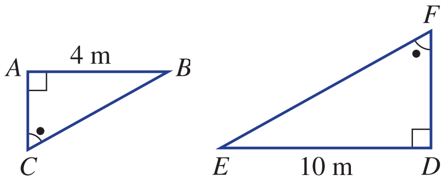
c Which side on ΔDEF corresponds to (matches) side AC on ΔABC?

d Which side on ΔABC corresponds to (matches) side EF on ΔDEF?

e Which angle on ΔABC corresponds to (matches) $\angle D$ on ΔDEF?

f Which angle on ΔDEF corresponds to (matches) $\angle B$ on ΔABC?

2 What is the scale factor on this pair of similar triangles which enlarges ΔABC to ΔDEF?



3 State the missing terms to complete the following sentences.

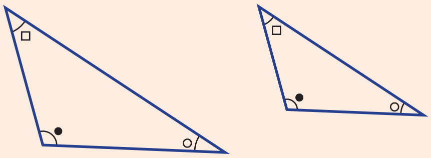
a The abbreviated tests for similar triangles are SSS, ______, ______ and ______.

b Similar figures have the same ______ but are not necessarily the same ______.

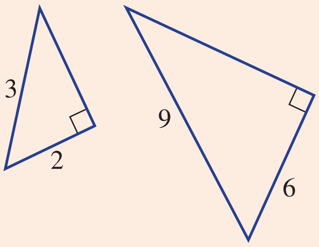
Example 15 Choosing a similarity test for triangles

Choose the similarity test which proves that these pairs of triangles are similar.

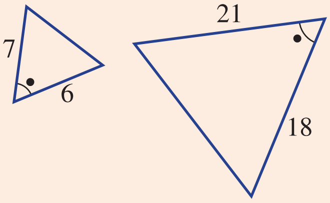
a



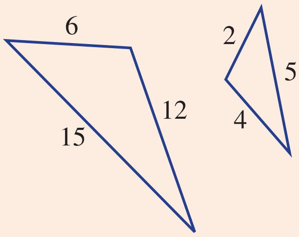
b



c



d



SOLUTION | **EXPLANATION**

a AAA

Three pairs of angles are equal.

b RHS

Both are right-angled triangles and the hypotenuses and another pair of sides are in the same ratio $\left(\frac{9}{3} = \frac{6}{2}\right)$.

c SAS

Two pairs of corresponding sides are in the same ratio $\left(\frac{21}{7} = \frac{18}{6}\right)$ and the included angles are equal.

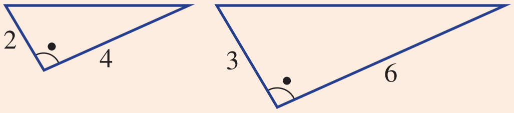
d SSS

Three pairs of corresponding sides are in the same ratio $\left(\frac{15}{5} = \frac{12}{4} = \frac{6}{2}\right)$.

Now you try

Choose the similarity test which proves that these pairs of triangles are similar.

a



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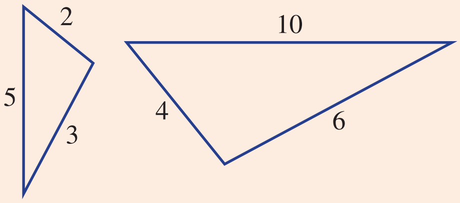

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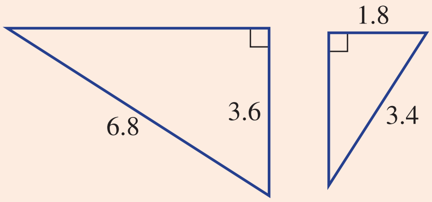
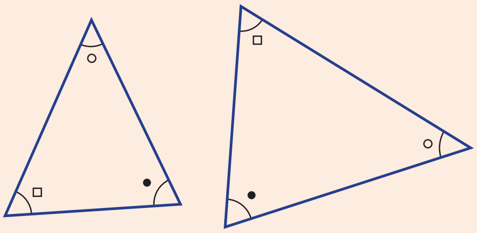


d



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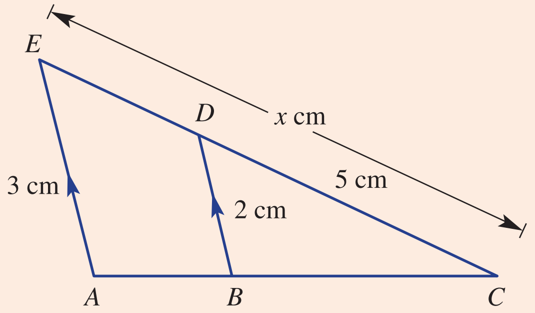
Example 16 Finding a missing length using similarity

For this pair of triangles:



a give a reason (SSS, SAS, AAA or RHS) why the two triangles are similar

b find the value of x.

SOLUTION

a AAA or just AA.

b Scale factor $= \frac{3}{2} = 1.5$

$$\therefore x = 5 \times 1.5$$
$$= 7.5$$

EXPLANATION

$\angle EAC = \angle DBC$ since AE is parallel to BD and $\angle C$ is common to both triangles. (Also $\angle AEC = \angle BDC$ since AE is parallel to BD).

$\frac{AE}{BD} = \frac{3}{2}$.

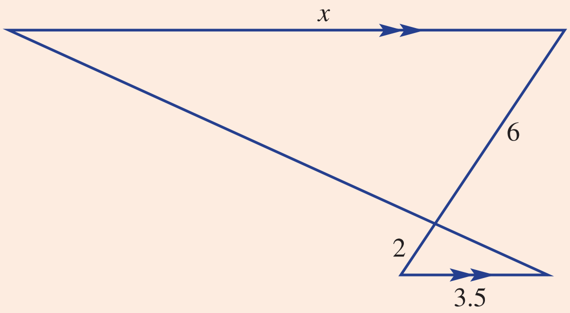
Multiply CD by the scale factor to find the length of the corresponding side CE.

Now you try

For this pair of triangles:

a give a reason (SSS, SAS, AAA or RHS) why the two triangles are similar

b find the value of x.



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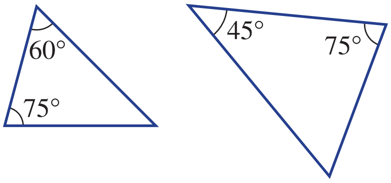
Exercise 7G

FLUENCY	1–4	2, 3(1/2), 4, 6	2, 3(1/2), 5–7

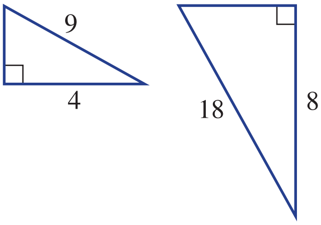
Example 15

1 Choose the similarity test which proves that these pairs of triangles are similar.

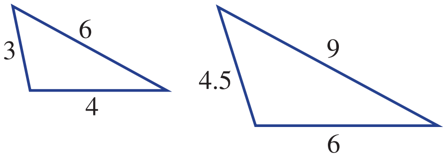
a



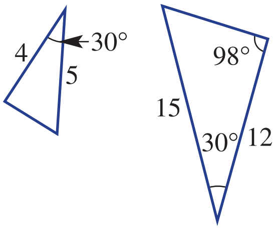
b



c



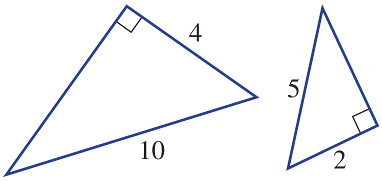
d



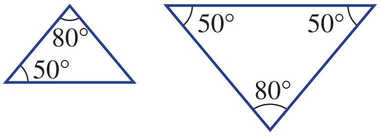
Example 15

2 Choose the similarity test which proves that these pairs of triangles are similar.

a



b



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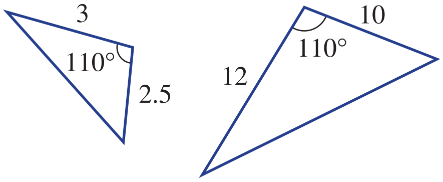

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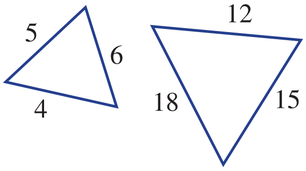
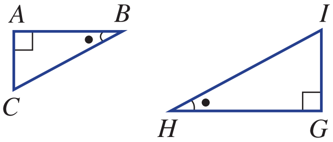
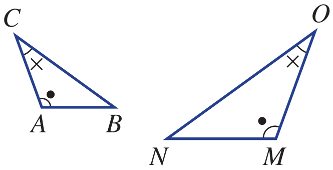


3 Write similarity statements for these pairs of similar triangles. Write letters in matching order.

a



b



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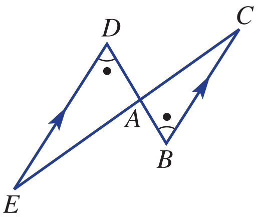

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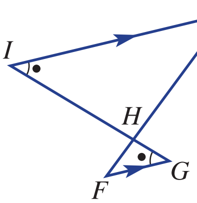
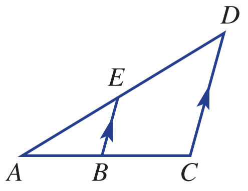
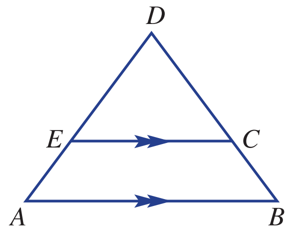


e



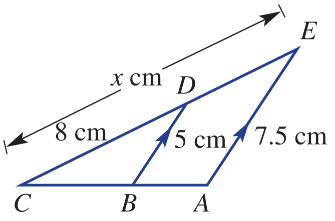
f



Example 16

4 For this pair of triangles:

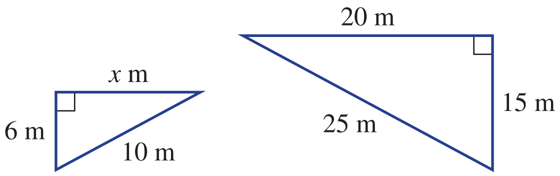
- **a** give a reason (SSS, SAS, AAA or RHS) why the two triangles are similar
- **b** find the value of x.



Example 16

5 For this pair of triangles:

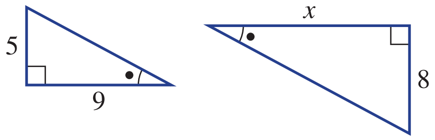
- **a** give a reason (SSS, SAS, AAA or RHS) why the two triangles are similar
- **b** find the value of x.



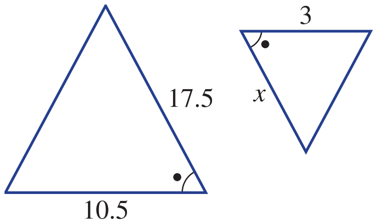
6 These pairs of triangles are similar. For each pair find:

- **i** the enlargement factor (scale factor) which enlarges the smaller triangle to the larger triangle
- **ii** the value of x.

a

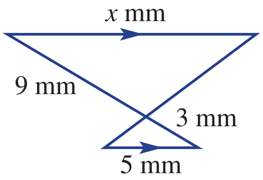


b



7 For this pair of triangles:

- **a** give a reason (SSS, SAS, AAA or RHS) why the two triangles are similar
- **b** find the value of x.

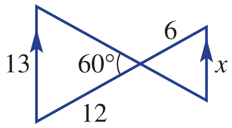


PROBLEM-SOLVING	8	8, 9	8–10

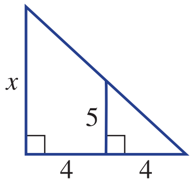
8 For each pair of similar triangles:

- **i** give a reason (SSS, SAS, AAA or RHS) why the two triangles are similar
- **ii** find the value of x.

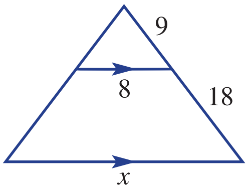
a



b

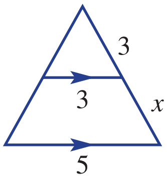


c

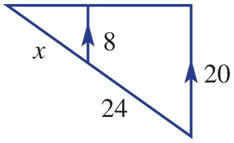


9 Find the value of x in these triangles.

a



b



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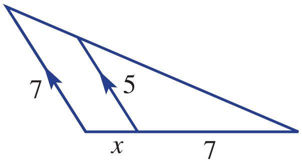
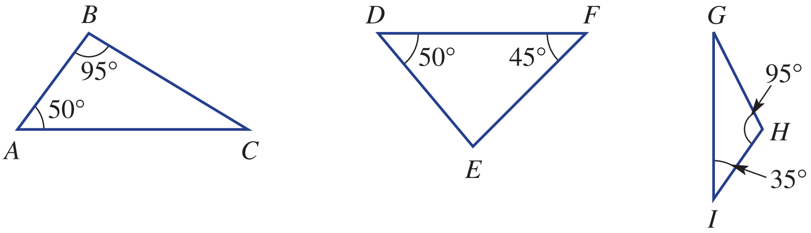


10 Name the triangle which is not similar to the other two in each group of three triangles.

a



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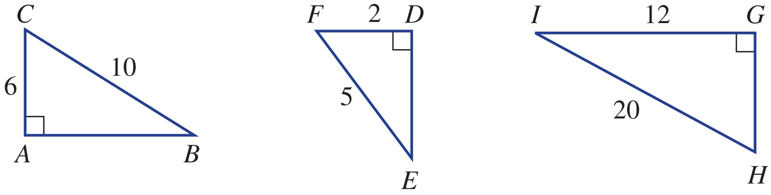

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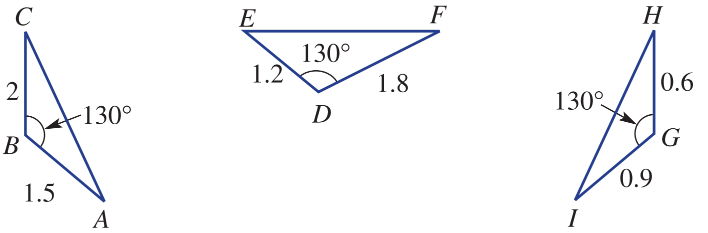

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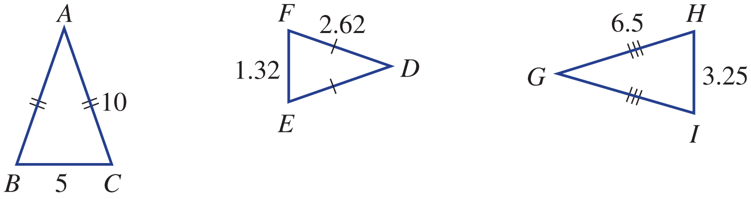
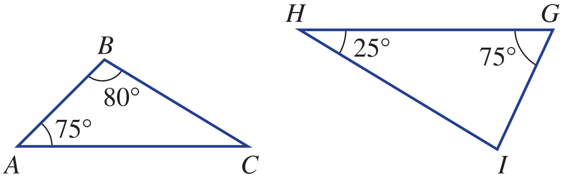
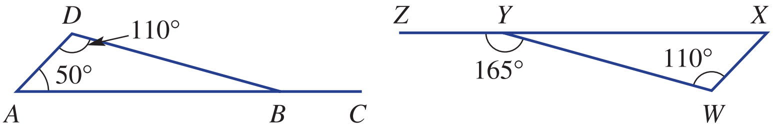


REASONING	11	11, 12	12–14

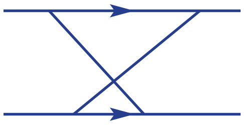
11 Give reasons why these two triangles are similar.



12 Give reasons why the two triangles in these diagrams are not similar.



13 When two intersecting transversals join parallel lines, two triangles are formed. Explain why these two triangles are similar.

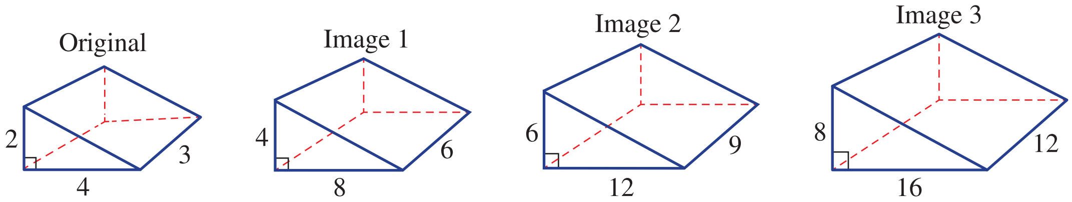


14 The four tests for similarity closely resemble the tests for congruence. Which similarity test closely matches the AAS congruence test? Explain the difference.

ENRICHMENT: Area and volume ratio

ENRICHMENT: Area and volume ratio	–	–	15

15 Consider these four similar triangular prisms (not drawn to scale).



a Complete this table.

Triangle	Original	Image 1	Image 2	Image 3
Length scale factor	1	2		
Area (cross-section)				
Area scale factor	1			
Volume				
Volume scale factor	1			

b What do you notice about the area scale factor compared to the length scale factor?

c What would be the area scale factor if the length scale factor is n?

d What would be the area scale factor if the length scale factor is:

i 10? **ii** 20? **iii** 100?

e What would be the area scale factor if the length scale factor is $\frac{1}{2}$?

f What do you notice about the volume scale factor compared to the length scale factor?

g What would be the volume scale factor if the length scale factor is n?

h What would be the volume scale factor if the length scale factor is:

i 5? **ii** 10? **iii** $\frac{1}{2}$?

7H Proving and applying similar triangles

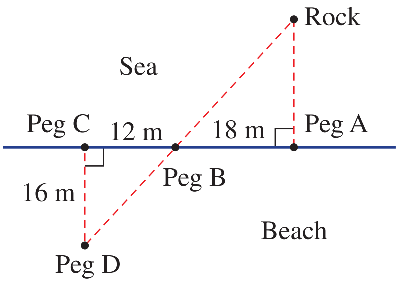
Learning intentions
- To be able to prove a pair of triangles is similar giving reasons
- To be able to identify a pair of similar triangles in a practical problem
- To know how to use similarity to solve a problem

Similar triangles can be used in many mathematical and practical problems. If two triangles are proved to be similar, then the properties of similar triangles can be used to find missing lengths or unknown angles. The approximate height of a tall object, or the width of a projected image, for example, can be found using similar triangles.

Similar triangles can be used to verify the height, 828 m, of the world's tallest building, the Burj Khalifa, Dubai. The Jeddah Tower, Saudi Arabia, is expected to be over 1 km tall when constructed.

LESSON STARTER How far is the rock?

Ali is at the beach and decides to estimate how far an exposed rock is from seashore. He places four pegs in the sand as shown and measures the distance between them.



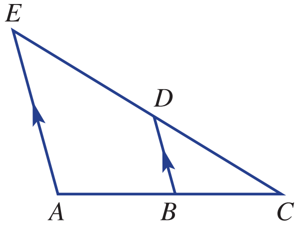
- Why do you think Ali has placed the four pegs in the way that is shown in the diagram?
- Why are the two triangles similar? Which test (SSS, SAS, AAA or RHS) could be used and why?
- How would Ali use the similar triangles to find the distance from the beach at peg A to the rock?

KEY IDEAS

- To prove triangles are similar, list any pairs of corresponding equal angles or pairs of sides in a given ratio.
  - Give reasons at each step.
  - Write a similarity statement, for example, $\Delta ABC \mathbin{|||} \Delta DEF$ or $\Delta ABC \sim \Delta DEF$.
  - Write the triangle similarity test in abbreviated form (SSS, SAS, AAA, RHS).
- To apply similarity in practical problems, follow these steps.
  - Prove two triangles are similar.
  - Find a scale factor.
  - Use the scale factor to find the value of any unknowns.

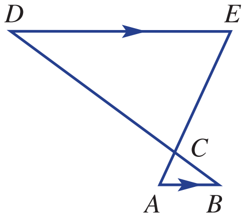
BUILDING UNDERSTANDING

1 In this diagram, name the angle which is common to both ΔACE and ΔBCD.



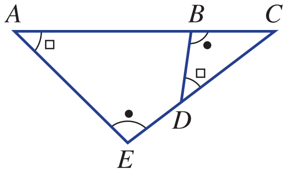
2 In this diagram:

- **a** name the pair of vertically opposite angles
- **b** name the two pairs of equal alternate angles.



3 In this diagram:

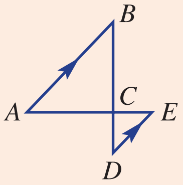
- **a** name the angle which is common for the two triangles
- **b** name the side that corresponds to side
  - **i** DC
  - **ii** AE.



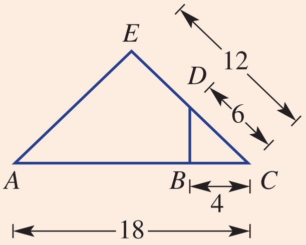
Example 17 Proving two triangles are similar

Prove that each pair of triangles is similar.

a



b



SOLUTION

a $\angle BAC = \angle DEC$

(alternate angles and $DE \parallel AB$)

$\angle ABC = \angle EDC$

(alternate angles and $DE \parallel AB$)

$\angle ACB = \angle ECD$

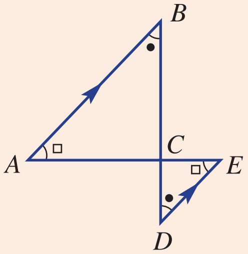
(vertically opposite angles)

$\therefore \Delta ABC \parallel\!\mid \Delta EDC$ (AAA)

EXPLANATION

Parallel lines cut by a transversal will create a pair of equal alternate angles.

Vertically opposite angles are also equal. Write the similarity statement and the abbreviated reason.



Continued on next page

b $\angle ACE = \angle DCB$ (common)

$$\frac{AC}{DC} = \frac{18}{6} = 3$$

$$\frac{EC}{BC} = \frac{12}{4} = 3 = \frac{AC}{DC}$$

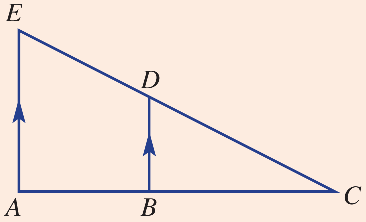
$\therefore \Delta ACE \ ||| \ \Delta DCB$ (SAS)

Note that there is a common angle and two pairs of corresponding sides. Find the scale factor for both pairs of sides to see if they are equal. Complete the proof with a similarity statement. The angle is the included angle.

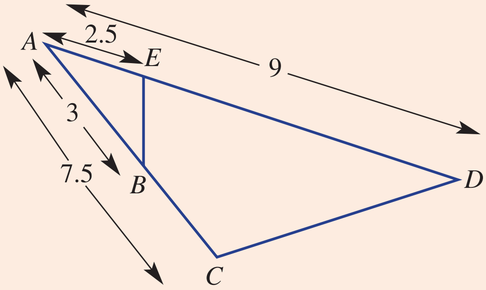
Now you try

Prove that each pair of triangles is similar.

a

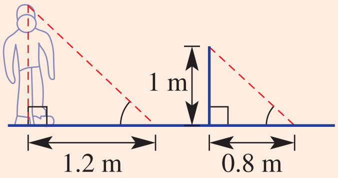


b



Example 18 Applying similarity

Chris' shadow is 1.2 m long when a 1 m vertical stick has a shadow 0.8 m long.



a Give a reason why the two triangles are similar.

b Determine Chris' height.

SOLUTION

a All angles are the same (AAA).

b $\text{Scale factor} = \frac{1.2}{0.8} = 1.5$

$\therefore \text{Chris' height} = 1 \times 1.5$

$= 1.5 \text{ m}$

EXPLANATION

The sun's rays will pass over Chris and the stick and hit the ground at approximately the same angle.

First find the scale factor.

Multiply the height of the stick by the scale factor to find Chris' height.

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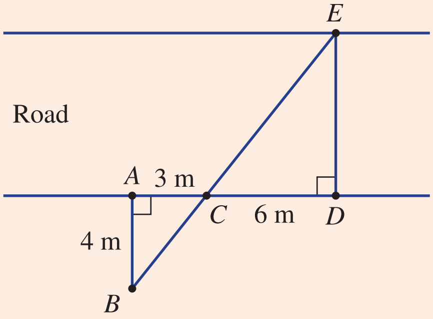
Now you try

Four pegs A, B, C and D are placed on one side of a road as shown.

a Give a reason why the two triangles are similar.

b Determine the distance across the road, DE.



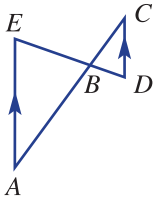
Exercise 7H

FLUENCY	1–3	2, 3	2, 3

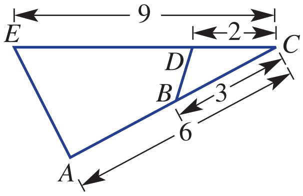
Example 17

1 Prove that each pair of triangles is similar.

a



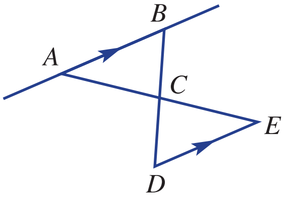
b



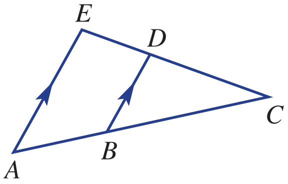
Example 17a

2 Prove that each pair of triangles is similar.

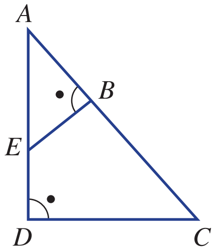
a



b



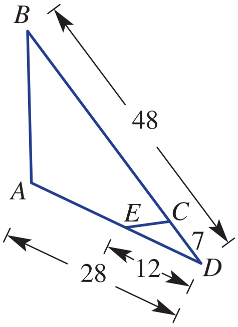
c



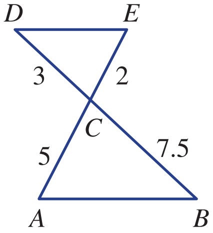
Example 17b

3 Prove that each pair of triangles is similar.

a



b



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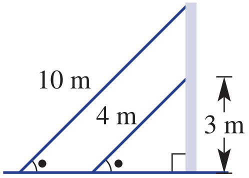
PROBLEM-SOLVING	4, 6, 7	5–8	6–9

Example 18

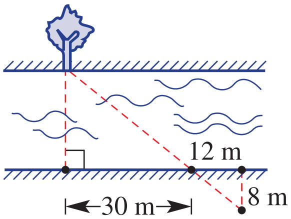
4 A tree’s shadow is 20 m long, while a 2 m vertical stick has a shadow 1 m long.
  a Give a reason why the two triangles formed by the objects and their shadows are similar.
  b Find the height of the tree.

5 Two cables support a steel pole at the same angle as shown. The two cables are 4 m and 10 m in length, and the shorter cable reaches 3 m up the pole.
  a Give a reason why the two triangles are similar.
  b Find the height of the pole.

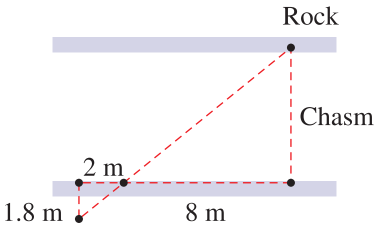


6 John stands 6 m from a vertical lamp post and casts a 2 m shadow. The shadow from the lamp post and from John end at the same place. Determine the height of the lamp post if John is 1.5 m tall.

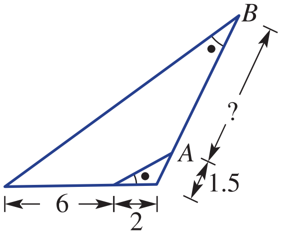
7 Joanne wishes to determine the width of a river without crossing it. She places four pegs as shown. Calculate the width of the river.



8 A deep chasm has a large rock sitting on one side as shown. Find the width of the chasm.



9 Find the length AB in this diagram if the two triangles are similar.

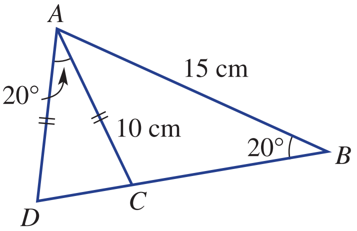


REASONING	10	10	10, 11

10 In this diagram, ΔADC is isosceles.

a There are two triangles which are similar. Identify them and complete a proof. You may need to find another angle first.

b Find the lengths DC and CB expressing your answers as fractions.



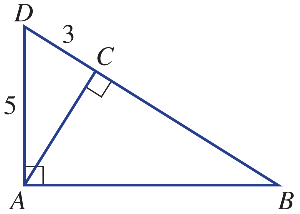
11 In this diagram, AC is perpendicular to BD and ΔABD is a right angle.

a Prove that ΔABD is similar to:

- **i** ΔCBA
- **ii** ΔCAD.

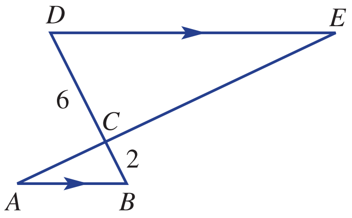
b Find these lengths.

- **i** BD
- **ii** AC
- **iii** AB

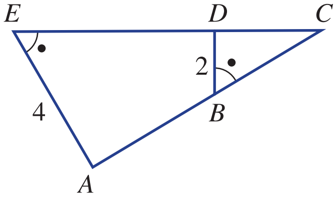


ENRICHMENT: Extended proofs	–	–	12

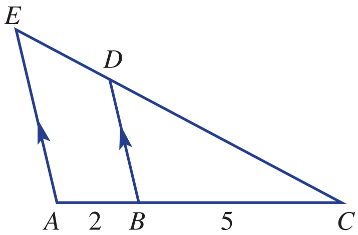
12 **a** Prove $AE = 4AC$.



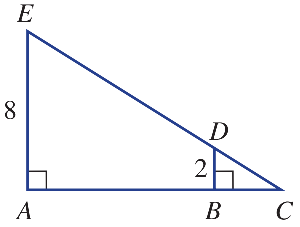
b Prove $BC = \frac{1}{2}CE$.

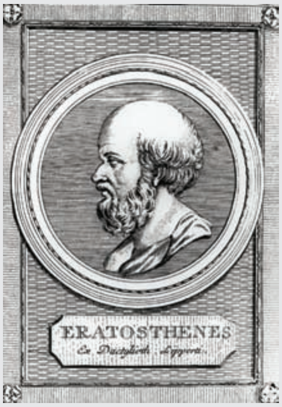


c Prove $CE = \frac{7}{5}CD$.



d Prove $AB = 3BC$.





Eratosthenes was a Greek mathematician, geographer and astronomer who, around 240 BCE, used similar triangles to quite accurately calculate Earth's circumference.

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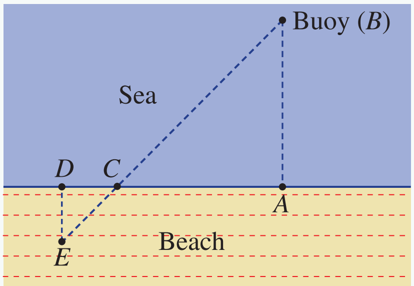
Modelling

Ocean ironman

The organisers of an ironman race want to estimate the distance from the beach to a floating buoy in the sea. Flags are placed on the beach by the organisers in an effort to create similar triangles as shown. The flag positions are at points A, C, D and E and the line segments AB and DE are assumed to be at right angles to the beach.



Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate.

Preliminary task

One of the organisers measures the following:

- $AC = 200$ metres
- $CD = 5$ metres
- $DE = 4$ metres.

a Find the scale factor between the two similar triangles.

b Hence, estimate the distance from the beach to the buoy.

Modelling task

Formulate

a The problem is to determine the distance between the beach and the buoy using similar triangles. Write down all the relevant information that will help solve this problem with the aid of a diagram.

b Explain why the given positions of the flags form a pair of similar triangles.

Solve

c Two more attempts at positioning flags and measuring distances are undertaken by the organisers.

Jimmy's placement:

- $AC = 360$ metres
- $CD = 8$ metres
- $DE = 3$ metres.

Olivia's placement:

- $AC = 90$ metres
- $CD = 5$ metres
- $DE = 8$ metres.

i Use Jimmy's model to estimate the distance between the beach and the buoy.

ii Use Olivia's model to estimate the distance between the beach and the buoy.

Evaluate and verity

d Another organiser attempts to place the flags such that:

- $AC = 100$ metres
- $CD = 50$ centimetres
- $DE = 1.2$ metres.

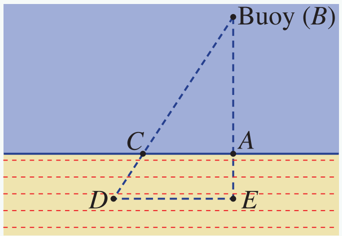
Do you think that arranging the flags in this way is a good idea? Explain why.

e If the actual distance from the beach to the buoy is 200 metres, determine an arrangement for the flags that would deliver the correct distance. Illustrate with a diagram including the measurements.

Communicate

f Summarise your results and describe any key findings.

Extension questions



a Another way to arrange the flags is to use the model shown here. Using this model, identify the pair of similar triangles and explain why they are similar.

b Using the following measurements, estimate the distance from the beach to the buoy.

- $AC = 100$ metres
- $AE = 60$ metres
- $DE = 140$ metres

c Compare the two models studied in this task using the different diagrams. Do you have a preferred model? Explain why.

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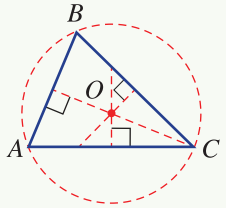
Triangle centres with technology

Use an interactive geometry package such as 'Geometers Sketchpad' or 'Cabri Geometry' to construct the following shapes.

The circumcentre of a triangle

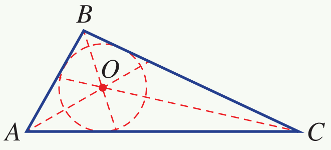
The point at which all perpendicular bisectors of the sides of a triangle meet is called the circumcentre.



- **a** Draw any triangle.
- **b** Label the vertices A, B and C.
- **c** Draw a perpendicular bisector for each side.
- **d** Label the intersection point of the bisectors O.
- **e** Using O as the centre, construct a circle that touches the vertices of the triangle.
- **f** Drag any of the vertices and describe what happens to your construction.

The incentre of a triangle

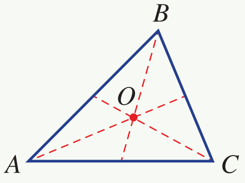
The point at which all angle bisectors of a triangle meet is called the incentre.



- **a** Draw any triangle.
- **b** Label the vertices A, B and C.
- **c** Draw the three angle bisectors through the vertices.
- **d** Label the intersection point of the bisectors O.
- **e** Using O as your centre, construct a circle that touches the sides of the triangle.
- **f** Drag any of the vertices and describe what happens to your construction.

The centroid of a triangle

The point of intersection of the three medians of a triangle is called the centroid. It can also be called the centre of gravity.



- **a** Draw a triangle and label the vertices A, B and C.
- **b** Find the midpoint of each line and draw a line segment from each midpoint to its opposite vertex.
- **c** Label the intersection point of these lines O. This is the centroid of the triangle.
- **d** Show your teacher the final construction and print it. Cut out the triangle and place a sharp pencil under the centroid. The triangle should balance perfectly.

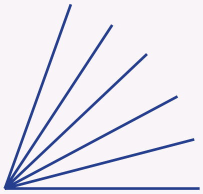
The equilateral triangle: The special triangle

- **a** Construct an equilateral triangle. Determine its incentre, circumcentre and centroid.
- **b** What do you notice?

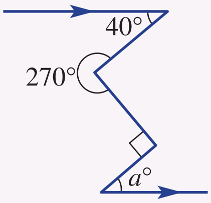
Problems and challenges

Up for a challenge? If you get stuck on a question, check out the 'Working with unfamiliar problems' poster at the end of the book to help you.

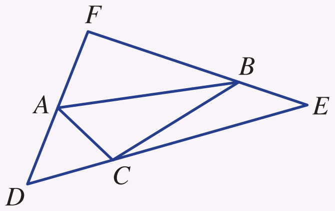
1. Use 12 matchsticks to make 6 equilateral triangles.
2. How many acute angles are there in the diagram shown below?



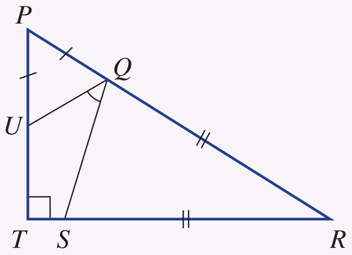
3. Find the value of a.



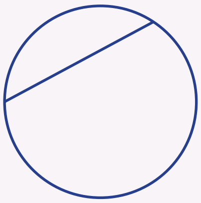
4. Explore (using interactive geometry software) where the points A, B and C should be on the sides of ΔDEF so that the perimeter of ΔABC is a minimum.



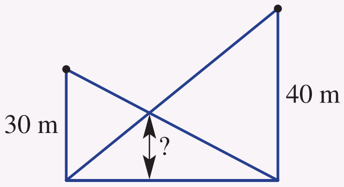
5. Find the size of angle UQS given the sides $PQ = PU$ and $QR = RS$. Angle PTR is 90°.



6. A circle is divided using chords (one chord is shown here). What is the maximum number of regions that can be formed if the circle is divided with 4 chords?



7. Two poles are 30 m and 40 m high. Cables connect the top of each vertical pole to the base of the other pole. How high is the intersection point of the cables above the ground?



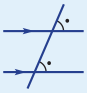
Angles

Acute $0^\circ < \theta < 90^\circ$
Right 90°
Obtuse $90^\circ < \theta < 180^\circ$
Straight 180°
Reflex $180^\circ < \theta < 360^\circ$
Revolution 360°
Complementary angles sum to 90°.
Supplementary angles sum to 180°.
Vertically opposite angles are equal.

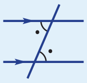
Parallel lines

If the lines are parallel:

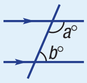
Corresponding angles are equal



Alternate angles are equal



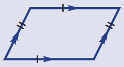
Cointerior angles are supplementary
$a + b = 180$



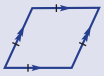
Quadrilaterals

Four-sided figures – sum of the interior angles is 360°

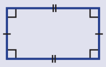
Parallelogram



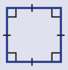
Rhombus



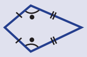
Rectangle



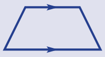
Square



Kite



Trapezium



Congruent triangles

These are identical in shape and size. They may need to be rotated or reflected. We write $\Delta ABC \equiv \Delta DEF$.
congruent to

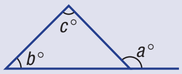
Tests for congruence:
SSS – three pairs of corresponding sides equal
SAS – two pairs of corresponding sides and the included angles are equal
AAS – two pairs of angles and any pair of corresponding sides are equal
RHS – a right angle, hypotenuse and one other pair of corresponding sides are equal

Polygons

The sum of the interior angles in a polygon with n sides is $S = 180(n - 2)$. Regular polygons have all sides and angles equal.

Triangles

Sum of angles is 180°.
Exterior angle equals sum of the two opposite interior angles.



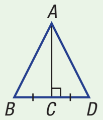
$a = b + c$
exterior angle

Types:
Acute angled – all angles $< 90^\circ$
Obtuse angled – 1 angle $> 90^\circ$
Right angled – 1 angle 90°
Equilateral – all angles 60° – all sides are equal
Isosceles – 2 angles and 2 sides are equal
Scalene – all sides and angles are different sizes

Geometry

Proving congruence (Ext)/similarity

List corresponding pairs of equal angles and pairs of sides that are equal/in same ratio. Give reasons for each pair. Write a congruence/similarity statement giving the abbreviated reason.
e.g.



AC (common)
$BC = DC$ (given)
$\angle ACD = \angle ACB = 90^\circ$ (given)
$\therefore \ \Delta ACD \equiv \Delta ACB$ (SAS)

Similar figures

These are the same shape but different size.
Two figures are similar if one can be enlarged to be congruent to the other.
Enlargement uses a scale factor.
Scale factor $= \dfrac{\text{image length}}{\text{original length}}$

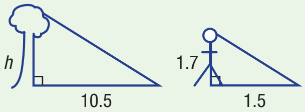
Similar triangles

Similar triangles have corresponding angles equal and corresponding sides are in the same ratio.
We say $\Delta ABC \,|||\, \Delta DEF$ or $\Delta ABC \sim \Delta DEF$.
similar to

Tests for similar triangles:
SSS – three pairs of corresponding sides in the same ratio
SAS – two pairs of corresponding sides in the same ratio and included angles equal
AAA or **AA** – all three pairs of corresponding angles are equal; two pairs is enough to prove this
RHS – the hypotenuses of right-angled triangles and another pair of corresponding sides are in the same ratio

Applying similar triangles

In practical problems, look to identify and prove pairs of similar triangles. Find a scale factor and use this to find the value of any unknowns.
e.g. shadow cast by a tree is 10.5 m while a person 1.7 m tall has a 1.5 m shadow. How tall is the tree?
Similar (AAA)
Scale factor $= \dfrac{10.5}{1.5} = 7$
$\therefore \ h = 1.7 \times 7$
$= 11.9$ m
Tree is 11.9 m tall.



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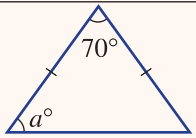
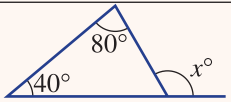
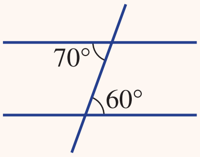
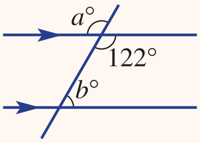
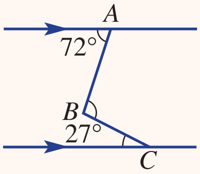
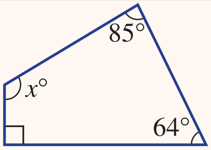
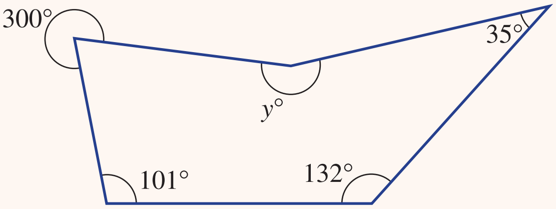
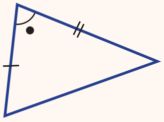
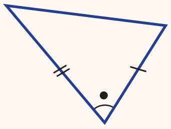
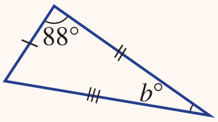
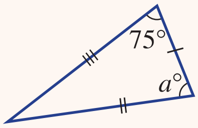
Chapter checklist

Chapter checklist: Success criteria

		✔
7A	**1. I can find supplementary and complementary angles.** e.g. For an angle of size 62° determine the size of its supplementary angle and its complementary angle.	
7A	**2. I can classify triangles and find an unknown angle.** e.g. Name the type of triangle shown and determine the value of the pronumeral.  	
7A	**3. I can find exterior angles.** e.g. Find the value of the pronumeral in the diagram giving reasons.  	
7B	**4. I can determine if lines are parallel.** e.g. Decide if the diagram contains a pair of parallel lines, giving a reason.  	
7B	**5. I can find angles in parallel lines.** e.g. Find the value of each of the pronumerals giving reasons.  	
7B	**6. I can find angles in parallel lines using a third parallel line.** e.g. Add a third parallel line to the diagram to help find ∠*ABC*.  	
7C	**7. I can find angles in quadrilaterals.** e.g. Find the value of the pronumeral in the quadrilateral.  	
7C	**8. I can find angles in polygons.** e.g. For the polygon shown, find the angle sum and use this to find the value of the pronumeral.  	
7D	**9. I can use the congruence tests to identify congruent triangles.** e.g. Which congruence test (SSS, SAS, AAS or RHS) would be used to show that these pairs of triangles are congruent?  	
7D	**10. I can use congruence to find missing side lengths or angles.** e.g. Find the value of the pronumerals in these congruent triangles.   	

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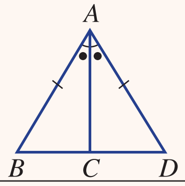
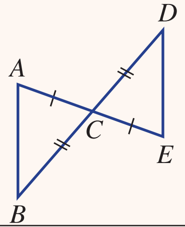
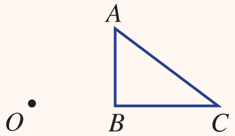
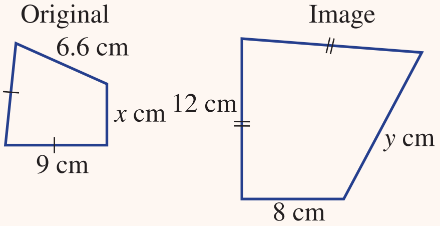
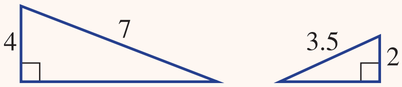
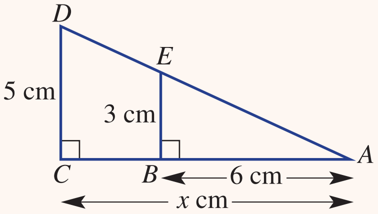
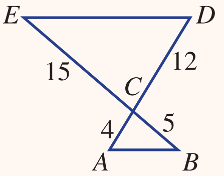
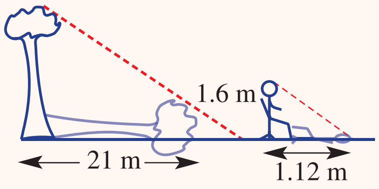
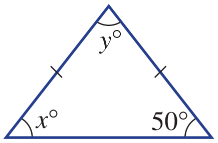
		✔
7E	**11. I can prove that two triangles are congruent.** e.g. Prove that $\Delta ABC \equiv \Delta ADC$.  Ext	
7E	**12. I can use congruence to prove geometrical results.** e.g. Prove that $\Delta ABC \equiv \Delta EDC$ and hence prove that $AB \parallel DE$.  Ext	
7F	**13. I can enlarge a figure using a scale factor.** e.g. Copy the diagram and use the centre of enlargement (O) and each of the scale factors 2 and $\frac{1}{3}$ to enlarge ΔABC.  	
7F	**14. I can find and use the scale factor in similar figures.** e.g. For the similar figures shown, find the scale factor and use this to find the value of x and y.  Original Image 6.6 cm x cm 12 cm y cm 9 cm 8 cm	
7G	**15. I can choose a similarity test for triangles.** e.g. Choose the similarity test which proves that these pairs of triangles are similar.  	
7G	**16. I can use similarity to find a missing length.** e.g. For the pair of triangles shown, give a reason why the two triangles are similar and find the value of x.  	
7H	**17. I can prove that two triangles are similar.** e.g. Prove that the two triangles are similar.  	
7H	**18. I can apply similarity.** e.g. A 1.6 m tall girl has a shadow that is 1.12 m long. A tree's shadow is 21 m long. Give a reason why the two triangles in the diagram are similar and determine the tree's height.  	

Short-answer questions

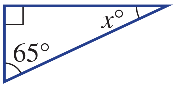
7A

1 Name the following triangles and find the value of the pronumerals.

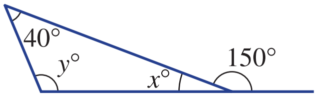
a



b



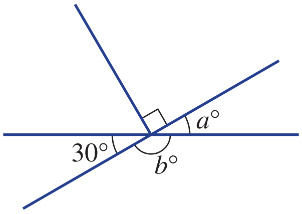
c



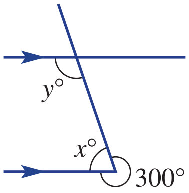
7A/B

2 Find the value of each pronumeral in the diagrams. Give reasons for your answers.

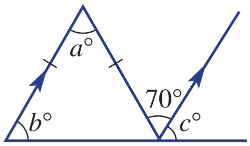
a



b

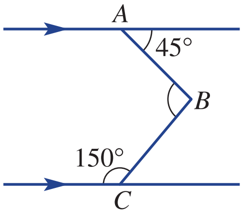


c



7B

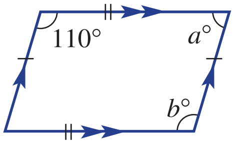
3 By adding a third parallel line to the diagram, find ∠ABC.



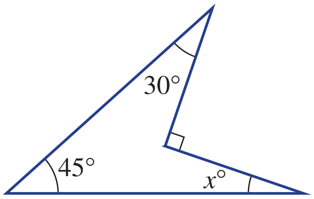
7C

4 Find the value of each pronumeral in the following polygons. The polygon in part **d** is regular.

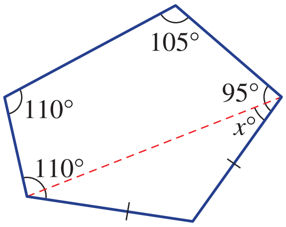
a



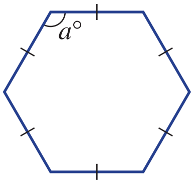
b



c



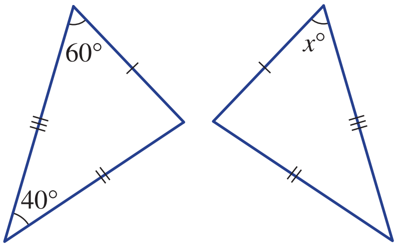
d



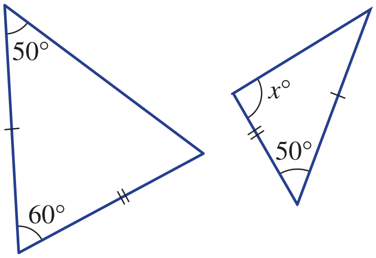
7D

5 Determine if the triangles in each pair are congruent. If congruent, give the abbreviated reason and state the value of any pronumerals.

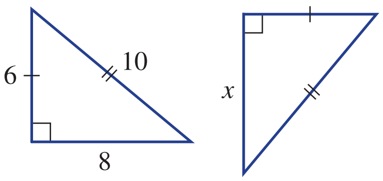
a



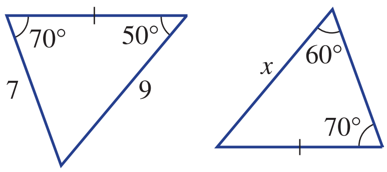
b



c

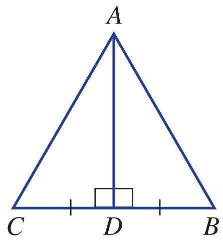


d



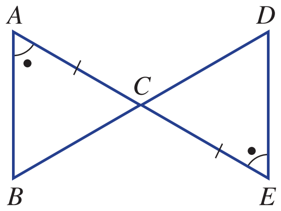
7E **Ext**

6 **a** For the diagram at right, prove that $\Delta ADB \equiv \Delta ADC$. List your reasons and give the abbreviated congruence test.



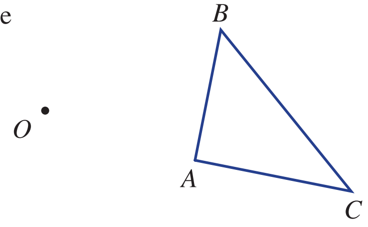
b **i** For the diagram at right, prove that $\Delta ACB \equiv \Delta ECD$. List your reasons and give the abbreviated congruence test.

ii Hence, prove that $AB \parallel DE$.



7F

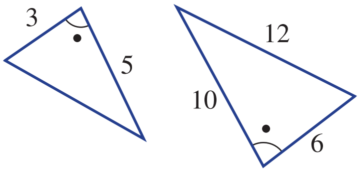
7 Copy the given diagram using plenty of space. Using the centre of enlargement (O) and a scale factor of 3, enlarge ΔABC.



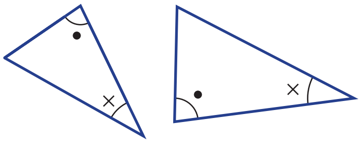
7G

8 Determine if the following pairs of triangles are similar, and state the similarity test which proves this.

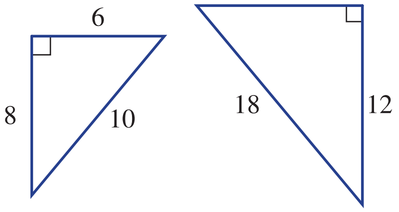
a



b



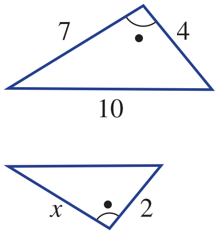
c



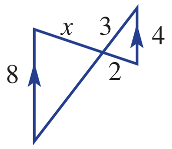
7G

9 For the following pairs of similar triangles, find the value of x.

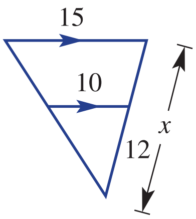
a



b



c

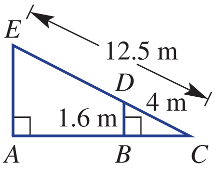


7H

10 A conveyor belt used to load luggage onto a plane is 12.5 m long. A vertical support 1.6 m high is placed under the conveyor belt such that it is 4 m along the conveyor belt as shown.

a Prove that $\Delta BCD \mathrel{|||} \Delta ACE$.

b Find the height (AE) of the luggage door above the ground.



Multiple-choice questions

7A

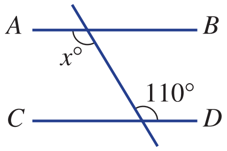
1 The angle that is supplementary to an angle of 55° is:

A 35° **B** 55° **C** 95°
D 125° **E** 305°

7B

2 What is the value of x if AB is parallel to CD?

A 110 **B** 70 **C** 20
D 130 **E** 120



7A

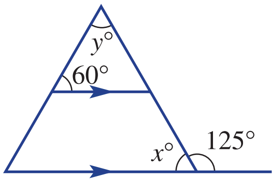
3 If two angles in a triangle are complementary, then the third angle is:

A acute **B** a right angle
C obtuse **D** reflex
E supplementary

7B

4 The values of x and y in the diagram are:

A $x = 35, y = 85$
B $x = 45, y = 45$
C $x = 55, y = 60$
D $x = 55, y = 65$
E $x = 65, y = 55$



7C

5 The sum of the interior angles in a hexagon is:

A 1078° **B** 360° **C** 720°
D 900° **E** 540°

7C

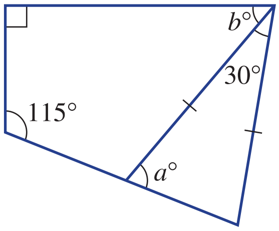
6 The quadrilateral with all sides equal, two pairs of opposite parallel sides and no right angles is:

A a kite **B** a trapezium **C** a parallelogram
D a rhombus **E** a square

7C

7 The values of a and b in the diagram are:

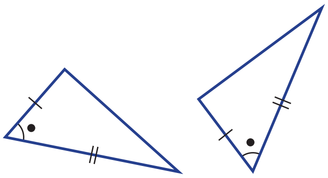
A $a = 85, b = 60$
B $a = 75, b = 80$
C $a = 80, b = 55$
D $a = 70, b = 55$
E $a = 75, b = 50$



7D

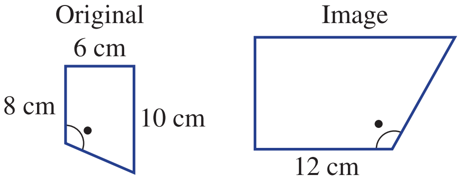
8 The abbreviated reason for congruence in the two triangles shown is:

A AA
B SAS
C SSS
D AAS
E RHS



7G

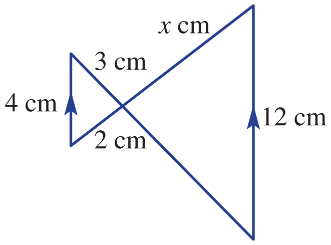
9 The scale factor that enlarges the original figure to its image is:



A $\frac{2}{3}$
B 2
C 1.2
D 1.5
E 0.5

7G

10 The value of x in the diagram is:

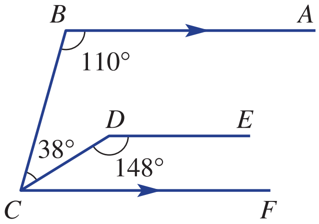


A 6
B 9
C 10
D 8
E 7.5

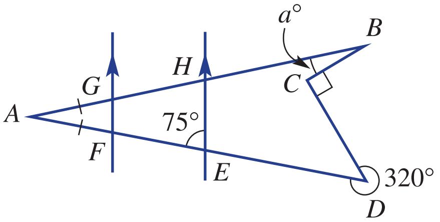
Extended-response questions

1 Complete the following.

a Prove that $DE \parallel CF$.

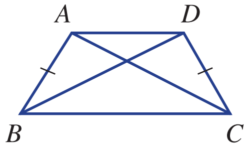


b Show, with reasons, that $a = 20$.

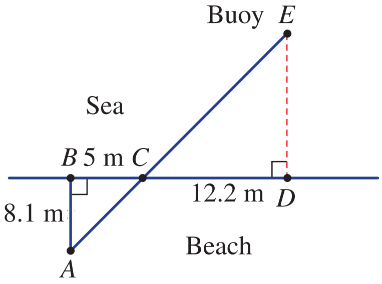


Ext

c Use congruence to prove that $AC = BD$ in the diagram, given $AB = CD$ and $\angle ABC = \angle DCB$.



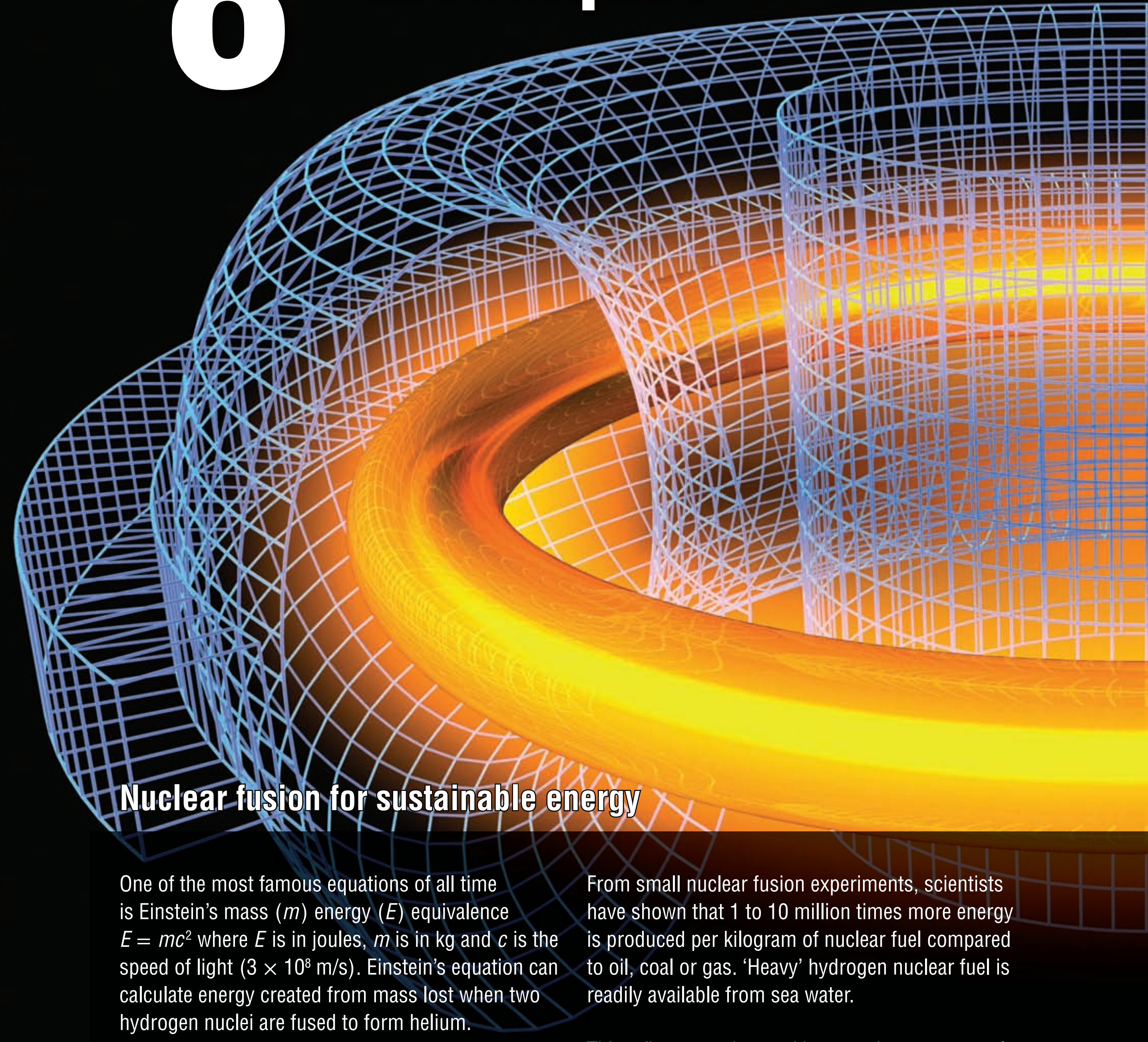
2 A buoy (E) is floating in the sea at some unknown distance from the beach as shown. The points A, B, C and D are measured and marked out on the beach as shown.



a Name the angle which is vertically opposite to $\angle ACB$.
b Explain, with reasons, why $\Delta ABC \parallel\!| \Delta EDC$.
c Find the distance from the buoy to the beach (ED), to one decimal place.

CHAPTER 8

Algebraic techniques



Nuclear fusion for sustainable energy

One of the most famous equations of all time is Einstein's mass (m) energy (E) equivalence $E = mc^2$ where E is in joules, m is in kg and c is the speed of light (3×10^8 m/s). Einstein's equation can calculate energy created from mass lost when two hydrogen nuclei are fused to form helium.

In the Sun's core, at 15 million °C and compressed by immense gravity, nuclear fusion reactions convert 600 million tonnes of hydrogen per second into helium, releasing enormous quantities of heat and light energy.

From small nuclear fusion experiments, scientists have shown that 1 to 10 million times more energy is produced per kilogram of nuclear fuel compared to oil, coal or gas. 'Heavy' hydrogen nuclear fuel is readily available from sea water.

Thirty-five countries working together are currently constructing the International Thermonuclear Experimental Reactor (ITER) in France. The large, doughnut shaped chamber, called a Tokamak, is where fuel will reach 150 million °C, the temperature needed for fusion reactions on Earth.

Nuclear fusion power offers a safe, reliable and unlimited source of clean energy. Development of technology to enable a fusion reaction to produce more energy than it uses is one of the world's greatest challenges for mathematicians, scientists and engineers to solve.

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

In this chapter

8A Expanding binomial products
8B Perfect squares and difference of perfect squares
8C Factorising algebraic expressions
8D Factorising the difference of perfect squares
8E Factorising by grouping
8F Factorising quadratic trinomials (EXTENDING)
8G Factorising trinomials of the form $ax^2 + bx + c$ (EXTENDING)
8H Simplifying algebraic fractions: Multiplication and division
8I Simplifying algebraic fractions: Addition and subtraction
8J Further simplification of algebraic fractions (EXTENDING)
8K Equations with algebraic fractions (EXTENDING)

Australian Curriculum

NUMBER AND ALGEBRA
Patterns and algebra

Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate (ACMNA213)

© ACARA

8A Expanding binomial products

Learning intentions

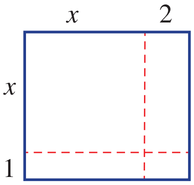
- To know how to apply the distributive law to binomial products
- To be able to expand and simplify binomial products

A binomial is an expression with two terms such as $x + 5$ or $x^2 + 3$. You will recall from Chapter 2 that we looked at the product of a single term with a binomial expression, e.g. $2(x - 3)$ or $x(3x - 1)$. The product of two binomial expressions can also be expanded using the distributive law. This involves multiplying every term in one expression by every term in the other expression.

Business analysts expand algebraic products to model profit, as sales and profit per item can both depend on the selling price.
E.g. $P = 50(20 - c) \times (c - 4)$ for chocolates selling at $\$c$ per box.

LESSON STARTER Rectangular expansions

If $(x + 1)$ and $(x + 2)$ are the side lengths of a rectangle as shown below, the total area can be found as an expression in two different ways.



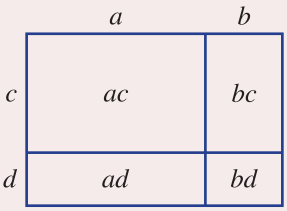
- Write an expression for the total area of the rectangle using length $= (x + 2)$ and width $= (x + 1)$.
- Now find the area of each of the four parts of the rectangle and combine to give an expression for the total area.
- Compare your two expressions above and complete this equation:

  $$(x + 2)(\_\_\_\_\_) = x^2 + \_\_\_\_\_ + \_\_\_\_\_.$$
- Can you explain a method for expanding the left-hand side to give the right-hand side?

KEY IDEAS

- Expanding **binomial products** uses the **distributive law**.

$$\begin{aligned}(a + b)(c + d) &= a(c + d) + b(c + d)\\ &= ac + ad + bc + bd\end{aligned}$$



- Diagrammatically $(a + b)(c + d) = ac + ad + bc + bd$

For example:
$$\begin{aligned}(x + 1)(x + 5) &= x^2 + 5x + x + 5\\ &= x^2 + 6x + 5\end{aligned}$$

ISBN 978-1-108-77288-4

Updated September 2021

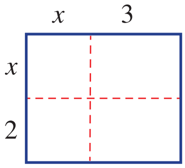
BUILDING UNDERSTANDING

1 The diagram on the right shows the area $(x + 2)(x + 3)$.

a State an expression for the area of each of the four regions inside the rectangle.

b State the missing terms.

$$\begin{aligned}(x + 2)(x + 3) &= \_\_\_ + 3x + \_\_\_ + 6\\ &= \_\_\_ + 5x + \_\_\_\end{aligned}$$

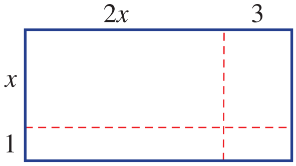


2 The diagram on the right shows the area $(2x + 3)(x + 1)$.

a State an expression for the area of each of the four regions inside the rectangle.

b State the missing terms.

$$\begin{aligned}(2x + 3)(\_\_\_) &= 2x^2 + \_\_\_ + 3x + \_\_\_\\ &= \_\_\_ + \_\_\_ + \_\_\_\end{aligned}$$



3 State the missing terms in these expansions.

a $(x + 1)(x + 5) = \_\_\_ + 5x + \_\_\_ + 5$
$= \_\_\_ + 6x + \_\_\_$

b $(x - 3)(x + 2) = \_\_\_ + \_\_\_ - 3x - \_\_\_$
$= \_\_\_ - x - \_\_\_$

c $(3x - 2)(7x + 2) = \_\_\_ + 6x - \_\_\_ - \_\_\_$
$= \_\_\_ - \_\_\_ + \_\_\_$

d $(4x - 1)(3x - 4) = \_\_\_ - \_\_\_ - 3x + \_\_\_$
$= \_\_\_ - 19x + \_\_\_$

Example 1 Expanding binomial products

Expand the following.

a $(x + 3)(x + 5)$ **b** $(x - 4)(x + 7)$ **c** $(2x - 1)(x - 6)$ **d** $(5x - 2)(3x + 7)$

SOLUTION	EXPLANATION
a $(x+3)(x+5) = x^2 + 5x + 3x + 15$ $= x^2 + 8x + 15$	Use the distributive law to expand the brackets and then collect the like terms $5x$ and $3x$.
b $(x-4)(x+7) = x^2 + 7x - 4x - 28$ $= x^2 + 3x - 28$	After expanding to get the four terms, collect the like terms $7x$ and $-4x$.
c $(2x-1)(x-6) = 2x^2 - 12x - x + 6$ $= 2x^2 - 13x + 6$	Remember $2x \times x = 2x^2$ and $-1 \times (-6) = 6$.
d $(5x-2)(3x+7) = 15x^2 + 35x - 6x - 14$ $= 15x^2 + 29x - 14$	Recall $5x \times 3x = 5 \times 3 \times x \times x = 15x^2$.

Now you try

Expand the following.

a $(x + 2)(x + 4)$ **b** $(x - 3)(x + 5)$ **c** $(2x - 3)(x - 4)$ **d** $(3x - 5)(4x + 3)$

Exercise 8A

FLUENCY	1, 2–3($\frac{1}{2}$)	2–3($\frac{1}{2}$), 3($\frac{1}{2}$)	2–4($\frac{1}{2}$)

1 Expand the following.

Example 1a
a **i** $(x + 3)(x + 4)$
ii $(x + 2)(x + 7)$

Example 1b
b **i** $(x - 2)(x + 6)$
ii $(x - 3)(x + 4)$

Example 1c
c **i** $(2x - 5)(x - 2)$
ii $(3x - 2)(x - 4)$

Example 1d
d **i** $(5x - 4)(2x + 3)$
ii $(7x - 3)(4x + 5)$

Example 1a,b
2 Expand the following.

a $(x + 2)(x + 5)$ **b** $(b + 3)(b + 4)$ **c** $(t + 8)(t + 7)$
d $(a + 1)(a + 7)$ **e** $(y + 10)(y + 2)$ **f** $(m + 4)(m + 12)$
g $(x + 3)(x - 4)$ **h** $(x + 4)(x - 8)$ **i** $(x - 6)(x + 2)$
j $(x - 1)(x + 10)$ **k** $(x - 1)(x - 2)$ **l** $(x - 4)(x - 5)$

Example 1c,d
3 Expand the following.

a $(4x + 3)(2x + 5)$ **b** $(3x + 2)(2x + 1)$ **c** $(3x + 1)(5x + 4)$
d $(2x - 3)(3x + 5)$ **e** $(8x - 3)(3x + 4)$ **f** $(3x - 2)(2x + 1)$
g $(5x + 2)(2x - 7)$ **h** $(2x + 3)(3x - 2)$ **i** $(4x + 1)(4x - 5)$
j $(3x - 2)(6x - 5)$ **k** $(5x - 2)(3x - 1)$ **l** $(7x - 3)(3x - 4)$

4 Expand these binomial products.

a $(a + b)(a + c)$ **b** $(a - b)(a + c)$ **c** $(y - x)(z - y)$
d $(2x + y)(x - 2y)$ **e** $(2a + b)(a - b)$ **f** $(3x - y)(2x + y)$
g $(2a - b)(3a + 2)$ **h** $(4x - 3y)(3x - 4y)$ **i** $(xy - yz)(z + 3x)$

PROBLEM-SOLVING	5	5, 6	6, 7

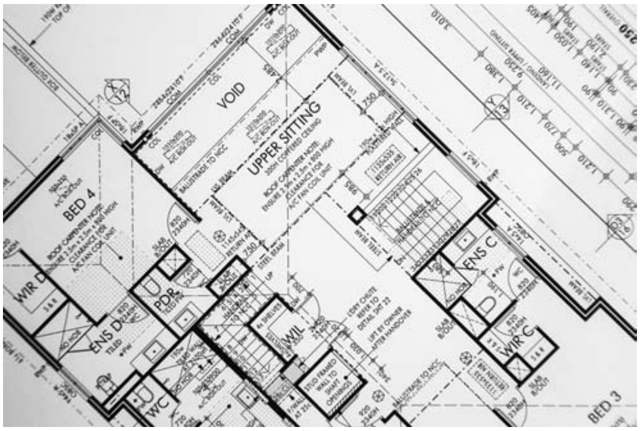
5 A rectangular room with dimensions 4 m by 5 m is to be extended. Both the length and the width are to be increased by x m.

a Find an expanded expression for the area of the new room.

b If $x = 3$:

i find the area of the new room.

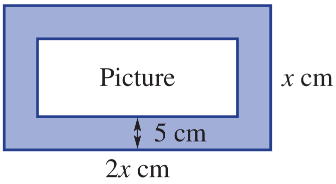
ii by how much has the area increased?



6 A rectangular picture frame 5 cm wide has a length which is twice the width x cm.

a Find an expression for the total area of the frame and picture.

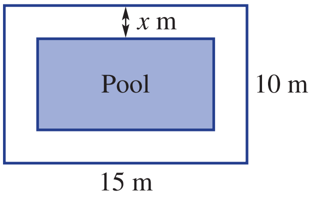
b Find an expression in expanded form for the area of the picture only.



7 The outside edge of a path around a swimming pool is 15 m long and 10 m wide. The path is x metres wide.

a Find an expression for the area of the pool in expanded form.

b Find the area of the pool if $x = 2$.



REASONING	8($\frac{1}{2}$)	8–9($\frac{1}{2}$)	8($\frac{1}{3}$), 9($\frac{1}{2}$)

8 Write the missing terms in these expansions.

a $(x + 2)(x + \_\_\_) = x^2 + 5x + 6$

b $(x + \_\_\_)(x + 5) = x^2 + 7x + 10$

c $(x + 1)(x + \_\_\_) = x^2 + 7x + \_\_\_$

d $(x + \_\_\_)(x + 9) = x^2 + 11x + \_\_\_$

e $(x + 3)(x - \_\_\_) = x^2 + x - \_\_\_$

f $(x - 5)(x + \_\_\_) = x^2 - 2x - \_\_\_$

g $(x + 1)(\_\_\_ + 3) = 2x^2 + \_\_\_ + \_\_\_$

h $(\_\_\_ - 4)(3x - 1) = 9x^2 - \_\_\_ + \_\_\_$

i $(x + 2)(\_\_\_ + \_\_\_) = 7x^2 + \_\_\_ + 6$

j $(\_\_\_ - \_\_\_)(2x - 1) = 6x^2 - \_\_\_ + 4$

9 Consider the binomial product $(x + a)(x + b)$. Find the possible integer values of a and b for each of the following.

a $(x + a)(x + b) = x^2 + 5x + 6$

b $(x + a)(x + b) = x^2 - 5x + 6$

c $(x + a)(x + b) = x^2 + x - 6$

d $(x + a)(x + b) = x^2 - x - 6$

ENRICHMENT: Trinomial expansions	–	–	10($\frac{1}{2}$), 11

10 Using the distributive law, $(a + b)(c + d + e) = ac + ad + ae + bc + bd + be$.
Use this knowledge to expand and simplify these products. (Note: $x \times x^2 = x^3$.)

a $(x + 1)(x^2 + x + 1)$

b $(x - 2)(x^2 - x + 3)$

c $(2x - 1)(2x^2 - x + 4)$

d $(x^2 - x + 1)(x + 3)$

e $(5x^2 - x + 2)(2x - 3)$

f $(2x^2 - x + 7)(4x - 7)$

g $(x + a)(x^2 - ax + a)$

h $(x - a)(x^2 - ax - a^2)$

i $(x + a)(x^2 - ax + a^2)$

j $(x - a)(x^2 + ax + a^2)$

11 Now try to expand $(x + 1)(x + 2)(x + 3)$.

ISBN 978-1-108-77288-4

8B Perfect squares and difference of perfect squares

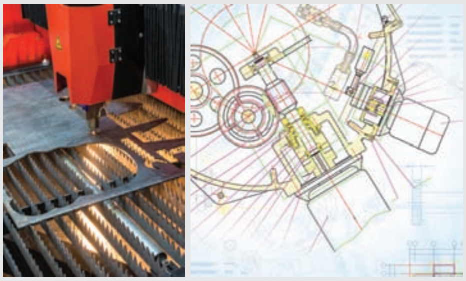
Learning intentions
- To be able to identify a perfect square
- To be able to expand a perfect square
- To understand what type of expansion forms a difference of perfect squares (DOPS)
- To be able to expand to form a difference of perfect squares

$2^2 = 4$, $15^2 = 225$, x^2 and $(a + b)^2$ are all examples of perfect squares. To expand $(a + b)^2$ we multiply $(a + b)$ by $(a + b)$ and use the distributive law:

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b)\\ &= a^2 + ab + ba + b^2\\ &= a^2 + 2ab + b^2\end{aligned}$$

A similar result is obtained for the square of $(a - b)$:

$$\begin{aligned}(a - b)^2 &= (a - b)(a - b)\\ &= a^2 - ab - ba + b^2\\ &= a^2 - 2ab + b^2\end{aligned}$$



Mechanical engineers design all kinds of equipment to be fabricated from steel sheets. Binomial products can be used when calculating the most efficient way to cut out the required shapes.

Another type of expansion involves the case that deals with the product of the sum and difference of the same two terms. The result is the difference of two perfect squares:

$$\begin{aligned}(a + b)(a - b) &= a^2 - ab + ba - b^2\\ &= a^2 - b^2 \text{ (since } ab = ba, \text{ the two middle terms cancel each other out)}\end{aligned}$$

LESSON STARTER Seeing the pattern

Using $(a + b)(c + d) = ac + ad + bc + bd$, expand and simplify the binomial products in the two sets below.

Set A	**Set B**
$(x + 1)(x + 1) = x^2 + x + x + 1$	$(x + 1)(x - 1) = x^2 - x + x - 1$
$=$	$=$
$(x + 3)(x + 3) =$	$(x - 3)(x + 3) =$
$=$	$=$
$(x - 5)(x - 5) =$	$(x - 5)(x + 5) =$
$=$	$=$

- Describe the patterns you see in both sets of expansions above.
- Generalise your observations by completing the following expansions.

a $(a + b)(a + b) = a^2 + \_\_ + \_\_ + \_\_$
$= a^2 + \_\_ + \_\_$
$(a - b)(a - b) =$
$=$

b $(a + b)(a - b) = a^2 - \_\_ + \_\_ - \_\_$
$=$

ISBN 978-1-108-77288-4

KEY IDEAS

- $3^2 = 9$, a^2, $(2y)^2$, $(x - 1)^2$ and $(3 - 2y)^2$ are all examples of **perfect squares**.
- Expanding perfect squares
  - $$\begin{aligned}(a + b)^2 &= (a + b)(a + b)\\ &= a^2 + ab + ba + b^2\\ &= a^2 + 2ab + b^2\end{aligned}$$
  - $$\begin{aligned}(a - b)^2 &= (a - b)(a - b)\\ &= a^2 - ab - ba + b^2\\ &= a^2 - 2ab + b^2\end{aligned}$$
- **Difference of perfect squares** (DOPS)
  - $$\begin{aligned}(a + b)(a - b) &= a^2 - ab + ba - b^2\\ &= a^2 - b^2\end{aligned}$$
  - $(a - b)(a + b)$ also expands to $a^2 - b^2$
  - The result is a difference of two perfect squares.

BUILDING UNDERSTANDING

1 State the missing terms in these expansions.

a $(x + 3)(x + 3) = x^2 + 3x + \_\_\_\_ + \_\_\_\_$
$= \_\_\_\_\_\_\_\_\_\_\_\_$

b $(x + 5)(x + 5) = x^2 + 5x + \_\_\_\_ + \_\_\_\_$
$= \_\_\_\_\_\_\_\_\_\_\_\_$

c $(x - 2)(x - 2) = x^2 - 2x - \_\_\_\_ + \_\_\_\_$
$= \_\_\_\_\_\_\_\_\_\_\_\_$

d $(x - 7)(x - 7) = x^2 - 7x - \_\_\_\_ + \_\_\_\_$
$= \_\_\_\_\_\_\_\_\_\_\_\_$

2 **a** Substitute the given value of b into $x^2 + 2bx + b^2$ and simplify.

i $b = 3$ **ii** $b = 11$ **iii** $b = 15$

b Substitute the given value of b into $x^2 - 2bx + b^2$ and simplify.

i $b = 2$ **ii** $b = 9$ **iii** $b = 30$

3 State the missing terms in these expansions.

a $(x + 4)(x - 4) = x^2 - 4x + \_\_\_\_ - \_\_\_\_$
$= \_\_\_\_\_\_\_\_\_\_\_\_$

b $(x - 10)(x + 10) = x^2 + 10x - \_\_\_\_ - \_\_\_\_ - \_\_\_\_$
$= \_\_\_\_\_\_\_\_\_\_\_\_$

c $(2x - 1)(2x + 1) = 4x^2 + \_\_\_\_ - \_\_\_\_ - \_\_\_\_$
$= \_\_\_\_\_\_\_\_\_\_\_\_$

d $(3x + 4)(3x - 4) = 9x^2 - \_\_\_\_ + \_\_\_\_ - \_\_\_\_$
$= \_\_\_\_\_\_\_\_\_\_\_\_$

ISBN 978-1-108-77288-4

Example 2 Expanding perfect squares

Expand each of the following.

a $(x-2)^2$ **b** $(2x+3)^2$

SOLUTION	EXPLANATION
a $(x-2)^2 = (x-2)(x-2)$	Write in expanded form.
$= x^2 - 2x - 2x + 4$	Use the distributive law.
$= x^2 - 4x + 4$	Collect like terms.
Alternative method:	
$(x-2)^2 = x^2 - 2 \times x \times 2 + 2^2$ $= x^2 - 4x + 4$	Expand using $(a-b)^2 = a^2 - 2ab + b^2$ where $a = x$ and $b = 2$.
b $(2x+3)^2 = (2x+3)(2x+3)$	Write in expanded form.
$= 4x^2 + 6x + 6x + 9$	Use the distributive law.
$= 4x^2 + 12x + 9$	Collect like terms.
Alternative method:	
$(2x+3)^2 = (2x)^2 + 2 \times 2x \times 3 + 3^2$ $= 4x^2 + 12x + 9$	Expand using $(a+b)^2 = a^2 + 2ab + b^2$ where $a = 2x$ and $b = 3$. Recall $(2x)^2 = 2x \times 2x = 4x^2$.

Now you try

Expand each of the following.

a $(x-3)^2$ **b** $(3x+4)^2$

Example 3 Forming a difference of perfect squares

Expand and simplify the following.

a $(x+2)(x-2)$ **b** $(3x-2y)(3x+2y)$

SOLUTION	EXPLANATION
a $(x+2)(x-2) = x^2 - \cancel{2x} + \cancel{2x} - 4$	Expand using the distributive law.
$= x^2 - 4$	$-2x + 2x = 0$
Alternative method:	
$(x+2)(x-2) = (x)^2 - (2)^2$ $= x^2 - 4$	$(a+b)(a-b) = a^2 - b^2$. Here $a = x$ and $b = 2$.
b $(3x-2y)(3x+2y) = 9x^2 + \cancel{6xy} - \cancel{6xy} - 4y^2$	Expand using the distributive law.
$= 9x^2 - 4y^2$	$6xy - 6xy = 0$
Alternative method:	
$(3x-2y)(3x+2y) = (3x)^2 - (2y)^2$ $= 9x^2 - 4y^2$	$(a+b)(a-b) = a^2 - b^2$ with $a = 3x$ and $b = 2y$ here.

Now you try

Expand and simplify the following.

a $(x+4)(x-4)$ **b** $(5x-3y)(5x+3y)$

Exercise 8B

FLUENCY	1, 2–5(1/2)	2–6(1/3)	2–4(1/4), 5–6(1/3)

1 Expand each of the following.

Example 2a **a** **i** $(x-4)^2$ **ii** $(x+3)^2$

Example 2b **b** **i** $(3x+2)^2$ **ii** $(4x-5)^2$

Example 2a **2** Expand each of the following perfect squares.

a $(x+1)^2$ **b** $(x+3)^2$ **c** $(x+2)^2$ **d** $(x+5)^2$
e $(x+4)^2$ **f** $(x+9)^2$ **g** $(x-2)^2$ **h** $(x-6)^2$
i $(x-1)^2$ **j** $(x-3)^2$ **k** $(x-9)^2$ **l** $(x-7)^2$

Example 2b **3** Expand each of the following perfect squares.

a $(2x+1)^2$ **b** $(2x+5)^2$ **c** $(3x+2)^2$
d $(3x+1)^2$ **e** $(5x+2)^2$ **f** $(4x+3)^2$
g $(7+2x)^2$ **h** $(5+3x)^2$ **i** $(2x-3)^2$
j $(3x-1)^2$ **k** $(4x-5)^2$ **l** $(2x-9)^2$
m $(3x+5y)^2$ **n** $(2x+4y)^2$ **o** $(7x+3y)^2$
p $(6x+5y)^2$ **q** $(4x-9y)^2$ **r** $(2x-7y)^2$
s $(3x-10y)^2$ **t** $(4x-6y)^2$ **u** $(9x-2y)^2$

4 Expand each of the following perfect squares.

a $(3-x)^2$ **b** $(5-x)^2$ **c** $(1-x)^2$
d $(6-x)^2$ **e** $(11-x)^2$ **f** $(8-2x)^2$
g $(2-3x)^2$ **h** $(9-2x)^2$ **i** $(10-4x)^2$

Example 3a **5** Expand and simplify the following to form a difference of perfect squares.

a $(x+1)(x-1)$ **b** $(x+3)(x-3)$ **c** $(x+8)(x-8)$
d $(x+4)(x-4)$ **e** $(x+12)(x-12)$ **f** $(x+11)(x-11)$
g $(x-9)(x+9)$ **h** $(x-5)(x+5)$ **i** $(x-6)(x+6)$
j $(5-x)(5+x)$ **k** $(2-x)(2+x)$ **l** $(7-x)(7+x)$

Example 3b **6** Expand and simplify the following.

a $(3x-2)(3x+2)$ **b** $(5x-4)(5x+4)$ **c** $(4x-3)(4x+3)$
d $(7x-3y)(7x+3y)$ **e** $(9x-5y)(9x+5y)$ **f** $(11x-y)(11x+y)$
g $(8x+2y)(8x-2y)$ **h** $(10x-9y)(10x+9y)$ **i** $(7x-5y)(7x+5y)$
j $(6x-11y)(6x+11y)$ **k** $(8x-3y)(8x+3y)$ **l** $(9x-4y)(9x+4y)$

PROBLEM-SOLVING	7	7, 8	7, 8

7 Lara is x years old and her two best friends are $(x-2)$ and $(x+2)$ years old.

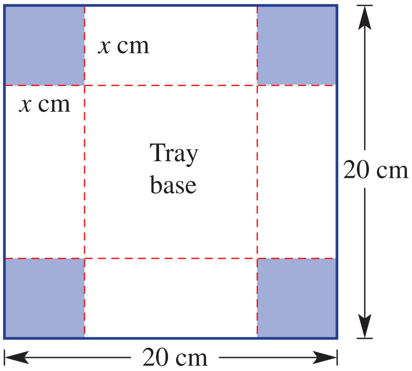
a Write an expression for:

i the square of Lara's age

ii the product of the ages of Lara's best friends (in expanded form).

b Are the answers from parts **a i** and **ii** equal? If not, by how much do they differ?

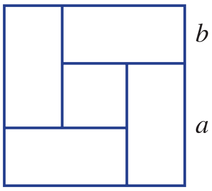
8 A square piece of tin of side length 20 cm has four squares of side length x cm removed from each corner. The sides are folded up to form a tray. The centre square forms the tray base.



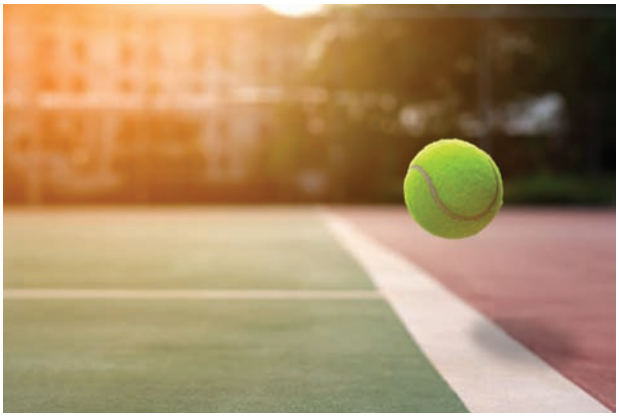
- **a** Write an expression for the side length of the base of the tray.
- **b** Write an expression for the area of the base of the tray. Expand your answer.
- **c** Find the area of the tray base if $x = 3$.
- **d** Find the volume of the tray if $x = 3$.

REASONING	9	9, 10	10, 11

9 Four tennis courts are arranged as shown with a square storage area in the centre. Each court area has the dimensions $a \times b$.



- **a** Write an expression for the side length of the total area.
- **b** Write an expression for the total area.
- **c** Write an expression for the side length of the inside storage area.
- **d** Write an expression for the area of the inside storage area.
- **e** Subtract your answer to part **d** from your answer to part **b** to find the area of the four courts.
- **f** Find the area of one court. Does your answer confirm that your answer to part **e** is correct?



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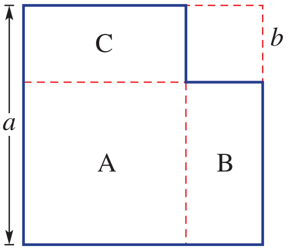
10 A square of side length x units has one side reduced by 1 unit and the other increased by 1 unit.

- **a** Find an expanded expression for the area of the resulting rectangle.
- **b** Is the area of the original square the same as the area of the resulting rectangle? Explain why/why not.

11 A square of side length b is removed from a square of side length a.

- **a** Using subtraction write down an expression for the remaining area.
- **b** Write expressions for the area of these regions.
  - **i** A
  - **ii** B
  - **iii** C
- **c** Add all the expressions from part **b** to see if you get your answer from part **a**.



ENRICHMENT: Extended expansions

– – 12(½)

12 Expand and simplify these expressions.

- **a** $(x + 2)^2 - 4$
- **b** $(2x - 1)^2 - 4x^2$
- **c** $(x + 3)(x - 3) + 6x$
- **d** $1 - (x + 1)^2$
- **e** $x^2 - (x + 1)(x - 1)$
- **f** $(x + 1)^2 - (x - 1)^2$
- **g** $(3x - 2)(3x + 2) - (3x + 2)^2$
- **h** $(5x - 1)^2 - (5x + 1)(5x - 1)$
- **i** $(x + y)^2 - (x - y)^2 + (x + y)(x - y)$
- **j** $(2x - 3)^2 + (2x + 3)^2$
- **k** $(2 - x)^2 - (2 + x)^2$
- **l** $(3 - x)^2 + (x - 3)^2$
- **m** $2(3x - 4)^2 - (3x - 4)(3x + 4)$
- **n** $2(x + y)^2 - (x - y)^2$

Engineers work with binomial products. A metal rectangle, with possible dimensions 16 cm by 24 cm, can have square corners removed and be folded to give a height of x cm. The box has base area $A = (16 - 2x)(24 - 2x)$.

ISBN 978-1-108-77288-4

8C Factorising algebraic expressions

Learning intentions

- To be able to identify a highest common factor of two or more terms
- To understand what it means to write an expression in factorised form
- To understand the relationship between factorised and expanded form
- To be able to factorise an expression involving a common factor

The process of factorisation is a key step in the simplification of many algebraic expressions and in the solution of equations. It is the reverse process of expansion and involves writing an expression as a product of its factors.

$$\underset{\xleftarrow[\text{factorising}]{\qquad\qquad}}{\overset{\xrightarrow{\text{expanding}}}{2(x-3) = 2x - 6}}$$

Factorising is a key step when solving many of the equations used in business, engineering, technology and science. Fashion business analysts develop and solve equations to model and predict sales, revenue and profit.

LESSON STARTER Which factorised form?

The product $x(4x + 8)$ when expanded gives $4x^2 + 8x$.

- Write down three other products that when expanded give $4x^2 + 8x$. (Do not use fractions.)
- Which of your products uses the highest common factor of $4x^2$ and $8x$? What is this highest common factor?

KEY IDEAS

- When **factorising** expressions with common factors, take out the highest common factor (HCF). The HCF could be:
  - a number
    For example: $2x + 10 = 2(x + 5)$
  - a pronumeral (or variable)
    For example: $x^2 + 5x = x(x + 5)$
  - the product of numbers and pronumerals
    For example: $2x^2 + 10x = 2x(x + 5)$
- A factorised expression can be checked by using expansion.

  For example: $2x(x + 5) = 2x^2 + 10x$

BUILDING UNDERSTANDING

1 State the highest common factor (HCF) of these pairs of numbers.

a 8, 12 **b** 24, 30 **c** 100, 75 **d** 16, 24

2 Give the missing factor.

a $5 \times \_\_\_ = 5x$ **b** $5a \times \_\_\_ = 10a^2$

c $\_\_\_ \times 12x = -36x^2$ **d** $\_\_\_ \times -7xy = 14x^2y$

3 **a** State the missing factor in each part.

i $\_\_\_(x^2 + 2x) = 6x^2 + 12x$ **ii** $\_\_\_(2x + 4) = 6x^2 + 12x$ **iii** $\_\_\_(x + 2) = 6x^2 + 12x$

b Which equation above uses the HCF of $6x^2$ and $12x$?

c By looking at the terms left in the brackets, how do you know you have taken out the HCF?

Example 4 Finding the HCF

Determine the highest common factor (HCF) of the following.

a $6a$ and $8ab$ **b** $3x^2$ and $6xy$

SOLUTION	EXPLANATION
a $2a$	HCF of 6 and 8 is 2. HCF of a and ab is a.
b $3x$	HCF of 3 and 6 is 3. HCF of x^2 and xy is x.

Now you try

Determine the highest common factor (HCF) of the following.

a $4x$ and $10xy$ **b** $5x^2$ and $15xy$

Example 5 Factorising expressions

Factorise the following.

a $40 - 16b$ **b** $-8x^2 - 12x$

SOLUTION	EXPLANATION
a $40 - 16b = 8(5 - 2b)$	The HCF of 40 and $16b$ is 8. Place 8 in front of the brackets and divide each term by 8.
b $-8x^2 - 12x = -4x(2x + 3)$	The HCF of the terms is $-4x$, including the common negative 4. Place the factor in front of the brackets and divide each term by $-4x$.

Now you try

Factorise the following.

a $28 - 21a$ **b** $-9x^2 - 15x$

Example 6 Taking out a binomial factor

Factorise the following.

a $3(x + y) + x(x + y)$ **b** $(7 - 2x) - x(7 - 2x)$

SOLUTION

a $3(x + y) + x(x + y)$
$= (x + y)(3 + x)$

EXPLANATION

HCF $= (x + y)$.
The second pair of brackets contains what remains when $3(x + y)$ and $x(x + y)$ are divided by $(x + y)$.

SOLUTION

b $(7 - 2x) - x(7 - 2x)$
$= 1(7 - 2x) - x(7 - 2x)$
$= (7 - 2x)(1 - x)$

EXPLANATION

Insert 1 in front of the first bracket.
HCF $= (7 - 2x)$.
The second bracket must contain $1 - x$ as $\frac{(7 - 2x)}{(7 - 2x)} = 1$ and $\frac{-x(7 - 2x)}{(7 - 2x)} = -x$.

Now you try

Factorise the following.

a $4(a + b) + a(a + b)$ **b** $(4x + 3) - x(4x + 3)$

Exercise 8C

FLUENCY	1, 2–5(1/2)	2–6(1/3)	2–6(1/4)

1 Determine the highest common factor (HCF) of the following.

Example 4a **a** **i** $5a$ and $15ab$ **ii** $9b$ and $12ab$

Example 4b **b** **i** $2x^2$ and $8xy$ **ii** $5x^2$ and $10x$

Example 4 **2** Determine the HCF of the following.

a $6x$ and $14xy$ **b** $12a$ and $18a$ **c** $10m$ and 4 **d** $12y$ and 8
e $15t$ and $6s$ **f** 15 and p **g** $9x$ and $24xy$ **h** $6n$ and $21mn$
i $10y$ and $2y$ **j** $8x^2$ and $14x$ **k** $4x^2y$ and $18xy$ **l** $5ab^2$ and $15a^2b$

Example 5a **3** Factorise the following.

a $7x + 7$ **b** $3x + 3$ **c** $4x - 4$ **d** $5x - 5$
e $4 + 8y$ **f** $10 + 5a$ **g** $3 - 9b$ **h** $6 - 2x$
i $12a + 3b$ **j** $6m + 6n$ **k** $10x - 8y$ **l** $4a - 20b$
m $x^2 + 2x$ **n** $a^2 - 4a$ **o** $y^2 - 7y$ **p** $x - x^2$
q $3p^2 + 3p$ **r** $8x - 8x^2$ **s** $4b^2 + 12b$ **t** $6y - 10y^2$
u $12a - 15a^2$ **v** $9m + 18m^2$ **w** $16xy - 48x^2$ **x** $7ab - 28ab^2$

Example 5b

4 Factorise the following by factoring out the negative sign as part of the HCF.

a $-8x-4$
b $-4x-2$
c $-10x-5y$
d $-7a-14b$
e $-9x-12$
f $-6y-8$
g $-10x-15y$
h $-4m-20n$
i $-3x^2-18x$
j $-8x^2-12x$
k $-16y^2-6y$
l $-5a^2-10a$
m $-6x-20x^2$
n $-6p-15p^2$
o $-16b-8b^2$
p $-9x-27x^2$

Example 6

5 Factorise the following which involve a binomial common factor.

a $4(x+3)+x(x+3)$
b $3(x+1)+x(x+1)$
c $7(m-3)+m(m-3)$
d $x(x-7)+2(x-7)$
e $8(a+4)-a(a+4)$
f $5(x+1)-x(x+1)$
g $y(y+3)-2(y+3)$
h $a(x+2)-x(x+2)$
i $t(2t+5)+3(2t+5)$
j $m(5m-2)+4(5m-2)$
k $y(4y-1)-(4y-1)$
l $(7-3x)+x(7-3x)$

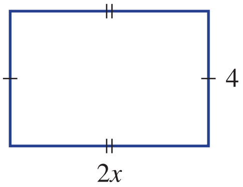
6 Factorise these mixed expressions.

a $6a+30$
b $5x-15$
c $8b+18$
d x^2-4x
e y^2+9y
f a^2-3a
g $x^2y-4xy+xy^2$
h $6ab-10a^2b+8ab^2$
i $m(m+5)+2(m+5)$
j $x(x+3)-2(x+3)$
k $b(b-2)+(b-2)$
l $x(2x+1)-(2x+1)$
m $y(3-2y)-5(3-2y)$
n $(x+4)^2+5(x+4)$
o $(y+1)^2-4(y+1)$

PROBLEM-SOLVING	7	7(½), 8	7(⅓), 8, 9

7 Write down the perimeter of these shapes in factorised form.

a



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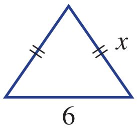

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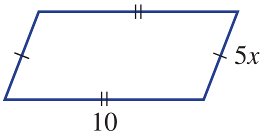

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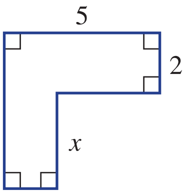
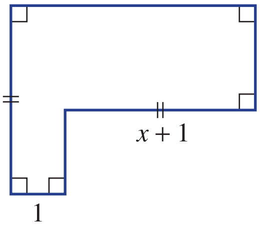
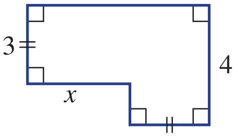


e



f



8 The expression for the area of a rectangle is $(4x^2+8x)$ square units. Find an expression for the width of the rectangle if its length is $(x+2)$ units.

9 The height, above its starting point in metres, of a ball thrown in the air is given by $5t-t^2$, where t is the time in seconds.

a Write an expression for the ball's height in factorised form.

b Find the ball's height above its starting point at these times:

i $t=0$
ii $t=4$
iii $t=2$.

c How long does it take for the ball's height to return to 0 metres (its starting height)? Use trial and error if required.

REASONING	10	10–11(½)	11–12(½)

10 $7 \times 9 + 7 \times 3$ can be evaluated by first factorising to $7(9 + 3)$. This gives $7 \times 12 = 84$. Use a similar technique to evaluate the following.

a $9 \times 2 + 9 \times 5$
b $6 \times 3 + 6 \times 9$
c $-2 \times 4 - 2 \times 6$
d $-5 \times 8 - 5 \times 6$
e $23 \times 5 - 23 \times 2$
f $63 \times 11 - 63 \times 8$

11 Common factors can also be removed from expressions with more than two terms.
For example: $2x^2 + 6x + 10xy = 2x(x + 3 + 5y)$
Factorise these expressions by taking out the HCF.

a $3a^2 + 9a + 12$
b $5z^2 - 10z + zy$
c $x^2 - 2xy + x^2y$
d $4by - 2b + 6b^2$
e $-12xy - 8yz - 20xyz$
f $3ab + 4ab^2 + 6a^2b$

12 Sometimes we can choose to factor out a negative HCF or a positive HCF. Both factorisations are correct. For example:

$$\begin{aligned} -13x + 26 &= -13(x - 2) \quad \text{(HCF is } -13) \\ \text{OR } -13x + 26 &= 13(-x + 2) \quad \text{(HCF is 13)} \\ &= 13(2 - x) \end{aligned}$$

Factorise in two different ways: the first by factoring out a negative HCF and the second by a positive HCF.

a $-4x + 12$
b $-3x + 9$
c $-8n + 8$
d $-3b + 3$
e $-5m + 5m^2$
f $-7x + 7x^2$
g $-5x + 5x^2$
h $-4y + 22y^2$

ENRICHMENT: Factoring out a negative	–	–	13(½)

13 Using the fact that $a - b = -(b - a)$, you can factorise $x(x - 2) - 5(2 - x)$ by following these steps.

$$\begin{aligned} x(x - 2) - 5(2 - x) &= x(x - 2) + 5(x - 2) \\ &= (x - 2)(x + 5) \end{aligned}$$

Use this idea to factorise these expressions.

a $x(x - 4) + 3(4 - x)$
b $x(x - 5) - 2(5 - x)$
c $x(x - 3) - 3(3 - x)$
d $3x(x - 4) + 5(4 - x)$
e $3(2x - 5) + x(5 - 2x)$
f $2x(x - 2) + (2 - x)$
g $-4(3 - x) - x(x - 3)$
h $x(x - 5) + (10 - 2x)$
i $x(x - 3) + (6 - 2x)$

ISBN 978-1-108-77288-4

Updated September 2021

8D Factorising the difference of perfect squares

Learning intentions

- To be able to recognise a difference of perfect squares (DOPS)
- To be able to express a DOPS in factorised form
- To know to check for a common factor first before further factorisation

Recall that a difference of two perfect squares is formed when expanding the product of the sum and difference of two terms. For example, $(x + 2)(x - 2) = x^2 - 4$. Reversing this process means that a difference of two perfect squares can be factorised into two binomial expressions of the form $(a + b)$ and $(a - b)$.

Engineers use algebra when designing products used for bridges, buildings and machinery, such as this steel turbine for a power station. For example, a circle area difference: $A = \pi R^2 - \pi r^2 = \pi(R^2 - r^2) = \pi(R - r)(R + r)$.

LESSON STARTER Expanding to understand factorising

Complete the steps in these expansions and then write the conclusion.

- $(x + 3)(x - 3) = x^2 - 3x + \_\_\_ - \_\_\_$
  $= x^2 - \_\_\_\_\_\_$
  $\therefore x^2 - 9 = (\_\_ + \_\_)(\_\_ - \_\_)$
- $(a + b)(a - b) = a^2 - ab + \_\_\_ - \_\_\_$
  $= \_\_\_\_\_ - \_\_\_\_\_$
  $\therefore a^2 - \_\_\_ = (\_\_\_ + \_\_\_)(\_\_\_ - \_\_\_)$
- $(2x - 5)(2x + 5) = 4x^2 + 10x - \_\_\_ - \_\_\_$
  $= \_\_\_\_\_\_ - \_\_\_\_\_\_$
  $\therefore 4x^2 - \_\_\_\_ = (\_\_ + \_\_)(\_\_ - \_\_)$

KEY IDEAS

- Factorising the difference of perfect squares (DOPS) uses the rule $a^2 - b^2 = (a + b)(a - b)$.
  - $x^2 - 16 = x^2 - 4^2$
    $= (x + 4)(x - 4)$
  - $9x^2 - 100 = (3x)^2 - 10^2$
    $= (3x + 10)(3x - 10)$
  - $25 - 4y^2 = 5^2 - (2y)^2$
    $= (5 + 2y)(5 - 2y)$
- First take out common factors where possible.
  - $2x^2 - 18 = 2(x^2 - 9)$
    $= 2(x + 3)(x - 3)$

ISBN 978-1-108-77288-4

Updated September 2021

BUILDING UNDERSTANDING

1 Expand these binomial products to form a difference of perfect squares.

a $(x + 2)(x - 2)$

b $(x - 7)(x + 7)$

c $(3x - y)(3x + y)$

2 State the missing term. Assume it is a positive number.

a $(\_\_)^2 = 9$

b $(\_\_)^2 = 121$

c $(\_\_)^2 = 25b^2$

d $(\_\_)^2 = 49y^2$

3 State the missing terms to complete these factorisations.

a $x^2 - 16 = x^2 - 4^2$
$= (x + 4)(\_\_ - \_\_)$

b $x^2 - 144 = x^2 - (\_\_)^2$
$= (\_\_ + 12)(x - \_\_)$

c $16x^2 - 1 = (\_\_)^2 - (\_\_)^2$
$= (4x + \_\_)(\_\_ - 1)$

d $9a^2 - 4b^2 = (\_\_)^2 - (\_\_)^2$
$= (3a + \_\_)(\_\_ - 2b)$

Example 7 Factorising DOPS

Factorise each of the following.

a $x^2 - 4$

b $9a^2 - 25$

c $81x^2 - y^2$

SOLUTION	EXPLANATION
a $x^2 - 4 = x^2 - 2^2$ $= (x + 2)(x - 2)$	Write as a DOPS (4 is the same as 2^2). Write in factorised form: $a^2 - b^2 = (a + b)(a - b)$ Here $a = x$ and $b = 2$.
b $9a^2 - 25 = (3a)^2 - 5^2$ $= (3a + 5)(3a - 5)$	Write as a DOPS. $9a^2$ is the same as $(3a)^2$. Write in factorised form.
c $81x^2 - y^2 = (9x)^2 - y^2$ $= (9x + y)(9x - y)$	$81x^2 = (9x)^2$ Use $a^2 - b^2 = (a + b)(a - b)$.

Now you try

Factorise each of the following.

a $x^2 - 16$

b $4a^2 - 9$

c $25a^2 - b^2$

Example 8 Factorising more complex DOPS

Factorise each of the following.

a $2b^2 - 32$ **b** $(x + 1)^2 - 4$

SOLUTION

a $2b^2 - 32 = 2(b^2 - 16)$
$= 2(b^2 - 4^2)$
$= 2(b + 4)(b - 4)$

EXPLANATION

First, factor out the common factor of 2.
Write as a DOPS and then factorise.

b $(x + 1)^2 - 4 = (x + 1)^2 - 2^2$
$= ((x + 1) + 2)((x + 1) - 2)$
$= (x + 3)(x - 1)$

Write as a DOPS. In $a^2 - b^2$ here, a is the expression $x + 1$ and $b = 2$.
Write in factorised form and simplify.

Now you try

Factorise each of the following.

a $3a^2 - 27$ **b** $(x + 2)^2 - 9$

Exercise 8D

FLUENCY	1, 2–4(½)	2–5(½)	2–3(¼), 4–5(½)

1 Factorise each of the following.

Example 7a **a** **i** $x^2 - 49$ **ii** $x^2 - 36$

Example 7b **b** **i** $16y^2 - 81$ **ii** $25y^2 - 4$

Example 7c **c** **i** $36a^2 - b^2$ **ii** $x^2 - 9y^2$

Example 7a **2** Factorise each of the following.

a $x^2 - 9$ **b** $y^2 - 25$ **c** $y^2 - 1$ **d** $x^2 - 64$
e $x^2 - 16$ **f** $b^2 - 49$ **g** $a^2 - 81$ **h** $x^2 - y^2$
i $a^2 - b^2$ **j** $16 - a^2$ **k** $25 - x^2$ **l** $1 - b^2$
m $36 - y^2$ **n** $121 - b^2$ **o** $x^2 - 400$ **p** $900 - y^2$

Example 7b,c **3** Factorise each of the following.

a $4x^2 - 25$ **b** $9x^2 - 49$ **c** $25b^2 - 4$ **d** $4m^2 - 121$
e $100y^2 - 9$ **f** $81a^2 - 4$ **g** $1 - 4x^2$ **h** $25 - 64b^2$
i $16 - 9y^2$ **j** $36x^2 - y^2$ **k** $4x^2 - 25y^2$ **l** $64a^2 - 49b^2$
m $4p^2 - 25q^2$ **n** $81m^2 - 4n^2$ **o** $25a^2 - 49b^2$ **p** $100a^2 - 9b^2$

Example 8a **4** Factorise each of the following by first taking out the common factor.

a $3x^2 - 108$ **b** $10a^2 - 10$ **c** $6x^2 - 24$
d $4y^2 - 64$ **e** $98 - 2x^2$ **f** $32 - 8m^2$
g $5x^2y^2 - 5$ **h** $3 - 3x^2y^2$ **i** $63 - 7a^2b^2$

ISBN 978-1-108-77288-4

Updated September 2021

Example 8b

5 Factorise each of the following.

a $(x + 5)^2 - 9$ **b** $(x + 3)^2 - 4$ **c** $(x + 10)^2 - 16$
d $(x - 3)^2 - 25$ **e** $(x - 7)^2 - 1$ **f** $(x - 3)^2 - 36$
g $49 - (x + 3)^2$ **h** $4 - (x + 2)^2$ **i** $81 - (x + 8)^2$

PROBLEM-SOLVING	6	6, 7	6, 7

6 The height above ground (in metres) of an object thrown off the top of a building is given by $36 - 4t^2$ where t is in seconds.

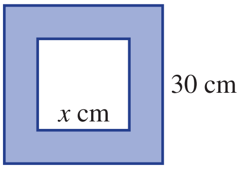
a Factorise the expression for the height of the object by first taking out the common factor.

b Find the height of the object:

i initially ($t = 0$) **ii** at 2 seconds ($t = 2$).

c How long does it take for the object to hit the ground? Use trial and error if you wish.

7 This ‘multisize’ square picture frame has side length 30 cm and can hold a square picture with any side length less than 26 cm.



a If the side length of the picture is x cm, write an expression for:

i the area of the picture

ii the area of the frame (in factorised form).

b Use your result from part **a ii** to find the area of the frame if:

i $x = 20$

ii the area of the picture is 225 cm^2.

REASONING	8(1/2)	8(1/2), 9	8(1/2), 9, 10

8 Initially it may not appear that an expression such as $-4 + 9x^2$ is a difference of perfect squares. However, swapping the position of the two terms makes $-4 + 9x^2 = 9x^2 - 4$, which can be factorised to $(3x + 2)(3x - 2)$. Use this idea to factorise these difference of perfect squares.

a $-9 + x^2$ **b** $-121 + 16x^2$ **c** $-25a^2 + 4$ **d** $-y^2 + x^2$
e $-25a^2 + 4b^2$ **f** $-36a^2b^2 + c^2$ **g** $-16x^2 + y^2z^2$ **h** $-900a^2 + b^2$

9 Olivia factorises $16x^2 - 4$ to get $(4x + 2)(4x - 2)$, but the answer says $4(2x + 1)(2x - 1)$.

- **a** What should Olivia do to get from her answer to the actual answer?
- **b** What should Olivia have done initially to avoid this issue?

10 Find and explain the error in this working and correct it.

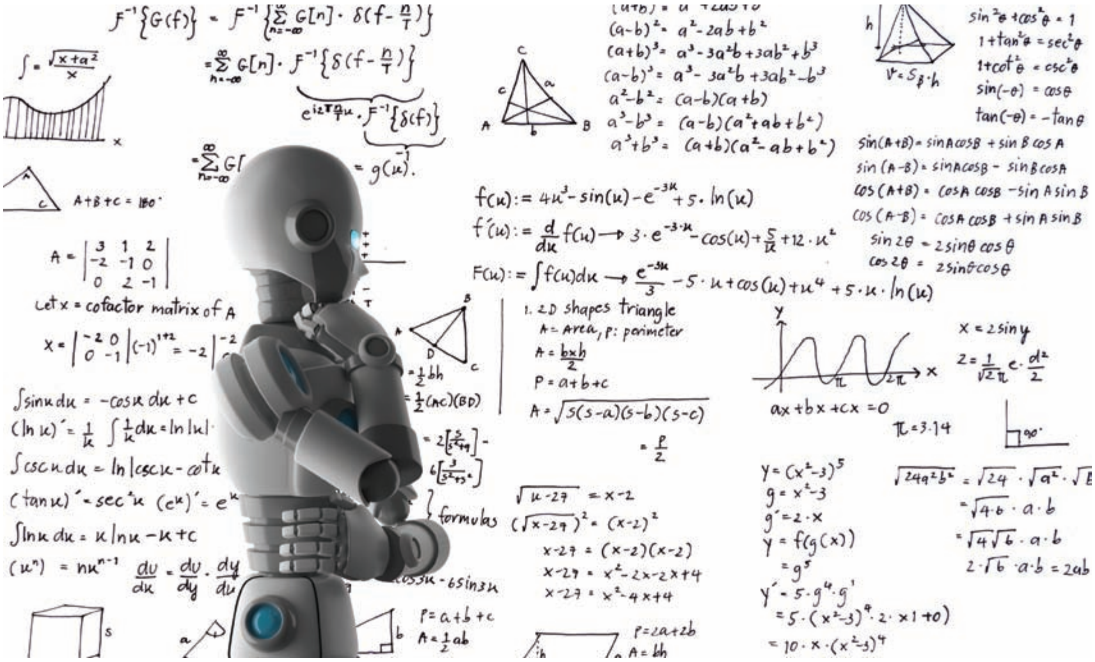
$$\begin{aligned} 9 - (x - 1)^2 &= (3 + x - 1)(3 - x - 1) \\ &= (2 + x)(2 - x) \end{aligned}$$

ENRICHMENT: Factorising with fractions and powers of 4

– – 11(½)

11 Some expressions with fractions or powers of 4 can be factorised in a similar way. Factorise these.

a $x^2 - \frac{1}{4}$	**b** $x^2 - \frac{4}{25}$	**c** $25x^2 - \frac{9}{16}$	**d** $\frac{x^2}{9} - 1$
e $\frac{a^2}{4} - \frac{b^2}{9}$	**f** $\frac{5x^2}{9} - \frac{5}{4}$	**g** $\frac{7a^2}{25} - \frac{28b^2}{9}$	**h** $\frac{a^2}{8} - \frac{b^2}{18}$
i $x^4 - y^4$	**j** $2a^4 - 2b^4$	**k** $21a^4 - 21b^4$	**l** $\frac{x^4}{3} - \frac{y^4}{3}$



8E Factorising by grouping

Learning intentions

- To know that a common factor can include a binomial term
- To be able to use the grouping method to factorise some four-term expressions
- To know that expressions can be rearranged to find common factors to use in the grouping process

When an expression can be written using four terms, such as $x^2 + 2x - x - 2$, it may be possible to factorise it into a product of two binomial terms such as $(x - 1)(x + 2)$. In such situations the method of grouping is often used.

Algebra is an essential tool for the engineers who work together making renewable energy possible, including: aerospace, civil, electrical, electronics, environmental, industrial, materials, mechanical and solar engineers.

LESSON STARTER Two methods: Same result

The four-term expression $x^2 - 3x - 3 + x$ is written on the board.

Tommy chooses to rearrange the terms to give $x^2 - 3x + x - 3$ and then factorises by grouping.

Sharon chooses to rearrange the terms to give $x^2 + x - 3x - 3$ and then also factorises by grouping.

- Complete Tommy and Sharon's factorisation working.

Tommy	Sharon
$x^2 - 3x + x - 3 = x(x - 3) + 1(\_\_\_\_)$ $= (x - 3)(\_\_\_\_)$	$x^2 + x - 3x - 3 = x(\_\_\_\_) - 3(\_\_\_\_)$ $= (x + 1)(\_\_\_\_)$

- Discuss the differences in the methods. Is there any difference in their answers?
- Whose method do you prefer?

KEY IDEAS

- **Factorisation by grouping** is a method that is often used to factorise a four-term expression.
  - Terms are grouped into pairs and factorised separately.
  - The common binomial factor is then taken out to complete the factorisation.
  - Terms can be rearranged to assist in the search of a common factor.

$$\begin{aligned} &x^2 + 3x - 2x - 6 \\ &= x(x + 3) - 2(x + 3) \\ &= (x + 3)(x - 2) \end{aligned}$$

BUILDING UNDERSTANDING

1 State each expanded expression.

a $2(x-1)$

b $-5(1-a)$

c $a(a+5)$

d $y(4-y)$

e $x(a+1)+2(a+1)$

f $b(x-2)-3(x-2)$

2 State the missing information.

a $2(x+1)+x(x+1)=(x+1)(\_\_\_\_)$

b $5(x+5)-x(x+5)=(x+5)(\_\_\_\_)$

c $a(x-3)+(x-3)=(x-3)(\_\_\_\_)$

d $(x-3)-a(x-3)=(x-3)(\_\_\_\_)$

3 Identify the common binomial term, and then factorise each expression.

a $x(x-3)-2(x-3)$

b $x(x+4)+3(x+4)$

c $3(2x+1)-x(2x+1)$

d $4(3x-2)-x(3x-2)$

e $3x(5-x)+2(5-x)$

f $x(x-2)+(x-2)$

Example 9 Factorising by grouping

Use the method of grouping to factorise these expressions.

a $x^2+2x+3x+6$

b $x^2+3x-5x-15$

SOLUTION	EXPLANATION
a $x^2+2x+3x+6=(x^2+2x)+(3x+6)$	Group the first and second pair of terms.
$=x(x+2)+3(x+2)$	Factorise each group.
$=(x+2)(x+3)$	Take the common factor $(x+2)$ out of both groups.
b $x^2+3x-5x-15=(x^2+3x)+(-5x-15)$	Group the first and second pair of terms.
$=x(x+3)-5(x+3)$	Factorise each group.
$=(x+3)(x-5)$	Take out the common factor $(x+3)$.

Now you try

Use the method of grouping to factorise these expressions.

a $x^2+4x+2x+8$

b $x^2+2x-3x-6$

Example 10 Rearranging an expression to factorise by grouping

Factorise $2x^2 - 9 - 18x + x$ using grouping.

SOLUTION

$$\begin{aligned} 2x^2 - 9 - 18x + x &= 2x^2 + x - 18x - 9 \\ &= x(2x + 1) - 9(2x + 1) \\ &= (2x + 1)(x - 9) \end{aligned}$$

EXPLANATION

Rearrange so that each group has a common factor.

Factorise each group and then take out $(2x + 1)$.

Alternative method:

$$\begin{aligned} 2x^2 - 9 - 18x + x &= 2x^2 - 18x + x - 9 \\ &= 2x(x - 9) + 1(x - 9) \\ &= (x - 9)(2x + 1) \end{aligned}$$

Alternatively, you can group in another order where each group has a comman factor. Then factorise.

The answer will be the same.

Now you try

Factorise $2x^2 - 15 - 10x + 3x$ using grouping.

Exercise 8E

FLUENCY	1, 2–3(1/2)	2–4(1/2)	2–4(1/3)

1 Use the method of grouping to factorise these expressions.

Example 9a **a** **i** $x^2 + 5x + 4x + 20$ **ii** $x^2 + 6x + 3x + 18$

Example 9b **b** **i** $x^2 + 4x - 5x - 20$ **ii** $x^2 + 6x - 2x - 12$

Example 9 **2** Use the method of grouping to factorise these expressions.

a $x^2 + 3x + 2x + 6$ **b** $x^2 + 4x + 3x + 12$ **c** $x^2 + 7x + 2x + 14$

d $x^2 - 6x + 4x - 24$ **e** $x^2 - 4x + 6x - 24$ **f** $x^2 - 3x + 10x - 30$

g $x^2 + 2x - 18x - 36$ **h** $x^2 + 3x - 14x - 42$ **i** $x^2 + 4x - 18x - 72$

j $x^2 - 2x - xa + 2a$ **k** $x^2 - 3x - 3xc + 9c$ **l** $x^2 - 5x - 3xa + 15a$

3 Use the method of grouping to factorise these expressions. The HCF for each pair includes a pronumeral. For example, $2ab - 2ad + bc - cd = 2a(b - d) + c(b - d) = (b - d)(2a + c)$

a $3ab + 5bc + 3ad + 5cd$ **b** $4ab - 7ac + 4bd - 7cd$ **c** $2xy - 8xz + 3wy - 12wz$

d $5rs - 10r + st - 2t$ **e** $4x^2 + 12xy - 3x - 9y$ **f** $2ab - a^2 - 2bc + ac$

4 Factorise these expressions. Remember to use a factor of 1 where necessary, for example, $x^2 - ax + x - a = x(x - a) + 1(x - a)$.

a $x^2 - bx + x - b$ **b** $x^2 - cx + x - c$ **c** $x^2 + bx + x + b$

d $x^2 + cx - x - c$ **e** $x^2 + ax - x - a$ **f** $x^2 - bx - x + b$

PROBLEM-SOLVING	5(½)	5–6(½)	5–7(½)

Example 10

5 Factorise these expressions by first rearranging the terms.

a $2x^2 - 7 - 14x + x$
b $5x + 2x + x^2 + 10$
c $2x^2 - 3 - x + 6x$
d $3x - 8x - 6x^2 + 4$
e $11x - 5a - 55 + ax$
f $12y + 2x - 8xy - 3$
g $6m - n + 3mn - 2$
h $15p - 8r - 5pr + 24$
i $16x - 3y - 8xy + 6$

6 What expanded expression factorises to the following?

a $(x - a)(x + 4)$
b $(x - c)(x - d)$
c $(x + y)(2 - z)$
d $(x - 1)(a + b)$
e $(3x - b)(c - b)$
f $(2x - y)(y + z)$
g $(3a + b)(2b + 5c)$
h $(m - 2x)(3y + z)$

7 Note that $x^2 + 5x + 6 = x^2 + 2x + 3x + 6$, which can be factorised by grouping. Use a similar method to factorise the following.

a $x^2 + 7x + 10$
b $x^2 + 8x + 15$
c $x^2 + 10x + 24$
d $x^2 - x - 6$
e $x^2 + 4x - 12$
f $x^2 - 11x + 18$

REASONING	8	8	8, 9

8 $xa - 21 + 7a - 3x$ could be rearranged in two different ways before factorising.

Method 1

$xa + 7a - 3x - 21 = a(x + 7) - 3(\_\_\_\_\_\_)$
$= \_\_\_\_\_\_\_\_\_\_\_\_$

Method 2

$xa - 3x + 7a - 21 = x(a - 3) + 7(\_\_\_\_\_\_)$
$= \_\_\_\_\_\_\_\_\_\_\_\_$

a Copy and complete both methods for the above expression.

b Use different arrangements of the four terms to complete the factorisation of the following in two ways. Show working using both methods.

i $xb - 6 - 3b + 2x$
ii $xy - 8 + 2y - 4x$
iii $4m^2 - 15n + 6m - 10mn$
iv $2m + 3n - mn - 6$
v $4a - 6b^2 + 3b - 8ab$
vi $3ab - 4c - b + 12ac$

9 Make up at least three of your own four-term expressions that factorise to a binomial product. Describe the method that you used to make up each four-term expression.

ENRICHMENT: Grouping with more than four terms	–	–	10(½)

10 Factorise by grouping.

a $2(a - 3) - x(a - 3) - c(a - 3)$
b $b(2a + 1) + 5(2a + 1) - a(2a + 1)$
c $x(a + 1) - 4(a + 1) - ba - b$
d $3(a - b) - b(a - b) - 2a^2 + 2ab$
e $c(1 - a) - x + ax + 2 - 2a$
f $a(x - 2) + 2bx - 4b - x + 2$
g $a^2 - 3ac - 2ab + 6bc + 3abc - 9bc^2$
h $3x - 6xy - 5z + 10yz + y - 2y^2$
i $8z - 4y + 3x^2 + xy - 12x - 2xz$
j $-ab - 4cx + 3aby + 2abx + 2c - 6cy$

Using a CAS calculator 8E: Expanding and factorising

This activity is in the Interactive Textbook in the form of a printable PDF.

8F Factorising quadratic trinomials EXTENDING

Learning intentions
- To know the form of a quadratic trinomial
- To understand how a quadratic trinomial is formed from a binomial expansion
- To know how to find the numbers required to express a quadratic trinomial in factorised form

An expression that takes the form $x^2 + bx + c$, where b and c are constants, is an example of a monic quadratic trinomial that has the coefficient of x^2 equal to 1. To factorise a quadratic expression, we need to use the distributive law in reverse. Consider the expansion shown below:

Quadratic equations are widely used in engineering, business and science, and factorisation is a key step when solving such equations. Self-driving cars are programmed to solve equations, including quadratic equations of motion.

$$\xrightarrow{\text{expanding}}$$

$$\underbrace{(x+2)(x-4)}_{\text{factorised form}} = \underbrace{x^2-2x-8}_{\text{expanded form}}$$

$$\xleftarrow{\text{factorising}}$$

If we examine the expansion above, we can see how each term of the product is formed.

Product of x and x is x^2

$$(x+2)(x-4) = x^2 - 2x - 8$$

Product of 2 and -4 is -8
$(2 \times (-4) = -8$, the constant term)

$$x \times (-4) = -4x$$

$$(x+2)(x-4) = x^2 - 2x - 8$$

$$2 \times x = 2x$$

Add $-4x$ and $2x$ to give the middle term, $-2x$
$(-4 + 2 = -2$, the coefficient of $x)$

LESSON STARTER So many choices

Mia says that as $-2 \times 3 = -6$, then $x^2 + 5x - 6$ must equal $(x - 2)(x + 3)$.

- Expand $(x - 2)(x + 3)$ to see if Mia is correct.
- What other pairs of numbers multiply to give -6?
- Which pair of numbers should Mia choose to correctly factorise $x^2 + 5x - 6$?
- What advice can you give Mia when trying to factorise these types of trinomials?

KEY IDEAS

- To factorise a **quadratic trinomial** of the form $x^2 + bx + c$, find two numbers that:
  - multiply to give c and
  - add to give b.

For example:
$x^2 - 3x - 10 = (x - 5)(x + 2)$
choose -5 and $+2$ since $-5 \times 2 = -10$ and $-5 + 2 = -3$

- Check factorisation steps by expanding.

check: $(x - 5)(x + 2) = x^2 + 2x - 5x - 10$
$= x^2 - 3x - 10$

- Write the factors in any order.

write $(x - 5)(x + 2)$ or $(x + 2)(x - 5)$

BUILDING UNDERSTANDING

1 Give the result of expanding these binomial products.

a $(x + 2)(x + 7)$
b $(x + 3)(x + 11)$
c $(x + 12)(x - 5)$
d $(x - 2)(x - 6)$

2 Decide which two numbers multiply to give the first number and add to give the second number.

a 6, 5
b 12, 13
c 20, 9
d −15, 2
e −30, −1
f 18, −11

Example 11 Factorising quadratic trinomials

Factorise each of the following quadratic expressions.

a $x^2 + 7x + 10$
b $x^2 + 2x - 8$
c $x^2 - 7x + 10$

SOLUTION	EXPLANATION
a $x^2 + 7x + 10 = (x + 5)(x + 2)$	Need two numbers that × to 10 and + to 7. Factors of 10 include (10, 1) and (5, 2). The pair that adds to 7 is (5, 2).
b $x^2 + 2x - 8 = (x + 4)(x - 2)$	Need two numbers that × to −8 and + to 2. Factors of −8 are (−8, 1) or (8, −1) or (4, −2) or (−4, 2) and $4 + (-2) = 2$, so choose (4, −2).
c $x^2 - 7x + 10 = (x - 2)(x - 5)$	Need two numbers that × to 10 and + to −7. Factors of 10 are (10, 1) or (−10, −1) or (5, 2) or (−5, −2). To add to a negative (−7), both factors must then be negative: $-5 + (-2) = -7$, so choose (−5, −2).

Now you try

Factorise each of the following quadratic expressions.

a $x^2 + 9x + 14$
b $x^2 + 4x - 12$
c $x^2 - 7x + 12$

Example 12 Factorising with a common factor

Factorise the quadratic expression $2x^2 - 2x - 12$.

SOLUTION

$$\begin{aligned}2x^2 - 2x - 12 &= 2(x^2 - x - 6)\\ &= 2(x - 3)(x + 2)\end{aligned}$$

EXPLANATION

First take out common factor of 2.
Factors of -6 are $(-6, 1)$ or $(6, -1)$ or $(-3, 2)$ or $(3, -2)$.
$-3 + 2 = -1$ so choose $(-3, 2)$.

Now you try

Factorise the quadratic expression $3x^2 - 6x - 45$.

Exercise 8F

FLUENCY	1, 2–5(1/2)	2–6(1/3)	2–6(1/3)

1 Factorise each of the following quadratic expressions.

Example 11a
a **i** $x^2 + 6x + 8$ **ii** $x^2 + 3x + 2$

Example 11b
b **i** $x^2 + 3x - 10$ **ii** $x^2 - 2x - 15$

Example 11c
c **i** $x^2 - 9x + 18$ **ii** $x^2 - 4x + 3$

Example 11a
2 Factorise each of the following quadratic expressions.

a $x^2 + 8x + 12$ **b** $x^2 + 10x + 9$ **c** $x^2 + 8x + 7$
d $x^2 + 15x + 14$ **e** $x^2 + 7x + 12$ **f** $x^2 + 10x + 16$
g $x^2 + 8x + 15$ **h** $x^2 + 9x + 20$ **i** $x^2 + 11x + 24$

Example 11b
3 Factorise each of the following quadratic expressions.

a $x^2 + 3x - 4$ **b** $x^2 + x - 2$ **c** $x^2 + 4x - 5$
d $x^2 + 5x - 14$ **e** $x^2 + 2x - 15$ **f** $x^2 + 8x - 20$
g $x^2 + 3x - 18$ **h** $x^2 + 7x - 18$ **i** $x^2 + x - 12$

Example 11c
4 Factorise each of the following quadratic expressions.

a $x^2 - 6x + 5$ **b** $x^2 - 2x + 1$ **c** $x^2 - 5x + 4$
d $x^2 - 9x + 8$ **e** $x^2 - 4x + 4$ **f** $x^2 - 8x + 12$
g $x^2 - 11x + 18$ **h** $x^2 - 10x + 21$ **i** $x^2 - 5x + 6$

5 Factorise each of the following quadratic expressions.

a $x^2 - 7x - 8$ **b** $x^2 - 3x - 4$ **c** $x^2 - 5x - 6$
d $x^2 - 6x - 16$ **e** $x^2 - 2x - 24$ **f** $x^2 - 2x - 15$
g $x^2 - x - 12$ **h** $x^2 - 11x - 12$ **i** $x^2 - 4x - 12$

Example 12

6 Factorise each of the following quadratic expressions by first taking out a common factor.

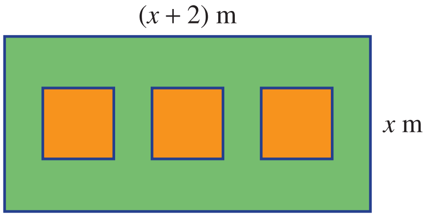
a $2x^2 + 10x + 8$ **b** $2x^2 + 22x + 20$ **c** $3x^2 + 18x + 24$
d $2x^2 + 14x - 60$ **e** $2x^2 - 14x - 36$ **f** $4x^2 - 8x + 4$
g $2x^2 + 2x - 12$ **h** $6x^2 - 30x - 36$ **i** $5x^2 - 30x + 40$
j $3x^2 - 33x + 90$ **k** $2x^2 - 6x - 20$ **l** $3x^2 - 3x - 36$

PROBLEM-SOLVING	7	7(1/2), 8	7(1/2), 8

7 Find the missing term in these trinomials if they are to factorise using integers. For example: the missing term in $x^2 + \square + 10$ could be $7x$ because $x^2 + 7x + 10$ factorises to $(x + 5)(x + 2)$ and 5 and 2 are integers. There may be more than one answer in each case.

a $x^2 + \square + 5$ **b** $x^2 - \square + 9$ **c** $x^2 - \square - 12$ **d** $x^2 + \square - 12$
e $x^2 + \square + 18$ **f** $x^2 - \square + 18$ **g** $x^2 - \square - 16$ **h** $x^2 + \square - 25$

8 A rectangular backyard has a length 2 metres more than its width (x metres). Inside the rectangle are three paved squares each of area 5 m^2 as shown. The remaining area is lawn.



a Find an expression for:
- **i** the total backyard area
- **ii** the area of lawn in expanded form
- **iii** the area of lawn in factorised form.

b Find the area of lawn if:
- **i** $x = 10$
- **ii** $x = 7$.

REASONING	9(1/2)	9(1/2)	9(1/2), 10

9 The expression $x^2 - 6x + 9$ factorises to $(x - 3)(x - 3) = (x - 3)^2$, which is a perfect square. Factorise these perfect squares.

a $x^2 + 8x + 16$ **b** $x^2 + 10x + 25$ **c** $x^2 + 30x + 225$
d $x^2 - 2x + 1$ **e** $x^2 - 14x + 49$ **f** $x^2 - 26x + 169$
g $2x^2 + 4x + 2$ **h** $5x^2 - 30x + 45$ **i** $-3x^2 + 36x - 108$

10 Sometimes it is not possible to factorise quadratic trinomials using integers. Decide which of the following cannot be factorised using integers.

A $x^2 - x - 56$ **B** $x^2 + 5x - 4$ **C** $x^2 + 7x - 6$
D $x^2 + 3x - 108$ **E** $x^2 + 3x - 1$ **F** $x^2 + 12x - 53$

ENRICHMENT: Completing the square	–	–	11

11 It is useful to be able to write a simple quadratic trinomial in the form $(x + b)^2 + c$. This involves adding (and subtracting) a special number to form the first perfect square. This procedure is called completing the square. Here is an example.

$$\left(-\tfrac{6}{2}\right)^2 = 9$$

$$\begin{aligned} x^2 - 6x - 8 &= \underbrace{x^2 - 6x + 9}\,\underbrace{-\,9 - 8} \\ &= (x - 3)(x - 3) - 17 \\ &= (x - 3)^2 - 17 \end{aligned}$$

Complete the square for these trinomials.

a $x^2 - 2x - 8$ **b** $x^2 + 4x - 1$ **c** $x^2 + 10x + 3$
d $x^2 - 16x - 3$ **e** $x^2 + 18x + 7$ **f** $x^2 - 32x - 11$

Progress quiz

8A

1 Expand the following.
- **a** $(x + 4)(x + 2)$
- **b** $(a - 5)(a + 8)$
- **c** $(2x - 3)(x + 6)$
- **d** $(3a - b)(a - 2b)$

8B

2 Expand each of the following perfect squares.
- **a** $(y + 4)^2$
- **b** $(x - 3)^2$
- **c** $(2a - 3)^2$
- **d** $(7k + 2m)^2$

8B

3 Expand and simplify the following.
- **a** $(x + 5)(x - 5)$
- **b** $(11x - 9y)(11x + 9y)$

8C

4 Factorise the following.
- **a** $25a - 15$
- **b** $x^2 - 7x$
- **c** $-12x^2 - 16x$
- **d** $2(a + 3) + a(a + 3)$
- **e** $7(8 + a) - a(8 + a)$
- **f** $k(k - 4) - (k - 4)$

8D

5 Factorise each of the following.
- **a** $x^2 - 81$
- **b** $16a^2 - 49$
- **c** $25x^2 - y^2$
- **d** $2a^2 - 50$
- **e** $12x^2y^2 - 12$
- **f** $(h + 3)^2 - 64$

8E

6 Use the method of grouping to factorise these expressions.
- **a** $x^2 + 7x + 2x + 14$
- **b** $a^2 + 5a - 4a - 20$
- **c** $x^2 - hx + x - h$

8E

7 Use grouping to factorise these expressions by first rearranging.
- **a** $2x^2 - 9 - 6x + 3x$
- **b** $3ap - 10 + 2p - 15a$

8F

Ext

8 Factorise these quadratic expressions.
- **a** $x^2 + 6x + 8$
- **b** $a^2 + 2a - 15$
- **c** $m^2 - 11m + 30$
- **d** $2k^2 + 2k - 24$

8G Factorising trinomials of the form $ax^2 + bx + c$ EXTENDING

Learning intentions

- To know the process for factorising a trinomial of the form $ax^2 + bx + c$ where a is not a common factor

So far we have factorised quadratic trinomials in which the coefficient of x^2 is 1, such as $x^2 - 3x - 40$. These are called monic trinomials. We will now consider non-monic trinomials in which the coefficient of x is not equal to 1 and is also not a factor common to all three terms, such as in $6x^2 + x - 15$. The method used in this section uses grouping, which was discussed in **Section 8E**.

Flight simulators are programmed to replicate an aircraft's response to pilot input and weather conditions. Modelling plane flight includes quadratic equations, as the upward force on a plane's wing is proportional to the square of its airspeed.

LESSON STARTER How the grouping method works

Consider the trinomial $2x^2 + 9x + 10$.

- First write $2x^2 + 9x + 10 = 2x^2 + 4x + 5x + 10$ and then factorise by grouping.
- Note that $9x$ was split to give $4x + 5x$ and the product of $2x^2$ and 10 is $20x^2$. Describe the link between the pair of numbers $\{4, 5\}$ and the pair of numbers $\{2, 10\}$.
- Why was $9x$ split to give $4x + 5x$ and not, say, $3x + 6x$?
- Describe how the $13x$ should be split in $2x^2 + 13x + 15$ so it can be factorised by grouping.
- Now try your method for $2x^2 - 7x - 15$.

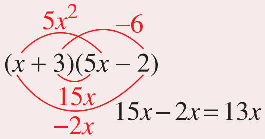
KEY IDEAS

- To factorise a trinomial of the form $ax^2 + bx + c$ by grouping, find two numbers which sum to give b and multiply to give $a \times c$.
  For example:

$$\begin{aligned} &5x^2 + 13x - 6\\ &= 5x^2 + 15x - 2x - 6\\ &= 5x(x + 3) - 2(x + 3)\\ &= (x + 3)(5x - 2) \end{aligned}$$

$a \times c = 5 \times (-6) = -30$ so the two numbers are 15 and -2 since $15 + (-2) = 13$ and $15 \times (-2) = -30$.

- Mentally check your factors by expanding your answer.



BUILDING UNDERSTANDING

① State the two numbers that satisfy the given criteria.

- **a** Multiply to give 12 and add to give 8
- **b** Multiply to give -24 and add to give 5
- **c** Multiply to give 35 and add to give -12
- **d** Multiply to give -28 and add to give -3

② State the missing terms for these factorisations.

a $$\begin{aligned}2x^2 + 7x + 5 &= 2x^2 + 2x + \_\_\_ + 5\\ &= 2x(\_\_\_\_\_\_) + 5(\_\_\_\_\_\_)\\ &= (\_\_\_\_\_\_)(\_\_\_\_\_\_)\end{aligned}$$

b $$\begin{aligned}2x^2 - 7x + 6 &= 2x^2 - 3x - \_\_\_ + 6\\ &= x(\_\_\_\_\_\_) - 2(\_\_\_\_\_\_)\\ &= (\_\_\_\_\_\_)(\_\_\_\_\_\_)\end{aligned}$$

c $$\begin{aligned}5x^2 + 9x - 2 &= 5x^2 + 10x - \_\_\_ - 2\\ &= 5x(\_\_\_\_\_\_) - 1(\_\_\_\_\_\_)\\ &= (\_\_\_\_\_\_)(\_\_\_\_\_\_)\end{aligned}$$

d $$\begin{aligned}4x^2 + 11x + 6 &= 4x^2 + \_\_\_ + 3x + 6\\ &= (\_\_\_\_\_\_)(x + 2) + 3(\_\_\_\_\_\_)\\ &= (\_\_\_\_\_\_)(\_\_\_\_\_\_)\end{aligned}$$

Example 13 Factorising trinomials of the form $ax^2 + bx + c$

Factorise $2x^2 + 7x + 3$.

SOLUTION

$$\begin{aligned}2x^2 + 7x + 3 &= 2x^2 + x + 6x + 3\\ &= x(2x + 1) + 3(2x + 1)\\ &= (2x + 1)(x + 3)\end{aligned}$$

EXPLANATION

$a \times c = 2 \times 3 = 6$, then ask what factors of this number (6) add to 7. The answer is 1 and 6, so split $7x$ into $x + 6x$ and factorise by grouping.

Now you try

Factorise $3x^2 + 11x + 6$.

Example 14 Factorising trinomials involving negative numbers

Factorise these quadratic trinomials.

a $10x^2 + 9x - 9$

b $6x^2 - 17x + 12$

SOLUTION

a $$\begin{aligned}10x^2 + 9x - 9 &= 10x^2 + 15x - 6x - 9\\ &= 5x(2x + 3) - 3(2x + 3)\\ &= (2x + 3)(5x - 3)\end{aligned}$$

b $$\begin{aligned}6x^2 - 17x + 12 &= 6x^2 - 9x - 8x + 12\\ &= 3x(2x - 3) - 4(2x - 3)\\ &= (2x - 3)(3x - 4)\end{aligned}$$

EXPLANATION

$10 \times (-9) = -90$, so ask what factors of -90 add to give 9. Choose 15 and -6. Then complete the factorisation by grouping.

$6 \times 12 = 72$, so ask what factors of 72 add to give -17. Choose -9 and -8.
Complete a mental check.

$$(2x - 3)(3x - 4)$$
$$-9x$$
$$-8x \quad -9x - 8x = -17x$$

Now you try

Factorise these quadratic trinomials.

a $4x^2 + 4x - 15$

b $8x^2 - 18x + 9$

Exercise 8G

FLUENCY	1, 2–3(1/2)	2–3(1/2)	2–3(1/3)

Example 13

1 Factorise:

a $2x^2 + 11x + 5$
b $3x^2 + 10x + 3$

Example 13

2 Factorise these quadratic trinomials.

a $2x^2 + 9x + 4$
b $3x^2 + 7x + 2$
c $2x^2 + 7x + 6$
d $3x^2 + 8x + 4$
e $5x^2 + 12x + 4$
f $2x^2 + 11x + 12$
g $6x^2 + 13x + 5$
h $4x^2 + 5x + 1$
i $8x^2 + 14x + 5$

Example 14

3 Factorise these quadratic trinomials.

a $3x^2 + 2x - 5$
b $5x^2 + 6x - 8$
c $8x^2 + 10x - 3$
d $6x^2 - 13x - 8$
e $10x^2 - 3x - 4$
f $5x^2 - 11x - 12$
g $4x^2 - 16x + 15$
h $2x^2 - 15x + 18$
i $6x^2 - 19x + 10$
j $12x^2 - 13x - 4$
k $4x^2 - 12x + 9$
l $7x^2 + 18x - 9$
m $9x^2 + 44x - 5$
n $3x^2 - 14x + 16$
o $4x^2 - 4x - 15$

PROBLEM-SOLVING	4(1/2)	4–5(1/2)	4(1/3), 5–6(1/2)

4 Factorise these quadratic trinomials.

a $10x^2 + 27x + 11$
b $15x^2 + 14x - 8$
c $20x^2 - 36x + 9$
d $18x^2 - x - 5$
e $25x^2 + 5x - 12$
f $32x^2 - 12x - 5$
g $27x^2 + 6x - 8$
h $33x^2 + 41x + 10$
i $54x^2 - 39x - 5$
j $12x^2 - 32x + 21$
k $75x^2 - 43x + 6$
l $90x^2 + 33x - 8$

5 Factorise by first taking out a common factor.

a $30x^2 - 14x - 4$
b $12x^2 + 18x - 30$
c $27x^2 - 54x + 15$
d $21x^2 - 77x + 42$
e $36x^2 + 36x - 40$
f $50x^2 - 35x - 60$

6 Factorise these trinomials.

a $-2x^2 + 7x - 6$
b $-5x^2 - 3x + 8$
c $-6x^2 + 13x + 8$
d $18 - 9x - 5x^2$
e $16x - 4x^2 - 15$
f $14x - 8x^2 - 5$

REASONING	7	7	7, 8

7 When splitting the $3x$ in $2x^2 + 3x - 20$, you could write:

A $2x^2 + 8x - 5x - 20$ or **B** $2x^2 - 5x + 8x - 20$

a Complete the factorisation using **A**.
b Complete the factorisation using **B**.
c Does the order matter when you split the $3x$?
d Factorise these trinomials twice each. Factorise once by grouping and then repeat but reverse the order of the two middle terms in the first line of working.
  i $3x^2 + 5x - 12$
  ii $5x^2 - 3x - 14$
  iii $6x^2 + 5x - 4$

8 Make up five non-monic trinomials in which the coefficient of x^2 is not equal to 1 and which factorise using the above method. Explain your method in finding these trinomials.

ENRICHMENT: The cross method

– – 9(1/3)

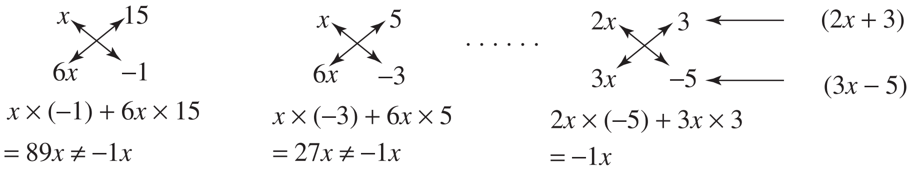
9 The cross method is another way to factorise trinomials of the form $ax^2 + bx + c$. It involves finding factors of ax^2 and factors of c and then choosing pairs of these factors that add to bx.

For example: Factorise $6x^2 - x - 15$.

Factors of $6x^2$ include $(x, 6x)$ and $(2x, 3x)$.

Factors of -15 include $(15, -1)$, $(-15, 1)$, $(5, -3)$ and $(-5, 3)$.

We arrange a chosen pair of factors vertically and then cross-multiply and add to get $-1x$.



$x \times (-1) + 6x \times 15$
$= 89x \neq -1x$

$x \times (-3) + 6x \times 5$
$= 27x \neq -1x$

$2x \times (-5) + 3x \times 3$
$= -1x$

You will need to continue until a particular combination works. The third cross-product gives a sum of $-1x$ so choose the factors $(2x + 3)$ and $(3x - 5)$, therefore:

$$6x^2 - x - 15 = (2x + 3)(3x - 5)$$

Try this method on the trinomials from Questions **3** and **4**.

Without air resistance, the trajectory (i.e. flight path) of a thrown or launched object, subject to the Moon's gravity, can be modelled accurately using quadratic equations.

ISBN 978-1-108-77288-4

8H Simplifying algebraic fractions: Multiplication and division

Learning intentions

- To understand that cancelling in algebraic fractions can only take place using a common factor
- To be able to simplify algebraic fractions by cancelling common factors
- To know to factorise expressions and cancel common factors before multiplying or dividing
- To be able to multiply algebraic fractions by multiplying numerators and denominators
- To be able to divide an algebraic fraction by multiplying by the reciprocal of the fraction after the division sign

With a numerical fraction such as $\frac{6}{9}$, the highest common factor of 6 and 9 is 3, which can be cancelled: $\frac{6}{9} = \frac{{}^1\cancel{3} \times 2}{{}_1\cancel{3} \times 3} = \frac{2}{3}$. For algebraic fractions the process is the same. If expressions are in a factorised form, common factors can be easily identified and cancelled.

LESSON STARTER Correct cancelling

Consider this cancelling attempt:

$$\frac{5x + \cancel{10}^1}{\cancel{20}_2} = \frac{5x + 1}{2}$$

- Substitute $x = 6$ into the left-hand side to evaluate $\frac{5x + 10}{20}$.
- Substitute $x = 6$ into the right-hand side to evaluate $\frac{5x + 1}{2}$.
- What do you notice about the two answers? How can you explain this?
- Decide how you might correctly cancel the expression on the left-hand side. Show your steps and check by substituting a value for x.

KEY IDEAS

- Simplify **algebraic fractions** by factorising and cancelling only common factors.

  Incorrect

  $$\frac{2x + \cancel{4}^2}{\cancel{2}^1} = 2x + 2$$

  Correct

  $$\begin{aligned}\frac{2x + 4}{2} &= \frac{{}^1\cancel{2}(x + 2)}{\cancel{2}^1} \\ &= x + 2\end{aligned}$$

- To multiply algebraic fractions:
  - factorise expressions where possible
  - cancel if possible
  - multiply the numerators and the denominators.
- To divide algebraic fractions:
  - multiply by the reciprocal of the fraction following the division sign
  - follow the rules for multiplication after converting to the reciprocal.
    - The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.

BUILDING UNDERSTANDING

1 State the result after simplifying these fractions using cancelling.

a $\frac{5}{15}$ **b** $\frac{8x}{12}$ **c** $\frac{24}{8x}$ **d** $\frac{3(x+1)}{6}$

2 Factorise by taking out common factors.

a $3x + 6$ **b** $20 - 40x$ **c** $x^2 - 7x$ **d** $6x^2 + 24x$

3 State the missing terms.

a $\frac{2x-4}{8} = \frac{2(\boxed{})}{8} = \frac{\boxed{}}{4}$

b $\frac{12-18x}{2x-3x^2} = \frac{6(\boxed{})}{x(\boxed{})} = \frac{6}{x}$

4 Simplify:

a $\frac{6}{7} \times \frac{21}{2}$ **b** $\frac{5}{8} \div \frac{15}{2}$

Example 15 Simplifying algebraic fractions

Simplify the following by cancelling.

a $\frac{3(x+2)(x-4)}{6(x-4)}$ **b** $\frac{20-5x}{8-2x}$ **c** $\frac{x^2-4}{x+2}$

SOLUTION	EXPLANATION
a $\frac{{}^{1}\cancel{3}(x+2)\cancel{(x-4)}^{1}}{{}^{2}\cancel{6}\cancel{(x-4)}^{1}} = \frac{x+2}{2}$	Cancel the common factors $(x - 4)$ and 3.
b $\frac{20-5x}{8-2x} = \frac{5\cancel{(4-x)}^{1}}{2\cancel{(4-x)}^{1}}$ $= \frac{5}{2}$	Factorise the numerator and denominator and then cancel common factor of $(4 - x)$.
c $\frac{x^2-4}{x+2} = \frac{{}^{1}\cancel{(x+2)}(x-2)}{\cancel{(x+2)}_{1}}$ $= x - 2$	Factorise the difference of squares in the numerator and then cancel the common factor.

Now you try

Simplify the following by cancelling.

a $\frac{4(x+1)(x-3)}{12(x+1)}$ **b** $\frac{3-6x}{5-10x}$ **c** $\frac{x^2-9}{x+3}$

Example 16 Multiplying and dividing algebraic fractions

Simplify the following.

a $\dfrac{3(x-1)}{(x+2)} \times \dfrac{4(x+2)}{9(x-1)(x-7)}$ **b** $\dfrac{(x-3)(x+4)}{x(x+7)} \div \dfrac{3(x+4)}{x+7}$ Ext **c** $\dfrac{x^2-4}{25} \times \dfrac{5x+5}{x^2-x-2}$

SOLUTION

a $\dfrac{{}^{1}\cancel{3}\cancel{(x-1)}^{1}}{\cancel{(x+2)}_{1}} \times \dfrac{4\cancel{(x+2)}^{1}}{{}_{3}\cancel{9}\cancel{(x-1)}_{1}(x-7)}$

$= \dfrac{1 \times 4}{1 \times 3(x-7)}$

$= \dfrac{4}{3(x-7)}$

EXPLANATION

First, cancel any factors in the numerators with a common factor in the denominators. Then multiply the numerators and the denominators.

b $\dfrac{(x-3)(x+4)}{x(x+7)} \div \dfrac{3(x+4)}{x+7}$

$= \dfrac{(x-3)\cancel{(x+4)}^{1}}{x\cancel{(x+7)}_{1}} \times \dfrac{\cancel{(x+7)}^{1}}{3\cancel{(x+4)}^{1}}$

$= \dfrac{x-3}{3x}$

Multiply by the reciprocal of the fraction after the division sign.

Cancel common factors and multiply remaining numerators and denominators.

c $\dfrac{x^2-4}{25} \times \dfrac{5x+5}{x^2-x-2}$

$= \dfrac{{}^{1}\cancel{(x-2)}(x+2)}{\cancel{25}^{5}} \times \dfrac{\cancel{5}^{1}\cancel{(x+1)}^{1}}{{}_{1}\cancel{(x-2)}\cancel{(x+1)}_{1}}$

$= \dfrac{x+2}{5}$

First factorise all the algebraic expressions. Note that $x^2 - 4$ is a difference of perfect squares. Then cancel as normal.

Now you try

Simplify the following.

a $\dfrac{2(x+1)}{(x-3)} \times \dfrac{9(x-3)}{10(x+1)(x-4)}$ **b** $\dfrac{4(x-2)(x+2)}{x(x-1)} \div \dfrac{2(x+2)}{x-1}$ **c** $\dfrac{x^2-5x+6}{18} \times \dfrac{6x+12}{x^2-4}$

ISBN 978-1-108-77288-4

Exercise 8H

FLUENCY	1, 2–6(1/2)	2–7(1/2)	2–7(1/2)

1 Simplify the following by cancelling.

Example 15a

a **i** $\frac{5(x+3)(x-2)}{15(x+3)}$ **ii** $\frac{8(x+4)}{12(x+4)(x-1)}$

Example 15b

b **i** $\frac{4x+8}{7x+14}$ **ii** $\frac{6-4x}{9-6x}$

Example 15c

c **i** $\frac{x^2-16}{x+4}$ **ii** $\frac{x^2-49}{x-7}$

Example 15a

2 Simplify the following by cancelling.

a $\frac{3(x+2)}{4(x+2)}$ **b** $\frac{x(x-3)}{3x(x-3)}$ **c** $\frac{20(x+7)}{5(x+7)}$

d $\frac{(x+5)(x-5)}{(x+5)}$ **e** $\frac{6(x-1)(x+3)}{9(x+3)}$ **f** $\frac{8(x-2)}{4(x-2)(x+4)}$

Example 15b

3 Simplify the following by factorising and then cancelling.

a $\frac{5x-5}{5}$ **b** $\frac{4x-12}{10}$ **c** $\frac{2x-4}{3x-6}$ **d** $\frac{12-4x}{6-2x}$

e $\frac{x^2-3x}{x}$ **f** $\frac{4x^2+10x}{5x}$ **g** $\frac{3x+3y}{2x+2y}$ **h** $\frac{4x-8y}{3x-6y}$

Example 15c

4 Simplify the following. These expressions involve difference of perfect squares.

a $\frac{x^2-100}{x+10}$ **b** $\frac{x^2-49}{x+7}$ **c** $\frac{x^2-25}{x+5}$

d $\frac{2(x-20)}{x^2-400}$ **e** $\frac{5(x-6)}{x^2-36}$ **f** $\frac{3x+27}{x^2-81}$

Example 16a

5 Simplify the following by cancelling.

a $\frac{2x(x-4)}{4(x+1)} \times \frac{(x+1)}{x}$ **b** $\frac{(x+2)(x-3)}{x-5} \times \frac{x-5}{x+2}$

c $\frac{x-3}{x+2} \times \frac{3(x+4)(x+2)}{x+4}$ **d** $\frac{2(x+3)(x+4)}{(x+1)(x-5)} \times \frac{(x+1)}{4(x+3)}$

Example 16b

6 Simplify the following by cancelling.

a $\frac{x(x+1)}{x+3} \div \frac{x+1}{x+3}$ **b** $\frac{x+3}{x+2} \div \frac{x+3}{2(x-2)}$

c $\frac{x-4}{(x+3)(x+1)} \div \frac{x-4}{4(x+3)}$ **d** $\frac{4x}{x+2} \div \frac{8x}{x-2}$

e $\frac{3(4x-9)(x+2)}{2(x+6)} \div \frac{9(x+4)(4x-9)}{4(x+2)(x+6)}$ **f** $\frac{5(2x-3)}{(x+7)} \div \frac{(x+2)(2x-3)}{x+7}$

Ext

7 Simplify by first factorising.

a $\frac{x^2-x-6}{x-3}$ **b** $\frac{x^2+8x+16}{x+4}$ **c** $\frac{x^2-7x+12}{x-4}$

d $\frac{x-2}{x^2+x-6}$ **e** $\frac{x+7}{x^2+5x-14}$ **f** $\frac{x-9}{x^2-19x+90}$

PROBLEM-SOLVING

8(1/2) 8(1/2) 8(1/2)

Example 16c

Ext

8 These expressions involve a combination of trinomials, difference of perfect squares and simple common factors. Simplify by first factorising where possible.

a $\frac{x^2+5x+6}{x+5} \div \frac{x+3}{x^2-25}$

b $\frac{x^2+6x+8}{x^2-9} \div \frac{x+4}{x-3}$

c $\frac{x^2+x-12}{x^2+8x+16} \times \frac{x^2-16}{x^2-8x+16}$

d $\frac{x^2+12x+35}{x^2-25} \times \frac{x^2-10x+25}{x^2+9x+14}$

e $\frac{9x^2-3x}{6x-45x^2}$

f $\frac{x^2-4x}{3x-x^2}$

g $\frac{3x^2-21x+36}{2x^2-32} \times \frac{2x+10}{6x-18}$

h $\frac{2x^2-18x+40}{x^2-x-12} \times \frac{3x+15}{4x^2-100}$

REASONING

9 9 9–10(1/2)

9 The expression $\frac{5-2x}{2x-5}$ can be written in the form $\frac{-1(-5+2x)}{2x-5} = \frac{-1(2x-5)}{2x-5}$, which can be cancelled to -1.

Use this idea to simplify these algebraic fractions.

a $\frac{7-3x}{3x-7}$

b $\frac{4x-1}{1-4x}$

c $\frac{8x+16}{-2-x}$

d $\frac{x+3}{-9-3x}$

e $\frac{5-3x}{18x-30}$

f $\frac{x^2-9}{3-x}$

10 Just as $\frac{a^2}{2a}$ can be cancelled to $\frac{a}{2}$, $\frac{(a+5)^2}{2(a+5)}$ cancels to $\frac{a+5}{2}$. Use this idea to cancel these fractions.

a $\frac{(a+1)^2}{(a+1)}$

b $\frac{5(a-3)^2}{(a-3)}$

c $\frac{7(x+7)^2}{14(x+7)}$

d $\frac{3(x-1)(x+2)^2}{18(x-1)(x+2)}$

Ext **e** $\frac{x^2+6x+9}{2x+6}$

Ext **f** $\frac{11x-22}{x^2-4x+4}$

ENRICHMENT: All in together

– – 11(1/2)

Ext

11 Use your knowledge of factorisation and the ideas in Questions **9** and **10** above to simplify these algebraic fractions.

a $\frac{2x^2-2x-24}{16-4x}$

b $\frac{x^2-14x+49}{21-3x}$

c $\frac{x^2-16x+64}{64-x^2}$

d $\frac{4-x^2}{x^2+x-6} \times \frac{2x+6}{x^2+4x+4}$

e $\frac{2x^2-18}{x^2-6x+9} \times \frac{6-2x}{x^2+6x+9}$

f $\frac{x^2-2x+1}{4-4x} \div \frac{1-x^2}{3x^2+6x+3}$

g $\frac{4x^2-9}{x^2-5x} \div \frac{6-4x}{15-3x}$

h $\frac{x^2-4x+4}{8-4x} \times \frac{-2}{4-x^2}$

i $\frac{(x+2)^2-4}{(1-x)^2} \times \frac{x^2-2x+1}{3x+12}$

j $\frac{2(x-3)^2-50}{x^2-11x+24} \div \frac{x^2-4}{3-x}$

8I Simplifying algebraic fractions: Addition and subtraction

Learning intentions

- To be able to find a lowest common denominator involving numbers and pronumerals
- To know to express each fraction as an equivalent fraction with a common denominator before combining under addition or subtraction
- To be able to add or subtract numerators by simplifying

The process required for adding or subtracting algebraic fractions is similar to that used for fractions without pronumerals.

To simplify $\frac{2}{3}+\frac{4}{5}$, for example, you would find the lowest common multiple of the denominators (15) and then express each fraction using this denominator. Adding the numerators completes the task. The same process is applied to $\frac{2x}{3}+\frac{4x}{5}$.

Financial analysts and economists use algebraic fractions when finding current amounts, such as a government's future income tax and future spending. The current value of two future amounts of \$$x$ at 4% p.a.:

$$= \frac{x}{1.04} + \frac{x}{1.04^2}$$

LESSON STARTER Compare the working

Here is the working for the simplification of the sum of a pair of numerical fractions and the sum of a pair of algebraic fractions.

$$\begin{aligned}\frac{2}{5}+\frac{3}{4} &= \frac{8}{20}+\frac{15}{20}\\ &= \frac{23}{20}\end{aligned} \qquad \begin{aligned}\frac{2x}{5}+\frac{3x}{4} &= \frac{8x}{20}+\frac{15x}{20}\\ &= \frac{23x}{20}\end{aligned}$$

- What type of steps were taken to simplify the algebraic fractions that are the same as for the numerical fractions?
- Write down the steps required to add (or subtract) algebraic fractions.

KEY IDEAS

- To add or subtract algebraic fractions:
  - determine the lowest common denominator (LCD)
  - express each fraction using the LCD
  - add or subtract the numerators.

BUILDING UNDERSTANDING

① Find the lowest common multiple of these pairs of numbers.

a 6, 8 **b** 3, 5 **c** 11, 13 **d** 12, 18

② State the missing terms in the following expressions.

a $\frac{7x}{3} = \frac{\Box}{9}$ **b** $\frac{3x+5}{11} = \frac{\Box(3x+5)}{22}$ **c** $\frac{4}{x} = \frac{\Box}{2x}$ **d** $\frac{30}{x+1} = \frac{\Box}{3(x+1)}$

③ State the missing terms in these simplifications.

a $\frac{x}{4} + \frac{2x}{3} = \frac{\Box}{12} + \frac{\Box}{12} = \frac{\Box}{12}$ **b** $\frac{5x}{7} - \frac{2x}{5} = \frac{\Box}{35} - \frac{\Box}{35} = \frac{\Box}{35}$

c $\frac{x+1}{2} + \frac{2x+3}{4} = \frac{\Box(x+1)}{4} + \frac{2x+3}{4} = \frac{\Box + 2x + 3}{4} = \frac{\Box}{4}$

④ State the LCD for these pairs of fractions.

a $\frac{x}{3}, \frac{2x}{5}$ **b** $\frac{3x}{7}, \frac{x}{2}$ **c** $\frac{2x}{3}, \frac{-5x}{6}$

Example 17 Adding and subtracting with numerals in the denominators

Simplify:

a $\frac{x}{4} - \frac{2x}{5}$ **b** $\frac{7x}{3} + \frac{x}{6}$ **c** $\frac{x+3}{2} + \frac{x-2}{5}$

SOLUTION

a $\frac{x}{4} - \frac{2x}{5} = \frac{5x}{20} - \frac{8x}{20}$

$= -\frac{3x}{20}$

b $\frac{7x}{3} + \frac{x}{6} = \frac{14x}{6} + \frac{x}{6}$

$= \frac{15x}{6}$

$= \frac{5x}{2}$

c $\frac{x+3}{2} + \frac{x-2}{5} = \frac{5(x+3)}{10} + \frac{2(x-2)}{10}$

$= \frac{5x + 15 + 2x - 4}{10}$

$= \frac{7x + 11}{10}$

EXPLANATION

Determine the LCD for 4 and 5, i.e. 20. Express each fraction as an equivalent fraction with a denominator of 20: $2x \times 4 = 8x$. Then subtract numerators.

Note the LCD for 3 and 6 is 6 not $3 \times 6 = 18$.

Simplify $\frac{15}{6}$ to $\frac{5}{2}$ in the final step.

The LCD for 2 and 5 is 10; write as equivalent fractions with denominator 10.

Expand the brackets and simplify the numerator by adding and collecting like terms.

Now you try

Simplify:

a $\frac{x}{3} - \frac{3x}{4}$ **b** $\frac{5x}{4} + \frac{x}{8}$ **c** $\frac{x+2}{3} + \frac{x-4}{2}$

Example 18 Adding and subtracting with algebraic terms in the denominators

Simplify:

a $\frac{2}{x} - \frac{5}{2x}$

b $\frac{2}{x} + \frac{3}{x^2}$

SOLUTION

a $\frac{2}{x} - \frac{5}{2x} = \frac{4}{2x} - \frac{5}{2x}$

$= -\frac{1}{2x}$

b $\frac{2}{x} + \frac{3}{x^2} = \frac{2x}{x^2} + \frac{3}{x^2}$

$= \frac{2x + 3}{x^2}$

EXPLANATION

The LCD for x and $2x$ is $2x$, so rewrite the first fraction in an equivalent form with a denominator also of $2x$.

The LCD for x and x^2 is x^2, so rewrite the first fraction in an equivalent form so its denominator is also x^2, and then add numerators.

Now you try

Simplify:

a $\frac{3}{x} - \frac{4}{3x}$

b $\frac{4}{x} + \frac{5}{x^2}$

Exercise 8I

FLUENCY	1, 2–5(1/2)	2–5(1/2)	2(1/4), 3(1/3), 4–5(1/2)

1 Simplify:

Example 17a

a **i** $\frac{x}{5} - \frac{x}{3}$ **ii** $\frac{x}{4} + \frac{2x}{7}$

Example 17b

b **i** $\frac{2x}{5} + \frac{x}{15}$ **ii** $\frac{5x}{6} - \frac{2x}{9}$

Example 17c

c **i** $\frac{x+2}{3} + \frac{x-1}{4}$ **ii** $\frac{x-2}{2} + \frac{x-3}{7}$

Example 17a,b

2 Simplify:

a $\frac{x}{7} + \frac{x}{2}$ **b** $\frac{x}{3} + \frac{x}{15}$ **c** $\frac{x}{4} - \frac{x}{8}$ **d** $\frac{x}{9} + \frac{x}{5}$

e $\frac{y}{7} - \frac{y}{8}$ **f** $\frac{a}{2} + \frac{a}{11}$ **g** $\frac{b}{3} - \frac{b}{9}$ **h** $\frac{m}{3} - \frac{m}{6}$

i $\frac{m}{6} + \frac{3m}{4}$ **j** $\frac{a}{4} + \frac{2a}{7}$ **k** $\frac{2x}{5} + \frac{x}{10}$ **l** $\frac{p}{9} - \frac{3p}{7}$

m $\frac{b}{2} - \frac{7b}{9}$ **n** $\frac{9y}{8} + \frac{2y}{5}$ **o** $\frac{4x}{7} - \frac{x}{5}$ **p** $\frac{3x}{4} - \frac{x}{3}$

Example 17c

3 Simplify:

a $\frac{x+1}{2}+\frac{x+3}{5}$ **b** $\frac{x+3}{3}+\frac{x-4}{4}$ **c** $\frac{a-2}{7}+\frac{a-5}{8}$

d $\frac{y+4}{5}+\frac{y-3}{6}$ **e** $\frac{m-4}{8}+\frac{m+6}{5}$ **f** $\frac{x-2}{12}+\frac{x-3}{8}$

g $\frac{2b-3}{6}+\frac{b+2}{8}$ **h** $\frac{3x+8}{6}+\frac{2x-4}{3}$ **i** $\frac{2y-5}{7}+\frac{3y+2}{14}$

j $\frac{2t-1}{8}+\frac{t-2}{16}$ **k** $\frac{4-x}{3}+\frac{2-x}{7}$ **l** $\frac{2m-1}{4}+\frac{m-3}{6}$

Example 18a

4 Simplify:

a $\frac{3}{x}+\frac{5}{2x}$ **b** $\frac{7}{3x}-\frac{2}{x}$ **c** $\frac{7}{4x}-\frac{5}{2x}$ **d** $\frac{4}{3x}+\frac{2}{9x}$

e $\frac{3}{4x}-\frac{2}{5x}$ **f** $\frac{2}{3x}+\frac{1}{5x}$ **g** $\frac{-3}{4x}-\frac{7}{x}$ **h** $\frac{-5}{3x}-\frac{3}{4x}$

Example 18b

5 Simplify:

a $\frac{3}{x}+\frac{2}{x^2}$ **b** $\frac{5}{x^2}+\frac{4}{x}$ **c** $\frac{7}{x}+\frac{3}{x^2}$ **d** $\frac{4}{x}-\frac{5}{x^2}$

e $\frac{3}{x^2}-\frac{8}{x}$ **f** $-\frac{4}{x^2}+\frac{1}{x}$ **g** $\frac{3}{x}-\frac{7}{2x^2}$ **h** $-\frac{2}{3x}+\frac{3}{x^2}$

PROBLEM-SOLVING	6	6(½)	6(½), 7, 8

6 Simplify these mixed algebraic fractions.

a $\frac{2}{x}+\frac{x}{4}$ **b** $\frac{-5}{x}+\frac{x}{2}$ **c** $\frac{-2}{x}-\frac{4x}{3}$ **d** $\frac{3}{2x}-\frac{5x}{4}$

e $\frac{3x}{4}-\frac{5}{6x}$ **f** $\frac{1}{3x}-\frac{x}{9}$ **g** $-\frac{2}{5x}+\frac{3x}{2}$ **h** $-\frac{5}{4x}-\frac{3x}{10}$

7 Find the missing algebraic fraction. The fraction should be in simplest form.

a $\frac{x}{2}+\frac{\square}{\square}=\frac{5x}{6}$ **b** $\frac{\square}{\square}+\frac{x}{4}=\frac{3x}{8}$

c $\frac{2x}{5}+\frac{\square}{\square}=\frac{9x}{10}$ **d** $\frac{2x}{3}-\frac{\square}{\square}=\frac{7x}{15}$

e $\frac{\square}{\square}-\frac{x}{3}=\frac{5x}{9}$ **f** $\frac{2x}{3}-\frac{\square}{\square}=\frac{5x}{12}$

8 A hiker walks x metres up a hill at $\frac{6}{5}$ m/s and walks back down the same path to where they started at 2 m/s.

a Give a simplified expression for the total time in seconds of the walk in terms of x.

b Use your answer to part **a** to find the time taken if $x = 150$.

REASONING	9	9, 10	9, 10

9 Find and describe the error in each set of working. Then find the correct answer.

a $\frac{4x}{5}-\frac{x}{3}=\frac{3x}{2}$

b $\frac{x+1}{5}+\frac{x}{2}=\frac{2x+1}{10}+\frac{5x}{10}$
$=\frac{7x+1}{10}$

c $\frac{5x}{3}+\frac{x-1}{2}=\frac{10x}{6}+\frac{3x-1}{6}$
$=\frac{13x-1}{6}$

d $\frac{2}{x}-\frac{3}{x^2}=\frac{2}{x^2}-\frac{3}{x^2}$
$=\frac{-1}{x^2}$

10 A student thinks that the LCD to use when simplifying $\frac{x+1}{2}+\frac{2x-1}{4}$ is 8.

a Complete the simplification using a common denominator of 8.

b Now complete the simplification using the actual LCD of 4.

c How does your working for parts **a** and **b** compare? Which method is preferable and why?

ENRICHMENT: More than two fractions!	–	–	11(½)

11 Simplify by first finding the LCD.

a $\frac{2x}{5}-\frac{3x}{2}-\frac{x}{3}$

b $\frac{x}{4}-\frac{2x}{3}+\frac{5x}{6}$

c $\frac{5x}{8}-\frac{5x}{6}+\frac{3x}{4}$

d $\frac{x+1}{4}+\frac{2x-1}{3}-\frac{x}{5}$

e $\frac{2x-1}{3}-\frac{2x}{7}+\frac{x-3}{6}$

f $\frac{1-2x}{5}-\frac{3x}{8}+\frac{3x+1}{2}$

g $\frac{2}{3x}+\frac{5}{x}-\frac{1}{x}$

h $-\frac{1}{2x}+\frac{2}{x}-\frac{4}{3x}$

i $-\frac{4}{5x}-\frac{1}{2x}+\frac{3}{4x}$

j $\frac{4}{x^2}+\frac{3}{2x}-\frac{5}{3x}$

k $\frac{5}{x}-\frac{3}{2x^2}-\frac{5}{7x}$

l $\frac{2}{x^2}-\frac{4}{9x}-\frac{5}{3x^2}$

m $\frac{2}{x}+\frac{x}{5}-\frac{x}{3}$

n $\frac{3x}{2}-\frac{1}{2x}+\frac{x}{3}$

o $-\frac{4x}{9}+\frac{2}{5x}+\frac{2x}{5}$

Astronomers and astrophysicists use algebraic fractions when applying equations to calculate the speed of a star or galaxy. As a star moves towards or away from Earth, the wavelength of its light is changed, altering the colour. This is known as the Doppler effect.

Applications and problem-solving

The following problems will investigate practical situations drawing upon knowledge and skills developed throughout the chapter. In attempting to solve these problems, aim to identify the key information, use diagrams, formulate ideas, apply strategies, make calculations and check and communicate your solutions.

Ice-cream profit

1 An ice-cream vendor has determined the profit equation for his business based on the selling price of his ice-cream cones. The price needs to be set high enough for the associated costs to be covered, but not too high such that sales start to decline.

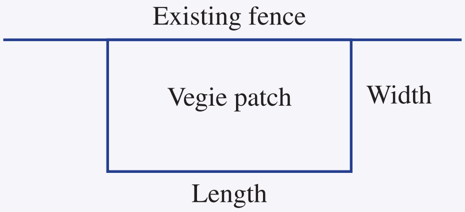
The profit of the business, $\$P$, for an ice-cream selling price of $\$x$ is given by the equation $P = -120x^2 + 840x - 1200$.

With this knowledge, the vendor wants to analyse the rule to find which selling price leads to a profit and how he can set the selling price to maximise the profit of the business.

- **a** Interpret the result from the rule for profit if the vendor gave the ice-cream cones away for free.
- **b** Use the rule to determine the profit when:
  - **i** $x = 3$
  - **ii** $x = 4.5$.
- **c** Consider the results from part **b**. Describe what has happened to the company's profit as the ice-cream price increased from \$3 to \$4.50.
- **d** By considering your results from part **b** and using increments of 0.5, find the selling price $\$x$ that gives the greatest profit. What is this profit?
- **e** The ice-cream prices are variable from day to day and the vendor will adjust to ensure that a profit is made each day.
  - **i** Factorise the rule for P by first removing the common factor.
  - **ii** Using your result from part **i**, consider which values of x would lead to a negative profit (i.e. a loss). Hence, between which values should the vendor maintain the ice-cream selling price to make a profit?

Vegie patch fencing

2 Farmer Jo is clearing an area of the farm for a new rectangular vegie patch. The area will need to be fenced off to keep some of the animals from trampling on it. Jo has 8 m of leftover chicken wire and some posts to use as fencing. She plans to use an existing fence on one side of the vegie patch as shown at right.



Jo is interested in looking at the different rectangular areas she can make with her 8 m of fencing material. In particular, she wishes to make the area for her vegie patch as large as possible.

- **a** For a vegie patch of width 1 m:
  - **i** find the maximum possible length of the vegie patch
  - **ii** determine the maximum area of the vegie patch.
- **b** If the width of the vegie patch is 3.2 m, repeat steps **i** and **ii** from part **a**. What do you notice about the area of this vegie patch compared to the one in part **a** for the same 8 m perimeter?
- **c** Let the width of the vegie patch be x m. Find expressions for the length and area of the vegie patch in terms of x.
- **d** Use your area expression from part **c** to find the area of the vegie patch if $x = 1.5$.
- **e** By considering the length and width dimensions of the vegie patch, what are the possible values that the width x m can take?
- **f** Use a suitable method to predict the maximum possible area of the vegie patch and the dimensions that produce this area.

Tracking running progress

3 Oscar is starting a running program and is running for 48 minutes three times a week. In the first week he records that he walks a warm-up lap of a nearby oval at 2 km/h and jogs 6 laps at 4 km/h. Let d km be distance around the oval.

Oscar is keen to analyse this data so that he can track his progress over time when he runs in different locations, and to set himself goals through the course of the program.

- **a** Find an expression in terms of d for the time taken in hours to:
  - **i** walk the warm-up lap
  - **ii** jog the 6 laps.
- **b** Hence, find a simplified expression in terms of d for the total time taken.
- **c** Given the warm-up lap and jog took 48 minutes, use your result from part **b** to help find the distance around the oval in metres.

As the training continues in week 2, Oscar increases his jogging speed to 5 km/h.

- **d** He goes for a run on a different oval, completing the warm-up lap at 2 km/h and then jogging 8 laps at 5 km/h, all in 48 minutes. Repeat parts **a**–**c** to find the distance around this oval to the nearest metre.
- **e** In week 3, Oscar returns to the first oval. The goal for the 48 minutes is to complete the regular warm-up lap and 8 laps jogging. Determine what his jogging speed in km/h will need to be to achieve this.

8J Further simplification of algebraic fractions EXTENDING

Learning intentions

- To be able to simplify algebraic fractions involving subtraction of binomial numerators
- To know how to find a lowest common denominator of binomial denominators
- To be able to simplify algebraic fractions with more complex numerators and denominators

More complex addition and subtraction of algebraic fractions involves expressions such as the following:

$$\frac{2x-1}{3}-\frac{x+4}{4} \quad \text{and} \quad \frac{2}{x-3}-\frac{5}{(x-3)^2}$$

In such examples, care needs to be taken at each step in the working to avoid common errors.

Electrical and biomedical engineers use algebraic fractions for many applications, including in the design of microcircuits and sensors for sophisticated medical equipment, such as MRI scanners.

LESSON STARTER Three critical errors

The following simplification of algebraic fractions has three critical errors. Can you find them?

$$\begin{aligned}\frac{2x+1}{3}-\frac{x+2}{2} &= \frac{2x+1}{6}-\frac{3(x+2)}{6}\\ &= \frac{2x+1-3x+6}{6}\\ &= \frac{x+7}{6}\end{aligned}$$

The correct answer is $\frac{x-4}{6}$.

Fix the solution to obtain the correct answer.

KEY IDEAS

- When combining algebraic fractions which involve subtraction signs, recall that:
  - the product of two numbers of opposite sign is a negative number
  - the product of two negative numbers is a positive number.

For example: $$\begin{aligned}\frac{2(x-1)}{6}-\frac{3(x+2)}{6} &= \frac{2(x-1)-3(x+2)}{6}\\ &= \frac{2x-2-3x\enclose{circle}{-6}}{6}\end{aligned}$$

and $$\begin{aligned}\frac{5(1-x)}{8}-\frac{2(x-1)}{8} &= \frac{5(1-x)-2(x-1)}{8}\\ &= \frac{5-5x-2x\enclose{circle}{+2}}{8}\end{aligned}$$

- A common denominator can be a product of two algebraic expressions.

For example: $\dfrac{2}{x+3}+\dfrac{3}{x-1}=\dfrac{2(x-1)}{(x+3)(x-1)}+\dfrac{3(x+3)}{(x+3)(x-1)}$

$$=\frac{2x-2+3x+9}{(x+3)(x-1)}$$

$$=\frac{5x+7}{(x+3)(x-1)}$$

BUILDING UNDERSTANDING

① State the result of expanding the following.

a $-2(x+3)$ **b** $-7(2+3x)$ **c** $-3(x-1)$ **d** $-10(3-2x)$

② State the LCD for these pairs of fractions.

a $\frac{1}{3}, \frac{5}{9}$ **b** $-\frac{5}{2x}, \frac{3}{2}$ **c** $\frac{3}{x}, \frac{5}{x^2}$ **d** $\frac{7}{x-2}, \frac{3}{x+3}$

③ Simplify:

a $2(x+1)-3(x+2)$ **b** $5(x+2)-2(x-3)$ **c** $4-2(x-1)$

Example 19 Simplifying with more complex numerators

Simplify:

a $\dfrac{x-1}{3}-\dfrac{x+4}{5}$

b $\dfrac{2x-3}{6}-\dfrac{3-x}{5}$

SOLUTION

a $\dfrac{x-1}{3}-\dfrac{x+4}{5}=\dfrac{5(x-1)}{15}-\dfrac{3(x+4)}{15}$

$$=\frac{5x-5-3x-12}{15}$$

$$=\frac{2x-17}{15}$$

b $\dfrac{2x-3}{6}-\dfrac{3-x}{5}=\dfrac{5(2x-3)}{30}-\dfrac{6(3-x)}{30}$

$$=\frac{10x-15-18+6x}{30}$$

$$=\frac{16x-33}{30}$$

EXPLANATION

The LCD for 3 and 5 is 15. Insert brackets around each numerator when multiplying. (Note: $-3(x+4)=-3x-12$ not $-3x+12$.)

Determine the LCD and express as equivalent fractions. Insert brackets.
Expand the brackets, recalling that $-6\times(-x)=6x$, and then simplify the numerator.

Now you try

Simplify:

a $\dfrac{x-4}{2}-\dfrac{x+2}{3}$

b $\dfrac{2x+1}{4}-\dfrac{2-x}{3}$

Example 20 Simplifying with more complex denominators

Simplify:

a $\dfrac{4}{x+1} + \dfrac{3}{x-2}$

b $\dfrac{3}{(x-1)^2} - \dfrac{2}{x-1}$

SOLUTION

a

$$\begin{aligned}&\frac{4}{x+1} + \frac{3}{x-2}\\ &= \frac{4(x-2)}{(x+1)(x-2)} + \frac{3(x+1)}{(x+1)(x-2)}\\ &= \frac{4x-8+3x+3}{(x+1)(x-2)}\\ &= \frac{7x-5}{(x+1)(x-2)}\end{aligned}$$

b

$$\begin{aligned}\frac{3}{(x-1)^2} - \frac{2}{x-1} &= \frac{3}{(x-1)^2} - \frac{2(x-1)}{(x-1)^2}\\ &= \frac{3-2x+2}{(x-1)^2}\\ &= \frac{5-2x}{(x-1)^2}\end{aligned}$$

EXPLANATION

a The LCD for $(x+1)$ and $(x-2)$ is $(x+1)(x-2)$. Rewrite each fraction as an equivalent fraction with this denominator, and then add numerators.

b Just as the LCD for 3^2 and 3 is 3^2, the LCD for $(x-1)^2$ and $x-1$ is $(x-1)^2$.

Remember that $-2(x-1) = -2x+2$.

Now you try

Simplify:

a $\dfrac{3}{x+2} + \dfrac{2}{x-3}$

b $\dfrac{5}{(x-2)^2} - \dfrac{3}{x-2}$

Exercise 8J

FLUENCY	1, 2–5(1/2)	2–5(1/2)	2–5(1/3)

1 Simplify:

Example 19a

a **i** $\dfrac{x-2}{4} - \dfrac{x+3}{5}$ **ii** $\dfrac{x+1}{2} - \dfrac{x-5}{5}$

Example 19b

b **i** $\dfrac{3x+2}{7} - \dfrac{4-x}{2}$ **ii** $\dfrac{2x+5}{4} - \dfrac{x-2}{3}$

Example 19a

2 Simplify:

a $\dfrac{x+3}{4} - \dfrac{x+2}{3}$ **b** $\dfrac{x-1}{3} - \dfrac{x+3}{5}$ **c** $\dfrac{x-4}{3} - \dfrac{x+1}{6}$

d $\dfrac{3-x}{5} - \dfrac{x+4}{2}$ **e** $\dfrac{5x-1}{4} - \dfrac{2+x}{8}$ **f** $\dfrac{3x+2}{14} - \dfrac{x+4}{4}$

g $\dfrac{1+3x}{4} - \dfrac{2x+3}{6}$ **h** $\dfrac{2-x}{5} - \dfrac{3x+1}{3}$ **i** $\dfrac{2x-3}{6} - \dfrac{4+x}{15}$

Example 19b

3 Simplify:

a $\frac{x+5}{3}-\frac{x-1}{2}$ **b** $\frac{x-4}{5}-\frac{x-6}{7}$ **c** $\frac{3x-7}{4}-\frac{x-1}{2}$

d $\frac{5x-9}{7}-\frac{2-x}{3}$ **e** $\frac{3x+2}{4}-\frac{5-x}{10}$ **f** $\frac{9-4x}{6}-\frac{2-x}{8}$

g $\frac{4x+3}{3}-\frac{5-2x}{9}$ **h** $\frac{2x-1}{4}-\frac{1-3x}{14}$ **i** $\frac{3x-2}{8}-\frac{4x-3}{7}$

Example 20a

4 Simplify:

a $\frac{3}{x-1}+\frac{4}{x+1}$ **b** $\frac{5}{x+4}+\frac{2}{x-3}$ **c** $\frac{3}{x-2}+\frac{4}{x+3}$

d $\frac{3}{x-4}+\frac{2}{x+7}$ **e** $\frac{7}{x+2}-\frac{3}{x+3}$ **f** $\frac{3}{x+4}-\frac{2}{x-6}$

g $\frac{-1}{x+5}+\frac{2}{x+1}$ **h** $\frac{-2}{x-3}-\frac{4}{x-2}$ **i** $\frac{3}{x-5}-\frac{5}{x-6}$

Example 20b

5 Simplify:

a $\frac{4}{(x+1)^2}-\frac{3}{x+1}$ **b** $\frac{2}{(x+3)^2}-\frac{4}{x+3}$ **c** $\frac{3}{x-2}+\frac{4}{(x-2)^2}$

d $\frac{-2}{x-5}+\frac{8}{(x-5)^2}$ **e** $\frac{-1}{x-6}+\frac{3}{(x-6)^2}$ **f** $\frac{2}{(x-4)^2}-\frac{3}{x-4}$

g $\frac{5}{(2x+1)^2}+\frac{2}{2x+1}$ **h** $\frac{9}{(3x+2)^2}-\frac{4}{3x+2}$ **i** $\frac{4}{(1-4x)^2}-\frac{5}{1-4x}$

PROBLEM-SOLVING 6(1/2) 6–7(1/2) 6–7(1/2)

6 Simplify:

a $\frac{3x}{(x-1)^2}+\frac{2}{x-1}$ **b** $\frac{3x+2}{3x}+\frac{7}{12}$ **c** $\frac{2x-1}{4}+\frac{2-3x}{10x}$

d $\frac{2x}{x-5}-\frac{x}{x+1}$ **e** $\frac{3}{4-x}-\frac{2x}{x-1}$ **f** $\frac{5x+1}{(x-3)^2}+\frac{x}{x-3}$

g $\frac{3x-7}{(x-2)^2}-\frac{5}{x-2}$ **h** $\frac{-7x}{2x+1}+\frac{3x}{x+2}$ **i** $\frac{x}{x+1}-\frac{5x+1}{(x+1)^2}$

7 Simplify:

a $\frac{1}{(x+3)(x+4)}+\frac{2}{(x+4)(x+5)}$ **b** $\frac{3}{(x+1)(x+2)}-\frac{5}{(x+1)(x+4)}$

c $\frac{4}{(x-1)(x-3)}-\frac{6}{(x-1)(2-x)}$ **d** $\frac{5x}{(x+1)(x-5)}-\frac{2}{x-5}$

e $\frac{3}{x-4}+\frac{8x}{(x-4)(3-2x)}$ **f** $\frac{3x}{(x+4)(2x-1)}-\frac{x}{(x+4)(3x+2)}$

REASONING	8	8, 9(1/2)	9(1/2)

8 One of the most common errors made when subtracting algebraic fractions is hidden in this working shown below.

$$\begin{aligned}\frac{7x}{2}-\frac{x-2}{5} &= \frac{35x}{10}-\frac{2(x-2)}{10}\\ &= \frac{35x-2x-4}{10}\\ &= \frac{33x-4}{10}\end{aligned}$$

a What is the error and in which step is it made?
b By correcting the error how does the answer change?

9 Use the fact that $a-b=-1(b-a)$ to help simplify these.

a $\frac{3}{1-x}-\frac{2}{x-1}$
b $\frac{4x}{5-x}+\frac{3}{x-5}$
c $\frac{2}{7x-3}-\frac{7}{3-7x}$
d $\frac{1}{4-3x}+\frac{2x}{3x-4}$
e $\frac{-3x}{5-3x}-\frac{5}{3x-5}$
f $\frac{4}{x-6}+\frac{4}{6-x}$

ENRICHMENT: Factorise first	–	–	10(1/2)

10 Factorising a denominator before further simplification is a useful step. Simplify these by first factorising the denominators if possible.

a $\frac{3}{x+2}+\frac{5}{2x+4}$
b $\frac{7}{3x-3}-\frac{2}{x-1}$
c $\frac{3}{8x-4}-\frac{5}{1-2x}$
d $\frac{4}{x^2-9}-\frac{3}{x+3}$
e $\frac{5}{2x+4}+\frac{2}{x^2-4}$
f $\frac{10}{3x-4}-\frac{7}{9x^2-16}$
g $\frac{7}{x^2+7x+12}+\frac{2}{x^2-2x-15}$
h $\frac{3}{(x+1)^2-4}-\frac{2}{x^2+6x+9}$
i $\frac{3}{x^2-7x+10}-\frac{2}{10-5x}$
j $\frac{1}{x^2+x}-\frac{1}{x^2-x}$

8K Equations with algebraic fractions EXTENDING

Learning intentions

- To know how to solve equations with algebraic fractions by multiplying both sides by the lowest common denominator or by expressing the fractions with a common denominator

For equations with more than one fraction, it is often best to try to simplify the equation by dealing with all the denominators at once. This involves finding and multiplying both sides by the lowest common denominator.

The study of optometry includes solving algebraic fractional equations that model the path of light through spectacle lenses.

LESSON STARTER Why use the LCD?

For this equation follow each instruction.

$$\frac{x+1}{3}+\frac{x}{4}=1$$

- Multiply every term in the equation by 3. What effect does this have on the fractions on the left-hand side?
- Starting with the original equation, multiply every term in the equation by 4. What effect does this have on the fractions on the left-hand side?
- Starting with the original equation, multiply every term in the equation by 12 and simplify.

Which instruction above does the best job in simplifying the algebraic fractions? Why?

KEY IDEAS

- For equations with more than one fraction, multiply both sides by the **lowest common denominator (LCD)**.
  - Multiply every term on both sides, not just the fractions.
  - Simplify the fractions and solve the equation using the methods learnt earlier.
- Alternatively, express each fraction using the same denominator, and then simplify by adding or subtracting.

ISBN 978-1-108-77288-4

BUILDING UNDERSTANDING

1 State the lowest common denominator of all the fractions in each equation.

a $\frac{x}{3} - \frac{2x}{5} = 1$

b $\frac{x+1}{3} - \frac{x}{6} = 5$

c $\frac{2x-1}{7} + \frac{5x+2}{4} = 6$

2 Simplify the fractions by cancelling.

a $\frac{12x}{3}$

b $\frac{4(x+3)}{2}$

c $\frac{15x}{5x}$

d $\frac{36(x-7)(x-1)}{9(x-7)}$

3 Describe a method for solving the following equations.

a $\frac{2x+3}{5} = 3$

b $\frac{5}{x} = 2$

Example 21 Solving equations involving algebraic fractions

Solve each of the following equations.

a $\frac{2x}{3} + \frac{x}{2} = 7$

b $\frac{x-2}{5} - \frac{x-1}{3} = 1$

SOLUTION

a

$$\frac{2x}{3} + \frac{x}{2} = 7$$

$$\begin{aligned} \frac{2x}{\cancel{3}_1} \times \cancel{6}^2 + \frac{x}{\cancel{2}_1} \times \cancel{6}^3 &= 7 \times 6 \\ 4x + 3x &= 42 \\ 7x &= 42 \\ x &= 6 \end{aligned}$$

EXPLANATION

Multiply each term by the LCD (LCD of 3 and 2 is 6) and cancel.

Simplify and solve for x.

OR

$$\begin{aligned} \frac{2x}{3} + \frac{x}{2} &= 7 \\ \frac{4x}{6} + \frac{3x}{6} &= 7 \\ \frac{7x}{6} &= 7 \\ 7x &= 42 \\ x &= 6 \end{aligned}$$

Alternatively, write each fraction on the left-hand side using the LCD = 6.

Simplify by adding the numerators and solve the remaining equation.

b

$$\frac{x-2}{5} - \frac{x-1}{3} = 1$$

$$\begin{aligned} \frac{\cancel{15}^3(x-2)}{\cancel{5}_1} - \frac{\cancel{15}^5(x-1)}{\cancel{3}_1} &= 1 \times 15 \\ 3(x-2) - 5(x-1) &= 15 \\ 3x - 6 - 5x + 5 &= 15 \\ -2x - 1 &= 15 \\ -2x &= 16 \\ x &= -8 \end{aligned}$$

Multiply each term on both sides by 15 (LCD of 3 and 5 is 15) and cancel.

Expand the brackets and simplify by combining like terms. (Note: $-5(x-1) = -5x + 5$ not $-5x - 5$.)

Alternatively, write each fraction using the LCD = 15 and then combine the numerators and solve $\frac{3(x-2)}{15} - \frac{5(x-1)}{15} = 1$.

Continued on next page

Now you try

Solve each of the following equations.

a $\frac{x}{4} + \frac{2x}{3} = 11$

b $\frac{x+3}{5} - \frac{x-2}{2} = -2$

Example 22 Solving equations that include algebraic denominators

Solve each of the following equations.

a $\frac{5}{2x} - \frac{4}{3x} = 2$

b $\frac{3}{x+1} = \frac{2}{x+4}$

SOLUTION

a

$$\frac{5}{2x} - \frac{4}{3x} = 2$$

$$\frac{5}{\cancel{2x}_1} \times \cancel{6x}^3 - \frac{4}{\cancel{3x}_1} \times \cancel{6x}^2 = 2 \times 6x$$

$$\begin{aligned} 15 - 8 &= 12x \\ 7 &= 12x \\ x &= \frac{7}{12} \end{aligned}$$

b

$$\frac{3}{x+1} = \frac{2}{x+4}$$

$$\frac{3\cancel{(x+1)}(x+4)}{\cancel{(x+1)}} = \frac{2(x+1)\cancel{(x+4)}}{\cancel{(x+4)}}$$

$$\begin{aligned} 3(x+4) &= 2(x+1) \\ 3x + 12 &= 2x + 2 \\ x + 12 &= 2 \\ x &= -10 \end{aligned}$$

OR

$$\begin{aligned} \frac{3}{x+1} &= \frac{2}{x+4} \\ 3(x+4) &= 2(x+1) \\ 3x + 12 &= 2x + 2 \\ x + 12 &= 2 \\ x &= -10 \end{aligned}$$

EXPLANATION

a

LCD of $2x$ and $3x$ is $6x$.

Multiply each term by $6x$. Cancel and simplify.
Solve for x, leaving the answer in fraction form.
(Alternative solution: $\frac{15}{6x} - \frac{8}{6x} = 2$)

b

Multiply each term by the common denominator $(x+1)(x+4)$.

Expand the brackets.
Subtract $2x$ from both sides to gather x terms on one side, and then subtract 12 from both sides.

Since each side is a single fraction you can 'cross-multiply': $\frac{3}{x+1} \times \frac{2}{x+4}$

This gives the same result as above.

Now you try

Solve each of the following equations.

a $\frac{3}{4x} - \frac{2}{5x} = 2$

b $\frac{3}{x+2} = \frac{2}{x-1}$

ISBN 978-1-108-77288-4

Updated September 2021

Exercise 8K

FLUENCY	1, 2–4($\frac{1}{2}$)	2–5($\frac{1}{3}$), 6($\frac{1}{2}$)	2–6($\frac{1}{3}$)

1 Solve each of the following equations.

Example 21a

a **i** $\frac{x}{7} + \frac{x}{3} = 20$ **ii** $\frac{3x}{2} + \frac{x}{4} = -7$

Example 21b

b **i** $\frac{x+1}{4} - \frac{x-3}{3} = 2$ **ii** $\frac{x-4}{2} - \frac{x-2}{6} = 3$

Example 21a

2 Solve each of the following equations.

a $\frac{x}{2} + \frac{x}{5} = 7$ **b** $\frac{x}{2} + \frac{x}{3} = 10$ **c** $\frac{y}{3} + \frac{y}{4} = 14$

d $\frac{x}{2} - \frac{3x}{5} = -1$ **e** $\frac{5m}{3} - \frac{m}{2} = 1$ **f** $\frac{3a}{5} - \frac{a}{3} = 2$

g $\frac{3x}{4} - \frac{5x}{2} = 14$ **h** $\frac{8a}{3} - \frac{2a}{5} = 34$ **i** $\frac{7b}{2} + \frac{b}{4} = 15$

Example 21b

3 Solve each of the following equations.

a $\frac{x-1}{2} + \frac{x+2}{3} = 11$ **b** $\frac{b+3}{2} + \frac{b-4}{3} = 1$ **c** $\frac{n+2}{3} + \frac{n-2}{2} = 1$

d $\frac{a+1}{5} - \frac{a+1}{6} = 2$ **e** $\frac{x+5}{2} - \frac{x-1}{4} = 3$ **f** $\frac{x+3}{2} - \frac{x+1}{3} = 2$

g $\frac{m+4}{3} - \frac{m-4}{4} = 3$ **h** $\frac{2a-8}{2} + \frac{a+7}{6} = 1$ **i** $\frac{2y-1}{4} - \frac{y-2}{6} = -1$

4 Solve each of the following equations.

a $\frac{x+1}{2} = \frac{x}{3}$ **b** $\frac{x-2}{3} = \frac{x}{2}$ **c** $\frac{n+3}{4} = \frac{n-1}{2}$

d $\frac{a+2}{3} = \frac{a+1}{2}$ **e** $\frac{3+y}{2} = \frac{2-y}{3}$ **f** $\frac{2m+4}{4} = \frac{m+6}{3}$

Example 22a

5 Solve each of the following equations.

a $\frac{3}{4x} - \frac{1}{2x} = 4$ **b** $\frac{2}{3x} - \frac{1}{2x} = 2$ **c** $\frac{4}{2m} - \frac{2}{5m} = 3$

d $\frac{1}{2x} - \frac{1}{4x} = 9$ **e** $\frac{1}{2b} + \frac{1}{b} = 2$ **f** $\frac{1}{2y} + \frac{1}{3y} = 4$

g $\frac{1}{3x} + \frac{1}{2x} = 2$ **h** $\frac{2}{3x} - \frac{1}{x} = 2$ **i** $\frac{7}{2a} - \frac{2}{3a} = 1$

Example 22b

6 Solve each of the following equations.

a $\frac{3}{x+1} = \frac{1}{x+2}$ **b** $\frac{2}{x+3} = \frac{3}{x+2}$

c $\frac{2}{x+5} = \frac{3}{x-2}$ **d** $\frac{1}{x-3} = \frac{1}{2x+1}$

e $\frac{2}{x-1} = \frac{1}{2x+1}$ **f** $\frac{1}{x-2} = \frac{2}{3x+2}$

ISBN 978-1-108-77288-4

PROBLEM-SOLVING	7, 8	7, 8, 9($1/2$)	8, 9($1/2$), 10

7 Half of a number (x) plus one-third of twice the same number is equal to 4.

- **a** Write an equation describing the situation.
- **b** Solve the equation to find the number.

8 Molly and Billy each have the same number of computer games (x computer games each). Hazel, who doesn't have any games, takes one-third of Molly's computer games and a quarter of Billy's computer games to give her a total of 77 computer games.

- **a** Write an equation describing the total number of computer games for Hazel.
- **b** Solve the equation to find how many computer games Molly and Billy each had.

9 Use your combined knowledge of all the methods learnt earlier to solve these equations with algebraic fractions.

- **a** $\dfrac{2x+3}{1-x} = 4$
- **b** $\dfrac{5x+2}{x+2} = 3$
- **c** $\dfrac{3x-2}{x-1} = 2$
- **d** $\dfrac{2x}{3} + \dfrac{x-1}{4} = 2x - 1$
- **e** $\dfrac{3}{x^2} - \dfrac{2}{x} = \dfrac{5}{x}$
- **f** $\dfrac{1-3x}{x^2} + \dfrac{3}{2x} = \dfrac{4}{x}$
- **g** $\dfrac{x-1}{2} + \dfrac{3x-2}{4} = \dfrac{2x}{3}$
- **h** $\dfrac{4x+1}{3} - \dfrac{x-3}{6} = \dfrac{x+5}{6}$
- **i** $\dfrac{1}{x+2} - \dfrac{2}{x-3} = \dfrac{5}{(x+2)(x-3)}$

10 One number is twice another number and the sum of their reciprocals is $\dfrac{3}{10}$.

- **a** Write an equation describing the situation.
- **b** Solve the equation to find the two numbers.

REASONING	11, 12	11, 12	12, 13($1/2$)

11 A common error when solving equations with algebraic fractions is made in this working. Find the error and explain how to avoid it.

$$\begin{aligned} \frac{3x-1}{4} + 2x &= \frac{x}{3} \quad \text{(LCD = 12)} \\ \frac{12(3x-1)}{4} + 2x &= \frac{12x}{3} \\ 3(3x-1) + 2x &= 4x \\ 9x - 3 + 2x &= 4x \\ 7x &= 3 \\ x &= \frac{3}{7} \end{aligned}$$

12 Another common error is made in this working. Find and explain how to avoid this error.

$$\begin{aligned} \frac{x}{2} - \frac{2x-1}{3} &= 1 \quad \text{(LCD = 6)} \\ \frac{6x}{2} - \frac{6(2x-1)}{3} &= 6 \\ 3x - 2(2x-1) &= 6 \\ 3x - 4x - 2 &= 6 \\ -x &= 8 \\ x &= -8 \end{aligned}$$

13 Some equations with decimals can be solved by first multiplying by a power of 10. Here is an example.

$$\begin{aligned} 0.8x - 1.2 &= 2.5 \quad \text{(Multiply both sides by 10 to remove all decimals.)} \\ 8x - 12 &= 25 \\ 8x &= 37 \\ x &= \frac{37}{8} \end{aligned}$$

Solve these decimal equations using the same idea. For parts **d–f** you will need to multiply by 100.

a $0.4x + 1.4 = 3.2$

b $0.3x - 1.3 = 0.4$

c $0.5 - 0.2x = 0.2$

d $1.31x - 1.8 = 2.13$

e $0.24x + 0.1 = 3.7$

f $2 - 3.25x = 8.5$

ENRICHMENT: Literal equations

– – 14(½)

14 Solve each of the following equations for x in terms of the other pronumerals. (*Hint:* You may need to use factorisation to make x the subject in your working.)

a $\frac{x}{a} - \frac{x}{2a} = b$

b $\frac{ax}{b} - \frac{cx}{2} = d$

c $\frac{x-a}{b} = \frac{x}{c}$

d $\frac{x+a}{b} = \frac{d+e}{c}$

e $\frac{ax+b}{4} = \frac{x+c}{3}$

f $\frac{x+a}{3b} + \frac{x-a}{2b} = 1$

g $\frac{2a-b}{a} + \frac{a}{x} = a$

h $\frac{1}{a} - \frac{1}{x} = \frac{1}{c}$

i $\frac{a}{x} = \frac{b}{c}$

j $\frac{a}{x} + b = \frac{c}{x}$

k $\frac{ax-b}{x-b} = c$

l $\frac{cx+b}{x+a} = d$

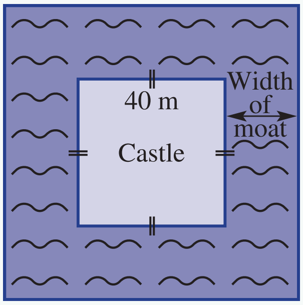
m $\frac{2a+x}{a} = b$

n $\frac{1}{x-a} = \frac{1}{ax+b}$

o $\frac{a}{b} - \frac{a}{a+x} = 1$

Modelling

Square castle moats



Moats were constructed around castles to help protect its occupants from invaders. A square castle of side length 40 metres is to be surrounded by a square moat of a certain width as shown.

Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate.

Preliminary task

a Find the area of land occupied by the 40-metre square castle.
b If the width of the moat is 10 metres, find the surface area of the water in the moat.
c Find the outside perimeter of the moat if the width of the moat is 8.5 metres.

Modelling task

Formulate

a The problem is to determine a width for the moat so that the surface area of water is at least 10 times the area of the castle. Write down all the relevant information that will help solve this problem with the aid of a diagram.
b If the width of the moat is x metres, find expressions in terms of x for the following:
  i the perimeter of the outside of the moat
  ii the total area occupied by the moat and the castle.

Solve

c Use your expressions from part **b ii** to show that the area occupied by the moat, not including the castle, can be written in the form $4x(40 + x)$.
d Using the expression from part **c**, find the area occupied by the moat for the following values of x.
  i 5 **ii** 8 **iii** 12
e Determine the smallest integer value of x such that the area of the moat is at least four times the area of the castle.

Evaluate and verify

f Assuming that the lord of the castle wants the area of the moat to be as close as possible to four times the area of the castle, investigate if this is possible using non-integer values of x.
g Instead, the lord wants to build a square castle of side length x metres inside a square lake of side length 50 metres.
  i Show that the remaining lake area forming the moat is given by the expression $(50 + x)(50 - x)$.
  ii Use a trial and error approach to determine the width of the castle that delivers a moat area which is four times the area of the castle. Give your answer correct to one decimal place.

Communicate

h Summarise your results and describe any key findings.

Extension questions

A square castle of width a metres is surrounded by a moat of width x metres as per the castle in the introduction of this task.

a Find an expression for the area of the moat in terms of a and x.
b Find the value of a such that a moat of width 10 metres occupies four times the area of the castle. Give your answer correct to one decimal place. Illustrate the solution with a drawing.

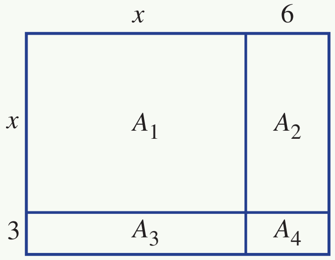
Expanding quadratics using areas

Consider the expansion of the quadratic $(x+3)(x+6)$. This can be represented by finding the area of the rectangle shown.

$$\begin{aligned}\text{Total area} &= A_1 + A_2 + A_3 + A_4\\ &= x^2 + 6x + 3x + 18\end{aligned}$$

Therefore:

$$(x+3)(x+6) = x^2 + 9x + 18$$



Expanding with positive signs

- **a** Draw a diagram and calculate the area to determine the expansion of the following quadratics.
  - **i** $(x+4)(x+5)$
  - **ii** $(x+7)(x+8)$
  - **iii** $(x+3)^2$
  - **iv** $(x+5)^2$
- **b** Using the same technique establish the rule for expanding $(a+b)^2$.

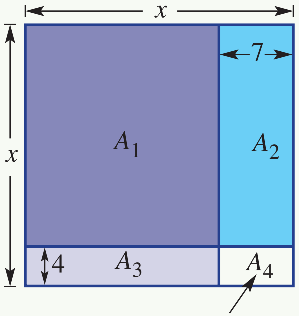
Expanding with negative signs

Consider the expansion of $(x-4)(x-7)$.

$$\begin{aligned}\text{Area required} &= \text{total area} - (A_2 + A_3 + A_4)\\ &= x^2 - [(A_2 + A_4) + (A_3 + A_4) - A_4]\\ &= x^2 - (7x + 4x - 28)\\ &= x^2 - 11x + 28\end{aligned}$$

Therefore:

$$(x-4)(x-7) = x^2 - 11x + 28$$



This area is counted twice when we add $7x + 4x$.

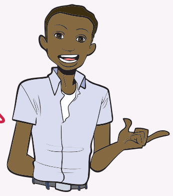
- **a** Draw a diagram and calculate the area to determine the expansion of the following quadratics.
  - **i** $(x-3)(x-5)$
  - **ii** $(x-6)(x-4)$
  - **iii** $(x-4)^2$
  - **iv** $(x-2)^2$
- **b** Using the same technique, establish the rule for expanding $(a-b)^2$.

Difference of perfect squares

Using a diagram to represent $(a-b)(a+b)$, determine the appropriate area and establish a rule for the expansion of $(a-b)(a+b)$.

ISBN 978-1-108-77288-4

Problems and challenges



Up for a challenge? If you get stuck on a question, check out the 'Working with unfamiliar problems' poster at the end of the book to help you.

1 **a** The difference between the squares of two consecutive numbers is 97.
What are the two numbers?

b The difference between the squares of two consecutive odd numbers is 136.
What are the two numbers?

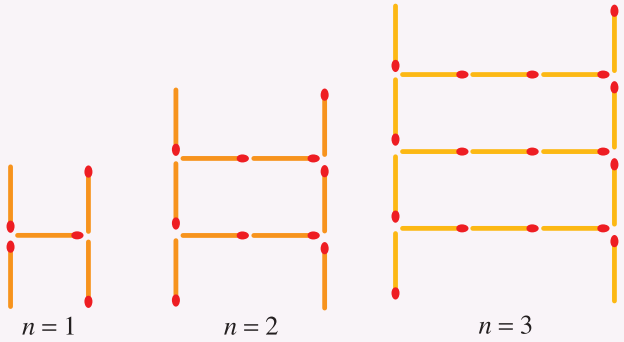
c The difference between the squares of two consecutive multiples of 3 is 81.
What are the two numbers?

2 **a** If $x^2 + y^2 = 6$ and $(x + y)^2 = 36$, find the value of xy.

b If $x + y = 10$ and $xy = 2$, find the value of $\frac{1}{x} + \frac{1}{y}$.

3 Find the values of the different digits a, b, c and d if the four-digit number $abcd$ is such that $abcd \times 4 = dcba$.

4 **a** Find the quadratic rule that relates the width, n, to the number of matches in the pattern below.



b Draw a possible pattern for these rules.

i $n^2 + 3$

ii $n(n - 1)$

5 Factorise $n^2 - 1$ and use the factorised form to explain why when n is prime and greater than 3, $n^2 - 1$ is:

a divisible by 4

b divisible by 3

c thus divisible by 12.

6 Prove that this expression is equal to 1.

$$\frac{2x^2 - 8}{5x^2 - 5} \div \frac{x - 2}{5x - 5} \div \frac{2x^2 - 10x - 28}{x^2 - 6x - 7}$$

7 Prove that $4x^2 - 4x + 1 \geqslant 0$ for all x.

8 Ryan and Susan competed in a race over 4 km. Ryan ran at a constant speed. Sophie, however, ran the first 2 km at a speed 1 km/h more than Ryan and ran the second 2 km at a speed 1 km/h less than Ryan. Who won the race?

Algebraic techniques

Monic quadratic trinomials (Ext)

These are of the form:

$x^2 + bx + c$

Require two numbers that multiply to c and add to b

e.g. $x^2 - 7x - 18 = (x - 9)(x + 2)$

since $-9 \times 2 = -18$

and $-9 + 2 = -7$

DOPS

Difference of perfect squares

$a^2 - b^2 = (a - b)(a + b)$

e.g. $x^2 - 16 = x^2 - 4^2$

$= (x - 4)(x + 4)$

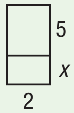
e.g. $4x^2 - 9 = (2x)^2 - 3^2$

$= (2x - 3)(2x + 3)$

Expansion

The process of removing brackets.

e.g. $2(x + 5) = 2x + 10$



$(x + 3)(2x - 4) = x(2x - 4) + 3(2x - 4)$

$= 2x^2 - 4x + 6x - 12$

$= 2x^2 + 2x - 12$

Special cases

DOPS (difference of perfect squares)

$(a - b)(a + b) = a^2 - b^2$

e.g. $(x - 5)(x + 5) = x^2 + \cancel{5x} - \cancel{5x} - 25$

$= x^2 - 25$

Perfect squares

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

e.g. $(2x + 3)^2 = (2x + 3)(2x + 3)$

$= 4x^2 + 6x + 6x + 9$

$= 4x^2 + 12x + 9$

Factorisation

The reverse process of expansion. Always remove the highest common factor first.

e.g. $2ab + 8b$ HCF $= 2b$

$= 2b(a + 4)$

e.g. $3(x + 2) + y(x + 2)$ HCF $= x + 2$

$= (x + 2)(3 + y)$

Algebraic fractions

These involve algebraic expressions in the numerator and/or denominator.

e.g. $\frac{2x}{x - 7}, \frac{3}{5x}, \frac{7}{(x - 1)^2}$

Add/subtract

Find the lowest common denominator (LCD) before applying operation.

e.g. $\frac{3x}{4} + \frac{x - 1}{3}$

$= \frac{9x}{12} + \frac{4(x - 1)}{12}$

$= \frac{9x + 4x - 4}{12}$

$= \frac{13x - 4}{12}$

By grouping

If there are four terms, it may be possible to group into two binomial terms.

e.g. $5x + 10 - 3x^2 - 6x$

$= 5(x + 2) - 3x(x + 2)$

$= (x + 2)(5 - 3x)$

Multiply/divide

To divide, multiply by the reciprocal. Factorise all expressions, cancel and then multiply.

e.g. $\frac{3}{x^2 - 9} \div \frac{15}{2x + 6}$

$= \frac{\cancel{3}^{1}}{(x - 3)\cancel{(x + 3)}_{1}} \times \frac{2\cancel{(x + 3)}^{1}}{\cancel{15}_{5}}$

$= \frac{2}{5(x - 3)}$

Trinomials of the form $ax^2 + bx + c$ (Ext)

Can be factorised using grouping also:

e.g. $3x^2 + 7x - 6$ $3 \times (-6) = -18$

$= 3x^2 + 9x - 2x - 6$ Use 9 and -2

$= 3x(x + 3) - 2(x + 3)$ since

$= (x + 3)(3x - 2)$ $9 \times (-2) = -18$

and $9 + (-2) = 7$

Solving equations with algebraic fractions (Ext)

Find the lowest common denominator (LCD) and multiply every term by the LCD or express each fraction using the LCD.

e.g. LCD of $2x$ and $3x$ is $6x$

LCD of $(x - 1)$ and $(x + 3)$ is $(x - 1)(x + 3)$

LCD of $2x$ and x^2 is $2x^2$

Chapter checklist: Success criteria

			✔
8A	**1. I can expand binomial products.** e.g. Expand $(2x-3)(x+4)$.		
8B	**2. I can expand a perfect square and form a difference of perfect squares.** e.g. Expand $(x-6)^2$ and $(4x-5)(4x+5)$.		
8C	**3. I can factorise using the HCF.** e.g. Factorise fully $12x^2-8x$ and $x(2x+3)+5(2x+3)$.		
8D	**4. I can factorise a DOPS.** e.g. Factorise x^2-100 and $4a^2-9$.		
8E	**5. I can factorise by grouping.** e.g. Use the method of grouping to factorise $x^2+6x-3x-18$.		
8E	**6. I can factorise using grouping by first rearranging.** e.g. Factorise $2x^2-9-6x+3x$.		
8F	**7. I can factorise a quadratic trinomial.** e.g. Factorise $x^2+4x-21$.	Ext	
8D/F	**8. I can factorise with trinomials or DOPS by first taking out a common factor.** e.g. Fully factorise $2x^2-18$ and $3x^2-21x+36$.	Ext	
8G	**9. I can factorise trinomials of the form ax^2+bx+c.** e.g. Factorise $3x^2+10x+9$ and $4x^2-4x-3$.	Ext	
8H	**10. I can simplify algebraic fractions by factorising and cancelling.** e.g. Simplify $\frac{3x+6}{8x+16}$.		
8H	**11. I can multiply and divide algebraic fractions by first cancelling.** e.g. Simplify $\frac{2(x+5)}{x+4}\times\frac{5(x+4)(x-1)}{6(x+5)}$.		
8H	**12. I can multiply and divide algebraic fractions by factorising.** e.g. Simplify $\frac{x^2-16}{x+4}\div\frac{6x-24}{3}$.		
8I	**13. I can add and subtract algebraic fractions with numeric denominators.** e.g. Simplify $\frac{x+4}{2}+\frac{x-2}{3}$.		
8I	**14. I can add and subtract algebraic fractions with algebraic terms in the denominator.** e.g. Simplify $\frac{4}{x}-\frac{2}{3x}$.		
8J	**15. I can subtract algebraic fractions.** e.g. Simplify $\frac{2x+2}{3}-\frac{x-5}{4}$.	Ext	
8J	**16. I can add or subtract algebraic fractions with binomial denominators.** e.g. Simplify $\frac{3}{x+2}+\frac{2}{x-4}$.	Ext	
8K	**17. I can solve equations involving algebraic fractions.** e.g. Solve $\frac{3x}{2}+\frac{x}{5}=3$ and $\frac{2}{x+1}=\frac{6}{x-5}$.	Ext	

Short-answer questions

8A

1 Expand the following binomial products.

a $(x-3)(x+4)$

b $(x-7)(x-2)$

c $(2x-3)(3x+2)$

d $3(x-1)(3x+4)$

8B

2 Expand the following.

a $(x+3)^2$

b $(x-4)^2$

c $(3x-2)^2$

d $(x-5)(x+5)$

e $(7-x)(7+x)$

f $(11x-4)(11x+4)$

8C

3 Write the following in fully factorised form by removing the highest common factor.

a $4a+12b$

b $6x-9x^2$

c $-5x^2y-10xy$

d $3(x-7)+x(x-7)$

e $x(2x+1)-(2x+1)$

f $(x-2)^2-4(x-2)$

8D

4 Factorise the following DOPS.

a x^2-100

b $3x^2-48$

c $25x^2-y^2$

d $49-9x^2$

e $(x-3)^2-81$

f $1-x^2$

8E

5 Factorise the following by grouping.

a $x^2-3x+6x-18$

b $4x^2+10x-2x-5$

c $3x-8b+2bx-12$

8F/G

Ext

6 Factorise the following trinomials.

a $x^2+8x+15$

b $x^2-3x-18$

c x^2-7x+6

d $3x^2+15x-42$

e $2x^2+16x+32$

f $5x^2+17x+6$

g $4x^2-4x-3$

h $6x^2-17x+12$

8H

7 Simplify the following.

a $\dfrac{3x+12}{3}$

b $\dfrac{2x-16}{3x-24}$

c $\dfrac{x^2-9}{5(x+3)}$

8H

8 Simplify the following algebraic fractions by first factorising and cancelling where possible.

a $\dfrac{3}{2x}\times\dfrac{x}{6}$

b $\dfrac{x(x-4)}{8(x+1)}\times\dfrac{4(x+1)}{x}$

c $\dfrac{x^2+3x}{3x+6}\times\dfrac{x+2}{x+3}$

d $\dfrac{2x}{5x+20}\div\dfrac{x}{x+4}$

e $\dfrac{4x^2-9}{10x^2}\div\dfrac{10x-15}{x}$

Ext **f** $\dfrac{x^2+5x+6}{x+3}\div\dfrac{x^2-4}{4x-8}$

8I/J

9 Simplify the following by first finding the lowest common denominator.

a $\dfrac{x}{4}+\dfrac{2x}{3}$

b $\dfrac{3}{4x}+\dfrac{1}{2x}$

c $\dfrac{7}{x}-\dfrac{2}{x^2}$

Ext **d** $\dfrac{x-1}{6}-\dfrac{x+3}{8}$

Ext **e** $\dfrac{3}{x+1}+\dfrac{5}{x+2}$

Ext **f** $\dfrac{7}{(x-4)^2}-\dfrac{2}{x-4}$

8K

Ext

10 Solve the following equations involving fractions.

a $\dfrac{x}{4}+\dfrac{2x}{5}=13$

b $\dfrac{4}{x}-\dfrac{2}{3x}=20$

c $\dfrac{x+3}{2}+\dfrac{x-4}{3}=6$

d $\dfrac{4}{1-x}=\dfrac{5}{x+4}$

ISBN 978-1-108-77288-4

Multiple-choice questions

8A

1 $(2x - 1)(x + 5)$ in expanded and simplified form is:

A $2x^2 + 9x - 5$ **B** $x^2 + 11x - 5$ **C** $4x^2 - 5$
D $3x^2 - 2x + 5$ **E** $2x^2 + 4x - 5$

8B

2 $(3a + 2b)^2$ is equivalent to:

A $9a^2 + 6ab + 4b^2$ **B** $9a^2 + 4b^2$ **C** $3a^2 + 6ab + 2b^2$
D $3a^2 + 12ab + 2b^2$ **E** $9a^2 + 12ab + 4b^2$

8D

3 $16x^2 - 49$ in factorised form is:

A $(4x - 7)^2$ **B** $(16x - 49)(16x + 49)$ **C** $(2x - 7)(8x + 7)$
D $(4x - 7)(4x + 7)$ **E** $4(4x^2 - 49)$

8E

4 The factorised form of $x^2 + 3x - 2x - 6$ is:

A $(x - 3)(x + 2)$ **B** $x - 2(x + 3)^2$ **C** $(x + 3)(x - 2)$
D $x - 2(x + 3)$ **E** $x(x + 3) - 2$

8F
Ext

5 If $(x - 2)$ is a factor of $x^2 + 5x - 14$ the other factor is:

A x **B** $x + 7$ **C** $x - 7$ **D** $x - 16$ **E** $x + 5$

8G
Ext

6 The factorised form of $3x^2 + 10x - 8$ is:

A $(3x + 1)(x - 8)$
B $(x - 4)(3x + 2)$
C $(3x + 2)(x + 5)$
D $(3x - 2)(x + 4)$
E $(x + 1)(3x - 8)$

8H

7 The simplified form of $\dfrac{3x + 6}{(x + 5)(x + 1)} \times \dfrac{x + 5}{x + 2}$ is:

A $\dfrac{3}{x + 1}$ **B** $\dfrac{15}{2x^2 + 5}$ **C** $x + 3$ **D** $\dfrac{5}{x + 2}$ **E** $3(x + 5)$

8I

8 $\dfrac{x + 2}{5} + \dfrac{2x - 1}{3}$ written as a single fraction is:

A $\dfrac{11x + 1}{15}$ **B** $\dfrac{11x + 9}{8}$ **C** $\dfrac{3x + 1}{8}$ **D** $\dfrac{11x + 7}{15}$ **E** $\dfrac{13x + 1}{15}$

8J
Ext

9 The LCD of $\dfrac{3x + 1}{2x}$ and $\dfrac{4}{x + 1}$ is:

A $8x$ **B** $2x(x + 1)$ **C** $(x + 1)(3x + 1)$
D $8x(3x + 1)$ **E** $x(x + 1)$

8K
Ext

10 The solution to $\dfrac{3}{1 - x} = \dfrac{4}{2x + 3}$ is:

A $x = -\dfrac{1}{2}$ **B** $x = -\dfrac{9}{11}$ **C** $x = \dfrac{1}{7}$ **D** $x = 2$ **E** $x = -\dfrac{4}{5}$

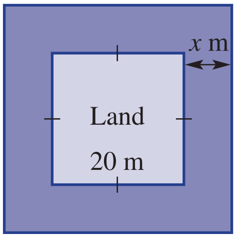
ISBN 978-1-108-77288-4

Extended-response questions

1 A pig pen for a small farm is being redesigned. It is originally a square of side length x m.

- **a** In the planning, the length is initially kept as x m and the width altered such that the area of the pen is $(x^2 + 3x)$ square metres. What is the new width?
- **b** Instead, it is determined that the original length will be increased by 1 metre and the original width will be decreased by 1 metre.
  - **i** What effect does this have on the perimeter of the pig pen compared with the original size?
  - **ii** Determine an expression in expanded form for the new area of the pig pen. How does this compare with the original area?
- **c** The final set of dimensions requires an extra 8 m of fencing to go around the pen compared with the original pen. If the length of the pen has been increased by 7 m, then the width of the pen must decrease. Find:
  - **i** the change that has been made to the width of the pen
  - **ii** the new area enclosed by the pen
  - **iii** what happens when $x = 3$.

2 The security tower for a palace is on a small square piece of land 20 m by 20 m with a moat of width x metres the whole way around it as shown.



- **a** State the area of the piece of land.
- **b** **i** Give expressions for the length and the width of the combined moat and land.
  - **ii** Find an expression, in expanded form, for the entire area occupied by the moat and the land.
- **c** If the tower occupies an area of $(x + 10)^2$ m^2, what fraction of the total area in part **b ii** is this?
- **d** Use your answers to parts **a** and **b** to give an expression for the area occupied by the moat alone, in factorised form.
- **e** Use trial and error to find the value of x such that the area of the moat alone is 500 m^2.

ISBN 978-1-108-77288-4

Updated September 2021

CHAPTER 9 Probability and statistics

Once in 100 years

Water from burst pipes, storms or floods can cause extensive damage to a person's home and contents. Different insurers offer different types of cover and differing premiums based on the calculated risk of damage. The higher the risk the higher the premiums, with some insurers offering cover for flood damage as an optional extra, while others do not offer cover at all.

After the Brisbane floods of 2011, Australian regulators introduced a standard definition of flood. All insurers have been made to adopt this definition into their policies since 19 June 2014. Flood damage, it should be noted, is different from storm damage or rainwater damage.

The Brisbane floods of 2011 resulted in a 45% increase in some people's premiums. In NSW it is predicted that 13% of properties will suffer flood damage at sometime, with 2% being of high risk.

Many people talk about the 'once in a hundred years' chance of a flood; this is misleading as it

ISBN 978-1-108-77288-4

Cambridge University Press
Updated September 2021

simply means that there is a 1% chance of this level of flood in any given year. Although unlikely, it is possible for such a flood to occur more than once in a hundred years. This level of flooding has occurred in Brisbane in 1893, 1974 and 2011.

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

In this chapter

9A Review of probability (CONSOLIDATING)
9B Venn diagrams and two-way tables
9C Using set notation
9D Using arrays for two-step experiments
9E Tree diagrams
9F Experimental probability
9G Summarising data: Measures of centre
9H Stem-and-leaf plots
9I Grouped data
9J Measures of spread
9K Box plots (EXTENDING)

Australian Curriculum

STATISTICS AND PROBABILITY

Chance

List all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Assign probabilities to outcomes and determine probabilities for events (ACMSP225)

Calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and' or 'or' (ACMSP226)

Investigate reports of surveys in digital media and elsewhere for information on how data were obtained to estimate population means and medians (ACMSP227)

Data representation and interpretation

Identify everyday questions and issues involving at least one numerical and at least one categorical variable and collect data directly and from secondary sources (ACMSP228)

Construct back-to-back stem-and-leaf plots and histograms and describe data, using terms including ‘skewed’, ‘symmetric’ and ‘bi-modal’ (ACMSP282)

Compare data displays using mean, median and range to describe and interpret numerical data sets in terms of location (centre) and spread (ACMSP283)

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9A Review of probability CONSOLIDATING

Learning intentions

- To review the key terms of probability
- To understand how a probability can be expressed numerically
- To be able to interpret the chance of an event based on its numerical probability
- To be able to find the probability of events for equally likely outcomes
- To be able to find the probability of the complement of an event

The mathematics associated with describing chance is called probability. We can precisely calculate the chance of some events occurring, such as rolling a sum of 12 from two dice or flipping three heads if a coin is tossed five times. To do this we need to know how many outcomes there are in total and how many of the outcomes are favourable (i.e. which match the result we are interested in). The number of favourable outcomes in comparison to the total number of outcomes will determine how likely it is that the favourable event will occur.

Tennis matches are won or lost on the average probability, p, of a player winning each point. Probability calculations show that the chance of winning a match increases rapidly if p increases slightly. E.g. 50% chance if $p = 0.5$; 75% if $p = 0.52$; and 95% if $p = 0.55$.

LESSON STARTER Choose an event

As a class group, write down and discuss at least three events which have the following chance of occurring.

- impossible chance
- very low chance
- medium to low chance
- even (50–50) chance
- medium to high chance
- very high chance
- certain chance

KEY IDEAS

- A **random experiment** results in a list of **outcomes** which occur without interference.
- The **sample space** is the list of all possible outcomes from an experiment.
- Set brackets $\{\ldots\}$ are used to list sets of numbers or other objects.
- An **event** is a collection of outcomes resulting from an experiment.
  For example, rolling a die is a random experiment with six possible outcomes: $\{1, 2, 3, 4, 5, 6\}$.
  The event 'rolling a number greater than 4' includes the outcomes 5 and 6. This is an example of a **compound event** because it contains more than one element from the sample space.

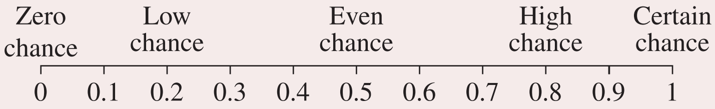
ISBN 978-1-108-77288-4

Cambridge University Press
Updated September 2021

- The probability of an event in which all outcomes are **equally likely** is given by:

$$\Pr(\text{Event}) = \frac{\text{Number of outcomes in which event occurs}}{\text{Total number of outcomes}}$$

- Probabilities are numbers between 0 and 1 inclusive, and can be written as a decimal, fraction or percentage. For example: 0.55 or $\frac{11}{20}$ or 55%
- For all events, $0 \leqslant \Pr(\text{Event}) \leqslant 1$.



- The **complement** of an event A is the event in which A does not occur.

$$\Pr(\text{not A}) = 1 - \Pr(\text{A})$$

BUILDING UNDERSTANDING

① Jim believes that there is a 1 in 4 chance that the flower on his prized rose will bloom tomorrow.

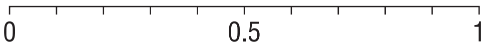
a Express the chance '1 in 4' as:

  i a fraction

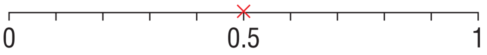
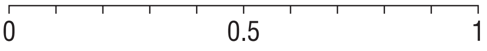
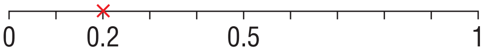
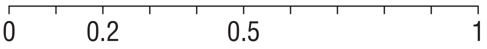
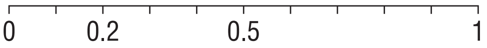
  ii a decimal

  iii a percentage.

b Indicate the level of chance described by Jim on this number line.



② State the components missing from this table.

	Percentage	Decimal	Fraction	Number line
	50%	0.5	$\frac{1}{2}$	
a	25%			
b				
c		0.6		
d			$\frac{17}{20}$	

③ Ten people make the following guesses of the chance that they will get a salary bonus this year.

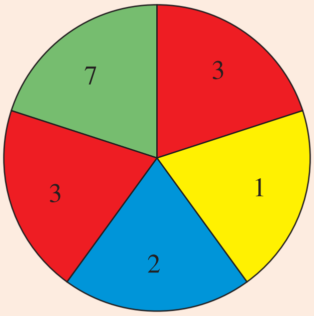
0.7, $\frac{2}{5}$, 0.9, $\frac{1}{3}$, 2 in 3, $\frac{3}{7}$, 1 in 4, 0.28, $\frac{2}{9}$, 0.15

Can you order their chances from lowest to highest? (*Hint*: Change each into a decimal.)

Example 1 Finding probabilities of events

This spinner has five equally divided sections.

a List the sample space using the given numbers.
b Find Pr(3).
c Find Pr(not a 3).
d Find Pr(a 3 or a 7).
e Find Pr(a number which is at least a 3).

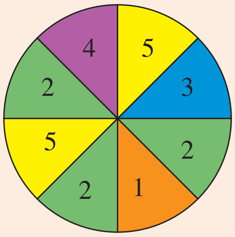


SOLUTION	EXPLANATION
a $\{1, 2, 3, 7\}$	Use set brackets and list all the possible outcomes in any order.
b $\text{Pr}(3) = \frac{2}{5}$ or 0.4	$\text{Pr}(3) = \frac{\text{number of sections labelled 3}}{\text{number of equal sections}}$
c $\begin{aligned}\text{Pr}(\text{not a }3) &= 1 - \text{Pr}(3)\\ &= 1 - \frac{2}{5} \text{ or } 1 - 0.4\\ &= \frac{3}{5} \text{ or } 0.6\end{aligned}$	‘Not a 3’ is the complementary event of obtaining a 3. Alternatively, count the number of sectors which are not 3.
d $\begin{aligned}\text{Pr}(\text{a 3 or a 7}) &= \frac{2}{5} + \frac{1}{5}\\ &= \frac{3}{5}\end{aligned}$	There are two 3s and one 7 in the five sections.
e $\text{Pr}(\text{at least a 3}) = \frac{3}{5}$	Three of the sections have the numbers 3 or 7, which are 3 or more.

Now you try

This spinner has eight equally divided sections.

a List the sample space using the given numbers.
b Find Pr(2).
c Find Pr(not a 2).
d Find Pr(a 2 or a 5).
e Find Pr(a number which is at least a 2).



Example 2 Choosing letters from a word

A letter is randomly chosen from the word PROBABILITY. Find the following probabilities.

- **a** Pr(L)
- **b** Pr(not L)
- **c** Pr(vowel)
- **d** Pr(consonant)
- **e** Pr(vowel or a B)
- **f** Pr(vowel or consonant)

SOLUTION	EXPLANATION
a $\Pr(\text{L}) = \dfrac{1}{11}$	One of the 11 letters in PROBABILITY is an L.
b $\Pr(\text{not L}) = 1 - \dfrac{1}{11} = \dfrac{10}{11}$	The event ‘not L’ is the complement of the event selecting an L. Complementary events sum to 1.
c $\Pr(\text{vowel}) = \dfrac{4}{11}$	There are 4 vowels: O, A and two letter Is.
d $\Pr(\text{consonant}) = 1 - \dfrac{4}{11} = \dfrac{7}{11}$	The events ‘vowel’ and ‘consonant’ are complementary.
e $\Pr(\text{vowel or a B}) = \dfrac{6}{11}$	There are 4 vowels and 2 letter Bs.
f $\Pr(\text{vowel or consonant}) = 1$	This event includes all possible outcomes.

Now you try

A letter is randomly chosen from the word AEROPLANE.
Find the following probabilities.

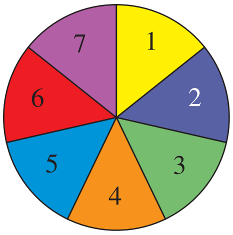
- **a** Pr(R)
- **b** Pr(not R)
- **c** Pr(vowel)
- **d** Pr(consonant)
- **e** Pr(consonant or an A)
- **f** Pr(vowel or consonant)

Exercise 9A

FLUENCY	1, 2–3($\frac{1}{2}$), 5, 6	2–4($\frac{1}{2}$), 5, 6	2–4($\frac{1}{2}$), 5, 6($\frac{1}{2}$)

Example 1

1 This spinner has seven equally divided sections.



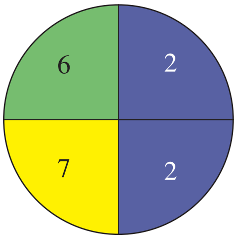
- **a** List the sample space using the given numbers.
- **b** Find Pr(2).
- **c** Find Pr(not a 2).
- **d** Find Pr(a 2 or a 3).
- **e** Find Pr(a number which is at least a 2).

Example 1

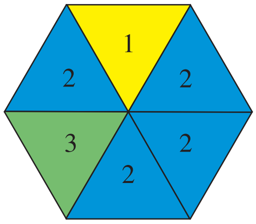
2 The spinners below have equally divided sections. Complete the following for each spinner.

- **i** List the sample space using the given numbers.
- **ii** Find Pr(2).
- **iii** Find Pr(not a 2).
- **iv** Find Pr(a 2 or a 3).
- **v** Find Pr(a number which is at least a 2).

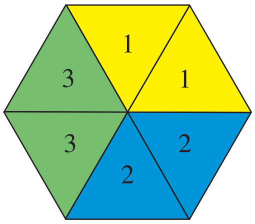
a



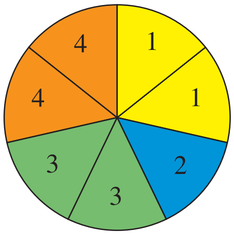
b



c



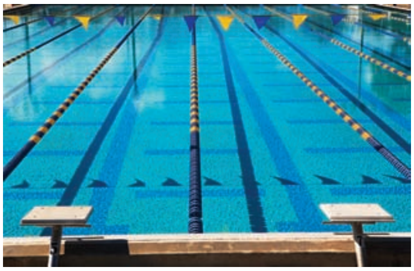
d



3 Find the probability of obtaining a blue ball when a ball is selected at random from a box which contains:

- **a** 4 blue balls and 4 red balls
- **b** 3 blue balls and 5 red balls
- **c** 1 blue ball, 3 red balls and 2 white balls
- **d** 8 blue balls, 15 black balls and 9 green balls
- **e** 15 blue balls only
- **f** 5 yellow balls and 2 green balls.

4 Find the probability of *not* selecting a blue ball when a ball is selected at random from a box containing the balls described in Question **3** parts **a** to **f** above.



5 A swimming pool has eight lanes and each of eight swimmers has an equal chance of being placed in lane 1. Find the probability that a particular swimmer:

- **a** will swim in lane 1
- **b** will not swim in lane 1.

Example 2

6 A letter is chosen at random from the word ALPHABET. Find the following probabilities.

- **a** Pr(L)
- **b** Pr(A)
- **c** Pr(A or L)
- **d** Pr(vowel)
- **e** Pr(consonant)
- **f** Pr(vowel or consonant)
- **g** Pr(Z)
- **h** Pr(A or Z)
- **i** Pr(not an A)
- **j** Pr(letter from the first half of the alphabet)

PROBLEM-SOLVING	7, 8	8, 9	8–9($\frac{1}{2}$), 10

7 The school captain is to be chosen at random from four candidates. Two are girls (Hayley and Alisa) and two are boys (Rocco and Stuart).

- **a** List the sample space.
- **b** Find the probability that the school captain will be:
  - **i** Hayley
  - **ii** male
  - **iii** neither Stuart nor Alisa.

8 A card is drawn at random from a deck of 52 playing cards. The deck includes 13 black spades, 13 black clubs, 13 red hearts and 13 red diamonds. This includes four of each of ace, king, queen, jack, 2, 3, 4, 5, 6, 7, 8, 9 and 10. Find the probability that the card will be:

- **a** the queen of diamonds
- **b** an ace
- **c** a red king
- **d** a red card
- **e** a jack or a queen
- **f** any card except a 2
- **g** any card except a jack or a black queen
- **h** not a black ace.

9 A six-sided die is rolled and the uppermost face is observed and recorded. Find the following probabilities.

- **a** Pr(6)
- **b** Pr(3)
- **c** Pr(not a 3)
- **d** Pr(1 or 2)
- **e** Pr(a number less than 5)
- **f** Pr(an even number or odd number)
- **g** Pr(a square number)
- **h** Pr(not a prime number)
- **i** Pr(a number greater than 1)

10 A letter is chosen at random from the word PROBABILITY. Find the probability that the letter will be:

- **a** a B
- **b** not a B
- **c** a vowel
- **d** not a vowel
- **e** a consonant
- **f** a letter belonging to one of the first five letters in the alphabet
- **g** a letter from the word RABBIT
- **h** a letter that is not in the word RABBIT.

REASONING	11, 12	11, 12	12, 13

11 Amanda selects a letter at random from the word SOLO and writes $\Pr(S) = \frac{1}{3}$. Explain her error.

12 A six-sided die is rolled. Which of the following events have a probability equal to $\frac{1}{3}$?

- **A** more than 4
- **B** at least 4
- **C** less than or equal to 3
- **D** no more than 2
- **E** at most 4
- **F** less than 3

13 A number is selected at random from the set $\{1, 2, 3, \ldots, 25\}$. Find the probability that the number chosen is:

- **a** a multiple of 2
- **b** a factor of 24
- **c** a square number
- **d** a prime number
- **e** divisible by 3
- **f** divisible by 3 or 2
- **g** divisible by 3 and 2
- **h** divisible by 2 or 3 or 7
- **i** divisible by 13 and 7.

ENRICHMENT: Faulty iPhone	–	–	14

14 Xavier selects his favourite album on his iPhone and taps the shuffle function so songs are randomly played. The time length for each track is as shown in the table on the right.

The shuffle function is faulty and begins playing randomly at an unknown place somewhere on the album, not necessarily at the beginning of a track.

Track	Time (minutes)
1	3
2	4
3	4
4	5
5	4
6	3
7	4
8	4

- **a** Find the total number of minutes of music available on this album.
- **b** Find the probability that the iPhone will begin playing on track 1.
- **c** Find the probability that the iPhone will begin on:
  - **i** track 2
  - **ii** track 3
  - **iii** a track that is 4 minutes long
  - **iv** track 4
  - **v** track 7 or 8
  - **vi** a track that is not 4 minutes long.

9B Venn diagrams and two-way tables

Learning intentions
- To understand how Venn diagrams and two-way tables are used to represent information
- To be able to construct a Venn diagram or two-way table based on collected data
- To be able to use a Venn diagram or two-way table to find the probability of events or the number in a particular category

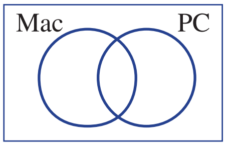
When the results of an experiment involve overlapping categories, it can be very helpful to organise the information into a Venn diagram or two-way table. Probabilities of particular events can easily be calculated from these types of diagrams.

LESSON STARTER Mac or PC

Twenty people were surveyed to find out whether they owned a Mac or PC computer at home. The survey revealed that 8 people owned a Mac and 15 people owned a PC. Everyone surveyed owned at least one type of computer.

- Do you think some people owned both a Mac and PC? Discuss.
- Use these diagrams to help organise the number of people who own Macs and PCs.
- Use your diagrams to describe the proportion (fraction) of people owning Macs and/or PCs for all the different areas in the diagrams.

Venn diagram



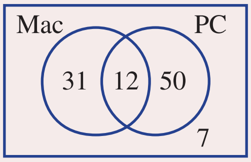
Two-way table

	Mac	No Mac	Total
PC			
No PC			
Total			

KEY IDEAS

- A **Venn diagram** and a **two-way table** help to organise outcomes into different categories. This example shows the type of computers owned by 100 people.

Venn diagram



Two-way table

	Mac	No Mac	Total
PC	12	50	62
No PC	31	7	38
Total	43	57	100

These diagrams show, for example, that:
- 12 people own both a Mac and a PC
- 62 people own a PC
- 57 people do not own a Mac
- $\Pr(\text{Mac}) = \dfrac{43}{100}$
- $\Pr(\text{only Mac}) = \dfrac{31}{100}$
- $\Pr(\text{Mac or PC}) = \dfrac{93}{100}$
- $\Pr(\text{Mac and PC}) = \dfrac{12}{100} = \dfrac{3}{25}$

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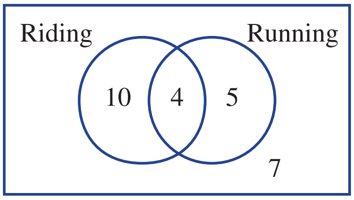
BUILDING UNDERSTANDING

1 This Venn diagram shows the number of people who enjoy riding and running.



a How many people in total are represented by this Venn diagram?

b How many people enjoy:
- **i** only riding?
- **ii** running (in total)?
- **iii** both riding and running?
- **iv** neither riding nor running?
- **v** riding or running?

c How many people do not enjoy:
- **i** riding?
- **ii** running?

2 Match the diagrams **A**, **B**, **C** or **D** with the given description.

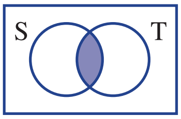
a S

b S only

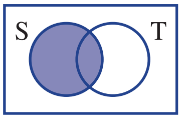
c S and T

d S or T

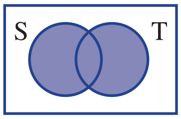
A



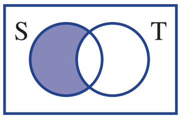
B



C



D



3 State the missing numbers in these two-way tables.

a

	A	Not A	Total
B	7	8	
Not B		1	
Total	10		

b

	A	Not A	Total
B	2		7
Not B		4	
Total			20

Example 3 Using a Venn diagram

A survey of 30 people found that 21 like AFL and 12 like soccer. Also, 7 people like both AFL and soccer and 4 like neither AFL nor soccer.

a Construct a Venn diagram for the survey results.

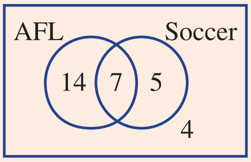
b How many people:
- **i** like AFL or soccer?
- **ii** do not like soccer?
- **iii** like only AFL?

c If one of the 30 people was randomly selected, find:
- **i** Pr(like AFL and soccer)
- **ii** Pr(like neither AFL nor soccer)
- **iii** Pr(like only soccer).

SOLUTION

EXPLANATION

a



Place the appropriate number in each category ensuring that:

- the total who like AFL is 21
- the total who like soccer is 12
- there are 30 in total.

b **i** 26

The total number of people who like AFL, soccer or both is 14 + 7 + 5 = 26.

ii $30 - 12 = 18$

12 like soccer so 18 do not.

iii 14

21 like AFL, but 7 of these also like soccer.

c **i** $\text{Pr(like AFL and soccer)} = \frac{7}{30}$

7 of the 30 people like AFL and soccer.

ii $\text{Pr(like neither AFL nor soccer)} = \frac{4}{30} = \frac{2}{15}$

The 4 people who like neither AFL nor soccer sit outside both categories.

iii $\text{Pr(like only soccer)} = \frac{5}{30} = \frac{1}{6}$

5 people like soccer but not AFL.

Now you try

A survey of 25 people found that 18 people like comedies and 12 like action movies. Also, 8 people like both comedies and action movies and 3 like neither comedies nor action movies.

a Construct a Venn diagram for the survey results.

b How many people:

- **i** like comedies or action movies?
- **ii** do not like action movies?
- **iii** like only comedies?

c If one of the 25 people was randomly selected, find:

- **i** Pr(likes comedies and action movies)
- **ii** Pr(likes neither comedies nor action movies)
- **iii** Pr(likes only action movies).

Example 4 Using a two-way table

At a car yard, 24 cars are tested for fuel used: 18 of the cars run on petrol, 8 cars run on gas and 3 cars can run on both petrol and gas.

a Illustrate the situation using a two-way table.

b How many of the cars:

- **i** do not run on gas?
- **ii** run on neither petrol nor gas?

c Find the probability that a randomly selected car:

- **i** runs on gas
- **ii** runs on only gas
- **iii** runs on gas or petrol.

SOLUTION

a

	Gas	Not gas	Total
Petrol	3	15	18
Not petrol	5	1	6
Total	8	16	24

b **i** 16

ii 1

c **i**

$$\Pr(\text{gas}) = \frac{8}{24} = \frac{1}{3}$$

ii

$$\Pr(\text{only gas}) = \frac{5}{24}$$

iii

$$\Pr(\text{gas or petrol}) = \frac{15 + 5 + 3}{24} = \frac{23}{24}$$

EXPLANATION

Set up a table as shown and enter the numbers (in black) from the given information.

Fill in the remaining numbers (in red) ensuring that each column and row adds to the correct total.

The total at the base of the 'Not gas' column is 16.

The number at the intersection of the 'Not gas' column and the 'Not petrol' row is 1.

8 cars in total run on gas out of the 24 cars.

Of the 8 cars that run on gas, 5 of them do not also run on petrol.

Of the 24 cars, some run on petrol only (15), some run on gas only (5) and some run on gas and petrol (3).

Now you try

Thirty people at a gym are asked about their gym usage. Of these, 18 people attend classes, 19 people use the gym equipment and 8 people attend classes and use the equipment.

a Illustrate the situation using a two-way table.

b How many of the people:

- **i** do not attend classes?
- **ii** neither attend classes nor use the equipment?

c Find the probability that a randomly selected person:

- **i** uses the gym equipment
- **ii** only attends classes
- **iii** attends classes or uses the gym equipment.

Exercise 9B

FLUENCY	1–5	2–5	2, 4, 5

Example 3

1 In a class of 30 students, 22 carried a phone and 9 carried an iPad, 3 carried both a phone and an iPad and 2 students carried neither.

- **a** Represent the information using a Venn diagram.
- **b** How many people:
  - **i** carried a phone or an iPad or both?
  - **ii** did not carry an iPad?
  - **iii** carried only an iPad?
- **c** If one of the 30 people was selected at random, find the following probabilities.
  - **i** Pr(carries a phone and an iPad)
  - **ii** Pr(carries neither a phone nor an iPad)
  - **iii** Pr(carries only a phone)

Example 3

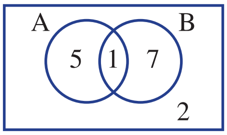
2 In a survey of 20 people, it is found that 13 people like toast for breakfast and 10 like cereal. Also 6 people like both toast and cereal and 3 like neither toast nor cereal.

- **a** Represent the information using a Venn diagram.
- **b** How many people:
  - **i** like toast or cereal for breakfast?
  - **ii** do not like toast?
  - **iii** like only toast?
- **c** If one of the 20 people was randomly selected, find:
  - **i** Pr(likes toast and cereal)
  - **ii** Pr(likes only cereal).

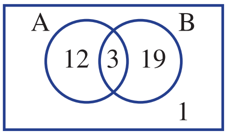
3 For each Venn diagram, find the following probabilities. You will need to calculate the total number in the sample first.

- **i** Pr(A)
- **ii** Pr(A only)
- **iii** Pr(not B)
- **iv** Pr(A and B)
- **v** Pr(A or B)
- **vi** Pr(neither A nor B)

a



b



Example 4

4 Of the 50 desserts served at a restaurant one evening, 25 were served with ice-cream, 21 were served with cream and 5 were served with both cream and ice-cream.

- **a** Illustrate the situation using a two-way table.
- **b** How many of the desserts:
  - **i** did not have cream?
  - **ii** had neither cream nor ice-cream?
- **c** Find the probability that a randomly chosen dessert:
  - **i** had cream
  - **ii** had only cream
  - **iii** had cream or ice-cream.

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5 Find the following probabilities using each of the given tables. First fill in the missing numbers.

i Pr(A)	**ii** Pr(not A)	**iii** Pr(A and B)
iv Pr(A or B)	**v** Pr(B only)	**vi** Pr(neither A nor B)

a

	A	Not A	Total
B	3	1	
Not B	2		4
Total			

b

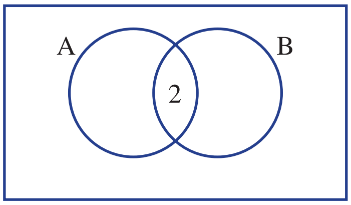
	A	Not A	Total
B		4	15
Not B	6		
Total			26

PROBLEM-SOLVING			
	6–8	6–9	6, 8, 10

6 For each two-way table, fill in the missing numbers and then transfer the information to a Venn diagram.

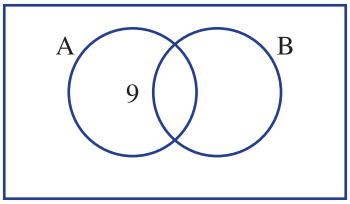
a

	A	Not A	Total
B	2		8
Not B			
Total		7	12



b

	A	Not A	Total
B		4	
Not B	9		13
Total	12		



7 Of a group of 10 people, 6 rented their house, 4 rented a car and 3 did not rent either a car or their house.

- **a** Draw a Venn diagram.
- **b** How many people rented both a car and their house?
- **c** Find the probability that one person rented only a car.

8 One hundred people were surveyed regarding their use of water for their garden. Of that group, 23 said that they used rainwater, 48 said that they used tap water and 41 said that they did not water at all.

- **a** Represent this information in a two-way table.
- **b** How many people used both rain and tap water?
- **c** What is the probability that one person selected at random uses only tap water?
- **d** What is the probability that one person selected at random uses tap water or rainwater?

9 All members of a ski club enjoy skiing and/or snowboarding. Of those members, 7 enjoy only snowboarding, 16 enjoy skiing and 4 enjoy both snowboarding and skiing. How many people are in the ski club?

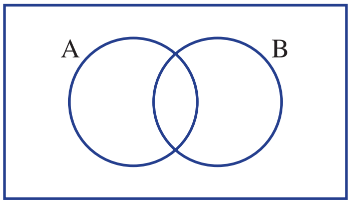
10 Of 30 cats, 24 eat tinned or dry food, 10 like dry food and 5 like both tinned and dry food. Find the probability that a randomly selected cat likes only tinned food.

REASONING	11	11, 12	12–14

11 Complete the two-way table and transfer to a Venn diagram using the pronumerals w, x, y and z.

	A	**Not A**	**Total**
B	x	y	
Not B	z	w	
Total			

→



12 The total number of people in a survey is T. The number of males in the survey is x and the number of doctors is y. The number of doctors who are males is z. Write algebraic expressions for the following using any of the variables x, y, z and T.

- **a** The number who are neither male nor a doctor
- **b** The number who are not males
- **c** The number who are not doctors
- **d** The number who are male but not a doctor
- **e** The number who are a doctor but not male
- **f** The number who are female and a doctor
- **g** The number who are female or a doctor

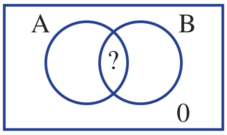
13 Explain what is wrong with this two-way table. Try to complete it to find out.

	A	**Not A**	**Total**
B		12	
Not B			7
Total	11		19

14 What is the minimum number of numbers that need to be given in a two-way table so that all numbers in the table can be calculated?

ENRICHMENT: Finding a rule for A and B	–	–	15

15 Two overlapping events, A and B, include 20 elements with 0 elements in the 'neither A nor B' region.



- **a** Draw a Venn diagram for the following situations.
  - **i** The number in A is 12 and the number in B is 10.
  - **ii** The number in A is 15 and the number in B is 11.
  - **iii** The number in A is 18 and the number in B is 6.
- **b** If the total number in A or B is now 100 (not 20), complete a Venn diagram for the following situations.
  - **i** The number in A is 50 and the number in B is 60.
  - **ii** The number in A is 38 and the number in B is 81.
  - **iii** The number in A is 83 and the number in B is 94.
- **c** Now describe a method that finds the number in the common area for A and B. Your method should work for all the above examples.

9C Using set notation

Learning intentions

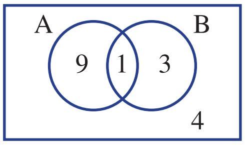
- To know the symbols and notation used to describe different sets of data
- To be able to list sets from a description or diagram
- To be able to interpret symbols used to represent sets of data and find associated probabilities

Using symbols to describe different sets of objects can make the writing of mathematics more efficient and easier to read. For example, *the probability that a randomly chosen person likes both apples and bananas* could be written Pr(A ∩ B), provided the events A and B are clearly defined.

Set notation is useful when analysing statistical data, such as: police data of car, bike and pedestrian accidents related to alcohol or speed or both; and energy company data of household numbers that use mains power or solar power or both.

LESSON STARTER English language meaning to mathematical meaning

The number of elements in two events called A and B are illustrated in this Venn diagram. Use your understanding of the English language meaning of the given words to match with one of the mathematical terms and a number from the Venn diagram. They are in jumbled order.



English	Mathematical	Number in Venn diagram
a not A	**A** A union B	**i** 1
b A or B	**B** sample space	**ii** 7
c A and B	**C** complement of A	**iii** 13
d anyone	**D** A intersection B	**iv** 17

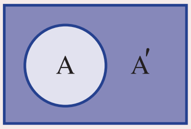
KEY IDEAS

- The **sample space** (list of all possible outcomes) is sometimes called the universal set and is given the symbol S, Ω, U or ξ.

  For example: $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

- Set A is a particular **subset** (⊂) of the sample space if all the elements in A are contained in the sample space.
  For example: $\{2, 3, 5, 7\} \subset \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{2, 3, 5, 7\}$

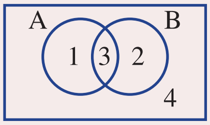


$A' = \{1, 4, 6, 8, 9, 10\}$

- A' is the complement of A and contains the elements not in A.
- 5 ∈ A means that 5 is **an element of** A.
- ∅ is the **null** or **empty** set and contains no elements; ∴ ∅ = { }.
- $n(A)$ is the **cardinal number** of A and means the number of elements in A, e.g. $n(A) = 4$.

- A Venn diagram can be used to illustrate how different subsets in the sample space are grouped.

  For example: A = $\{2, 3, 5, 7\}$
  B = $\{1, 3, 5, 7, 9\}$



	A	**A′**	**Total**
B	3	2	5
B′	1	4	5
Total	4	6	10

  - We usually write the cardinal numbers inside a Venn diagram.
  - A ∩ B means A **and** B, which means the **intersection** of A and B and includes the elements in common with both sets: ∴ A ∩ B = $\{3, 5, 7\}$.
  - A ∪ B means A **or** B, which means the **union** of A and B and includes the elements in either A or B or both: ∴ A ∪ B = $\{1, 2, 3, 5, 7, 9\}$.
- A **only** is the elements in A but not in B: ∴ A only = $\{2\}$ and n(A only) = 1.

BUILDING UNDERSTANDING

1 Match each of the terms in the first list with the symbols in the second list.

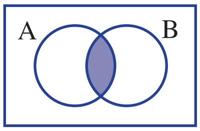
a complement
b union
c element of
d intersection
e empty or null set
f number of elements

A ∩
B n(A)
C A′
D ∪
E ∈
F ∅

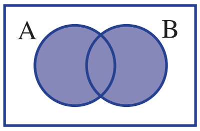
2 Choose a diagram that matches each of these sets.

a A ∪ B
b A′
c A ∩ B
d B only

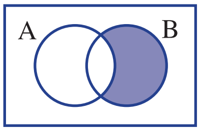
A



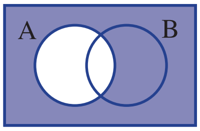
B



C



D



3 Using the given two-way table, state the following.

a $n(\text{A} \cap \text{B})$
b $n(\text{B})$
c $n(\text{A})$
d $n(\text{A}')$

	A	**A′**	**Total**
B	2	8	10
B′	5	1	6
Total	7	9	16

Example 5 Using set notation

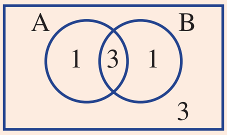
A number is chosen from the set of positive integers between 1 and 8 inclusive. A is the set of odd numbers between 1 and 8 inclusive and B is the set of prime numbers between 1 and 8 inclusive.

a List these sets.
  i the sample space **ii** A **iii** B

b Draw a Venn diagram.

c List these sets.
  i $A \cap B$ **ii** $A \cup B$ **iii** A' **iv** B only

d Find:
  i $n(A)$ **ii** $\Pr(A)$ **iii** $n(A \cap B)$ **iv** $\Pr(A \cap B)$

SOLUTION	EXPLANATION
a **i** $\{1, 2, 3, 4, 5, 6, 7, 8\}$	List all the numbers, using set brackets.
ii $A = \{1, 3, 5, 7\}$	A includes all the odd numbers.
iii $B = \{2, 3, 5, 7\}$	B includes all the prime numbers. 1 is not prime.
b 	Place each cardinal number into the appropriate region, i.e. there are 3 numbers common to sets A and B so 3 is placed in the overlapping region.
c **i** $A \cap B = \{3, 5, 7\}$	$\{3, 5, 7\}$ are common to both A and B.
ii $A \cup B = \{1, 2, 3, 5, 7\}$	$\{1, 2, 3, 5, 7\}$ are in either A or B or both.
iii $A' = \{2, 4, 6, 8\}$	A' means the elements not in A.
iv B only $= \{2\}$	B only means the elements in B but not in A.
d **i** $n(A) = 4$	$n(A)$ is the cardinal number of A. There are four elements in A.
ii $\Pr(A) = \frac{4}{8} = \frac{1}{2}$	$\Pr(A)$ means the chance that the element will belong to A. There are 4 numbers in A compared with 8 in the sample space.
iii $n(A \cap B) = 3$	There are three elements in $A \cap B$.
iv $\Pr(A \cap B) = \frac{3}{8}$	Three of eight elements are in $A \cap B$.

Now you try

A number is chosen from the set of positive integers between 1 and 15 inclusive. A is the set of odd numbers between 1 and 15 inclusive and B is the set of prime numbers between 1 and 15 inclusive.

a List these sets.
  - **i** the sample space
  - **ii** A
  - **iii** B

b Draw a Venn diagram.

c List these sets.
  - **i** $A \cap B$
  - **ii** $A \cup B$
  - **iii** A'
  - **iv** B only

d Find:
  - **i** $n(A)$
  - **ii** $\Pr(A)$
  - **iii** $n(A \cap B)$
  - **iv** $\Pr(A \cap B)$

Exercise 9C

FLUENCY	1–3	2–4	2–4

Example 5 **1** A number is chosen from the set of positive integers between 1 and 10 inclusive. A is the set of odd numbers between 1 and 10 inclusive and B is the set of prime numbers between 1 and 10 inclusive.

a List these sets.
  - **i** the sample space
  - **ii** A
  - **iii** B

b Draw a Venn diagram.

c List these sets.
  - **i** $A \cap B$
  - **ii** $A \cup B$
  - **iii** A'
  - **iv** B only

d Find:
  - **i** $n(A)$
  - **ii** $\Pr(A)$
  - **iii** $n(A \cap B)$
  - **iv** $\Pr(A \cap B)$.

Example 5 **2** A number is chosen from the set of positive integers between 1 and 20 inclusive. A is the set of multiples of 3 that are less than 20 and B is the set of factors of 15.

a List sets A and B, and then draw a Venn diagram.

b List these sets.
  - **i** $A \cap B$
  - **ii** $A \cup B$
  - **iii** A'
  - **iv** B only

c Find:
  - **i** $n(B)$
  - **ii** $\Pr(B)$
  - **iii** $n(A \cap B)$
  - **iv** $\Pr(A \cap B)$
  - **v** $n(A \cup B)$
  - **vi** $\Pr(A \cup B)$.

3 Consider the sample space $\Omega = \{1, 2, 3, 4, 5, 6\}$ and the sets $A = \{1, 4, 6\}$ and $B = \{2, 3, 5\}$.

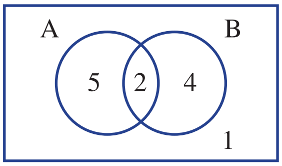
a State whether the following are true (T) or false (F).
  - **i** $A \subset \Omega$
  - **ii** $B \subset \Omega$
  - **iii** $3 \in A$
  - **iv** $5 \in A$
  - **v** $n(B) = 3$
  - **vi** $n(B') = 2$
  - **vii** $A \cap B = \varnothing$
  - **viii** $A \cup B = \varnothing$

b Find these probabilities.
  - **i** $\Pr(B)$
  - **ii** $\Pr(B')$
  - **iii** $\Pr(A \cap B)$

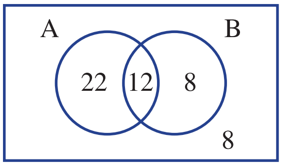
4 For each diagram or table, find the following probabilities.

i $\text{Pr}(A \cap B)$ **ii** $\text{Pr}(A \cup B)$ **iii** $\text{Pr}(A')$

a



b



c

	A′	A′	Total
B	5	2	
B′			
Total	9		15

d

	A	A′	Total
B		8	13
B′	4		
Total		12	

PROBLEM-SOLVING 5–7 5–7 6, 7

5 Four students have the names FRED, RON, RACHEL and HELEN while the sets A, B and C are defined by:

$A = \{\text{students with a name including the letter R}\}$
$B = \{\text{students with a name including the letter E}\}$
$C = \{\text{students with a name including the letter Z}\}$

a List these sets.

i A **ii** B **iii** C **iv** $A \cap B$

b If a student is chosen at random from the group, find these probabilities.

i $\text{Pr}(A)$ **ii** $\text{Pr}(A')$ **iii** $\text{Pr}(C)$
iv $\text{Pr}(C')$ **v** $\text{Pr}(A \cap B)$ **vi** $\text{Pr}(A \cup B)$

6 Consider all the letters of the alphabet. Let $A = \{\text{the set of vowels}\}$ and $B = \{\text{different letters of the word MATHEMATICS}\}$. Find:

a $n(\text{sample space})$ **b** $n(A)$ **c** $n(A \cap B)$ **d** $n(B')$
e $\text{Pr}(A)$ **f** $\text{Pr}(A')$ **g** $\text{Pr}(A \cap B)$ **h** $\text{Pr}(A \cup B)$.

7 For 50 people who all have at least a cat or a dog, let A be the set of all pet owners who have a dog and B be the set of all pet owners who have a cat. If $n(A) = 32$ and $n(B) = 29$, find the following.

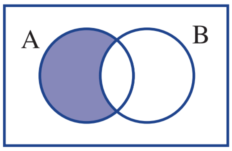
a $n(A \cap B)$ **b** $n(A\,\text{only})$ **c** $\text{Pr}(A \cup B)$ **d** $\text{Pr}(A')$

REASONING	8, 9	8–10	10–12

8 From Question **7**, A is the set of dog owners and B is the set of cat owners. Write a brief description of the following groups of people.

a $A \cap B$ **b** $A \cup B$ **c** B' **d** B only

9 The region 'A only' could be thought of as the intersection of A and the complement of B so, A only $= A \cap B'$. Use set notation to describe the region 'B only'.



10 **a** Is $n(A) + n(B)$ ever equal to $n(A \cup B)$? If so, show an example using a Venn diagram.

b Is $n(A) + n(B)$ ever less than $n(A \cup B)$? Explain.

11 The four inner regions of a two-way table can be described using intersections. The A only region is described by $n(A \cap B')$. Describe the other three regions using intersections.

	A	**A′**	**Total**
B			$n(B)$
B′	$n(A \cap B')$		$n(B')$
Total	$n(A)$	$n(A')$	$n(\Omega)$

12 Research what it means if we say that two events A and B are **mutually exclusive**. Give a brief description.

ENRICHMENT: How are $A' \cap B'$ and $(A \cup B)'$ related?	–	–	13

13 Consider the set of integers $\{1, 2, 3, \ldots, 20\}$. Let $A = \{\text{prime numbers less than } 20\}$ and $B = \{\text{factors of } 12\}$.

a List these sets.

i A **ii** B **iii** $A \cap B$

iv $A \cup B$ **v** $A' \cap B$ **vi** $A \cap B'$

vii $A' \cup B$ **viii** $A \cup B'$ **ix** $A' \cup B'$

b Find the following probabilities.

i $\Pr(A \cup B)$ **ii** $\Pr(A \cap B)$ **iii** $\Pr(A' \cap B)$ **iv** $\Pr(A' \cap B')$

v $\Pr(A \cup B')$ **vi** $\Pr((A \cap B)')$ **vii** $\Pr(A' \cup B')$ **viii** $\Pr((A \cup B)')$

c Draw Venn diagrams to shade the regions $A' \cap B'$ and $(A \cup B)'$. What do you notice?

9D Using arrays for two-step experiments

Learning intentions

- To understand how an array is used to list the sample space for experiments with two steps
- To understand the difference between experiments that are carried out with replacement and without replacement
- To be able to use an array to find the sample space for experiments carried out with replacement or without replacement and determine the probability of associated events

When an experiment consists of two steps such as rolling two dice or selecting two people from a group, we can use an array (or table) to systematically list the sample space.

Genetic arrays find probabilities of inherited traits, such as for the melanin pigments that control hair, eye and skin colour. Fairer skin has a higher risk of skin cancer: 30% of melanomas arise from moles, 70% from normal skin.

LESSON STARTER Is it a 1 in 3 chance?

Billy tosses two coins on the kitchen table at home and asks what the chance is of getting two tails.

- Dad says that there are 3 outcomes: two heads, two tails or one of each, so there is a 1 in 3 chance.
- Mum says that with coins, all outcomes have a 1 in 2 chance of occurring.
- Billy's sister Betty says that there are 4 outcomes so it's a 1 in 4 chance.

Can you explain who is correct and why?

KEY IDEAS

- An **array** or table is often used to list the sample space for experiments with two steps.
- When listing outcomes it is important to be consistent with the order for each outcome.
For example: the outcome (heads, tails) should be distinguished from the outcome (tails, heads).
- Some experiments are conducted **without replacement**, which means some outcomes that may be possible **with replacement** are not possible.
For example: Two letters are chosen from the word CAT.

With replacement

		1st		
		C	**A**	**T**
2nd	**C**	(C, C)	(A, C)	(T, C)
	A	(C, A)	(A, A)	(T, A)
	T	(C, T)	(A, T)	(T, T)

9 outcomes

Without replacement

		1st		
		C	**A**	**T**
2nd	**C**	×	(A, C)	(T, C)
	A	(C, A)	×	(T, A)
	T	(C, T)	(A, T)	×

6 outcomes

BUILDING UNDERSTANDING

1 State the missing outcomes, and then count the total number of outcomes. Part **a** is *with replacement* and part **b** is *without replacement*.

a

		1st		
		1	**2**	**3**
2nd	**1**	(1, 1)	(2, 1)	
	2			
	3			

b

		1st		
		A	**B**	**C**
2nd	**A**	×	(B, A)	(C, A)
	B		×	
	C			×

2 These two tables list the outcomes for the selection of two letters at random from the word MAT.

	Table A		
	M	**A**	**T**
M	(M, M)	(A, M)	(T, M)
A	(M, A)	(A, A)	(T, A)
T	(M, T)	(A, T)	(T, T)

	Table B		
	M	**A**	**T**
M	×	(A, M)	(T, M)
A	(M, A)	×	(T, A)
T	(M, T)	(A, T)	×

a Which table shows selection where replacement is allowed (with replacement)?

b Which table shows selection where replacement is not allowed (without replacement)?

c What is the probability of choosing the outcome (T, M) from:

- **i** Table A?
- **ii** Table B?

d How many outcomes include the letter A using:

- **i** Table A?
- **ii** Table B?

Example 6 Finding the sample space for events with replacement

Two coins are tossed.

a Draw a table to list the sample space.

b Find the probability of obtaining (H, T).

c Find Pr(1 head).

SOLUTION

a

		Toss 1	
		H	**T**
Toss 2	**H**	(H, H)	(T, H)
	T	(H, T)	(T, T)

Sample space =
{(H, H), (H, T), (T, H), (T, T)}

b $\Pr(\text{H, T}) = \frac{1}{4}$

c $\Pr(1 \text{ head}) = \frac{2}{4} = \frac{1}{2}$

EXPLANATION

Represent the results of each coin toss.

The table shows four possible outcomes.

One of the four outcomes is (H, T).

Two outcomes have one head: (H, T), (T, H).

Continued on next page

Now you try

Two identical counters have a red side (R) and a blue side (B). They are each tossed.

a Draw a table to list the sample space.

b Find the probability of obtaining (B, B).

c Find Pr(1 red side).

Example 7 Finding the sample space for events without replacement

Two letters are chosen at random from the word TREE without replacement.

a List the outcomes in a table.

b Find the probability that the two letters chosen are both E.

c Find the probability that at least one of the letters is an E.

SOLUTION

a

		1st			
		T	**R**	**E**	**E**
2nd	**T**	×	(R, T)	(E, T)	(E, T)
	R	(T, R)	×	(E, R)	(E, R)
	E	(T, E)	(R, E)	×	(E, E)
	E	(T, E)	(R, E)	(E, E)	×

b $\Pr(\text{E, E}) = \frac{2}{12} = \frac{1}{6}$

c $\Pr(\text{at least one E}) = \frac{10}{12} = \frac{5}{6}$

EXPLANATION

List all the outcomes maintaining a consistent order. Note that the same letter cannot be chosen twice.

Both Es need to be listed so that each outcome in the sample space is equally likely.

As there are 2 Es in the word TREE, it is possible to obtain the outcome (E, E) in two ways.

10 of the 12 outcomes contain at least one E.

Now you try

Two letters are chosen at random from the word DATA without replacement.

a List the outcomes in a table.

b Find the probability that the two letters chosen are both A.

c Find the probability that at least one of the letters is an A.

Exercise 9D

FLUENCY	1–4	2–5	2, 3, 5

Example 6

1 Two four-sided dice (numbered 1, 2, 3, 4) are rolled.

- **a** Complete a table like the one shown and list the sample space.
- **b** Find the probability of obtaining (2, 3).
- **c** Find Pr(double). A double is an outcome with two of the same number.

		1st			
		1	**2**	**3**	**4**
2nd	**1**	(1, 1)	(2, 1)		
	2				
	3				
	4				

Example 6

2 A six-sided die is rolled twice.

- **a** Complete a table like the one shown.
- **b** What is the total number of outcomes?
- **c** Find the probability that the outcome is:
  - **i** (1, 1)
  - **ii** a double
  - **iii** (3, 1), (2, 2) or (1, 3)
  - **iv** any outcome containing a 1 or a 6.

		1st					
		1	**2**	**3**	**4**	**5**	**6**
2nd	**1**	(1, 1)	(2, 1)				
	2						
	3						
	4						
	5						
	6						

Example 7

3 Two letters are chosen at random from the word DOG *without replacement*.

- **a** Complete a table like the one shown.
- **b** Find the probability of obtaining the (G, D) outcome.
- **c** Find the probability of obtaining an outcome with an O in it.

		1st		
		D	**O**	**G**
2nd	**D**	×	(O, D)	(G, D)
	O		×	
	G			×

4 Two dots are selected, one from each of the sets A and B, where A = {•, ∘} and B = {•, ∘, ∘}.

- **a** Complete a table like this one, showing all the possible outcomes.
- **b** State the total number of outcomes.
- **c** Find the probability that the outcome will:
  - **i** be (•, ∘)
  - **ii** contain one black dot
  - **iii** contain two of the same dots.

		A	
		•	∘
B	•	(•, •)	(∘, •)
	∘		
	∘		

5 Two digits are selected at random *without replacement* from the set {1, 2, 3, 4}.

- **a** Draw a table to show the sample space. Remember doubles such as (1, 1), (2, 2) are not allowed.
- **b** Find:
  - **i** Pr(1, 2)
  - **ii** Pr(4, 3).
- **c** Find the probability that:
  - **i** both numbers will be at least 3
  - **ii** the outcome will contain a 1 or a 4
  - **iii** the outcome will contain a 1 and a 4
  - **iv** the outcome will not contain a 3.

PROBLEM-SOLVING	6, 7	6, 7	7, 8

6 The total sum is recorded from rolling two four-sided dice.

a Copy and complete this table, showing all possible totals that can be obtained.

b Find the probability that the total sum is:

- i 2
- ii 2 or 3
- iii less than or equal to 4
- iv more than 6
- v at most 6.

		Roll 1			
		1	**2**	**3**	**4**
Roll 2	**1**	2	3		
	2				
	3				
	4				

7 Jill guesses the answers to two multiple-choice questions with options A, B, C, D or E.

a Copy and complete this table, showing all possible guesses that can be obtained.

		Guess 1				
		A	**B**	**C**	**D**	**E**
Guess 2	**A**	(A, A)	(B, A)			
	B					
	C					
	D					
	E					

b Find the probability that she will guess:

- i (D, A)
- ii the same letter
- iii different letters.

c Find the probability that Jill will get:

- i exactly one of her answers correct
- ii both of her answers correct.

8 Many board games involve the rolling of two six-sided dice.

a Use a table to help find the probability that the sum of the two dice is:

- i 12
- ii 2 or 3
- iii 11 or 12
- iv less than or equal to 7
- v less than 7
- vi at least 10
- vii at most 4
- viii 1.

b Which total sum has the highest probability? What is the probability of rolling that sum?

ISBN 978-1-108-77288-4

REASONING	9, 10	10–12	11–13

9 Two letters are chosen at random from the word MATHEMATICIAN.

- **a** How many outcomes sit in the sample space if selections are made:
  - **i** with replacement?
  - **ii** without replacement?
- **b** How many of the outcomes contain the same letter if selection is made:
  - **i** with replacement?
  - **ii** without replacement?

10 Two letters are chosen from the word WOOD without replacement. Is it possible to obtain the outcome (O, O)? Explain why.

11 In a bag are five counters each of a different colour: green (G), yellow (Y), red (R), blue (B) and purple (P).

- **a** A counter is drawn from the bag, replaced and then a second is selected. Find the probability that a green counter then a blue counter are selected; that is, find $\Pr(G, B)$.
- **b** The counter selected is not replaced before the second is selected. Find the probability that a green counter then a blue counter are selected.

12 A six-sided die and a ten-sided die have been rolled simultaneously. What total sum(s) has/have the highest probability?

13 A spinner numbered 1 to 50 with equal size sectors is spun twice. Find the probability that the total for the two spins is:

- **a** 100
- **b** 51
- **c** 99
- **d** 52
- **e** 55.

ENRICHMENT: Two cards from the deck	–	–	14

14 Two cards are dealt to you from a pack of playing cards that includes four suits (Hearts, Diamonds, Clubs and Spades), with each suit containing $\{2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A\}$. You keep both cards.

- **a** Does this situation involve with replacement or without replacement?
- **b** How many outcomes would there be in your sample space (all possible selections of two cards)?
- **c** What is the probability of receiving an ace of diamonds and an ace of hearts?
- **d** Find the probability of obtaining two cards that:
  - **i** are both twos
  - **ii** are both hearts.

9E Tree diagrams

Learning intentions

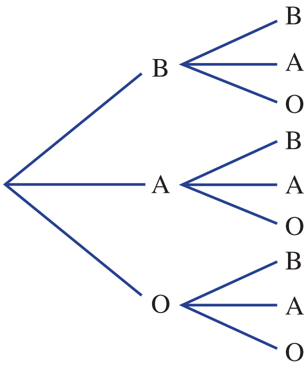
- To understand how tree diagrams are used to list the sample space for experiments with two or more steps where each outcome is equally likely
- To be able to construct a tree diagram to show all possible outcomes and find the probability of events

When experiments consist of two or more steps, a tree diagram can be used to list the sample space. Tables are often used for two-step experiments, but a tree diagram can be extended for experiments with any number of steps.

The spread of an infectious disease can be simply modelled with a tree diagram. If each ill person infects 4 other people, after 10 infection 'waves', over 1 million people are infected. Fortunately, vaccination strengthens the immune system, reducing infection rates.

LESSON STARTER What's the difference?

You are offered a choice of two pieces of fruit from a banana, an apple and an orange. You choose two at random. This tree diagram shows selection with replacement.

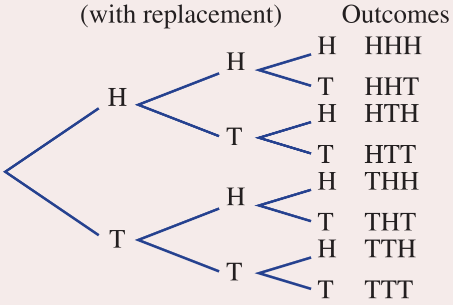


- How many outcomes will there be?
- How many of the outcomes contain two round fruits?
- How would the tree diagram change if the selection was completed without replacement? Would there be any difference in the answers to the above two questions? Discuss.

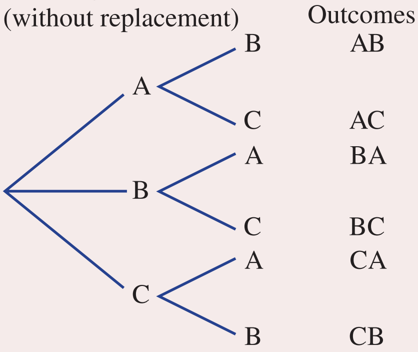
KEY IDEAS

- **Tree diagrams** are used to list the sample space for experiments with two or more steps.
  - The outcomes for each stage of the experiment are listed vertically and each stage is connected with branches.
    For example:

Tossing a coin 3 times (with replacement)



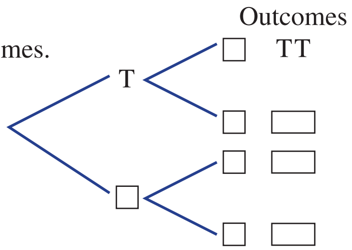
Selecting 2 letters from {A, B, C} (without replacement)



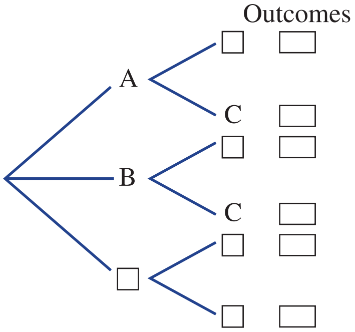
In these examples, each set of branches produces outcomes that are all equally likely.

BUILDING UNDERSTANDING

1 State the missing components for this tree diagram for selecting two letters *with replacement* from the word TO and list the outcomes.



2 State the missing components for this tree diagram to find the outcomes of selecting two people from a group of three (A, B and C) *without replacement*.



Example 8 Constructing a tree diagram

An experiment involves tossing two coins.

a Complete a tree diagram to show all possible outcomes.

b What is the total number of outcomes?

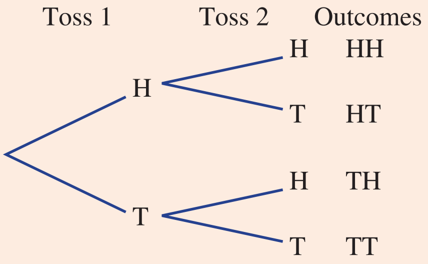
c Find the probability of tossing:

i two tails **ii** one tail **iii** at least one head.

SOLUTION

EXPLANATION

a



The tree diagram shows two coin tosses one after the other resulting in $2 \times 2 = 4$ outcomes.

b The total number of outcomes is 4.

There are four possibilities in the outcomes column.

c **i** $\Pr(\text{TT}) = \frac{1}{4}$

One of the four outcomes is TT.

ii $\Pr(1\text{ tail}) = \frac{2}{4} = \frac{1}{2}$

Two outcomes have one tail: $\{\text{HT}, \text{TH}\}$.

iii $\Pr(\geqslant 1\,\text{head}) = \frac{3}{4}$

Three outcomes have at least one head: $\{\text{HH}, \text{HT}, \text{TH}\}$.

Continued on next page

Now you try

Two regular six-sided die are rolled. It is noted if they show odd (O) or even (E) numbers.

a Complete a tree diagram to show all possible odd (O) and even (E) outcomes.

b What is the total number of outcomes?

c Find the probability of rolling:

- **i** two even numbers
- **ii** one even number
- **iii** at least one odd number.

Example 9 Constructing a tree diagram without replacement

Two people are selected at random without replacement from a group of three: Annabel (A), Brodie (B) and Chris (C).

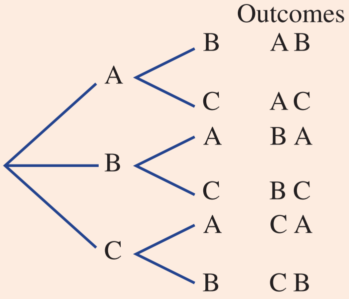
a List all the possible outcomes for the selection using a tree diagram.

b Find the probability that the selection will contain:

- **i** Annabel and Brodie
- **ii** Chris
- **iii** Chris or Brodie.

SOLUTION

a



EXPLANATION

On the first choice there are three options (A, B or C), but on the second choice there are only two remaining.

b **i** $\text{Pr(Annabel and Brodie)} = \frac{2}{6} = \frac{1}{3}$

2 of the 6 outcomes contain Annabel and Brodie (A B) and (B A).

ii $\text{Pr(Chris)} = \frac{4}{6} = \frac{2}{3}$

4 out of the 6 outcomes contain Chris.

iii $\text{Pr(Chris or Brodie)} = \frac{6}{6} = 1$

All of the outcomes contain at least one of Chris or Brodie.

Now you try

Two lollipops are selected at random without replacement from a group of three: strawberry (S), grape (G) and cola (C).

a List all the possible outcomes for the selection using a tree diagram.

b Find the probability that the selection will contain:

i strawberry and cola **ii** grape **iii** grape or cola.

Exercise 9E

FLUENCY	1–4	2–5	2, 4, 5

Example 8

1 A coin is tossed twice.

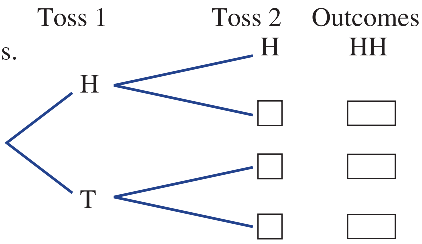
a Complete this tree diagram to show all the possible outcomes.

b What is the total number of outcomes?

c Find the probability of obtaining:

i two heads **ii** one head

iii at least one head **iv** at least one tail.



Example 8

2 A spinner with three numbers, 1, 2 and 3, is spun twice.

a List the set of possible outcomes, using a tree diagram.

b What is the total number of possible outcomes?

c Find the probability of spinning:

i two 3s **ii** at least one 3

iii no more than one 2 **iv** two odd numbers.

Example 9

3 Two people are selected at random without replacement from a group of three: Donna (D), Elle (E) and Fernando (F).

a List all the possible outcomes for the selection using a tree diagram.

b Find the probability that the selection will contain:

i Donna and Elle **ii** Fernando **iii** Fernando or Elle.

4 A drawer contains two red socks (R), one blue sock (B) and one yellow sock (Y). Two socks are selected at random without replacement.

a Copy and complete this tree diagram.

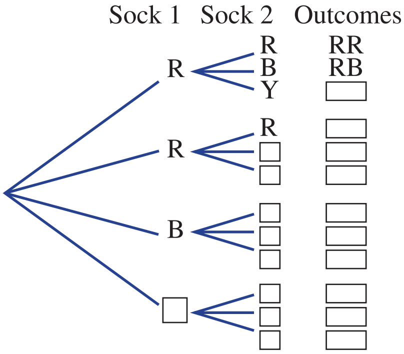
b Find the probability of obtaining:

i a red sock and a blue sock

ii two red socks

iii any pair of socks of the same colour

iv any pair of socks of different colour.



5 A student who has not studied for a multiple-choice test decides to guess the answers for every question. There are three questions, and three choices of answer (A, B and C) for each question. Given that only one of the possible choices (A, B or C) is correct for each question, state the probability that the student guesses:

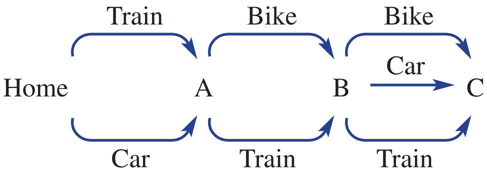
a 1 correct answer **b** 2 correct answers **c** 3 correct answers **d** 0 correct answers.

PROBLEM-SOLVING	6	6, 7	7

6 A shelf in a discount supermarket contains an equal number of tins of tomatoes and tins of peaches – all mixed together and without labels. You select four tins in a hurry. Use a tree diagram to help find the probability of selecting the correct number of tins of tomatoes and/or peaches for each of these recipe requirements. Assume that the shelves are continually refilled so that probabilities remain constant.

- **a** You need four tins of tomatoes for a stew.
- **b** You need four tins of peaches for a peach crumble.
- **c** You need at least three tins of tomatoes for a bolognese.
- **d** You need at least two tins of peaches for a fruit salad.
- **e** You need at least one tin of tomatoes for a vegetable soup.

7 Michael needs to deliver parcels to three places (A, B and C in order) in the city. This diagram shows the different ways in which he can travel.



- **a** Draw a tree diagram showing all the possible outcomes of transportation options.
- **b** What is the total number of possible outcomes?
- **c** If Michael randomly chooses one of these outcomes, find the probability that he will use:
  - **i** the train all three times
  - **ii** the train exactly twice
  - **iii** his bike exactly once
  - **iv** different transport each time
  - **v** a car at least once.

REASONING	8	8, 9	9, 10

8 You toss a coin four times. Use a tree diagram to find the probability that you toss:

- **a** 0 tails
- **b** 1 tail
- **c** 2 tails
- **d** 3 tails
- **e** 4 tails.

9
- **a** A coin is tossed 5 times. How many outcomes will there be?
- **b** A coin is tossed n times. Write a rule for the number of outcomes in the sample space.

10 Use a tree diagram to investigate the probability of selecting two counters from a bag of three black and two white counters if the selection is drawn:

- **a** with replacement
- **b** without replacement.

Is there any difference?

ENRICHMENT: Selecting matching clothes	–	–	11

11 A man randomly selects a tie from his collection of one green and two red ties, a shirt from a collection of one red and two white shirts, and either a red or a black hat. Use a tree diagram to help find the probability that he selects a tie, shirt and hat according to the following descriptions.

- **a** red tie, red shirt and black hat
- **b** all three items red
- **c** one item red
- **d** two items red
- **e** at least two items red
- **f** a green hat
- **g** a green tie and a black hat
- **h** a green tie or a black hat
- **i** not a red item
- **j** a red tie or white shirt or black hat

9F Experimental probability

Learning intentions

- To know how to calculate the experimental probability from an experiment or survey
- To understand the effect on the experimental probability if the number of trials is increased
- To be able to find the expected number of occurrences of an outcome using probability

In some situations it may not be possible to list all the outcomes in the sample space and find theoretical probabilities. If this is the case then an experiment, survey or simulation can be conducted to find experimental probabilities. Provided the experiment includes a sufficient number of trials, these probabilities can be used to estimate the chance of particular events. Experimental probability is frequently used in science experiments and for research in medicine and economics.

An actuary uses past data to calculate the probability of insurance claims for property loss, accidents, sickness or death. Based on these risk probabilities, actuaries determine the insurance premiums for different age groups and various activities.

LESSON STARTER Newspaper theories

A tabloid newspaper reports that of 10 people interviewed in the street, five had a dose of the flu. At about the same time a medical student tested 100 people and found that 21 had the flu.

- What is the experimental probability of having the flu, according to the newspaper's survey?
- What is the experimental probability of having the flu, according to the medical student's results?
- Which of the two sets of results would be more reliable and why? Discuss the reasons.
- Using the medical student's results, how many people would you expect to have the flu in a group of 1000 and why?

KEY IDEAS

- **Experimental probability** or relative frequency is calculated using the results of an experiment or survey.

  $$\text{Experimental probability} = \frac{\text{number of times the outcome occurs}}{\text{total number of trials in the experiment}} = \text{relative frequency}$$

- The **long-run proportion** is the experimental probability for a sufficiently large number of trials.
- The **expected number of occurrences** = probability × number of trials.

ISBN 978-1-108-77288-4

Cambridge University Press
Updated September 2021

BUILDING UNDERSTANDING

① This table shows the results of three different surveys of how many people in Perth use public transport (PT).

Survey	Number who use PT	Survey size	Experimental probability
A	2	10	$\frac{2}{10} = 0.2$
B	5	20	___________
C	30	100	___________

a What are the two missing numbers in the experimental probability list?

b Which survey should be used to estimate the probability that a person uses public transport and why?

② The experimental probability of Jess hitting a bullseye on a dartboard is 0.05 (or $\frac{5}{100}$). How many bullseyes would you expect Jess to get if she threw the following number of darts?

a 100 darts **b** 400 darts **c** 80 darts

Example 10 Finding the experimental probability

A box contains an unknown number of coloured balls and a ball is drawn from the box and then replaced. The procedure is repeated 100 times and the colour of the ball drawn is recorded each time. Twenty-five red balls were recorded.

a Find the experimental probability for selecting a red ball.

b Find the expected number of red balls drawn if the procedure was repeated 500 times.

SOLUTION	EXPLANATION
a $\Pr(\text{red ball}) = \frac{25}{100}$ $= 0.25$	$\Pr(\text{red ball}) = \frac{\text{number of red balls drawn}}{\text{total number of balls drawn}}$ There are 25 red balls drawn and 100 balls in total.
b Expected number of red balls in 500 $= 0.25 \times 500$ $= 125$	Expected number of occurrences $= \text{probability} \times \text{number of trials}$

Now you try

A biscuit jar contains an unknown number of biscuits and a biscuit is drawn from the jar and then replaced. The procedure is repeated 50 times and the type of biscuit drawn is recorded each time. Thirty choc-chip biscuits were recorded.

a Find the experimental probability for selecting a choc-chip biscuit.

b Find the expected number of choc-chip biscuits drawn if the procedure was repeated 200 times.

Exercise 9F

FLUENCY	1, 2	2, 3	2, 3

Example 10

1 A bag contains an unknown number of counters, and a counter is selected from the bag and then replaced. The procedure is repeated 100 times and the colour of the counter is recorded each time. Sixty of the counters drawn were blue.

- a Find the experimental probability for selecting a blue counter.
- b Find the expected number of blue counters drawn if the procedure was repeated:
  - i 100 times
  - ii 200 times
  - iii 600 times.

Example 10

2 In an experiment involving 200 people chosen at random, 175 people said that they owned a home computer.

- a Calculate the experimental probability of choosing a person who owns a home computer.
- b Find the expected number of people who own a home computer from the following group sizes.
  - i 400 people
  - ii 5000 people
  - iii 40 people

3 By calculating the experimental probability, estimate the chance that each of the following events will occur.

- a Nat will walk to work today, given that she walked to work five times in the last 75 working days.
- b Mike will win the next game of cards, given that of the last 80 games he has won 32.
- c Brett will hit the bullseye on the dartboard with his next attempt, given that of the last 120 attempts he hit the bullseye 22 times.

PROBLEM-SOLVING	4, 5	5, 6	5–7

4 A six-sided die is rolled 120 times. How many times would you expect the following events to occur?

- a a 6
- b a 1 or a 2
- c a number less than 4
- d a number that is at least 5

5 The colour of cars along a highway was noted over a short period of time and summarised in this frequency table.

Colour	White	Silver	Blue	Green
Frequency	7	4	5	4

- a How many cars had their colour recorded?
- b Find the experimental probability that a car's colour is:
  - i blue
  - ii white.
- c If the colour of 100 cars was recorded, what would be the expected number of:
  - i blue cars?
  - ii green cars?
  - iii blue or green cars?

6 The letters from a three-letter word are written separately onto cards. A card is chosen at random and replaced and this is repeated 100 times. The results are shown in the table.

Letter	E	S
Frequency	64	36

- a Find the experimental probability of obtaining an E.
- b Based on these results, what is the three-letter word likely to be?

7 A spinner is divided into three regions not necessarily of equal size. The regions are numbered 1, 2 and 3 and the spinner is spun 50 times. The table shows the results.

Number	1	2	3
Frequency	26	11	13

a Find the experimental probability of obtaining:
  i a 1 **ii** at least a 2 **iii** a 1 or a 3.

b Based on these results, how many 3s would you expect if the spinner is spun 300 times?

c In fact, the spinner is divided up using simple and common fractions. Give a likely example describing how the regions might be divided.

REASONING	8–10	8–11	10–12

8 The results of throwing a drawing pin and observing how many times the pin lands with the spike pointing up are shown in the table. Results are recorded at different stages of the experiment.

Number of throws	Frequency (spike up)	Experimental probability
1	1	1.00
5	2	0.40
10	5	0.50
20	9	0.45
50	18	0.36
100	41	0.41

Which experimental probability would you choose to best represent the probability that the pin will land spike up? Why?

9 Phil rolled a fair six-sided die 10 times and, incredibly, receives 9 sixes.

a Find the experimental probability of rolling a six.

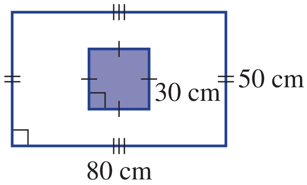
b Is it likely that Phil will receive the same number of sixes if he rolled the die 10 more times? Explain.

10 Do you think that a coin is fair or biased given the following experimental results? Give reasons.

a 47 tails out of 100 tosses **b** 23 heads out of 24 tosses **c** 1 tail out of 1 toss

11 One hundred darts are randomly thrown at the given dartboards. No darts miss the dartboard entirely. How many darts do you expect to hit the blue shaded region? Give reasons.

a



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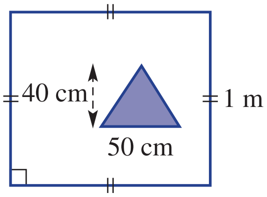

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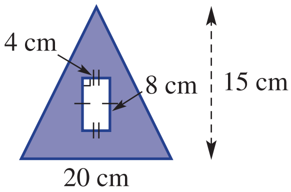
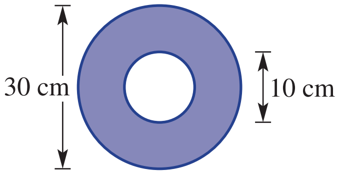


d



12 Decide if the following statements are true.

- **a** The experimental probability is always equal to the theoretical probability.
- **b** The experimental probability can be greater than the theoretical probability.
- **c** If the experimental probability is zero, then the theoretical probability is zero.
- **d** If the theoretical probability is zero, then the experimental probability is zero.

ENRICHMENT: More than a guessing game	–	–	13, 14

13 A bag of 10 counters includes counters of four different colours. The results from drawing and replacing one counter at a time for 80 trials are shown in this table.
Use the given information to find out how many counters of each colour were likely to be in the bag.

Colour	Total
Blue	26
Red	17
Green	29
Yellow	8

14 A box of 12 chocolates of the same size and shape includes five different centres. The results from selecting and replacing one chocolate at a time for 60 trials are shown in this table.
Use the given information to find out how many chocolates of each type were likely to be in the box.

Centre	Total
Strawberry	11
Caramel	14
Coconut	9
Nut	19
Mint	7

9A **1** List the sample space for:
- **a** rolling a standard die
- **b** the gender of a set of twins.

9A **2** Find the probability of choosing a vowel when choosing one letter at random from the word MATHEMATICS.

9B **3** A survey of 40 sporty people found that 28 liked tennis, 25 liked squash and 13 liked both.
- **a** Construct a Venn diagram for these results.
- **b** How many people liked both tennis and squash?
- **c** If one of the 40 people is chosen at random, what is the probability that they like only one sport?

9B **4** Use this two-way table to find the probability that a randomly selected car has both a sunroof and air-conditioning.

	Aircon	**No aircon**	**Total**
Sunroof		9	
No sunroof	55	6	61
Total		15	100

9C **5** A number is chosen from the set of positive integers between 10 and 20, inclusive. A is the set of even numbers between 10 and 20, inclusive, and B is the set of numbers divisible by 3 between 10 and 20, inclusive.
- **a** List these sets.
  - **i** the sample space
  - **ii** A
  - **iii** B
  - **iv** $A \cap B$
- **b** Find:
  - **i** $n(A \cap B)$
  - **ii** $\Pr(A \cap B)$
  - **iii** $\Pr(A')$.

9E **6** Find the probability of:
- **a** obtaining a head and a tail if two coins are tossed
- **b** choosing two vowels from the word MATE if two letters are chosen at random, without replacement
- **c** rolling a double 6 or a double 3 if two standard dice are rolled.

9D **7** Two letters are chosen, without replacement, from the letters of the word WAY.
- **a** Use a tree diagram to list all the possible combinations for the selection.
- **b** Find the probability that the selection will contain:
  - **i** the letter A
  - **ii** a W or a Y
  - **iii** not a Y
  - **iv** a W as the first letter.

9F **8** A drawer contains an unknown number of blue, black and white single socks. One sock is chosen at random from the drawer and its colour noted before the sock is returned to the drawer. The outcomes of this experiment are recorded in the table below.

Colour	Blue	Black	White
Frequency	16	44	40

Find the experimental probability of choosing:
- **a** a white sock
- **b** a blue or a white sock.

9F **9** If a regular six-sided die is rolled 420 times, how many times would you expect a 1 or a 2 to appear?

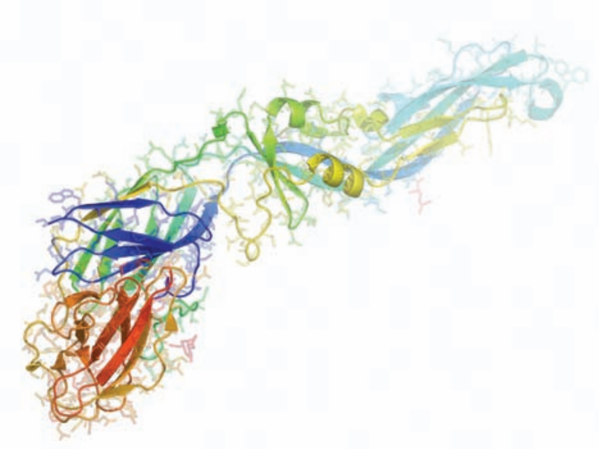
ISBN 978-1-108-77288-4

9G Summarising data: Measures of centre

Learning intentions

- To know the meaning of the terms mean, median, mode and outlier
- To be able to find the mean, median and mode of a data set

The discipline of statistics involves collecting and summarising data. It also involves drawing conclusions and making predictions, which is why many of the decisions we make today are based on statistical analysis. The type and amount of product stocked on supermarket shelves, for example, is determined by the sales statistics and other measures such as average cost and price range.



The Dengue fever virus (pictured) infects an average of 390 million people per year. Control of viral diseases improves when the molecular structures are known. Bioinformatics combines statistics, biology and information technology and analyses molecular biological data in the search for curing viral diseases.

LESSON STARTER Game purchase

Arathi purchases 7 computer games at a sale: 3 games cost \$20 each, 2 games cost \$30, 1 game costs \$50 and the last game costs \$200.

- Recall and discuss the meaning of the words mean, median and mode.
- Can you work out the mean, median or mode for the cost of Arathi's games?
- Which of the mean, median or mode gives the best 'average' for the cost of Arathi's games?
- Why is the mean greater than the median in this case?

KEY IDEAS

- The **mean** and **median** are called **measures of centre** because they give some idea of the 'average' or middle of the data set.
- **Mean** ($\bar{x}$)

  If there are n values, $x_1, x_2, x_3, \dots x_n$, then the mean is calculated as follows:

$$\bar{x} = \frac{\text{sum of all data values}}{\text{number of data values}}$$

$$= \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$$

ISBN 978-1-108-77288-4

- **Median**
  The median is the middle value if the data values are placed in order.
  - If there are two middle values, the median is calculated as the mean of these two values.

Odd number of values

$$1\quad 3\quad 5\quad \underset{\text{median}}{\textcircled{5}}\quad 6\quad 7\quad 10$$

Even number of values

$$13\quad 17\quad 17\quad \underbrace{20\quad 21}_{\substack{20.5\\ \text{median}}}\quad 27\quad 27\quad 28$$

- **Mode**
  The mode is the most common value.
  - There can be more than one mode.
  - If there are two modes, we say that the data set is **bi-modal**.
  - If each data value occurs just once, there is no mode.
- An **outlier** is a score that is much larger or smaller than the rest of the data.

BUILDING UNDERSTANDING

1 State the missing word.

a The mode is the most ________ value.

b The median is the ________ value.

c To calculate the ________, you add up all the values and divide by the number of values.

2 State the mean, median and mode for these simple ordered data sets.

a 1 2 2 2 4 4 6

b 7 11 14 18 20 20

Example 11 Finding measures of centre

For the given data sets, find the following:

i the mean **ii** the median **iii** the mode.

a 5 2 4 10 6 1 2 9 6

b 17 13 26 15 9 10

SOLUTION	EXPLANATION
a **i** $\text{Mean} = \dfrac{5+2+4+10+6+1+2+9+6}{9}$ $= 5$	Find the sum of all the numbers and divide by the number of values.
ii $1\ 2\ 2\ 4\ \textcircled{5}\ 6\ 6\ 9\ 10$ Median = 5	First, order the data. The median is the middle value.
iii Mode = 2 and 6	The data set is bi-modal as there are two numbers with the highest frequency.

b **i** Mean $= \dfrac{17 + 13 + 26 + 15 + 9 + 10}{6}$ $= 15$	The sum is 90 and there are 6 values.
ii $9 \quad 10 \quad \underbrace{13 \quad 15}_{14} \quad 17 \quad 26$	First, order the data.
Median $= \dfrac{13 + 15}{2}$ $= 14$	Since there are two values in the middle find the mean of them.
iii No mode	None of the values are repeated so there is no mode.

Now you try

For the given data sets, find the following:

- **i** the mean
- **ii** the median
- **iii** the mode.

a 4 8 10 2 3 6 2

b 12 15 21 24 18 16 17 13

Example 12 Finding a data value for a required mean

The hours a shop assistant spends cleaning the store in eight successive weeks are:
8, 9, 12, 10, 10, 8, 5, 10.

a Calculate the mean for this set of data.

b How many hours would the shop assistant need to clean in the ninth week for the mean to equal 10?

SOLUTION	EXPLANATION
a Mean $= \dfrac{8 + 9 + 12 + 10 + 10 + 8 + 5 + 10}{8}$ $= 9$	Sum of the 8 data values is 72
b Let a be the number of hours in the ninth week.	
Require $\dfrac{72 + a}{8 + 1} = 10$ $\dfrac{72 + a}{9} = 10$	$72 + a$ is the total of the new data and $8 + 1$ is the new total number of data values. Set this equal to the required mean of 10.
$72 + a = 90$ $a = 18$	Solve for a.
The new score would need to be 18.	Write the answer.

Continued on next page

Now you try

The amount of money a teenager earned in her last six babysitting appointments is:
\$15, \$25, \$35, \$18,\$20, \$25.

a Calculate the mean for this set of data.

b How much would the teenager need to earn on the next babysitting appointment for the mean to equal \$25?

Exercise 9G

FLUENCY	1, 2–3($\frac{1}{2}$), 4	2–4($\frac{1}{2}$)	2–4($\frac{1}{2}$)

Example 11 **1** For the given data sets, find:

i the mean **ii** the median **iii** the mode.

a 7 2 3 8 5 9 8 **b** 12 9 2 5 8 7 2 3

Example 11 **2** For the given data sets, find:

i the mean **ii** the median **iii** the mode.

a 6 13 5 4 16 10 3 5 10 **b** 10 17 5 16 4 14

c 3.5 2.1 4.0 8.3 2.1 **d** 0.7 3 2.9 10.4 6 7.2 1.3 8.5

e 6 0 -3 8 2 -3 9 5 **f** 3 -7 2 3 -2 -3 4

3 These data sets include an outlier. State the outlier, and then calculate the mean and the median. Include the outlier in your calculations.

a 5 7 7 8 12 33 **b** 1.3 1.1 1.0 1.7 1.5 1.6 -1.1 1.5

c -58 -60 -59 -4 -64 **d** -7.5 -2.4 -5.6 -1.2 10

4 Decide if the following data sets are bi-modal.

a 2 7 9 5 6 2 8 7 4 **b** 1 6 2 3 3 1 5 4 1 9

c 10 15 12 11 18 13 9 16 17 **d** 23 25 26 23 19 24 28 26 27

PROBLEM-SOLVING	5–7	6, 7, 9($\frac{1}{2}$)	6, 8, 9

5 In three races Paula recorded the times 25.1 seconds, 24.8 seconds and 24.1 seconds.

a What is the mean time of the races? Round to two decimal places.

b Find the median time.

Example 12 **6** A netball player scored the following number of goals in her 10 most recent games:

15 14 16 14 15 12 16 17 16 15

a What is her mean score?

b What number of goals does she need to score in the next game for the mean of her scores to be 16?

Example 12 **7** Stevie obtained the following scores on her first five Mathematics tests:

92 89 94 82 93

a What is her mean test score?

b If there is one more test left to complete, and she wants to achieve an average of at least 85, what is the lowest score Stevie can obtain for her final test?

8 Seven numbers have a mean of 8. Six of the numbers are 9, 7, 6, 4, 11 and 10. Find the seventh number.

9 Write down a set of 5 numbers which has the following values:

a a mean of 5, median of 6 and mode of 7 **b** a mean of 5, median of 4 and mode of 8

c a mean of 4, median of 4 and mode of 4 **d** a mean of 4.5, median of 3 and mode of 2.5

e a mean of 1, median of 0 and mode of 5 **f** a mean of 1, median of $1\frac{1}{4}$ and mode of $1\frac{1}{4}$.

REASONING	10, 11	10–12	11–13

10 The prices of six houses in Darwin are listed.

$324 000 $289 000 $431 000 $295 000 $385 000 $1 700 000

a What is the median house price?

b The data set includes an outlier. Which price would be considered the outlier?

c If the outlier was removed from the data set, by how much would the median change?

d Find the mean house price.

e If the outlier was removed from the data set, by how much would the mean change, to the nearest dollar?

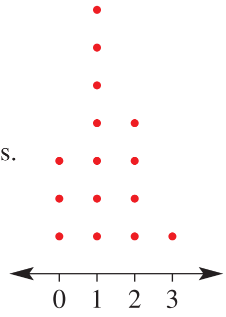
11 Explain why outliers significantly affect the mean but not the median.

12 This dot plot shows the frequency of households with 0, 1, 2 or 3 pets.

a How many households were surveyed? **b** Find the mean number of pets.

c Find the median number of pets. **d** Find the mode.

e Another household with 7 pets is added to the list. Does this change the median? Explain.



13 This simple data set contains nine numbers: 1 2 2 2 2 2 3 4 5

a Find the median.

b How many numbers greater than 5 need to be added to the list to change the median? (Give the least number.)

c How many numbers less than 1 need to be added to the list to change the median? (Give the least number.)

ENRICHMENT: Formula to get an A	–	–	14

14 A school awards grades in Mathematics each semester according to this table. Ryan has scored the following results for four topics this semester and has one topic to go: 75 68 85 79

Average score	Grade
90–100	A^+
80–	A
70–	B^+
60–	B
50–	C^+
0–	C

a What is Ryan's mean score so far?

b What grade will Ryan get for the semester if his fifth score is:

i 50? **ii** 68? **iii** 94?

c Find the maximum average score Ryan can receive for the semester. Is it possible for him to get an A^+?

d Find the least score that Ryan needs in his fifth topic for him to receive an average of:

i B^+ **ii** A.

e Write a rule for the mean score M for the semester if Ryan receives a mark of m in the fifth topic.

f Write a rule for the mark m in the fifth topic if Ryan scores an average of M for the semester.

Using CAS and non-CAS calculators 9G: Finding measures of centre

The activity is in the Interactive Textbook in the form of a printable PDF.

9H Stem-and-leaf plots

Learning intentions
- To know how a stem-and-leaf plot is used to display values in a data set
- To be able to display data in a stem-and-leaf plot and use it to find summary statistics and describe the data set
- To be able to construct and use back-to-back stem-and-leaf plots to compare data sets

Stem-and-leaf plots (or stem plots) are commonly used to display a single data set or two related data sets. They help to show how the data is distributed, as in a histogram, but retain all the individual data elements so no detail is lost. The median and mode can be easily read from a stem-and-leaf plot because all the data values sit in order.

Japanese train timetables use a stem-and-leaf plot to display departure times. The 'stems' are hours, and each 'leaf' represents a train's departure time in minutes past the hour.

LESSON STARTER Ships vs Chops

At a school, Ms Ships' class and Mr Chops' class sit the same exam. The scores are displayed using this back-to-back stem-and-leaf plot. Discuss the questions that follow.

Ms Ships' class		Mr Chops' class
3 1	5	0 1 1 3 5 7
8 8 7 5	6	2 3 5 5 7 9 9
6 4 4 2 1	7	8 9 9
7 4 3	8	0 3
6	9	1

7 | 8 means 78

- Which class had the greater number of students?
- What were the lowest and highest scores for each class?
- What were the median scores for each class?
- The results of which class could be described as symmetrical and which as skewed?
- Which class had the better results?

KEY IDEAS

- A **stem-and-leaf plot** uses a stem number and a leaf number to represent data.
  - The data is shown in two parts: a stem and a leaf.
  - The 'key' tells you how the plot is to be read.
  - The graph is similar to a histogram on its side or a bar graph with class intervals, but there is no loss of detail of the original data.

Ordered stem-and-leaf plot

Stem	Leaf
1	2 6
2	2 3 4 7
3	1 2 4 7 8 9
4	2 3 4 5 8
5	7 9

2 | 4 represents 24 people

A key is added to show the place value of the stems and leaves.

- **Back-to-back stem-and-leaf plots** can be used to compare two sets of data. The stem is drawn in the middle with the leaves on either side.

Scores for the last 30 football games

Winning scores		Losing scores
	7	4 5 8 8 9
1	8	0 0 3 3 6 7
7 5	9	1 2 3 6
8 4 4 1	10	3 9
9 5 0	11	1
3 1	12	

10 | 9 represents 109

81 lowest winning score

Symmetrical

Skewed

111 highest losing score

- **Symmetrical data** will produce a graph which is symmetrical about the centre.
- **Skewed data** will produce a graph which includes data bunched to one side of the centre.

BUILDING UNDERSTANDING

① This stem-and-leaf plot shows the length of time, in minutes, Alexis spoke on her phone for a number of calls.

a How many calls are represented by the stem-and-leaf plot?

b What is the length of the:

- **i** shortest phone call?
- **ii** longest phone call?

c What is the mode (the most common call time)?

d What is the median call time (middle value)?

Stem	Leaf
0	8
1	5 9
2	1 1 3 7
3	4 5

2 | 1 means 21 minutes

② This back-to-back stem-and-leaf plot shows the thickness of tyre tread on a selection of cars from the city and the country.

a How many car tyres were tested altogether?

b Find the median tyre tread thickness for tyres in:

  i the city

  ii the country.

c Is the distribution of tread thickness for city cars more symmetrical or skewed?

d Is the distribution of tread thickness for country cars more symmetrical or skewed?

City		Country
8 7 3 1 0 0 0	0	6 8 8 9
8 6 3 1 1 0	1	4 5 5 6 9
1	2	3 4 4

1 | 3 means 13 mm

Example 13 Constructing and using a stem-and-leaf plot

Consider this set of data.

0.3 2.5 4.1 3.7 2.0 3.3 4.8 3.3 4.6 0.1 4.1 7.5 1.4 2.4
5.7 2.3 3.4 3.0 2.3 4.1 6.3 1.0 5.8 4.4 0.1 6.8 5.2 1.0

a Organise the data into an ordered stem-and-leaf plot.

b Find the median.

c Find the mode.

d Describe the data as symmetrical or skewed.

SOLUTION

a

Stem	Leaf
0	1 1 3
1	0 0 4
2	0 3 3 4 5
3	0 3 3 4 7
4	1 1 1 4 6 8
5	2 7 8
6	3 8
7	5

3 | 4 means 3.4

b Median $= \dfrac{3.3 + 3.4}{2}$

$= 3.35$

c Mode is 4.1.

d Data is approximately symmetrical.

EXPLANATION

The minimum is 0.1 and the maximum is 7.5 so stems range from 0 to 7.

Place leaves in order from smallest to largest. As some numbers appear more than once, e.g. 0.1, their leaf (1) appears the same number of times.

There are 28 data values. The median is the average of the two middle values (the 14th and 15th values).

The most common value is 4.1.

The distribution of numbers is approximately symmetrical about the stem containing the median.

Now you try

Consider this set of data.

1.4 2.0 6.4 0.2 3.5 2.3 4.7 3.1 5.6 4.8
5.1 0.3 4.3 3.4 1.4 2.5 3.8 4.4 3.9 5.7

a Organise the data into an ordered stem-and-leaf plot.
b Find the median.
c Find the mode.
d Describe the data as symmetrical or skewed.

Example 14 Constructing back-to-back stem-and-leaf plots

A shop owner has two jeans shops. The daily sales in each shop over a 16-day period are monitored and recorded as follows.

Shop A: 3 12 12 13 14 14 15 15 21 22 24 24 24 26 27 28
Shop B: 4 6 6 7 7 8 9 9 10 12 13 14 14 16 17 27

a Draw a back-to-back stem-and-leaf plot with an interval of 10.
b Compare and comment on differences between the sales made by the two shops.

SOLUTION

a

Shop A		Shop B
3	0	4 6 6 7 7 8 9 9
5 5 4 4 3 2 2	1	0 2 3 4 4 6 7
8 7 6 4 4 4 2 1	2	7

1 | 3 means 13 sales

b Shop A has the highest number of daily sales. Its sales are generally between 12 and 28, with one day of very low sales of 3. Shop B sales are generally between 4 and 17 with only one high sale day of 27.

EXPLANATION

The data for each shop is already ordered. Stems are in intervals of 10. Record leaf digits for Shop A on the left and Shop B on the right.

Look at both sides of the plot for the similarities and differences.

Now you try

Two basketball teams scored the following points in their 10-game season:

Rockets: 71 62 75 88 73 67 64 72 68 78
Warriors: 87 76 82 94 88 75 81 76 97 89

a Draw a back to back stem-and-leaf plot with an interval of 10.
b Compare and comment on differences between the scores of the two teams.

Exercise 9H

FLUENCY	$1(\frac{1}{2})$, 2–4	$1(\frac{1}{2})$, 2–4	$1(\frac{1}{2})$, 3–5

Example 13

1 For each of the following data sets:

- **i** organise the data into an ordered stem-and-leaf plot
- **ii** find the median
- **iii** find the mode
- **iv** describe the data as symmetrical or skewed.

a 2.3 4.5 6.2 2.4 4.1 3.8 3.2 2.3 5.5 5.9 1.8 3.6 4.8 3.7

b 41 33 28 24 19 32 54 35 26 28 19 23 32 26 28

c 31 33 23 35 15 23 48 50 35 42 15 21 51 31 34 23 42 50 26 30 45 37 39 45 45

d 34.5 34.9 33.7 34.5 35.8 33.8 34.3 35.2 37.0 34.7
35.2 34.4 35.5 36.5 36.1 33.3 35.4 32.0 36.3 34.8

e 167 159 159 193 161 164 167 157 158 175 177 185
177 202 185 187 159 189 167 159 173 198 200

2 The number of vacant rooms in a motel each week over a 20-week period is shown below.

12 8 11 10 21 12 6 11 12 16 14 22 5 15 20 6 17 8 14 9

a Draw a stem-and-leaf plot of this data.

b In how many weeks were there fewer than 12 vacant rooms?

c Find the median number of vacant rooms.

Example 14

3 For each of the following sets of data:

- **i** draw a back-to-back stem-and-leaf plot
- **ii** compare and comment on the difference between the two data sets.

a **Set A:** 46 32 40 43 45 47 53 54 40 54 33 48 39 43
Set B: 48 49 31 40 43 47 48 41 49 51 44 46 53 44

b **Set A:** 0.7 0.8 1.4 8.8 9.1 2.6 3.2 0.3 1.7 1.9 2.5 4.1 4.3 3.3 3.4
3.6 3.9 3.9 4.7 1.6 0.4 5.3 5.7 2.1 2.3 1.9 5.2 6.1 6.2 8.3

Set B: 0.1 0.9 0.6 1.3 0.9 0.1 0.3 2.5 0.6 3.4 4.8 5.2 8.8 4.7 5.3
2.6 1.5 1.8 3.9 1.9 0.1 0.2 1.2 3.3 2.1 4.3 5.7 6.1 6.2 8.3

4 **a** Draw back-to-back stem-and-leaf plots for the final scores of St Kilda and Collingwood in the 24 games given here.

St Kilda:
126 68 78 90 87 118 88 125 111 117 82 82
80 66 84 138 109 113 122 80 94 83 106 68

Collingwood:
104 80 127 88 103 95 78 118 89 82 103 115
98 77 119 91 71 70 63 89 103 97 72 68

b In what percentage of games did each team score more than 100 points?

c Comment on the distribution of the scores for each team.

5 The data below gives the maximum temperature each day for a three-week period in spring.

18 18 15 17 19 17 21
20 15 17 15 18 19 19
20 22 19 17 19 15 17

Use a stem-and-leaf plot to determine:

a the number of days on which the temperature was higher than 18°C
b the median temperature
c the difference between the minimum and maximum temperatures.

PROBLEM-SOLVING

PROBLEM-SOLVING	6	6	6, 7

6 This stem-and-leaf plot shows the time taken, in seconds, by Helena to run 100 m in her last 25 races.

Stem	Leaf
14	9
15	4 5 6 6 7 7 7 8 9
16	0 0 1 1 2 2 3 4 4 5 5 5 7 7
17	2

14 | 9 represents 14.9 seconds

a Find Helena's median time.
b What is the difference between the slowest and fastest times?
c In her 26th race her time was 14.8 seconds. If this was added to the stem-and-leaf plot, would her median time change? If so, by how much?

7 Two brands of batteries were tested to determine their lifetimes in hours. The data below shows the lifetimes of 20 batteries of each brand.

Brand A: 7.3 8.2 8.4 8.5 8.7 8.8 8.9 9.0 9.1 9.2
9.3 9.4 9.4 9.5 9.5 9.6 9.7 9.8 9.9 9.9

Brand B: 7.2 7.3 7.4 7.5 7.6 7.8 7.9 7.9 8.0 8.1
8.3 9.0 9.1 9.2 9.3 9.4 9.5 9.6 9.8 9.8

a Draw a back-to-back stem-and-leaf plot for this data.
b How many batteries of each brand lasted more than 9 hours?
c Compare the two sets of data and comment on any similarities or differences.

REASONING

REASONING	8, 9	8–10	8, 10, 11

8 This ordered stem-and-leaf plot has some unknown digits.

Stem	Leaf
0	2 3 8
1	1 5 a 8
c	0 b 6 6 2
3	2 5 9

3 | 5 means 0.35

a What is the value of c?
b What is the smallest number in the data set?
c What values could the following pronumerals take?
i a **ii** b

9 The back-to-back stem-and-leaf plot below shows the birth weight in kilograms of babies born to mothers who do or don't smoke.

Birth weight of babies

Smoking mothers		Non-smoking mothers
4 3 2 2	2	4
9 9 8 7 6 6 5 5	2*	8 9
4 3 2 1 1 1 0 0 0	3	0 0 1 2 2 3
6 5 5	3*	5 5 5 6 6 7 7 8
1	4	
	4*	5 5 6

2 | 4 means 2.4 kg
2* | 5 means 2.5 kg

a What percentage of babies born to smoking mothers have a birth weight of less than 3 kg?
b What percentage of babies born to non-smoking mothers have a birth weight of less than 3 kg?
c Compare and comment on the differences between the birth weights of babies born to mothers who smoke and those born to mothers who don't smoke.

10 Explain why in a symmetrical distribution the mean is close to the median.

11 Find the median if all the data in each back-to-back stem-and-leaf plot was combined.

a

5	3	8 9
9 7 7 1	4	0 2 2 3 6 8
8 6 5 2 2	5	3 3 7 9
7 4 0	6	1 4

4 | 2 means 42

b

3	16	0 3 3 6 7 9
9 6 6 1	17	0 1 1 4 8 8
8 7 5 5 4 0	18	2 2 6 7
2	19	0 1

16 | 3 means 16.3

ENRICHMENT: How skewed?	–	–	12

12 Skewness can be positive or negative. If the tail of the distribution is pointing up in a stem-and-leaf plot (towards the smaller numbers), then we say the data is negatively skewed. If the tail is pointing in the reverse direction, then the data is positively skewed.

Stem	Leaf
1	3
2	1 4
3	0 2 7
4	1 1 3 8 9 9
5	0 4 4 5 7

3 | 2 means 32

Negatively skewed data

a Find the mean (correct to two decimal places) and the median for the above data.
b Which of the mean or median is higher for the given data? Can you explain why?
c Which of the mean or median would be higher for a set of positively skewed data? Why?
d What type of distribution would lead to the median and mean being quite close?

Applications and problem-solving

The following problems will investigate practical situations drawing upon knowledge and skills developed throughout the chapter. In attempting to solve these problems, aim to identify the key information, use diagrams, formulate ideas, apply strategies, make calculations and check and communicate your solutions.

Medical trials

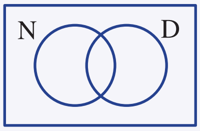
1 A team of medical researchers is involved in two trials. One trial will record the effect of a new drug on a group of patients with a certain condition and the second trial involves testing for a particular condition in its early stages.

The researchers need to be able to approach pharmaceutical companies to fund their drug trials and testing. To do so they need to be able to show some evidence of the impact of each trial to show that it is worth pursuing.

a For the first phase of the trial, 60 patients with the condition were involved in a drug trial. Half were given the new drug and half were given a placebo. The result recorded indicated whether or not a patient was still showing symptoms after one week.

After one week it was found that 32 patients showed no symptoms, with 10 of those on the placebo.

i Use this Venn diagram to summarise the results. D represents the set of patients taking the drug and N represents the set of patients with no symptoms after a week.



ii Find the probability that a randomly selected patient took the drug and had no symptoms after one week.

iii Find the probability that a randomly selected patient did not take the drug and had no symptoms after one week.

iv Based on your results to parts **ii** and **iii**, can you hypothesise whether you think the drug had an impact?

v If this drug was used to treat all patients who have the condition, based on the data what percentage could be expected to show no symptoms within a week? Round to the nearest per cent.

vi If you had to promote this drug to a pharmaceutical company or in an advertising campaign, which two statistics would you use based on the data?

The drug was also trialled on a group of patients who did not have the condition and some side effects were noted. The researchers determine that to be prescribed the drug, a test must be used in the early stages to try to determine if a patient's symptoms indicate they have the condition.

The researchers must first consider the accuracy of the test.

b The test was carried out on a group of 40 patients – some who went on to develop the condition and some who did not. The results of the test were that 29 people tested positive, and of those, 27 people had the condition. In total, 30 people had the condition.

i Summarise these findings in a two-way table where C is the set of patients who did have the condition and T is the set of patients who tested positive to the condition.

	T	**T′**	**Total**
C			
C′			
Total			

ii Based on the results, determine the probability of 'C and T' and the probability of 'C or T' and interpret these sets of patients.

iii A 'false positive' is a result in which a person tests positive but does not have the condition. From this data, what percentage of tests resulted in a false positive?

iv Based on this data, in what percentage of patients did the test not detect the condition when it was actually present?

v The researchers will receive extra funding to refine the test if fewer than 10% of patients get an incorrect result. Will the researchers get the extra funding?

Raising the average

2 Statisticians are now commonly connected to both sporting and commentary teams. Statistics are used to compare performances and to analyse opposition players and teams.

For batsmen in cricket, averages are particularly important. In cricket, if you finish an innings 'not out', the runs are added to your total runs but the average is calculated by dividing by the number of 'outs', not innings. For example, for the scores 30, 26 and 10 the average is 22, but for scores of 30, 26 and 10 not out the average is 33.

In a season of cricket in Australia, sports journalists spend time analysing statistics to highlight under-performing players and discuss possible selection issues by looking at scores achieved and averages, and by comparing players.

a A batsman is under pressure heading into the final Test match. He only gets to bat once in the match and he is keen to raise his average above 30. From the six previous completed innings he has played, he has averaged 26.5. What is the lowest score he can get to achieve this average if:

- **i** he gets out in the innings?
- **ii** he is not out at the end of the innings?

b Give a possible set of scores the batsman in part **a** had for his 7 innings if his mode score was 22, his median was 25, he achieved his average of at least 30 and he was out in each innings.

The opening batsman combination is considered not to have worked throughout the series. The team batted 10 times, but one of the batsman was injured in one game and only batted in 9 innings. Both openers were out each time they batted. Their scores are shown below.

Opener 1:	18	6	21	18	31	42	23	25	31	
Opener 2:	4	4	11	24	107	52	13	27	8	6

c **i** Display the data using a back-to-back stem-and-leaf plot and compare/comment on their performances.

- **ii** The commentators are calling for opener 1 to be left out of the team. Do you think this is reasonable? What statistic might they be basing this on?
- **iii** The data for opener 1 was incorrectly entered. The score of 21 is incorrect. Once the new score is entered, the median changes to 25 but the range is unchanged. What are the possible scores that opener 1 obtained in this innings?

d After the next three Test match series, a batsman is dropped from the team. In the two series this batsman had averaged 23 from 8 completed innings in the first series and 18.2 from 5 completed innings in the second series. What was the combined average for the two series, correct to one decimal place?

ISBN 978-1-108-77288-4

Cambridge University Press
Updated September 2021

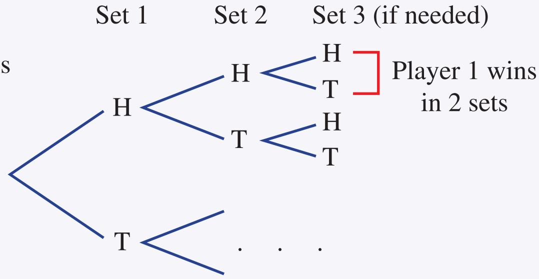
Winning odds

3 A betting agency sets odds on a range of sports around the world. They do so by analysing past performances and then setting the odds – trying to keep them in their favour!

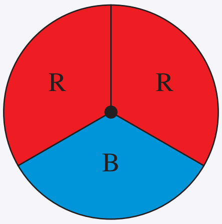
A particular up-and-coming betting agency is analysing the previous head-to-head performances of tennis players, in order to set the odds for the winner of the match, as well as updating the odds throughout a match. They use various simulations to predict the chances of different outcomes.

Two particular players have split the sets they have played against each other in the past 50–50. The match can be simulated with a coin, as each player is equally likely to win a set.
In women's matches, the winner is the first to win two sets.

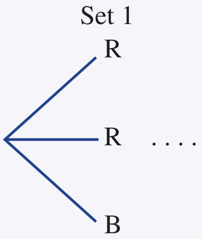
a Complete the tree diagram of a coin toss simulating the three possible sets. Player 1 wins the set if heads and player 2 wins if tails.
Use the tree diagram to find:
  i the probability that the match is completed in two sets
  ii the probability the winner of the match lost the first set.



In the next round, the head-to-head battle shows that one player has won two-thirds of the sets she has played against her opponent.
This can be simulated with a spinner which has three equal regions, two coloured red and one coloured blue.
The spinner is spun to determine the winner of each set. The third set will not be played if a winner has been determined.



b Complete a tree diagram to find the probability that:
  i the match lasted three sets
  ii the non-favourite wins at least one set.



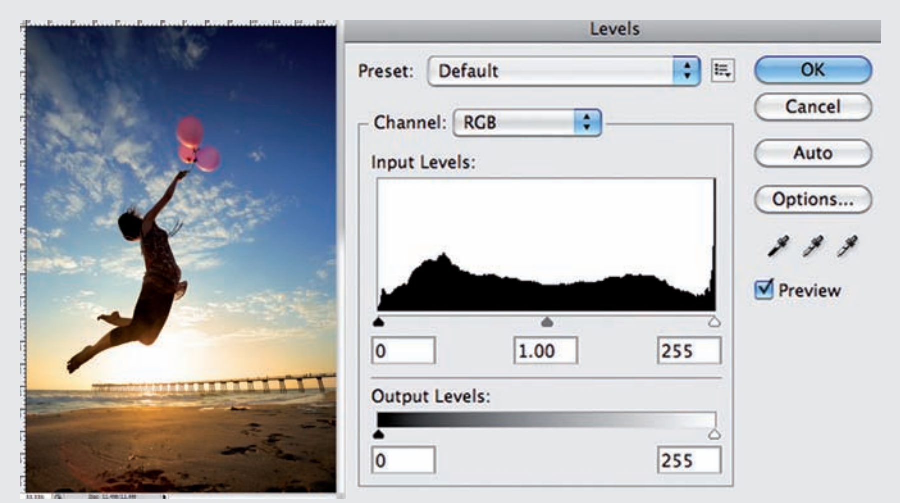
c At tournaments around the world, the scoring system in the final set and the tiebreaker is being altered. At a particular tournament the women's game is trialling a first to 7 tiebreaker that doesn't require a two-point advantage. Two close rivals are considered equally likely to win a point in a tiebreaker whether they are serving or not. Calculate the percentage chance of each player winning from the following positions:
  i 6 points to 5 (*Hint*: At most two points will need to be played.)
  ii 6 points to 4 (*Hint*: Consider how many points might be needed and how to simulate this.)

9I Grouped data

Learning intentions

- To know how to use a frequency table to record data in class intervals
- To be able to calculate the percentage frequency of an interval in a frequency table
- To know the types of data that can be displayed in a histogram
- To be able to display grouped numerical data in a histogram and interpret the graph

For some data, especially large sets, it makes sense to group the data and then record the frequency for each group to produce a frequency table. For numerical data, a graph generated from a frequency table gives a histogram. Like a stem-and-leaf plot, a histogram shows how the data is distributed across the full range of values. A histogram, for example, is used to display the level of exposure of the pixels in an image in digital photography. It uses many narrow columns to show how the luminance values are distributed across the scale from black to white.



Preset: Default

Channel: RGB

Input Levels: 0 1.00 255

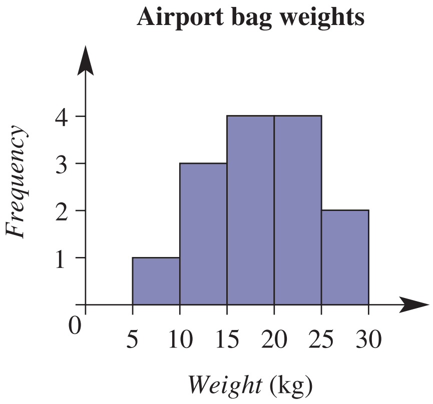
Output Levels: 0 255

A luminance value histogram is shown as used in digital photography software. The horizontal axis shows brightness levels in a range starting at zero (pure black) on the left and finishing at 255 (pure white) on the right. The vertical axis shows the number of pixels of each brightness value that the photo contains.

LESSON STARTER Baggage check

The histogram shows the distribution of the weight of a number of bags checked at the airport.

- How many bags had a weight in the range 10 to 15 kg?
- How many bags were checked in total?
- Is it possible to determine the exact mean, median or mode of the weight of the bags by looking at the histogram? Discuss.
- Describe the distribution of checked bag weights for the given graph.

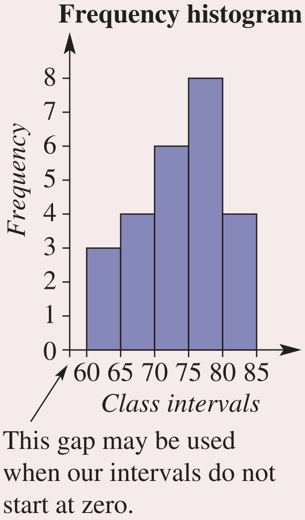

Airport bag weights

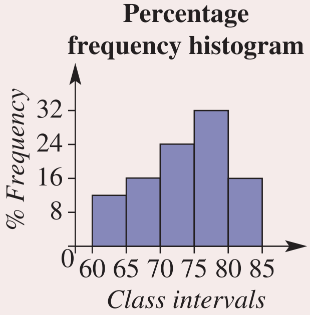
KEY IDEAS

- A **frequency table** shows the number of values within a set of categories or **class intervals**.
- Grouped numerical data can be illustrated using a **histogram**.
  - The height of a column corresponds to the frequency of values in that class interval.
  - There are usually no gaps between columns.
  - The scales are evenly spread with each bar spreading across the boundaries of the class interval.
  - A **percentage frequency histogram** shows the frequencies as percentages of the total.
- Like a stem-and-leaf plot, a histogram can show if the data is **skewed** or **symmetrical**.

Frequency table

Class interval	Frequency	Percentage frequency
60–	3	12
65–	4	16
70–	6	24
75–	8	32
80–85	4	16
Total	25	100





- In this frequency table, 70– includes numbers from 70 to less than 75.

BUILDING UNDERSTANDING

1 State the missing numbers in this frequency table, i.e. find the values of the pronumerals.

Class interval	Frequency	Percentage frequency
0–4	1	10
5–9	3	c
10–14	4	d
a–19	b	e
Total	10	f

2 This frequency histogram shows the number of books in which the number of pages is within a given interval for some textbooks selected from a school library.

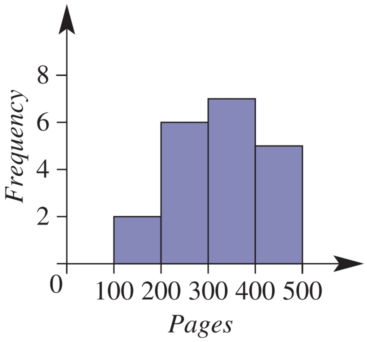
a How many textbooks had between 100 and 200 pages?

b How many textbooks were selected from the library?

c What percentage of textbooks had between:

  i 200 and 300 pages?

  ii 200 and 400 pages?



Example 15 Constructing frequency tables and histograms

The data below shows the number of hamburgers sold each hour by a 24-hour fast-food store during a 50-hour period.

1	10	18	14	20	11	19	10	17	21
5	16	7	15	21	15	10	22	11	18
12	12	3	12	8	12	6	5	14	14
14	4	9	15	17	19	6	24	16	17
14	11	17	18	19	19	19	18	18	20

a Set up and complete a grouped frequency table, using class intervals 0–, 5–, 10–, etc. Include a percentage frequency column.

b Construct a frequency histogram.

c In how many hours did the fast-food store sell:

- **i** fewer than 10 hamburgers?
- **ii** at least 15 hamburgers?

SOLUTION

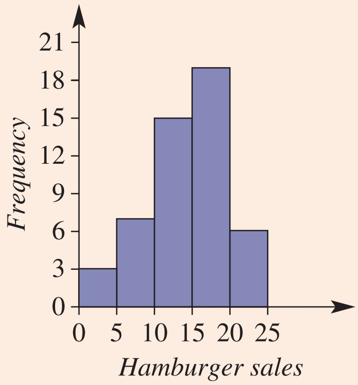
a

Class interval	Frequency	Percentage frequency
0–	3	6%
5–	7	14%
10–	15	30%
15–	19	38%
20–25	6	12%
Total	50	100%

EXPLANATION

Create class intervals of 5 from 0 up to 25, since 24 is the maximum number. Record the number of data values in each interval in the frequency column. Convert to a percentage by dividing by the total (50) and multiplying by 100.

b



Create a frequency histogram with frequency on the vertical axis and the class intervals on the horizontal axis. The height of the column shows the frequency of that interval.

c **i** 3 + 7 = 10 hours

Fewer than 10 hamburgers covers the 0–4 and 5–9 intervals.

ii 19 + 6 = 25 hours

At least 15 hamburgers covers the 15–19 and 20–24 intervals.

Now you try

The heights of 40 students were measured to the nearest centimetre and recorded.

160	149	153	143	179	159	152	161	169	157
167	163	148	175	173	150	175	160	170	162
145	158	163	164	172	178	169	171	153	165
152	161	170	174	166	165	158	157	152	148

a Set up and complete a grouped frequency table, using class intervals 140–, 145–, 150– etc. Include a percentage frequency column.

b Construct a frequency histogram.

c How many students are:

- **i** shorter than 150 cm?
- **ii** at least 170 cm tall?

Exercise 9I

FLUENCY	1, 2, 4	1, 3, 4	1, 3, 4

Example 15

1 The data below shows the number of ice-creams sold from an ice-cream van over a 50-day period.

0	5	0	35	14	15	18	21	21	36
45	2	8	2	2	3	17	3	7	28
35	7	21	3	46	47	1	1	3	9
35	22	7	18	36	3	9	2	2	11
37	37	45	11	12	14	17	22	1	2

a Set up and complete a grouped frequency table using class intervals 0–, 10–, 20–, etc. Include a percentage frequency column.

b Construct a frequency histogram.

c On how many days did the ice-cream van sell:

- **i** fewer than 20 ice-creams?
- **ii** at least 30 ice-creams?

Example 15

2 The data below shows the mark out of 100 on the Science exam for 60 Year 9 students.

50	67	68	89	82	81	50	50	89	52	60	82	52	60	87	89	71	73	75	83
86	50	52	71	80	95	87	87	87	74	60	60	61	63	63	65	82	86	96	88
50	94	87	64	64	72	71	72	88	86	89	69	71	80	89	92	89	89	60	83

a Set up and complete a grouped frequency table, using class intervals 50–, 60–, 70–, etc. Include a percentage frequency column, rounding to two decimal places where necessary.

b Construct a frequency histogram.

c **i** How many marks were less than 70 out of 100?

- **ii** What percentage of marks were at least 80 out of 100? Answer correct to one decimal place.

ISBN 978-1-108-77288-4

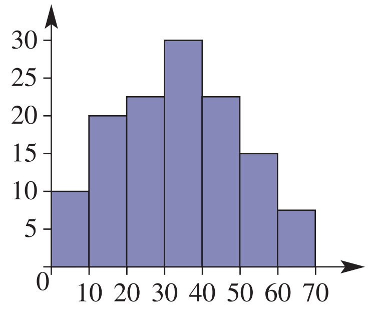
3 The number of goals kicked by a country footballer in each of his last 30 football matches is given below.

8 9 3 6 12 14 8 3 4 5 2 5 6 4 13
8 9 12 11 7 12 14 10 9 8 12 10 11 4 5

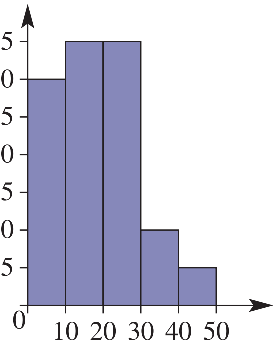
a Organise the data into a grouped frequency table using a class interval width of 3 starting at 0.
b Draw a frequency histogram for the data.
c In how many games did the player kick fewer than six goals?
d In how many games did he kick more than 11 goals?

4 Which one of these histograms illustrates a symmetrical data set and which one shows a skewed data set?

a



b



PROBLEM-SOLVING	5, 6	5, 6	5–7

5 Find the unknown numbers (pronumerals) in these frequency tables.

a

Class interval	Frequency	Percentage frequency
10–	*a*	15
15–	11	*b*
20–	7	*c*
25–	*d*	10
30–34	*e*	*f*
Total	40	*g*

b

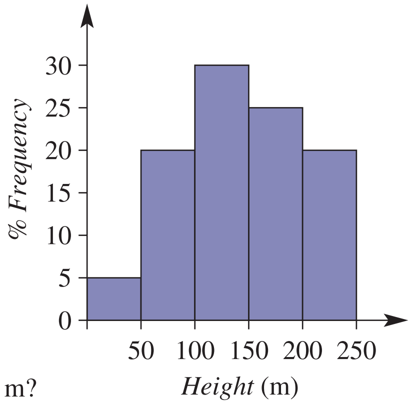
Class interval	Frequency	Percentage frequency
0–	*a*	2
3–	9	*b*
6–	*c*	16
9–	12	*d*
12–14	*e*	*f*
Total	50	*g*

6 This percentage frequency histogram shows the heights of office towers in a city.

a What percentage of office towers have heights of:
  i between 50 m and 100 m?
  ii less than 150 m?
  iii no more than 200 m?
  iv at least 100 m?
  v between 100 m and 150 m or greater than 200 m?

b If there are 100 office towers in the city, how many have a height of:
  i between 100 m and 150 m?
  ii at least 150 m?

c If the city had 40 office towers, how many have a height of:
  i between 0 m and 50 m?
  ii no more than 150 m?



7 The data below shows the length of overseas phone calls (in minutes) made by a particular household over a six-week period.

1.5	1	1.5	1.4	8	4	4	10.1	9.5	1	3
8	5.9	6	6.4	7	3.5	3.1	3.6	3	4.2	4.3
4	12.5	10.2	10.3	4.5	4.5	3.4	3.5	3.5	5	3.5
3.6	4.5	4.5	12	11	12	14	14	12	13	10.8
12.1	2.4	3.8	4.2	5.6	10.8	11.2	9.3	9.2	8.7	8.5

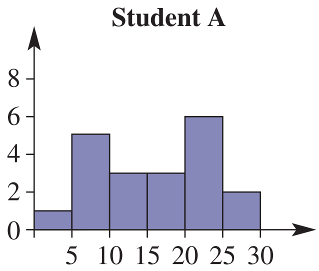
What percentage of phone calls were more than 3 minutes in length? Answer to one decimal place.

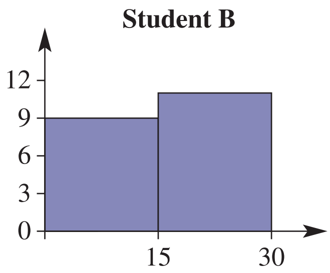
REASONING	8	8, 9	8, 10

8 Explain why you cannot work out the exact value for the mean, median and mode for a set of data presented in a histogram.

9 What can you work out from a frequency histogram that you cannot work out from a percentage frequency histogram? Completing Question **6** will provide a clue.

10 Two students show different histograms for the same set of data.


Student A


Student B

- **a** Which histogram is more useful in helping to analyse the data?
- **b** What would you advise student B to do differently when constructing the histogram?

ENRICHMENT: The distribution of weekly wages	–	–	11

11 The data below shows the weekly wages of 50 people in dollars.

400	500	552	455	420	424	325	204	860	894	464	379	563
940	384	370	356	345	380	720	540	654	678	628	656	670
740	750	730	766	760	700	700	768	608	576	890	920	874
450	674	725	612	605	600	548	670	230	725	860		

- **a** What is the minimum weekly wage and the maximum weekly wage?
- **b** **i** Organise the data into up to 10 class intervals.
  - **ii** Draw a frequency histogram for the data.
- **c** **i** Organise the data into up to 5 class intervals.
  - **ii** Draw a frequency histogram for the data.
- **d** Discuss the shapes of the two graphs. Which graph better represents the data and why?

Using CAS and non-CAS calculators 9I: Graphing grouped data

The activity is in the Interactive Textbook in the form of a printable PDF.

ISBN 978-1-108-77288-4

9J Measures of spread

Learning intentions

- To know how the range and interquartile range describe the spread of a data set
- To be able to find the quartiles of a data set with an odd or even number of data values
- To be able to find the range and interquartile range of a set of data

The mean, median and mode are three numbers that help define the centre of a data set; however, it is also important to describe the spread. Two teams of swimmers from different countries, for example, might have similar mean race times but the spread of race times for each team could be very different.

Planning future food supply requires statistics. Weather statisticians analyse rainfall and temperatures, providing measures of centre and spread, and climate trends. Agricultural statisticians analyse land use data, such as these irrigated vineyards near Mildura, Victoria.

LESSON STARTER Swim times

Two Olympic swimming teams are competing in the 4 × 100 m relay. The 100 m personal best times, in seconds, for the four members of each team are given.

Team A: 48.3 48.5 48.9 49.2

Team B: 47.4 48.2 49.0 51.2

- Find the mean time for each team.
- Which team's times are more spread out?
- What does the difference in the range of values for each team tell you about the spread?

KEY IDEAS

- Two measures that help to describe how data is spread are the **range** and **interquartile range**. These are called measures of spread.
- The **range** is the difference between the maximum and minimum values.
  - Range = maximum value − minimum value
- If a set of numerical data is placed in order from smallest to largest, then:
  - the middle number of the lower half is called the **lower quartile** (Q_1)
  - the middle number of the data is called the median (Q_2)
  - the middle number of the upper half is called the **upper quartile** (Q_3)
  - the difference between the upper quartile and lower quartile is called the **interquartile range** (IQR).

$$\text{IQR} = Q_3 - Q_1$$

- If there is an odd number of values, remove the middle value (the median) before calculating Q_1 and Q_3.
- An **outlier** is a value that is not in the vicinity of the rest of the data.

BUILDING UNDERSTANDING

1 State the missing words.

- **a** To find the range, you subtract the __________ from the __________.
- **b** To find the interquartile range, you subtract the __________ from the __________.

2 This ordered data set shows the number of hours of sleep 11 people had the night before.

3 5 5 6 7 7 7 8 8 8 9

- **a** State the range (the difference between the maximum and minimum values).
- **b** State the median (Q_2, the middle number).
- **c** Remove the middle number, and then find:
  - **i** the lower quartile (Q_1, the middle of the lower half)
  - **ii** the upper quartile (Q_3, the middle of the upper half).
- **d** Find the interquartile range (IQR, the difference between the upper quartile and lower quartile).

3 This ordered data set shows the number of fish Daniel caught in the 12 weekends that he went fishing during the year.

1 2 3 3 4 4 6 7 7 9 11 13

- **a** Find the range (the difference between the maximum and minimum values).
- **b** Find the median (Q_2, the middle number).
- **c** Split the data in half, and then find:
  - **i** the lower quartile (Q_1, the middle of the lower half)
  - **ii** the upper quartile (Q_3, the middle of the upper half).
- **d** Find the interquartile range (IQR, the difference between the upper quartile and lower quartile).

Example 16 Finding the range and quartiles for an odd number of data values

The following data values are the results for a school Mathematics test.

67 96 62 85 73 56 79 19 76 23 68 89 81

- **a** List the data in order from smallest to largest.
- **b** Find the range.
- **c** Find:
  - **i** the median (Q_2)
  - **ii** the lower quartile (Q_1)
  - **iii** the upper quartile (Q_3)
  - **iv** the interquartile range (IQR).

SOLUTION	EXPLANATION
a $19\ 23\ \underbrace{56\ 62}_{Q_1}\ 67\ 68\,]\,\underset{Q_2}{\textcircled{73}}\,[\,76\ 79\ \underbrace{81\ 85}_{Q_3}\ 89\ 96$	Order the data.
b Range $= 96 - 19$ $= 77$	Range $=$ maximum value $-$ minimum value.

Continued on next page

c **i** $Q_2 = 73$

The middle number is 73.

ii $Q_1 = \dfrac{56 + 62}{2}$
$= 59$

Q_1 (the middle value of the lower half) is halfway between 56 and 62. As there is an odd number of values, exclude the number 73 before finding Q_1 and Q_3.

iii $Q_3 = \dfrac{81 + 85}{2}$
$= 83$

Q_3 (the middle value of the upper half) is halfway between 81 and 85.

iv $\text{IQR} = 83 - 59$
$= 24$

$\text{IQR} = Q_3 - Q_1$

Now you try

The following data values are the percentage results for a student's spelling tests over a 9-week term.

83 90 76 81 72 84 92 78 86

a List the data in order from smallest to largest.
b Find the range.
c Find:
  i the median (Q_2)
  ii the lower quartile (Q_1)
  iii the upper quartile (Q_3)
  iv the interquartile range (IQR).

Example 17 Finding quartiles for an even number of data values

Here is a set of measurements collected by measuring the lengths, in metres, of 10 long-jump attempts.

6.7 9.2 8.3 5.1 7.9 8.4 9.0 8.2 8.8 7.1

a List the data in order from smallest to largest.
b Find the range.
c Find:
  i the median (Q_2)
  ii the lower quartile (Q_1)
  iii the upper quartile (Q_3)
  iv the interquartile range (IQR).
d Interpret the IQR.

SOLUTION

EXPLANATION

a $$5.1\quad 6.7\quad \underset{Q_1}{\boxed{7.1}}\quad 7.9\quad \underbrace{8.2\,|\,8.3}_{Q_2}\quad 8.4\quad \underset{Q_3}{\boxed{8.8}}\quad 9.0\quad 9.2$$

Order the data to locate Q_1, Q_2 and Q_3.

b $\text{Range} = 9.2 - 5.1$
$= 4.1\text{ m}$

Range = maximum value − minimum value.

c **i** $Q_2 = \dfrac{8.2 + 8.3}{2}$ $= 8.25$ m	Q_2 is halfway between 8.2 and 8.3.
ii $Q_1 = 7.1$ m	The middle value of the lower half is 7.1.
iii $Q_3 = 8.8$ m	The middle value of the upper half is 8.8.
iv IQR $= 8.8 - 7.1$ $= 1.7$ m	IQR is the difference between Q_1 and Q_3.
d The middle 50% of jumps differed by less than 1.7 m.	The IQR is the range of the middle 50% of the data.

Now you try

The weights, in kg, of 14 parcels lodged at the post office an hour before closing are listed:

1.6 0.8 2.3 2.6 3.1 1.2 1.8 2.0 2.3 3.5 1.7 1.1 0.4 2.8

a List the data in order from smallest to largest.
b Find the range.
c Find:
  i the median (Q_2)
  ii the lower quartile (Q_1)
  iii the upper quartile (Q_3)
  iv the interquartile range (IQR).
d Interpret the IQR.

Exercise 9J

FLUENCY	1(½), 2, 3	1(½), 2, 3	1(⅓), 3

Example 16

1 Consider the sets of data given below.
  i List the set of data in order from smallest to largest.
  ii Find the range.
  iii Find the median (Q_2).
  iv Find the lower quartile (Q_1).
  v Find the upper quartile (Q_3).
  vi Find the interquartile range (IQR).

a 5 7 3 6 2 1 9 7 11 9 0 8 5
b 38 36 21 18 27 41 29 35 37 30 30 21 26
c 180 316 197 176 346 219 183 253 228
d 256 163 28 520 854 23 367 64 43 787 12 343 76 3 28
e 1.8 1.9 1.3 1.2 2.1 1.2 0.9 1.7 0.8
f 10 35 0.1 2.3 23 12 0.02

Example 17

2 The top 10 highest box office takings for films, in millions of dollars, are given:

1405 1519 1672 2049 2187 2788 2068 1342 1516 1347

a Find the range.

b Find:

i the median (Q_2)

ii the lower quartile (Q_1)

iii the upper quartile (Q_3)

iv the interquartile range (IQR).

c Interpret the IQR.

3 The running time, in minutes, of 16 movies at the cinema were as follows:

123 110 98 120 102 132 112 140 120 139 42 96 152 115 119 128

a Find the range.

b Find:

i the median (Q_2)

ii the lower quartile (Q_1)

iii the upper quartile (Q_3)

iv the interquartile range (IQR).

c Interpret the IQR.

PROBLEM-SOLVING	4, 5	4–6	5, 6

4 The following set of data represents the sale price, in thousands of dollars, of 14 vintage cars.

89 46 76 41 12 52 76 97 547 59 67 76 78 30

a For the 14 vintage cars, find:

i the lowest price paid

ii the highest price paid

iii the median price

iv the lower quartile of the data

v the upper quartile of the data

vi the IQR.

b Interpret the IQR for the price of the vintage cars.

c If the price of the most expensive vintage car increased, what effect would this have on Q_1, Q_2 and Q_3? What effect would it have on the mean price?

5 Find the interquartile range for the data in these stem-and-leaf plots.

a

Stem	Leaf
3	4 8 9
4	1 4 8 8
5	0 3 6 9
6	2 6

5 | 2 means 52

b

Stem	Leaf
17	5 8
18	0 4 6 7
19	1 1 2 9 9
20	4 4 7 8
21	2 6 8

21 | 3 means 21.3

6 Over a period of 30 days, Lara records how many fairies she sees in the garden each day. The data is organised into this frequency table.

Number of fairies each day	0	1	2	3	4	5
Frequency	7	4	8	4	6	1

a Find the median number of fairies seen in the 30 days.

b Find the interquartile range.

c If Lara changes her mind about the day she saw 5 fairies and now says that she saw 10 fairies, would this change the IQR? Explain.

REASONING	7	7, 8	8, 9

7 Two data sets have the same range. Does this mean they have the same lowest and highest values? Explain.

8 A car yard has more than 10 cars listed for sale. One expensive car is priced at \$600 000 while the remaining cars are all priced near \$40 000. The salesperson realises that there is an error in the price for the expensive car – there is one too many zeros printed on the price.

- **a** Does the change in price for the expensive car change the value of the range? Give reasons.
- **b** Does the change in price for the expensive car change the value of the median? Give reasons.
- **c** Does the change in price for the expensive car change the value of the IQR? Give reasons.

9 **a** Is it possible for the range to equal the IQR? If so, give an example.

b Is it possible for the IQR to equal zero? If so, give an example.

ENRICHMENT: How many lollies in the jar?	–	–	10

10 The following two sets of data represent the number of jelly beans found in 10 jars purchased from two different confectionery stores, A and B.

Shop A: 25 26 24 24 28 26 27 25 26 28
Shop B: 22 26 21 24 29 19 25 27 31 22

- **a** Find Q_1, Q_2 and Q_3 for:
  - **i** shop A
  - **ii** shop B.
- **b** The top 25% of the data is above which value for shop A?
- **c** The lowest 25% of the data is below which value for shop B?
- **d** Find the interquartile range (IQR) for the number of jelly beans in the 10 jars from:
  - **i** shop A
  - **ii** shop B.
- **e** By looking at the given sets of data, why should you expect there to be a significant difference between the IQR of shop A and the IQR of shop B?
- **f** Which shop offers greater consistency in the number of jelly beans in each jar it sells?

9K Box plots EXTENDING

Learning intentions
- To understand how a box plot is used to display key values of a data set
- To be able to read summary statistics from a box plot and interpret quartiles
- To be able to construct a box plot for a data set

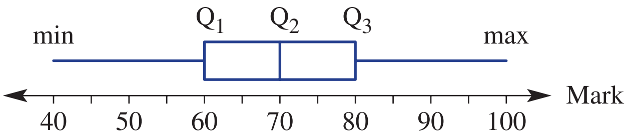
A box plot is a commonly used graph for a data set showing the maximum and minimum values, the median and the upper and lower quartiles. Box plots are often used to show how a data set is distributed and how two sets compare. Box plots are used, for example, to compare a school's examination performance against the performance of all schools in a state.

Every business seeks a competitive advantage from the statistical analysis of its data, including marketing, production, sales and profit. Data scientists who have computer algorithm skills in statistics and analytics are in high demand.

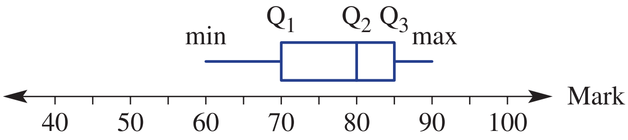
LESSON STARTER School performance

These two box plots show the performance of two schools on their English exams.

Box High School



Plot Secondary College

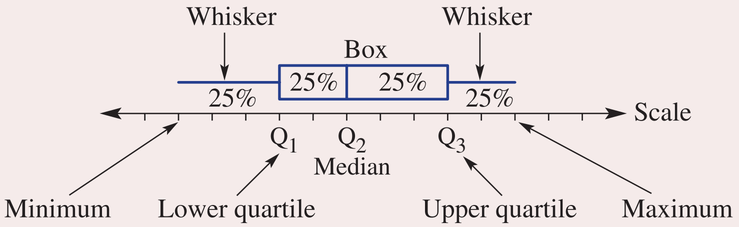


- Which school produced the highest mark?
- Which school produced the highest median (Q_2)?
- Which school produced the highest IQR?
- Which school produced the highest range?
- Describe the performance of Box High School against Plot Secondary College. Which school has better overall results and why?

ISBN 978-1-108-77288-4

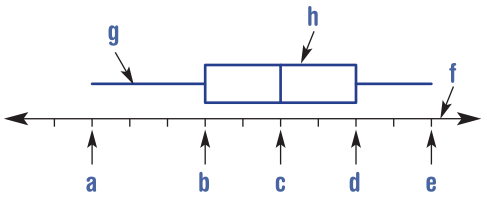
KEY IDEAS

- A **box plot** (also called a box-and-whisker plot) is a graph which shows:
  - maximum and minimum values
  - median (Q_2)
  - lower and upper quartiles (Q_1 and Q_3).
- A quarter (25%) of the data is spread across each of the four sections of the graph.



BUILDING UNDERSTANDING

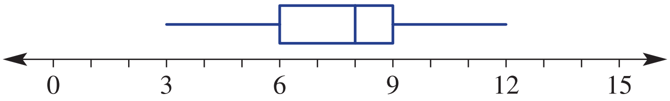
① State the names of the features labelled **a** to **h** on this box plot.



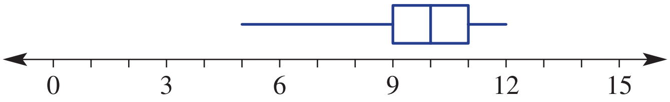
② Choose the correct box plot for a data set which has all these measures:

- minimum = 3
- maximum = 12
- median = 9
- lower quartile = 5
- upper quartile = 11.

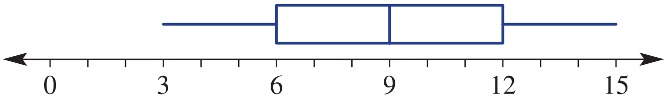
A



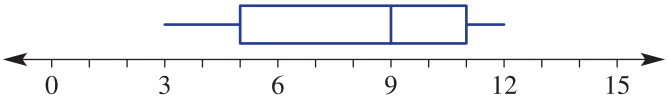
B



C



D



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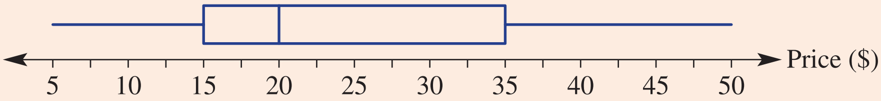
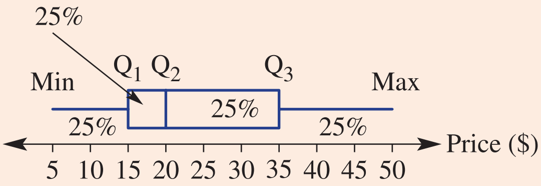
Example 18 Interpreting a box plot

This box plot summarises the price of all the books in a book shop.

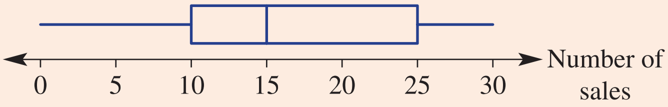


a State the minimum and maximum book prices.
b Find the range of the book prices.
c State the median book price.
d Find the interquartile range.
e Fifty per cent of the books are priced below what amount?
f Twenty-five per cent of the books are priced above what amount?
g If there were 1000 books in the store, how many would be priced below $15?

SOLUTION	EXPLANATION
a Minimum book price = \$5 Maximum book price = \$50	
b Range = \$50 − \$5 = \$45	Range = maximum price − minimum price
c Median book price = \$20	The median is Q_2.
d Interquartile range = \$35 − \$15 = \$20	Interquartile range = $Q_3 - Q_1$
e \$20	50% of books are below Q_2.
f \$35	The top 25% of books are above Q_3.
g 0.25 × 1000 = 250 250 books would be below \$15.	25% of books are priced below \$15.

Now you try

This box plot summarises the number of air-conditioning units sold at an electrical store each day in summer.



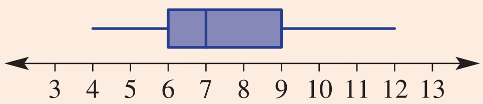
a State the minimum and maximum number of units sold in a day.
b Find the range of the number of sales.
c State the median number of sales.
d Find the interquartile range.
e Fifty per cent of days had sales below what number?
f Twenty-five per cent of days had sales above what number?
g If there were 80 days of sales, on how many days were there fewer than 10 sales?

Example 19 Constructing a box plot

The following set of data represents the 11 scores resulting from throwing two dice and adding their scores.

7 10 7 12 8 9 6 6 5 4 8

a Find:
  i the minimum value **ii** the maximum value **iii** the median
  iv the lower quartile **v** the upper quartile.
b Draw a box plot to represent the data.

SOLUTION	EXPLANATION
a $4\quad 5\quad \overset{Q_1}{\textcircled{6}}\quad 6\quad 7\quad \overset{Q_2}{\textcircled{7}}\quad 8\quad 8\quad \overset{Q_3}{\textcircled{9}}\quad 10\quad 12$	Order the data.
i Min. value = 4 **ii** Max. value = 12	Determine the minimum and maximum value.
iii $Q_2 = 7$ **iv** $Q_1 = 6$	The median is the middle value.
v $Q_3 = 9$	Remove the middle value and then locate the lower quartile and upper quartile.
b Box plot: Throwing two dice	Draw a scaled horizontal axis.
	Place the box plot above the axis marking in the five key statistics from part **a**.

Now you try

The following set of data represents the hours worked by a shift worker in their last 15 shifts.

6 8 4 2 6 6 7 3 5 5 6 8 7 3 5

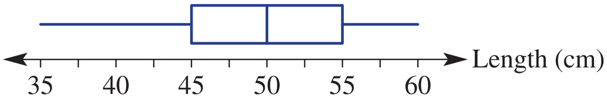
a Find:
  i the minimum value **ii** the maximum value **iii** the median
  iv the lower quartile **v** the upper quartile.
b Draw a box plot to represent the data.

Exercise 9K

FLUENCY	1, 2, 3(1/2)	1, 3(1/2)	2, 3(1/2)

Example 18

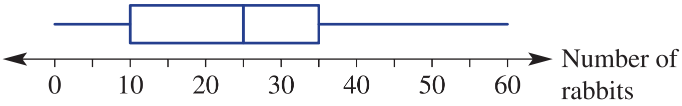
1 This box plot summarises the lengths of babies born in a particular week in a hospital.



a State the minimum and maximum lengths of babies.
b Find the range of the length of the babies.
c State the median baby length.
d Find the interquartile range.
e Fifty per cent of the babies were born shorter than what length?
f Twenty-five per cent of the babies were born longer than what length?
g If there were 80 babies born in the week, how many would be expected to be less than 45 cm in length?

Example 18

2 This box plot summarises the number of rabbits spotted per day in a paddock over a 100-day period.



a State the minimum and maximum number of rabbits spotted.
b Find the range of the number of rabbits spotted.
c State the median number of rabbits spotted.
d Find the interquartile range.
e On 75% of the days, the number of rabbits spotted is below what number?
f On 50% of the days, the number of rabbits spotted is more than what number?
g On how many days was the number of rabbits spotted less than 10?

Example 19

3 Consider the sets of data below.
- **i** State the minimum value.
- **ii** State the maximum value.
- **iii** Find the median (Q_2).
- **iv** Find the lower quartile (Q_1).
- **v** Find the upper quartile (Q_3).
- **vi** Draw a box plot to represent the data.

a 2 2 3 3 4 6 7 7 7 8 8 8 8 9 11 11 13 13 13
b 43 21 65 45 34 42 40 28 56 50 10 43 70 37 61 54 88 19
c 435 353 643 244 674 364 249 933 523 255 734
d 0.5 0.7 0.1 0.2 0.9 0.5 1.0 0.6 0.3 0.4 0.8 1.1 1.2 0.8 1.3 0.4 0.5

PROBLEM-SOLVING	4, 5	4, 5	5, 6

4 The following set of data describes the number of cars parked in a street on 18 given days.

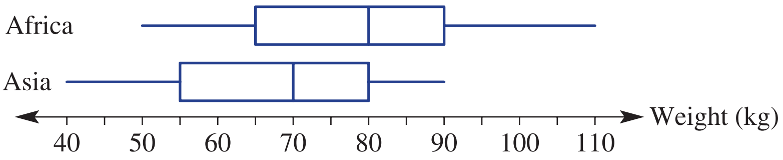
14 26 39 46 13 30 5 46 37 26 39 8 8 9 17 48 29 27

a Represent the data as a box plot.

b State the percentage of days on which the number of cars parked on the street was between:

i 4 and 48 **ii** 13 and 39 **iii** 5 and 39 **iv** 39 and 48.

5 The weights of a sample of adult leopards from Africa and Asia are summarised in these box plots.



a Which leopard population sample has the highest minimum weight?

b What is the difference between the ranges for the two population samples?

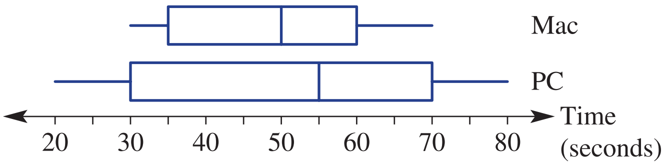
c Is the IQR the same for both leopard samples? If so, what is it?

d **i** What percentage of African leopards have a weight less than 80 kg?

ii What percentage of Asian leopards have a weight less than 80 kg?

e A leopard has a weight of 90 kg. Is it likely to be an Asian or African leopard?

6 The time that it takes for a sample of computers to start up is summarised in these box plots.



a What type of computer has the lowest median?

b What percentage of Mac computers started up in less than 1 minute?

c What percentage of PC computers took longer than 55 seconds to start up?

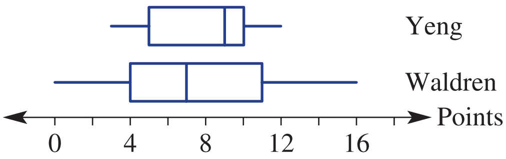
d What do you notice about the range for Mac computers and the IQR for PC computers? What does this mean?

ISBN 978-1-108-77288-4

REASONING

7 7–9 8–10

7 The number of points per game for two basketball players over a season is summarised in these box plots.



a Which player has the highest maximum?
b Which player has the highest median?
c Which player has the smallest IQR?
d Which player is a more consistent basketball scorer? Give reasons.
e Which player is more likely to have scored the greatest number of points? Give reasons.

8 Give an example of a small data set which has the following.
a Maximum = upper quartile
b Median = lower quartile

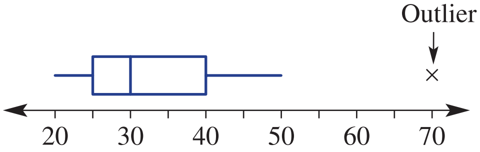
9 Does the median always sit exactly in the middle of a box on a box plot? Explain.

10 Could the mean of a data set be greater than Q_3? Explain.

ENRICHMENT: Outliers and battery life

– – 11

11 Outliers on box plots are shown as separate points, as shown in the following box plot.



The life, in months, of a particular battery used in a high-powered calculator is shown in this data set.

3 3 3 4 4 5 6 6 6 7 8 8 9 17

a Use all the values to calculate Q_1, Q_2 and Q_3 for the data set.
b Do any of the values appear to be outliers?
c Not including the outlier, what is the next highest value?
d Draw a box plot for the data using a cross (×) to show the outlier.
e Can you give a logical reason for the outlier?

Using CAS and non-CAS calculators 9K: Finding measures of spread and drawing box plots

The activity is in the Interactive Textbook in the form of a printable PDF.

The missing data

Gregory is a marine biologist and is collecting and analysing data regarding shark and dolphin numbers in a particular area of the Great Barrier Reef.

He collects data each day for 14 days, but unfortunately two pieces of data are lost due to a computer glitch as shown.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Sharks	15	12	4	$?_1$	15	11	19	5	1	10	12	6	24	11
Dolphins	4	7	2	3	2	6	4	2	$?_2$	4	7	2	4	6

Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate.

Preliminary task

a If Gregory assumes that $?_1 = 10$, find the following correct to one decimal place.
  i The mean and the median number of sharks recorded.
  ii The range and interquartile range for the number of sharks recorded.

b If Gregory assumes that $?_2 = 5$, find the following correct to one decimal place.
  i The mean and the median number of dolphins recorded.
  ii The range and interquartile range for the number of dolphins recorded.

c Which set of data (dolphin or shark) appears to be more inconsistent and spread out?

Modelling task

Formulate

a The problem is to determine the possible values of $?_1$ and $?_2$ so that Gregory can complete his report based on some statistics that he can remember. Write down all the relevant information that will help solve this problem.

Solve

b Gregory seems to remember that the range for the number of dolphins was 7. Determine the value of $?_2$ if Gregory's memory is correct.

c From memory, Gregory thought that the mean number of sharks over the 14 days was approximately 12. Determine the likely value for $?_1$.

d From memory, Gregory thought that the mean number of dolphins over the 14 days was approximately 5. Determine the likely value for $?_2$.

e If the median for the number of sharks was equal to 11, find the possible values of $?_1$.

f If the interquartile range of the number of dolphins is 4, determine the possible values of $?_2$

Evaluate and verity

g If in fact the mean number of dolphins over the 14-day period was 4 after rounding to the nearest integer, find all the possible values of $?_2$.

h If in fact the interquartile range for the number of sharks was 9, find all the possible values of $?_1$.

Communicate

i Summarise your results and describe any key findings.

Extension questions

a Decide if it is possible for the median of either set of data to be anything other than an integer. If so, determine what values of $?_1$ and $?_2$ to allow this to happen.

b Is it possible to choose values for $?_1$ and $?_2$ so that the interquartile range for sharks is exactly double that for dolphins? If so, explain how this is possible.

Investigation

How many in the bag?

For this activity you will need:

- a bag or large pocket
- different-coloured counters
- paper and pen.

Five counters

a Form pairs and then, without watching, have a third person (e.g. a teacher) place five counters of two different colours into the bag or pocket. An example of five counters could be two red and three blue, but at this point in the activity you will not know this.

b Without looking, one person selects a counter from the bag while the other person records its colour. Replace the counter in the bag.

c Repeat part **b** for a total of 100 trials. Record the results in a table similar to this one.

Colour	Tally	Frequency
Red	𝍸 III …	…
Blue	𝍸 𝍸 II …	…
Total	100	100

d Find the experimental probability for each colour. For example, if 42 red counters were recorded, then the experimental probability $= \frac{42}{100} = 0.42$.

e Use these experimental probabilities to help estimate how many of each colour of counter are in the bag. For example, 0.42 is close to $0.4 = \frac{2}{5}$, therefore guess two red and three blue counters. Use this table to help.

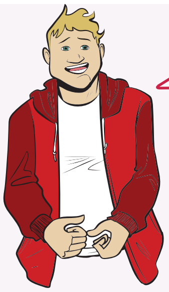
Colour	Frequency	Experimental probability	Closest multiple of 0.2, e.g. 0.2, 0.4, …	Guess of how many counters of this colour
Total	**100**	**1**	**1**	**5**

f Now take the counters out of the bag to see if your estimate is correct.

More colours and counters

a Repeat the steps above but this time use three colours and 8 counters.

b Repeat the steps above but this time use four colours and 12 counters.



Up for a challenge? If you get stuck on a question, check out the 'Working with unfamiliar problems' poster at the end of the book to help you.

1 A fair coin is tossed 5 times.
  - **a** How many outcomes are there?
  - **b** Find the probability of obtaining at least 4 tails.
  - **c** Find the probability of obtaining at least 1 tail.

2 Three cards, A, B and C, are randomly arranged in a row.
  - **a** Find the probability that the cards will be arranged in the order A B C from left to right.
  - **b** Find the probability that the B card will take the right-hand position.

3 Four students, Rick, Belinda, Katie and Chris, are the final candidates for the selection of school captain and vice-captain. Two of the four students will be chosen at random to fill the positions.
  - **a** Find the probability that Rick will be chosen for:
    - **i** captain
    - **ii** captain or vice-captain.
  - **b** Find the probability that Rick and Belinda will be chosen for the two positions.
  - **c** Find the probability that Rick will be chosen for captain and Belinda will be chosen for vice-captain.
  - **d** Find the probability that the two positions will be filled by two boys or two girls.

4 State what would happen to the mean, median and range of a data set in these situations.
  - **a** Five is added to each value in the data set.
  - **b** Each value in the data set is doubled.
  - **c** Each value in the data set is doubled and then decreased by 1.

5 Three pieces of fruit have an average weight of m grams. After another piece of fruit is added, the average weight doubles. Find the weight of the extra piece of fruit in terms of m.

6 **a** Five different data values have a range and median equal to 7. If two of the values are 3 and 5, what are the possible combinations of values?
  - **b** Four data values have a range of 10, a mode of 2 and a median of 5. What are the four values?

7 Five integer scores out of 10 are all greater than 0. If the median is x, the mode is one more than the median and the mean is one less than the median, find all the possible sets of values if $x < 7$.

8 Thomas works in the school office for work experience. He is given four letters and four addressed envelopes. What is the probability that Thomas, who is not very good at his job, places none of the four letters into its correct envelope?

Chapter summary

Probability and statistics

Probability review

The sample space is the list of all possible outcomes of an experiment.

For equally likely outcomes:

$$\Pr(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

e.g. roll a normal six-sided die

$\Pr(>4) = \frac{2}{6} = \frac{1}{3}$

$0 \le \Pr(\text{event}) \le 1$

$\Pr(\text{not } A) = 1 - \Pr(A)$

Measures of spread

Range = maximum value − minimum value

Interquartile range (IQR) = upper quartile (Q_3) − lower quartile (Q_1)

IQR is the range of the middle 50% of data.

e.g. To find Q_1 and Q_3, first locate median (Q_2) and then find middle values of upper and lower half.

e.g. $$1 \quad 2 \quad \underset{Q_1}{\textcircled{5}} \quad 6 \quad \underbrace{7 \mid 9}_{Q_2 = 8} \quad 10 \quad \underset{Q_3}{\textcircled{12}} \quad 12 \quad 15$$

$\text{IQR} = 12 - 5$

$= 7$

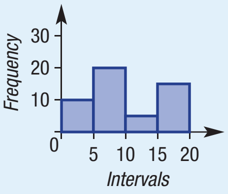
For odd number of values exclude median from each half.

Grouped data

Data values can be grouped into class intervals, e.g. 0–4, 5–9, etc. and recorded in a frequency table.

The frequency or percentage frequency of each interval can be recorded in a histogram.

e.g.



Stem-and-leaf plots

These display all the data values using a stem and a leaf.

An ordered back-to-back stem-and-leaf plot compares two data sets:

Leaf	Stem	Leaf
9 8 7 2	1	0 3
7 4 3 3	2	2 2 4
5 2 1	3	3 6 7 8
7	4	4 5 9
0	5	0

skewed

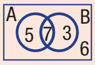
symmetrical

3 | 5 means 35

key

Venn diagrams and two-way tables

These organise data from two or more categories.



Venn diagram

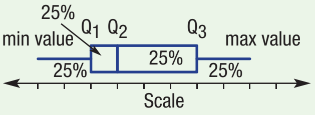
	A	A′	
B	7	3	10
B′	5	6	11
	12	9	21

i.e. 6 is in neither category, 7 is in both categories

$\Pr(A) = \frac{12}{21} = \frac{4}{7}$

$\Pr(B \text{ only}) = \frac{3}{21} = \frac{1}{7}$

Box plots (Ext)



25% of data is in each of the four sections.

Summarising data: Measures of centre

Mode is the most common value (there can be more than one). Two modes means the data is bi-modal.

Mean = average

$$= \frac{\text{sum of all values}}{\text{number of values}}$$

Median is the middle value of data that is ordered.

odd data set

$$2 \quad 4 \quad \underset{\text{median}}{\textcircled{7}} \quad 10 \quad 12$$

even data set

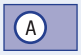
$$2 \quad 4 \quad \underbrace{6 \quad 10}_{\substack{8 \\ \text{median}}} \quad 15 \quad 18$$

An outlier is a value that is not in the vicinity of the rest of the data.

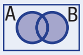
Set notation

Within a sample space are a number of subsets.

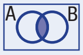
A′ = not A



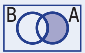
A ∪ B means A or B; the union of A and B.



A ∩ B means A and B; the intersection of A and B.



A only is the elements in A but not in B.



$n(A)$ is the number of elements in A

ϕ is the empty or null set

e.g. $A = \{1, 2, 3, 4, 5\}$ $B = \{2, 4, 6, 8\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 8\}$

$A \cap B = \{2, 4\}$ $n(B) = 4$

$B \text{ only} = \{6, 8\}$

$3 \in A$ means 3 is an element of A

Two-step experiments

These can be represented by tables or tree diagrams. They involve more than one component and can occur with or without replacement.

e.g. a bag contains 2 blue counters and 1 green; 2 are selected at random.

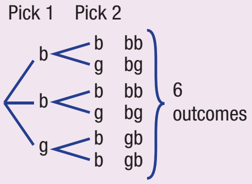
(I) Table with replacement

		Pick 1		
		b	b	g
Pick 2	b	(b, b)	(b, b)	(g, b)
	b	(b, b)	(b, b)	(g, b)
	g	(b, g)	(b, g)	(g, g)

Sample space of 9 outcomes

$\Pr(2 \text{ of same colour}) = \frac{5}{9}$

(II) Tree without replacement



$\Pr(2 \text{ of same colour}) = \frac{2}{6} = \frac{1}{3}$

Experimental probability

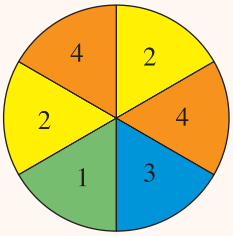
This is calculated from results of an experiment or survey.

Experimental probability

$$= \frac{\text{number of times event occurs}}{\text{total number of trials}}$$

Expected number of occurrences = probability × number of trials

Chapter checklist: Success criteria

		✔
9A	**1. I can find probabilities of events.** e.g. This spinner has six equally divided sections. List the sample space and find both Pr(not a 2) and Pr(a 1 or a 2).  	
9B	**2. I can use a Venn diagram.** e.g. A survey of 30 people in 2019 found that 12 thought Nick Kyrgios would win a grand slam tennis event in his career and 16 thought Alex De Minaur would win a grand slam tennis event in his career. Of those, 6 thought both would win a grand slam event while 8 thought neither would win one. Construct a Venn diagram. Use this to state how many people do not think Nick Kyrgios will win one as well as the probability that a randomly chosen person thinks only Alex De Minaur will win one.	
9B	**3. I can use a two-way table.** e.g. On a particular day at a car service station, 20 cars are serviced: 12 cars need a wheel alignment, 10 cars need their brake fluid replaced and 4 require both. Illustrate the situation using a two-way table. Use the table to find how many cars require neither, and the probability that a randomly selected car needs only a wheel alignment.	
9C	**4. I can use set notation.** e.g. A number is chosen from the set of integers between 1 and 10 inclusive. Let A be the set of even numbers between 1 and 10 inclusive and B the set of factors of 24 between 1 and 10 inclusive. Draw a Venn diagram showing the number of elements in sets A and B. **i.** List the sets $A \cup B$ and B only. **ii.** Find $n(A')$ and $\Pr(A \cap B)$.	
9D	**5. I can use an array to find the sample space for events with replacement.** e.g. Two four-sided dice numbered 1, 2, 3, 4 are rolled. Draw a table to list the sample space and find the probability of obtaining two even numbers.	
9D	**6. I can use an array to find the sample space for events without replacement.** e.g. Two letters are chosen at random from the word GREEN without replacement. List the outcomes in a table and find the probability that at least one of the letters is an E.	
9E	**7. I can construct a tree diagram.** e.g. A game involves tossing three coins. Complete a tree diagram to show all possible outcomes and find the probability of tossing exactly 2 heads.	
9E	**8. I can construct a tree diagram involving without replacement.** e.g. Two donuts are selected from a box containing three donuts with strawberry (S), chocolate (C) and banana (B) icing. List all the possible outcomes for the selection in a tree diagram and find the probability that the selection will contain the chocolate iced donut.	
9F	**9. I can find the experimental probability.** e.g. A container contains an unknown number of coloured counters. A counter is drawn from the container and then replaced. This is repeated 50 times and the colour of the counter is recorded each time. Given that 20 green counters were recorded, find the experimental probability for selecting a green counter, and find the expected number of green counters if there were 300 counters in total in the container.	
9G	**10. I can find the mean, median and mode.** e.g. Find the mean, median and mode for this data set: 8 8 3 6 4 4	

Chapter checklist

✔

9G **11. I can find a data value for a required mean.**
e.g. A student's quiz marks for the first 7 weeks of term were: 4, 8, 6, 5, 6, 7, 6. Calculate the mean of these marks and find the score needed in the 8th week for the mean to equal 6.5.

9H **12. I can construct and use a stem-and-leaf plot.**
e.g. Organise the data below into an ordered stem-and-leaf plot and find the median and mode. State whether the data is symmetrical or skewed.
0.4 3.2 2.4 1.7 4.4 2.1 4.1 2.7 3.2

9H **13. I can construct a back-to-back stem-and-leaf plot.**
e.g. The maximum daily temperatures for two world cities over a two-week period are recorded as:
City A: 21 18 24 26 30 26 22 19 18 24 19 28 27 32
City B: 12 12 9 15 10 13 20 21 21 17 22 18 17 9
Draw a back-to-back stem-and-leaf plot with an interval of 10 and compare and comment on the differences between the maximum temperatures for the two cities.

9I **14. I can construct frequency tables and histograms.**
e.g. For the data below showing the ice-cream sales each hour of operation over a week, set up and complete a grouped frequency table using class intervals 0–, 5–, etc.

15 22 1 0 20 3 10 2 23 3 11 14 19 3
16 14 24 7 1 15 5 7 2 29 28 3 2 13
26 20 22 25 6 7 12 6 13 7 7 20 11 11

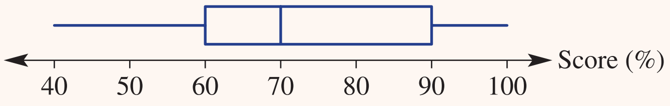
From the table, construct a frequency histogram and state for how many hours there were sales of 15 or more ice-creams.

9J **15. I can find the range.**
e.g. Find the range for this data set: 7 17 21 34 22 18 8 3 16

9J **16. I can find the quartiles for an odd number of data values.**
e.g. List the data below in order from smallest to largest and find the median (Q_2), the lower quartile (Q_1), the upper quartile (Q_3) and the interquartile range (IQR).
7 17 21 34 22 18 8 3 16

9J **17. I can find the quartiles for an even number of data values.**
e.g. List the data below in order from smallest to largest and find the median (Q_2), the lower quartile (Q_1), the upper quartile (Q_3) and the interquartile range (IQR). Interpret the IQR.
12 21 18 8 4 16 24 11 27 10

9K **18. I can interpret a box plot.**
e.g. This box plot summarises the test results (%) of a class of students.



Ext

i. Find the range and interquartile range of the results.
ii. Twenty-five per cent of results were above what number?
iii. If there were 24 students in the class, how many scored above 70%?

9K **19. I can construct a box plot.**
e.g. For the data shown below, find the minimum and maximum values, the median and lower and upper quartiles and draw a box plot to represent the data.
18 15 13 14 21 25 17 11 15 16 18 20 24 19

Ext

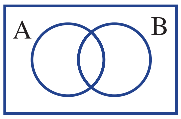
Short-answer questions

9A **1** Determine the probability of each of the following.
- **a** Rolling more than 2 on a normal six-sided die
- **b** Selecting a vowel from the word EDUCATION
- **c** Selecting a pink or white jelly bean from a packet containing 4 pink, 2 white and 4 black jelly beans

9B **2** From a survey of 50 people, 30 have the newspaper delivered, 25 read it online, 10 do both and 5 do neither.
- **a** Construct a Venn diagram for the survey results.
- **b** How many people only read the newspaper online?
- **c** If one of the 50 people was randomly selected, find:
  - **i** Pr(have paper delivered and read it online)
  - **ii** Pr(don’t have it delivered)
  - **iii** Pr(only read it online).

9B/C **3**
- **a** Copy and complete this two-way table.
- **b** Convert the information into a Venn diagram like that shown.

	A	**A′**	**Total**
B		16	
B′	8		20
Total	17		



- **c** Find the following probabilities.
  - **i** $\Pr(B')$
  - **ii** $\Pr(A \cap B)$
  - **iii** $n(A \text{ only})$
  - **iv** $n(A \cup B)$

9D **4** A spinner with equal areas of red, green and blue is spun and a four-sided die numbered 1 to 4 is rolled.
- **a** Complete a table like the one shown and state the number of outcomes in the sample space.
- **b** Find the probability that the outcome:
  - **i** is red and an even number
  - **ii** is blue or green and a 4
  - **iii** does not involve blue.

		Die			
		1	**2**	**3**	**4**
Spinner	**red**	(red, 1)	(red, 2)		
	green				
	blue				

9E **5** Libby randomly selects two coins from her pocket *without replacement*. Her pocket contains a \$1 coin, and two 10-cent coins.
- **a** List all the possible combinations using a tree diagram.
- **b** If a chocolate bar costs \$1.10, find the probability that she can hand over the two coins to pay for it.

9F

6 A quality controller records the frequency of the types of chocolates from a sample of 120 from its production line.

Centre	Soft	Hard	Nut
Frequency	50	22	48

a What is the experimental probability of randomly selecting a nut centre?
b In a box of 24 chocolates, how many would be expected to have a non-soft centre?

9G

7 Claudia records the number of emails she receives each weekday for two weeks as follows.

30 31 33 23 29 31 21 15 24 23

Find:

a the mean
b the median
c the mode.

9H

8 Two mobile phone salespeople are both aiming for a promotion to be the new assistant store manager. The best salesperson over a 15-week period will get the promotion. The number of mobile phones they sold each week is recorded below.

Employee 1: 21 34 40 38 46 36 23 51 35 25 39 19 35 53 45
Employee 2: 37 32 29 41 24 17 28 20 37 48 42 38 17 40 45

a Draw an ordered back-to-back stem-and-leaf plot for the data.
b For each employee, find:
  i the median number of sales
  ii the mean number of sales.
c By comparing the two sets of data, state, with reasons, who you think would get the promotion.
d Describe each employee's data as approximately symmetrical or skewed.

9I

9 The data below represents the finish times, in minutes, of 25 competitors in a local car rally.

134 147 162 164 145 159 151 143 136 155 163 157 168
128 144 161 158 136 178 152 167 154 161 152 171

a Record the data in a frequency table in class intervals of 10 minutes. Include a percentage frequency column.
b Construct a frequency histogram.
c Determine:
  i the number of competitors who finished in less than 140 minutes
  ii the percentage of competitors who finished with times of between 130 and 160 minutes.

9J/K Ext

10 Scott scores the following runs in each of his innings across the course of a cricket season:

20 5 34 42 10 3 29 55 25 37 51 12 34 22

a Find the range.

b Construct a box plot to represent the data by first finding the quartiles.

c From the box plot, 25% of his innings were above what number of runs?

Multiple-choice questions

9A

1 A letter is randomly chosen from the word XYLOPHONE. The probability that it is an O is:

A $\frac{1}{8}$ **B** $\frac{2}{9}$ **C** $\frac{1}{4}$ **D** $\frac{1}{9}$ **E** $\frac{1}{3}$

9B

2 The values of x and y in the two-way table are:

A $x = 12, y = 8$

B $x = 12, y = 11$

C $x = 16, y = 4$

D $x = 10, y = 1$

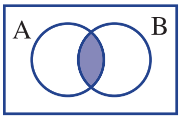
E $x = 14, y = 6$

	A	**not A**	**Total**
B		5	9
not B	8	y	
Total	x		25

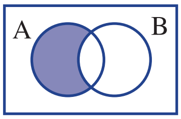
9C

3 Which shaded region represents $A \cup B$?

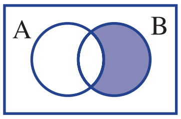
A



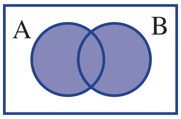
B



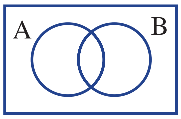
C



D



E



9D/E

4 A bag contains 2 green balls and 1 red ball. Two balls are randomly selected *without replacement*. The probability of selecting one of each colour is:

A $\frac{1}{2}$ **B** $\frac{2}{3}$ **C** $\frac{5}{6}$ **D** $\frac{1}{3}$ **E** $\frac{3}{4}$

9F

5 Students roll a biased die and find that the experimental probability of rolling a 5 is 0.3. From 500 rolls of the die, the expected number of 5s would be:

A 300 **B** 167 **C** 180 **D** 150 **E** 210

9H

6 The median of the data in this stem-and-leaf plot is:

A 74 **B** 71 **C** 86

D 65 **E** 70

Stem	Leaf
5	3 5 8
6	1 4 7
7	0 2 4 7 9
8	2 6 6
7	4 means 74

9G

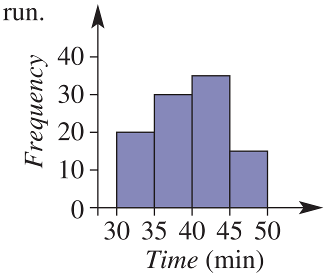
7 If Jacob achieved scores of 12, 9, 7 and 12 on his last four language vocabulary tests, what score must he get on the fifth test to have a mean of 11?

A 16 **B** 14 **C** 11 **D** 13 **E** 15

9I

8 This frequency histogram shows the times of competitors in a fun run. The percentage of competitors who finished in better than 40 minutes was:

A 55% **B** 85% **C** 50%
D 62.5% **E** 60%



9J

9 Consider the set of ordered data below.

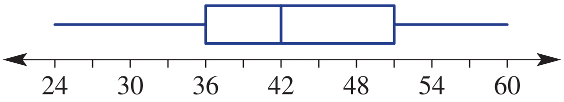
1.1 2.3 2.4 2.8 3.1 3.4 3.6 3.8 3.8 4.1 4.5 4.7 4.9

The interquartile range is:

A 2–5 **B** 1.7 **C** 3.8 **D** 1.3 **E** 2.1

9K Ext

10 Choose the incorrect statement about the box plot below.



A The range is 36.
B Fifty per cent of values are between 36 and 51.
C The median is 42.
D Twenty-five per cent of values are below 36.
E The interquartile range is 20.

Extended-response questions

1 The local Sunday market has a number of fundraising activities.

a For $1 you can spin a spinner numbered 1–5 twice. If you spin two even numbers you receive $2 (your dollar back plus an extra dollar), if you spin two odd numbers you get your dollar back and otherwise you lose your dollar.

		First spin				
		1	**2**	**3**	**4**	**5**
Second spin	**1**	(1, 1)	(2, 1)			
	2					
	3					
	4					
	5					

i Complete the table shown to list the sample space.
ii What is the probability of losing your dollar?
iii What is the probability of making a dollar profit?
iv In 50 attempts, how many times would you expect to lose your dollar?
v If you start with $100 and have 100 attempts, how much money would you expect to end up with?

b Forty-five people were surveyed as they walked through the market as to whether they bought a sausage and/or a drink from the sausage sizzle. Twenty-five people bought a sausage, 30 people bought a drink, with 15 buying both. Let S be the set of people who bought a sausage and D the set of people who bought a drink.

- **i** Construct a Venn diagram to represent this information.
- **ii** How many people did not buy either a drink or a sausage?
- **iii** How many people bought a sausage only?
- **iv** If a person was randomly selected from the 45 people surveyed, what is the probability they bought a drink but not a sausage?
- **v** Find $\Pr(S')$.
- **vi** Find $\Pr(S' \cup D)$ and state what this probability represents.

2 The data below represents the data collected over a month of 30 consecutive days of the delay time (in minutes) of the flight departures of the same evening flight for two rival airlines.

Airline A

2	11	6	14	18	1	7	4	12	14	9	2	13	4	19
13	17	3	52	24	19	12	14	0	7	13	18	1	23	8

Airline B

6	12	9	22	2	15	10	5	10	19	5	12	7	11	18
21	15	10	4	10	7	18	1	18	8	25	4	22	19	26

a Does the data for airline A appear to have any outliers (numbers not near the majority of data elements)?

b Remove any outliers listed in part **a**, and find the following values for airline A:

- **i** median (Q_2)
- **ii** lower quartile (Q_1)
- **iii** upper quartile (Q_3).

Ext **c** Hence, complete a box plot of the delay times for airline A.

d Airline A reports that half its flights for that month had a delay time of less than 10 minutes. Is this claim correct? Explain.

Ext **e** On the same axis as in part **c**, construct a box plot for the delay times for airline B.

f Find the range and interquartile range of the data for the two airlines and comment on the spread of the delay times for each company.

g Use your answers to the previous question parts to explain which airline you would choose on the basis of their delay times and why.

CHAPTER 10

Introduction to quadratic equations and graphs

Flight paths in sport

Objects launched into the air follow a parabola-shaped curve due to the effects of gravity. The flight path will be a wider or thinner parabola shape depending on the force applied and the launching angle. Generally, a ball thrown or hit at an angle of 45° will have maximum range (i.e. covers the longest horizontal distance). However, the ideal launching angle varies with the specific conditions of each sport.

For a basketball throw, launch angles are about 70° for a steep, thin parabola required when close to the goal; 50° at the free-throw line; and 45° for a 3-point shot needing a longer parabolic path. Shorter players use steeper launching angles than taller players.

Snowboarders and skiers follow a parabolic flight path when jumping. To reduce landing impact, engineers can use quadratic equations to design curved landing slopes that extend the parabolic shape of the jump.

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

In this chapter

10A Quadratic equations (EXTENDING)
10B Solving $ax^2 + bx = 0$ and $x^2 = d$ (EXTENDING)
10C Solving $x^2 + bx + c = 0$ (EXTENDING)
10D Applications of quadratic equations (EXTENDING)
10E The parabola
10F Sketching $y = ax^2$ with dilations and reflections
10G Sketching translations of $y = x^2$
10H Sketching parabolas using intercept form (EXTENDING)

Australian Curriculum

NUMBER AND ALGEBRA
Linear and non-linear relationships

Graph simple non-linear relations with and without the use of digital technologies and solve simple related equations (ACMNA296)

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In many ball sports, such as tennis or soccer, ball spin will modify the parabolic pathway, causing the ball to drop steeply and/or curve sideways. Famous soccer goals, such as Brazilian Roberto Carlos' 1997 'banana' goal against France, show how a powerful kick and spin can drive the ball into a sideways curve, out of reach of the goal keeper.

10A Quadratic equations EXTENDING

Learning intentions

- To be able to identify a quadratic equation
- To know how to write a quadratic equation in standard form
- To understand the Null Factor Law
- To be able to apply the Null Factor Law to solve an equation in factorised form

Quadratic equations are commonplace in theoretical and practical applications of mathematics. They are used to solve problems in geometry and measurement as well as in number theory and physics. The path that a projectile takes while flying through the air, for example, can be analysed using quadratic equations.

Quadratic equations are widely used in many areas of work including: sports scientists analysing the trajectory of a javelin or discus, electronic engineers finding currents and voltages, and financial analysts modelling future revenue and profit.

A quadratic equation can be written in the form $ax^2 + bx + c = 0$ where a, b and c are constants and $a \neq 0$. Examples include $x^2 - 2x + 1 = 0$, $5x^2 - 3 = 0$ and $-0.2x^2 + 4x = 0$. Unlike linear equations which have a single solution, quadratic equations can have zero, one or two solutions. For example, $x = 2$ and $x = -1$ are solutions to the quadratic equation $x^2 - x - 2 = 0$ since $2^2 - 2 - 2 = 0$ and $(-1)^2 - (-1) - 2 = 0$. One method for finding the solutions to quadratic equations involves the use of the Null Factor Law where each factor of a factorised quadratic expression is equated to zero.

LESSON STARTER Exploring the Null Factor Law

$x = 1$ is not a solution to the quadratic equation $x^2 - x - 12 = 0$ since $1^2 - 1 - 12 \neq 0$.

- Use trial and error to find at least one of the two numbers that are solutions to $x^2 - x - 12 = 0$.
- Rewrite the equation in factorised form.

$$x^2 - x - 12 = 0 \text{ becomes } (\qquad)(\qquad) = 0$$

- Now repeat the first task above to find solutions to the equation using the factorised form.
- Was the factorised form easier to work with? Discuss.

KEY IDEAS

- A **quadratic equation** can be written in the form $ax^2 + bx + c = 0$.
  - This is called **standard form**.
  - a, b and c are constants and $a \neq 0$.
- The **Null Factor Law** states that if the product of two numbers is zero, then either or both of the two numbers is zero.
  - If $a \times b = 0$ then $a = 0$ or $b = 0$.
  - If $(x + 1)(x - 3) = 0$ then $x + 1 = 0$ (so $x = -1$) or $x - 3 = 0$ (so $x = 3$).
  - $x + 1$ and $x - 3$ are the linear factors of $x^2 - 2x - 3$, which factorises to $(x + 1)(x - 3)$.

ISBN 978-1-108-77288-4

BUILDING UNDERSTANDING

1 Evaluate these quadratic expressions by substituting the x-value given in the brackets.

a $x^2 - 5\ (x = 1)$ **b** $x^2 + 2x\ (x = -1)$ **c** $3x^2 - x + 2\ (x = 5)$

2 Decide if the following equations are quadratics (yes or no).

a $1 - 2x = 0$ **b** $x^2 + 3x + 1 = 0$ **c** $x^2 - 1 = 0$

d $x^3 + x^2 - 2 = 0$ **e** $x^5 + 1 - x = 0$ **f** $x + 2 = 4x^2$

3 Solve these linear equations.

a $x - 1 = 0$ **b** $x + 3 = 0$ **c** $2x + 4 = 0$

4 State the missing components to complete the following working, which uses the Null Factor Law.

a $x(x - 5) = 0$
$x = 0$ or _____ $= 0$
$x = 0$ or $x =$ _____

b $(2x + 1)(x - 3) = 0$
_____ $= 0$ or $x - 3 = 0$
$2x =$ _____ or $x =$ _____
$x =$ _____ or $x = 3$

Example 1 Writing in standard form

Write these quadratic equations in standard form: $ax^2 + bx + c = 0,\ a > 0$.

a $x^2 = 2x + 7$ **b** $2(x^2 - 3x) = 5$ **c** $2x - 7 = -3x^2$

SOLUTION	EXPLANATION
a $x^2 = 2x + 7$ $x^2 - 2x - 7 = 0$	We require the form $ax^2 + bx + c = 0$. Subtract $2x$ and 7 from both sides to move all terms to the left-hand side.
b $2(x^2 - 3x) = 5$ $2x^2 - 6x = 5$ $2x^2 - 6x - 5 = 0$	First expand brackets and then subtract 5 from both sides.
c $2x - 7 = -3x^2$ $3x^2 + 2x - 7 = 0$	Add $3x^2$ to both sides.

Now you try

Write these quadratic equations in standard form: $ax^2 + bx + c = 0,\ a > 0$.

a $x^2 = 3x + 5$ **b** $2(x^2 + 4x) = 7$ **c** $3x - 4 = -2x^2$

Example 2 Testing for a solution

Substitute the given x-value into the equation and say whether or not it is a solution.

a $x^2 + x - 6 = 0 \quad (x = 2)$ **b** $2x^2 + 5x - 3 = 0 \quad (x = -4)$

SOLUTION	EXPLANATION
a $x^2 + x - 6 = 2^2 + 2 - 6$ $= 6 - 6$ $= 0$ $\therefore\ x = 2$ is a solution	Substitute $x = 2$ into the equation to see if the left-hand side equals zero. $x = 2$ satisfies the equation, so $x = 2$ is a solution.

Continued on next page

b $2x^2 + 5x - 3 = 2(-4)^2 + 5(-4) - 3$ $= 2 \times 16 + (-20) - 3$ $= 32 - 20 - 3$ $= 9$ $\therefore\ x = -4$ is not a solution	Substitute $x = -4$. Recall that $(-4)^2 = 16$ and $5 \times (-4) = -20$. The equation is not satisfied so $x = -4$ is not a solution.

Now you try

Substitute the given x-value into the equation and say whether or not it is a solution.

a $x^2 + x - 3 = 0 \quad (x = 1)$

b $3x^2 + 4x - 15 = 0 \quad (x = -3)$

Example 3 Using the Null Factor Law

Use the Null Factor Law to solve these equations.

a $x(x + 2) = 0$

b $(x - 1)(x + 5) = 0$

c $(2x - 1)(5x + 3) = 0$

SOLUTION	EXPLANATION
a $x(x + 2) = 0$ $x = 0$ or $x + 2 = 0$ $x = 0$ or $x = -2$	The factors are x and $(x + 2)$. Solve each linear factor equal to zero. Check each solution by substituting into the orginal equation.
b $(x - 1)(x + 5) = 0$ $x - 1 = 0$ or $x + 5 = 0$ $x = 1$ or $x = -5$	Solve each factor $(x - 1)$ and $(x + 5)$ equal to zero. Check your solutions using substitution.
c $(2x - 1)(5x + 3) = 0$ $2x - 1 = 0$ or $5x + 3 = 0$ $2x = 1$ or $5x = -3$ $x = \frac{1}{2}$ or $x = -\frac{3}{5}$	The two factors are $(2x - 1)$ and $(5x + 3)$. Each one results in a two-step linear equation. Check your solutions using substitution.

Now you try

Use the Null Factor Law to solve these equations.

a $x(x + 5) = 0$

b $(x + 2)(x - 3) = 0$

c $(3x - 2)(2x + 5) = 0$

ISBN 978-1-108-77288-4

Updated September 2021

Exercise 10A

FLUENCY	1, 2–3(1/2), 4, 6(1/2)	2–3(1/2), 5, 6–7(1/2)	2(1/2), 3(1/3), 5, 6–7(1/3)

Example 1

1 Write these quadratic equations in standard form: $ax^2 + bx + c = 0$, $a > 0$.

a $x^2 = 4x + 3$ **b** $2(x^2 + 5x) = 3$ **c** $x - 2 = -4x^2$

Example 1

2 Write these quadratic equations in standard form: $ax^2 + bx + c = 0$, $a > 0$.

a $x^2 = 7x + 2$ **b** $x^2 - 5x = -2$ **c** $x^2 = 4x - 1$
d $3(x^2 - 2x) = 4$ **e** $2(x^2 + x) + 1 = 0$ **f** $3(x^2 - x) = -4$
g $4 = -3x^2$ **h** $3x = x^2 - 1$ **i** $5x = 2(-x^2 + 5)$

Example 2

3 Substitute the given x-value into the quadratic equation and say whether or not it is a solution.

a $x^2 - 1 = 0$ $(x = 1)$ **b** $x^2 - 25 = 0$ $(x = 5)$ **c** $x^2 - 4 = 0$ $(x = 1)$
d $2x^2 + 1 = 0$ $(x = 0)$ **e** $x^2 - 9 = 0$ $(x = -3)$ **f** $x^2 - x - 12 = 0$ $(x = 5)$
g $x^2 + 2x + 1 = 0$ $(x = -1)$ **h** $2x^2 - x + 3 = 0$ $(x = -1)$ **i** $5 - 2x + x^2 = 0$ $(x = -2)$

4 Substitute $x = -2$ and $x = 5$ into the equation $x^2 - 3x - 10 = 0$. What do you notice?

5 Substitute $x = -3$ and $x = 4$ into the equation $x^2 - x - 12 = 0$. What do you notice?

Example 3a,b

6 Use the Null Factor Law to solve these equations.

a $x(x + 1) = 0$ **b** $x(x + 5) = 0$ **c** $x(x - 2) = 0$
d $x(x - 7) = 0$ **e** $(x + 1)(x - 3) = 0$ **f** $(x - 4)(x + 2) = 0$
g $(x + 7)(x - 3) = 0$ **h** $\left(x + \frac{1}{2}\right)\left(x - \frac{1}{2}\right) = 0$ **i** $2x(x + 5) = 0$
j $5x\left(x - \frac{2}{3}\right) = 0$ **k** $\frac{x}{3}\left(x + \frac{2}{3}\right) = 0$ **l** $\frac{2x}{5}(x + 2) = 0$

Example 3c

7 Use the Null Factor Law to solve these equations.

a $(2x - 1)(x + 2) = 0$ **b** $(x + 2)(3x - 1) = 0$ **c** $(5x + 2)(x + 4) = 0$
d $(x - 1)(3x - 1) = 0$ **e** $(x + 5)(7x + 2) = 0$ **f** $(3x - 2)(5x + 1) = 0$
g $(11x - 7)(2x - 13) = 0$ **h** $(4x + 9)(2x - 7) = 0$ **i** $(3x - 4)(7x + 1) = 0$

PROBLEM-SOLVING	8	8–9(1/2)	9–10(1/2)

8 Find the numbers which satisfy the given condition.

a The product of x and a number 3 more than x is zero.
b The product of x and a number 7 less than x is zero.
c The product of a number 1 less than x and a number 4 more than x is zero.
d The product of a number 1 less than twice x and 6 more than x is zero.
e The product of a number 3 more than twice x and 1 less than twice x is zero.

9 Write these equations as quadratics in standard form. Remove any brackets and fractions.

a $5x^2 + x = x^2 - 1$ **b** $2x = 3x^2 - x$
c $3(x^2 - 1) = 1 + x$ **d** $2(1 - 3x^2) = x(1 - x)$
e $\frac{x}{2} = x^2 - \frac{3}{2}$ **f** $\frac{4x}{3} - x^2 = 2(1 - x)$
g $\frac{5}{x} + 1 = x$ **h** $\frac{3}{x} + \frac{5}{2} = 2x$

ISBN 978-1-108-77288-4

10 These quadratic equations have two integer solutions between -5 and 5. Use trial and error to find them.

a $x^2 - x - 2 = 0$ **b** $x^2 - 4x + 3 = 0$ **c** $x^2 - 4x = 0$
d $x^2 + 3x = 0$ **e** $x^2 + 3x - 4 = 0$ **f** $x^2 - 16 = 0$

REASONING	11	11, 12	12, 13

11 Consider the quadratic equation $(x + 2)^2 = 0$.

a Write the equation in the form $(\_\_\_\_)(\_\_\_\_) = 0$.

b Use the Null Factor Law to find the solutions to the equation. What do you notice?

c Now solve these quadratic equations.

i $(x + 3)^2 = 0$ **ii** $(x - 5)^2 = 0$ **iii** $(2x - 1)^2 = 0$ **iv** $(5x - 7)^2 = 0$

12 Consider the equation $3(x - 1)(x + 2) = 0$.

a First divide both sides of the equation by 3. Write down the new equation.

b Solve the equation using the Null Factor Law.

c Compare the given original equation with the equation found in part **a**. Explain why the solutions are the same.

d Solve these equations.

i $7(x + 2)(x - 3) = 0$ **ii** $11x(x + 2) = 0$
iii $\frac{(x + 1)(x - 3)}{4} = 0$ **iv** $-2(x + 2)(x - 5) = 0$

13 Consider the equation $(x - 3)^2 + 1 = 0$.

a Substitute these x-values to decide if they are solutions to the equation.

i $x = 3$ **ii** $x = 4$ **iii** $x = 0$ **iv** $x = -2$

b Do you think the equation will have a solution? Explain why.

ENRICHMENT: Polynomials	–	–	14, 15

14 Polynomials are sums of integer powers of x. They are given names according to the highest power of x in the polynomial expression.

Example	Polynomial name
2	Constant
$2x + 1$	Linear
$x^2 - 2x + 5$	Quadratic
$x^3 - x^2 + 6x - 1$	Cubic
$7x^4 + x^3 + 2x - x + 4$	Quartic
$4x^5 - x + 1$	Quintic

Name these polynomial equations.

a $3x - 1 = 0$ **b** $x^2 + 2 = 0$ **c** $x^5 - x^4 + 3 = 0$
d $5 - 2x + x^3 = 0$ **e** $3x - 2x^4 + x^2 = 0$ **f** $5 - x^5 = x^4 + x$

15 Solve these polynomial equations using the Null Factor Law.

a $(x + 1)(x - 3)(x + 2) = 0$ **b** $(x - 2)(x - 5)(x + 11) = 0$
c $(2x - 1)(3x + 2)(5x - 1) = 0$ **d** $(3x + 2)(5x + 4)(7x + 10)(2x - 13) = 0$

10B Solving $ax^2 + bx = 0$ and $x^2 = d$ EXTENDING

Learning intentions

- To know the steps required to solve a quadratic equation
- To be able to factorise a quadratic equation in order to apply the Null Factor Law
- To be able to find the solutions of quadratic equations involving common factors or perfect squares

When using the Null Factor Law, we notice that equations must first be expressed as a product of two factors. Hence, any equation not in this form must first be factorised. Two types of quadratic equations are studied here. The first is of the form $ax^2 + bx = 0$, where x is a common factor, and the second is of the form $x^2 = d$.

Aerospace engineers use quadratic equations, such as for the trajectory of a rocket launching a communications satellite. An object launched vertically with velocity, u, moving under gravity, with acceleration, g, reaches a maximum height, h, where $u^2 = 2gh$.

LESSON STARTER Which factorisation technique?

These two equations may look similar but they are not the same: $x^2 - 9x = 0$ and $x^2 - 9 = 0$.

- Discuss how you could factorise each expression on the left-hand side of the equations.
- How does the factorised form help to solve the equations? What are the solutions? Are the solutions the same for both equations?
- By rearranging $x^2 - 9 = 0$ into $x^2 = 9$, can you explain how to find the two solutions you found above?

KEY IDEAS

- When solving an equation of the form $ax^2 + bx = 0$, factorise by taking out common factors including x.

$$\begin{aligned}2x^2 - 8x &= 0\\ 2x(x - 4) &= 0\\ 2x = 0 &\text{ or } x - 4 = 0\\ x = 0 &\text{ or } x = 4\end{aligned}$$

- When solving an equation of the form $ax^2 = d$, divide both sides by a and then take the square root of each side.

$$\begin{aligned}5x^2 &= 20\\ x^2 &= 4\\ x = 2 &\text{ or } x = -2\end{aligned}$$

- $x^2 = d^2$ could also be rearranged to $x^2 - d^2 = 0$ and then factorised as a difference of perfect squares.

$$\begin{aligned}x^2 - 4 &= 0\\ (x + 2)(x - 2) &= 0\\ x = -2 &\text{ or } x = 2\end{aligned}$$

ISBN 978-1-108-77288-4

BUILDING UNDERSTANDING

1 State the highest common factor of these pairs of terms.

a $16x$ and 24 **b** x^2 and $2x$ **c** $3x^2$ and $6x$

2 Factorise these expressions fully by first taking out the highest common factor.

a $x^2 - 3x$ **b** $6x^2 + 4x$

c $4x - 16x^2$ **d** $2x^2 - 8$

3 Use the Null Factor Law to state the solutions to these equations.

a $x(x - 3) = 0$ **b** $4x(x + 1) = 0$

4 Explain why $x = 3$ and $x = -3$ are solutions to $x^2 = 9$.

Example 4 Solving quadratic equations in which *x* is a common factor

Solve each of the following equations.

a $x^2 + 4x = 0$ **b** $2x^2 = 8x$

SOLUTION	EXPLANATION
a $x^2 + 4x = 0$ $x(x + 4) = 0$ $x = 0$ or $x + 4 = 0$ $x = 0$ or $x = -4$	Factorise by taking out the common factor x. Using the Null Factor Law, set each factor, x and $(x + 4)$, equal to 0. Solve for x. Check your solutions using substitution.
b $2x^2 = 8x$ $2x^2 - 8x = 0$ $2x(x - 4) = 0$ $2x = 0$ or $x - 4 = 0$ $x = 0$ or $x = 4$	Make the right-hand side equal to zero by subtracting $8x$ from both sides. Factorise by taking out the common factor of $2x$ and apply the Null Factor Law to solve. For $2 \times x = 0$, x must equal 0.

Now you try

Solve each of the following equations.

a $x^2 + 6x = 0$ **b** $3x^2 = 15x$

Example 5 Solving equations of the form $ax^2 = d$

Solve these equations.

a $x^2 = 16$ **b** $3x^2 = 18$

SOLUTION	EXPLANATION
a $x^2 = 16$ $x = 4$ or $x = -4$	As this is a perfect square, take the square root of both sides. $x = 4$ or -4 since $4^2 = 16$ and $(-4)^2 = 16$.

Alternative method:

$x^2 = 16$	Rearrange into standard form.
$x^2 - 16 = 0$	Factorise using $a^2 - b^2 = (a + b)(a - b)$ and then use the Null Factor Law to find the solutions.
$(x + 4)(x - 4) = 0$	
$x + 4 = 0$ or $x - 4 = 0$	
$x = -4$ or $x = 4$	
b $3x^2 = 18$	Divide both sides by 3 and then take the square root of each side.
$x^2 = 6$	
$x = \sqrt{6}$ or $x = -\sqrt{6}$	As 6 is not a square number leave answers in exact form as $\sqrt{6}$ and $-\sqrt{6}$.
Alternative method:	
$3x^2 = 18$	Alternatively, express in standard form and then note the common factor of 3.
$3x^2 - 18 = 0$	
$3(x^2 - 6) = 0$	Treat $x^2 - 6 = x^2 - (\sqrt{6})^2$ as a difference of squares.
$3(x + \sqrt{6})(x - \sqrt{6}) = 0$	Apply the Null Factor Law and solve for x.
$x + \sqrt{6} = 0$ or $x - \sqrt{6} = 0$	
$x = -\sqrt{6}$ or $x = \sqrt{6}$	

Now you try

Solve these equations.

a $x^2 = 49$ **b** $2x^2 = 14$

Exercise 10B

FLUENCY	1, 2–5(1/2)	2–6(1/2)	2–6(1/2)

1 Solve these equations.

Example 4a **a** **i** $x^2 + 5x = 0$ **ii** $x^2 - 3x = 0$

Example 4b **b** **i** $4x^2 = 12x$ **ii** $5x^2 = 20x$

Example 4a **2** Solve these equations.

a $x^2 + 3x = 0$ **b** $x^2 + 7x = 0$ **c** $x^2 + 4x = 0$ **d** $x^2 - 5x = 0$

e $x^2 - 8x = 0$ **f** $x^2 - 2x = 0$ **g** $x^2 + \frac{1}{3}x = 0$ **h** $x^2 - \frac{1}{2}x = 0$

3 Solve these equations by first taking out the highest common factor.

a $2x^2 - 6x = 0$ **b** $3x^2 - 12x = 0$ **c** $4x^2 + 20x = 0$

d $6x^2 - 18x = 0$ **e** $-5x^2 + 15x = 0$ **f** $-2x^2 - 8x = 0$

Example 4b **4** Solve the following equations.

a $x^2 = 3x$ **b** $5x^2 = 10x$ **c** $4x^2 = 16x$

d $3x^2 = -9x$ **e** $2x^2 = -8x$ **f** $7x^2 = -21x$

Example 5a **5** Solve the following equations.

a $x^2 = 9$ **b** $x^2 = 36$ **c** $x^2 = 25$

d $x^2 - 144 = 0$ **e** $x^2 - 81 = 0$ **f** $x^2 - 400 = 0$

ISBN 978-1-108-77288-4

Example 5b

6 Solve each of the following equations by first noting the common factor.

a $7x^2 = 28$ **b** $5x^2 = 45$ **c** $2x^2 = 50$ **d** $6x^2 = 24$

e $2x^2 = 12$ **f** $3x^2 = 15$ **g** $5x^2 - 35 = 0$ **h** $8x^2 - 24 = 0$

PROBLEM-SOLVING

7(½)	7(½), 9	8, 9

7 Solve these equations by first rearranging where necessary.

a $4 = x^2$ **b** $-x^2 + 25 = 0$ **c** $-x^2 = -100$

d $-3x^2 = 21x$ **e** $-5x^2 + 35x = 0$ **f** $1 - x^2 = 0$

g $12x = 18x^2$ **h** $2x^2 + x = 7x^2 - 2x$ **i** $9 - 2x^2 = x^2$

8 Remove brackets or fractions to help solve these equations.

a $x - \frac{4}{x} = 0$ **b** $\frac{36}{x} - x = 0$ **c** $\frac{3}{x^2} = 3$

d $5(x^2 + 1) = 3x^2 + 7$ **e** $x(x - 3) = 2x^2 + 4x$ **f** $3x(5 - x) = x(7 - x)$

9 Write an equation and solve it to find the number.

a The square of the number is 7 times the same number.

b The difference between the square of a number and 64 is zero.

c 3 times the square of a number is equal to -12 times the number.

REASONING

10	10	10, 11

10 Consider the equation $x^2 + 4 = 0$.

a Explain why it cannot be written in the form $(x + 2)(x - 2) = 0$.

b Are there any solutions to the equation $x^2 + 4 = 0$? Why/Why not?

11 An equation of the form $ax^2 + bx = 0$ will always have two solutions if a and b are not zero.

a Explain why one of the solutions will always be $x = 0$.

b Write the rule for the second solution in terms of a and b.

ENRICHMENT: More quadratic equation forms

–	–	12–13(½)

12 Note, for example, that $4x^2 = 9$ becomes $x^2 = \frac{9}{4}$ and then $x = \frac{3}{2}$ or $x = -\frac{3}{2}$, since $\sqrt{\frac{9}{4}} = \frac{3}{2}$. Now solve these equations.

a $9x^2 = 16$ **b** $25x^2 = 36$ **c** $4 = 100x^2$

d $81 - 25x^2 = 0$ **e** $64 - 121x^2 = 0$ **f** $-49x^2 + 144 = 0$

13 Note, for example, that $(x - 1)^2 - 4 = 0$ becomes $(x - 1)^2 = 4$ with $x - 1 = 2$ or $x - 1 = -2$, giving $x = 3$ and $x = -1$. Now solve these equations.

a $(x - 2)^2 = 9$ **b** $(x + 5)^2 = 16$ **c** $(2x + 1)^2 = 1$

d $(5x - 3)^2 - 25 = 0$ **e** $(4 - x)^2 - 9 = 0$ **f** $(3 - 7x)^2 - 100 = 0$

Using a CAS calculator 10B: Solving quadratic equations

This activity is in the Interactive Textbook in the form of a printable PDF.

10C Solving $x^2 + bx + c = 0$ EXTENDING

Learning intentions

- To be able to solve quadratic equations involving monic trinomials
- To understand that trinomials that factorise to perfect squares will have only one solution

Earlier in Chapter 8 we learnt to factorise quadratic trinomials with three terms. For example, $x^2 + 5x + 6$ factorises to $(x + 2)(x + 3)$. This means that the Null Factor Law can be used to solve equations of the form $x^2 + bx + c = 0$.

Mechanical engineers design industrial ventilation systems and can solve quadratic equations to calculate dimensions. For a duct with a rectangular cross-section area 1500 cm^2, a possible equation is: $x(x - 20) = 1500$, i.e. $x^2 - 20x - 1500 = 0$.

LESSON STARTER Remembering how to factorise quadratic trinomials

First expand these quadratics using the distributive law.

Distributive law

$(a + b)(c + d) = ac + ad + bc + bd$

- $(x + 1)(x + 2)$
- $(x - 3)(x + 4)$
- $(x - 5)(x - 2)$

Now factorise these expressions.

- $x^2 + 5x + 6$
- $x^2 - x - 12$
- $x^2 - 8x + 7$

Discuss your method for finding the factors of each quadratic above.

KEY IDEAS

- Solve quadratics of the form $x^2 + bx + c = 0$ by factorising the quadratic trinomial.
  - Ask 'What factors of c add to give b?'
  - Then use the Null Factor Law.

$$\begin{aligned} x^2 - 3x - 28 &= 0 \quad \text{and} \quad -7 \times 4 = -28 \\ (x - 7)(x + 4) &= 0 \quad \qquad\quad -7 + 4 = -3 \\ x - 7 = 0 &\text{ or } x + 4 = 0 \\ x = 7 &\text{ or } x = -4 \end{aligned}$$

- Equations involving perfect squares will give only one solution.

$$\begin{aligned} x^2 - 6x + 9 &= 0 \\ (x - 3)(x - 3) &= 0 \\ x - 3 &= 0 \\ x &= 3 \end{aligned}$$

- $x^2 + bx + c$ is called a **monic** quadratic since the coefficient of x^2 is 1.

BUILDING UNDERSTANDING

1 Decide which two factors of the first number add to give the second number.

a 6, 5 **b** 10, −7 **c** −5, 4 **d** −12, −1

2 Factorise each of the following.

a $x^2 + 12x + 35$ **b** $x^2 + 4x - 45$ **c** $x^2 - 10x + 16$

3 State the missing components to complete the working to solve each equation.

a $x^2 + 9x + 20 = 0$
$(x + 5)(\_\_\_\_) = 0$
$x + 5 = 0$ or $\_\_\_\_ = 0$
$x = \_\_\_\_$ or $x = \_\_\_\_$

b $x^2 - 2x - 24 = 0$
$(x - 6)(\_\_\_\_) = 0$
$x - 6 = 0$ or $\_\_\_\_ = 0$
$x = \_\_\_\_$ or $x = \_\_\_\_$

Example 6 Solving equations with quadratic trinomials

Solve these quadratic equations.

a $x^2 + 7x + 12 = 0$
b $x^2 - 2x - 8 = 0$
c $x^2 - 8x + 15 = 0$

SOLUTION	EXPLANATION
a $x^2 + 7x + 12 = 0$ $(x + 3)(x + 4) = 0$ $x + 3 = 0$ or $x + 4 = 0$ $x = -3$ or $x = -4$	Factors of 12 which add to 7 are 3 and 4. $3 \times 4 = 12$, $3 + 4 = 7$ Use the Null Factor Law to solve the equation.
b $x^2 - 2x - 8 = 0$ $(x - 4)(x + 2) = 0$ $x - 4 = 0$ or $x + 2 = 0$ $x = 4$ or $x = -2$	Factors of −8 which add to −2 are −4 and 2. $-4 \times 2 = -8$, $-4 + 2 = -2$ Solve using the Null Factor Law.
c $x^2 - 8x + 15 = 0$ $(x - 5)(x - 3) = 0$ $x - 5 = 0$ or $x - 3 = 0$ $x = 5$ or $x = 3$	The factors of 15 must add to give −8. $-5 \times (-3) = 15$ and $-5 + (-3) = -8$, so −5 and −3 are the two numbers.

Now you try

Solve these quadratic equations.

a $x^2 + 6x + 8 = 0$
b $x^2 - 3x - 10 = 0$
c $x^2 - 8x + 12 = 0$

Example 7 Solving with perfect squares and trinomials not in standard form

Solve these quadratic equations.

a $x^2 - 8x + 16 = 0$

b $x^2 = x + 6$

SOLUTION	EXPLANATION
a $x^2 - 8x + 16 = 0$ $(x-4)(x-4) = 0$ $x - 4 = 0$ $x = 4$	Factors of 16 which add to −8 are −4 and −4. $(x-4)(x-4) = (x-4)^2$ is a perfect square so there is only one solution.
b $x^2 = x + 6$ $x^2 - x - 6 = 0$ $(x-3)(x+2) = 0$ $x - 3 = 0$ or $x + 2 = 0$ $x = 3$ or $x = -2$	First make the right-hand side equal zero by subtracting x and 6 from both sides. This is now in standard form. (When rearranging to standard form, keep the x^2 term positive.) Factors of −6 which add to −1 are −3 and 2.

Now you try

Solve these quadratic equations.

a $x^2 - 10x + 25 = 0$

b $x^2 = 3x + 40$

Exercise 10C

FLUENCY	1, 2–3(1/2)	2–4(1/3)	2–4(1/3)

Example 6

1 Solve these quadratic equations.

a $x^2 + 10x + 24 = 0$ **b** $x^2 - 8x - 33 = 0$ **c** $x^2 - 13x + 36 = 0$

Example 6

2 Solve these quadratic equations.

a $x^2 + 8x + 12 = 0$ **b** $x^2 + 11x + 24 = 0$ **c** $x^2 + 7x + 10 = 0$

d $x^2 + 5x - 14 = 0$ **e** $x^2 + 4x - 12 = 0$ **f** $x^2 + 7x - 30 = 0$

g $x^2 - 12x + 32 = 0$ **h** $x^2 - 9x + 18 = 0$ **i** $x^2 - 10x + 21 = 0$

j $x^2 - 2x - 15 = 0$ **k** $x^2 - 6x - 16 = 0$ **l** $x^2 - 4x - 45 = 0$

m $x^2 - 10x + 24 = 0$ **n** $x^2 - x - 42 = 0$ **o** $x^2 + 5x - 84 = 0$

p $x^2 + 4x + 3 = 0$ **q** $x^2 - 6x - 27 = 0$ **r** $x^2 - 12x + 20 = 0$

Example 7a

3 Solve these quadratic equations which include perfect squares.

a $x^2 + 6x + 9 = 0$ **b** $x^2 + 4x + 4 = 0$ **c** $x^2 + 14x + 49 = 0$

d $x^2 + 24x + 144 = 0$ **e** $x^2 - 10x + 25 = 0$ **f** $x^2 - 16x + 64 = 0$

g $x^2 - 12x + 36 = 0$ **h** $x^2 - 18x + 81 = 0$ **i** $x^2 - 20x + 100 = 0$

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Example 7b

4 Solve these quadratic equations by first rearranging to standard form.

a $x^2 = 3x + 10$
b $x^2 = 7x - 10$
c $x^2 = 6x - 9$
d $x^2 = 4 - 3x$
e $14 - 5x = x^2$
f $x^2 + 16 = 8x$
g $x^2 - 12 = -4x$
h $6 - x^2 = 5x$
i $15 = 8x - x^2$
j $16 - 6x = x^2$
k $-6x = x^2 + 8$
l $-x^2 - 7x = -18$

PROBLEM-SOLVING	5(1/2), 8	5(1/2), 7(1/2), 8	5–7(1/2), 8

5 Solve these equations by taking out a common factor as the first factorising step.

a $2x^2 - 2x - 12 = 0$
b $3x^2 + 24x + 45 = 0$
c $4x^2 - 24x - 64 = 0$
d $4x^2 - 20x + 24 = 0$
e $2x^2 - 8x + 8 = 0$
f $3x^2 + 6x + 3 = 0$
g $7x^2 - 70x + 175 = 0$
h $2x^2 + 12x = -18$
i $5x^2 = 35 - 30x$

6 Remove brackets, decimals or fractions and write in standard form to help solve these equations.

a $x^2 = 5(x - 1.2)$
b $2x(x - 3) = x^2 - 9$
c $3(x^2 + x - 10) = 2x^2 - 5(x + 2)$
d $x - 2 = \frac{35}{x}$
e $2 + \frac{1}{x} = -x$
f $\frac{x}{4} = 1 - \frac{1}{x}$

7 Write down a quadratic equation in standard form which has the following solutions.

a $x = 1$ and $x = 2$
b $x = 3$ and $x = -2$
c $x = -4$ and $x = 1$
d $x = -3$ and $x = 10$
e $x = 5$ only
f $x = -11$ only

8 The temperature in °C inside a room at a scientific base in Antarctica after 10:00 a.m. is given by the expression $t^2 - 9t + 8$ where t is in hours. Find the times in the day when the temperature is 0 °C.

REASONING	9	9, 10	10, 11

9 Consider this equation and solution.

a Explain what is wrong with the solution.
b Find the correct solutions to the equation.

$$\begin{aligned} x^2 - x - 12 &= -6 \\ (x - 4)(x + 3) &= -6 \\ x - 4 = -6 \text{ or } x + 3 &= -6 \\ x = -2 \text{ or } x &= -9 \end{aligned}$$

10 Explain why $x^2 - 2x + 1 = 0$ only has one solution.

11 Write down a quadratic equation that has these solutions. Write in factorised form.

a $x = a$ only
b $x = a$ and $x = b$

ENRICHMENT: Solving non-monics	–	–	12(1/2)

12 Many quadratic equations have a coefficient of x^2 not equal to 1. These are called non-monic quadratics. Factorisation can be trickier but is still possible. You may have covered this in Chapter 8. Here is an example.

$$\begin{aligned} 2x^2 + 5x - 3 &= 0 \\ 2x^2 - x + 6x - 3 &= 0 \\ x(2x - 1) + 3(2x - 1) &= 0 \\ (2x - 1)(x + 3) &= 0 \\ 2x - 1 = 0 \text{ or } x + 3 &= 0 \\ x = \frac{1}{2} \text{ or } x &= -3 \end{aligned}$$

(Factors of $2 \times (-3) = -6$ that add to give 5 are -1 and 6.)
$(6x - 1x = 5x)$

Solve these quadratic equations.

a $5x^2 + 16x + 3 = 0$
b $3x^2 + 4x - 4 = 0$
c $6x^2 + x - 1 = 0$
d $10x^2 + 5x - 5 = 0$
e $2x^2 = 15 - 7x$
f $4x^2 = 12x - 9$
g $x = 3x^2 - 14$
h $17x = 12 - 5x^2$
i $25 = -9x^2 - 30x$

10D Applications of quadratic equations EXTENDING

Learning intentions

- To be able to form an equation from a word problem using a variable for the unknown
- To be able to recognise a quadratic equation and apply the appropriate steps to solve
- To understand that solutions will need to be checked for validity in the context of the problem

When using mathematics to solve problems, we often arrive at a quadratic equation. In solving the quadratic equation we obtain the solutions to the problem. Setting up an equation and then interpreting the solution are important parts of the problem-solving process.

When dividing up land for new suburbs, surveyors apply area formulas which form quadratic equations when dimensions are expressed using the same variable. Surveyors rearrange the quadratic to equal zero and solve it for the unknown variable.

LESSON STARTER Solving for the unknown number

The product of a positive number and 6 more than the same number is 16.

- Using x as the unknown number, write an equation describing the given condition.
- Solve your equation for x.
- Are both solutions feasible (allowed)?
- Discuss how this method compares with the method of trial and error.

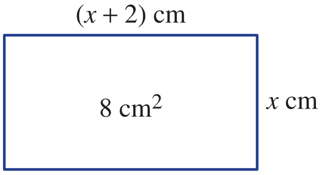
KEY IDEAS

- When using quadratic equations to solve problems, follow these steps.
  - Define your variable.
    - Write 'Let x be …' e.g. Let x be the width in cm.
  - Use your variable to define any other quantity, e.g. length $= x + 7$.
  - Write an equation relating the facts given in the question.
  - Solve your equation using the Null Factor Law.
  - Check that your solutions are feasible.
    - Some problems may not allow solutions that are negative numbers or fractions.
  - Answer the original question in words and check that your answer seems reasonable.

BUILDING UNDERSTANDING

1. Solve these equations for x by first expanding and producing a zero on the right-hand side.
   - **a** $x(x+3)=18$
   - **b** $x(x-1)=20$
   - **c** $(x-1)(x+4)=6$

2. This rectangle has an area of 8 cm^2 and a length that is 2 cm more than its width.



   - **a** Using length × width = area, give an equation.
   - **b** Solve your equation by expanding and subtracting 8 from both sides. Then use the Null Factor Law.
   - **c** Which of your two solutions is feasible for the width of the rectangle?
   - **d** State the dimensions (width and length) of the rectangle.

Example 8 Solving area problems

The length of a book is 4 cm more than its width and the area of the face of the book is 320 cm^2. Find the dimensions of the face of the book.

SOLUTION	EXPLANATION
Let x cm be the width of the book face. Length $=(x+4)$ cm	Define a variable for width and then write the length in terms of the width.
Area: $x(x+4)=320$ $x^2+4x-320=0$ $(x+20)(x-16)=0$	Write an equation to suit the given situation. Expand and then subtract 320 from both sides.
$x+20=0$ or $x-16=0$ $x=-20$ or $x=16$	Solve using the Null Factor Law, but note that a width of -20 cm is not feasible.
$\therefore x=16$ since $x>0$	Finish by writing the dimensions – width and length – as required.
$\therefore$ width $=16$ cm and length $=20$ cm	Length $=x+4=16+4=20$ cm

Now you try

The length of a rectangular jigsaw puzzle is 12 cm more than its width and the area of the jigsaw is 640 cm^2. Find the dimensions of the jigsaw puzzle.

Exercise 10D

FLUENCY	1–4, 6	2, 3, 5, 6	2, 4–6

Example 8 **1** The length of a rectangular magazine is 8 cm more than its width and the area of the magazine is 240 cm^2. Find the dimensions of the magazine.

Example 8 **2** The length of a rectangular brochure is 5 cm more than its width and the area of the face of the brochure is 36 cm^2. Find the dimensions of the face of the brochure.

3 The product of a number and 2 more than the same number is 48. Write an equation and solve to find the two possible solutions.

4 The product of a number and 7 less than the same number is 60. Write an equation and solve to find the two possible solutions.

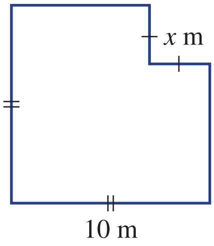
5 The product of a number and 13 less than the same number is 30. Write an equation and solve to find the two possible solutions.

6 The length of a small kindergarten play area is 20 metres less than its width and the area is 69 m^2. Find the dimensions of the play area.

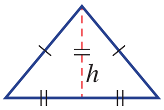
PROBLEM-SOLVING	7, 8	7–9	8–10

7 A square of side length 10 metres has a square of side length x metres removed from one corner.

- **a** Write an expression for the area remaining after the square of side length x metres is removed. (*Hint*: Use subtraction.)
- **b** Find the value of x if the area remaining is 64 m^2.

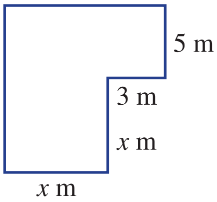


8 An isosceles triangle has height (h) equal to half its base length. Find the value of h if the area is 25 square units.

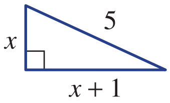


9 A rectangular farm shed (3 m by 5 m) is to be extended to form an 'L' shape as shown.

- **a** Write an expression for the total area of the extended farm shed.
- **b** Find the value of x if the total area is to be 99 m^2.



10 Use Pythagoras' theorem to find the value of x in these right-angled triangles.

a


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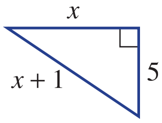

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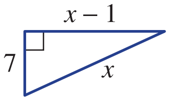
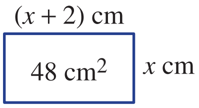


REASONING	11, 12	12, 13	12–14



11 The equation for the area of this rectangle is $x^2 + 2x - 48 = 0$, which has solutions $x = -8$ and $x = 6$. Which solution do you ignore and why?

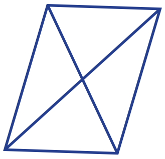
12 The product of an integer and one less than the same integer is 6. The equation for this is $x^2 - x - 6 = 0$. How many different solutions are possible and why?

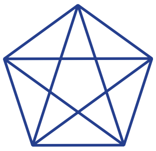
13 This table shows the sum of the first n positive integers. If $n = 3$ then the sum is $1 + 2 + 3 = 6$.

n	1	2	3	4	5	6
Sum	1	3	6			

- **a** Write the sum for $n = 4$, $n = 5$ and $n = 6$.
- **b** The expression for the sum is given by $\frac{n(n + 1)}{2}$. Use this expression to find the sum if:
  - **i** $n = 7$
  - **ii** $n = 20$.
- **c** Use the expression to find n for these sums. Write an equation and solve it.
  - **i** sum = 45
  - **ii** sum = 120

14 The number of diagonals in a polygon with n sides is given by $\frac{n}{2}(n - 3)$. Shown here are the diagonals for a quadrilateral and a pentagon.



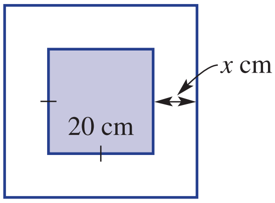


n	4	5	6	7
Diagonals	2	5		

- **a** Use the given expression to find the two missing numbers in the table.
- **b** Find the value of n if the number of diagonals is:
  - **i** 20
  - **ii** 54

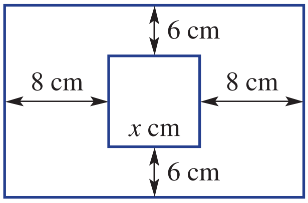
ENRICHMENT: Picture frames	–	–	15, 16

15 A square picture is to be edged with a border of width x cm. The inside picture has side length of 20 cm.



- **a** Write an expression for the total area.
- **b** Find the width of the frame if the total area of picture and frame is to be 1600 cm^2.

16 A square picture is surrounded by a rectangular frame as shown. The total area is to be 320 cm^2. Find the side length of the picture.



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10A Ext

1 Write these quadratic equations in standard form.

a $x^2 - 3x = 8$

b $3(x^2 - 2) = 4x$

c $2(x^2 - 5) = 2x - 7$

d $2x(x - 4) = 12 - 5x$

10A Ext

2 Substitute the given x-value into the equation and say whether or not it is a solution.

a $x^2 - x - 6 = 0 \quad (x = 2)$

b $x^2 - x - 6 = 0 \quad (x = -2)$

c $x^2 - 9 = 0 \quad (x = 3)$

d $x^2 - 9 = 0 \quad (x = -3)$

10A Ext

3 Use the Null Factor Law to solve these equations.

a $x(x - 7) = 0$

b $(x - 5)(x + 2) = 0$

c $3x\left(x + \frac{2}{5}\right) = 0$

d $(3x - 1)(4x - 3) = 0$

10B Ext

4 Solve the following equations.

a $x^2 + 11x = 0$

b $3x^2 - 18x = 0$

c $-2x^2 - 10x = 0$

d $5x^2 = 20x$

10B Ext

5 Solve these equations.

a $x^2 = 49$

b $x^2 - 1 = 0$

c $3x^2 = 12$

d $7x^2 - 63 = 0$

10C Ext

6 Solve these quadratic equations.

a $x^2 + 11x + 24 = 0$

b $x^2 - 15x + 36 = 0$

c $x^2 - 2x - 35 = 0$

d $x^2 + 5x - 36 = 0$

10C Ext

7 Solve these quadratic equations.

a $x^2 + 8x + 16 = 0$

b $x^2 - 14x + 49 = 0$

c $x^2 + 100 = 20x$

d $x^2 - 24 = 5x$

10D Ext

8 A rectangular lawn is 3 m longer than it is wide and has an area of 54 m^2. Write an equation and solve it to find the dimensions of this lawn.

10D Ext

9 The product of a number and 5 more than the same number is 84. Write an equation and solve to find the two possible values of the number.

10C Ext

10 Write down a quadratic equation in standard form which has the following solutions.

a $x = 3$ and $x = 2$

b $x = 6$ and $x = -1$

c $x = 4$ and $x = -4$

d $x = -3$ only

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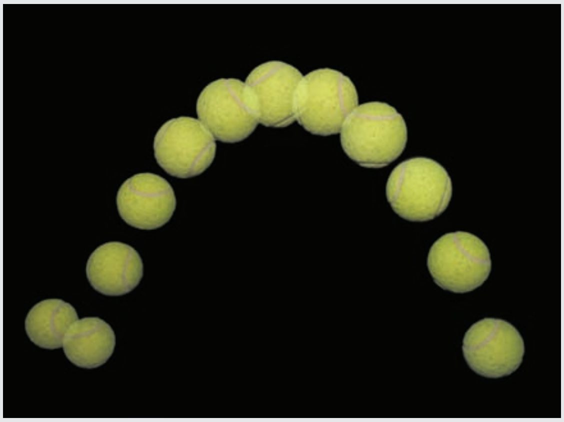
10E The parabola

Learning intentions

- To know that a quadratic relation produces a graph called a parabola
- To know the key features of a parabola including symmetry and the turning point
- To understand that a parabola can have a minimum or maximum turning point
- To be able to identify the key features of a parabola from a graph
- To be able to plot a parabola from a rule by creating a table of values and plotting points

Relations that have rules in which the highest power of x is 2, such as $y = x^2$, $y = 2x^2 - 3$ and $y = 3x^2 + 2x - 4$, are called quadratics and their graphs are called parabolas. Parabolic shapes can be seen in many modern-day objects or situations such as the arches of bridges, the paths of projectiles and the surfaces of reflectors.



When hitting a ball, tennis players, cricketers and golfers can vary the force and launching angle, hence modifying the direction, maximum height and range of the ball's parabolic flight path.

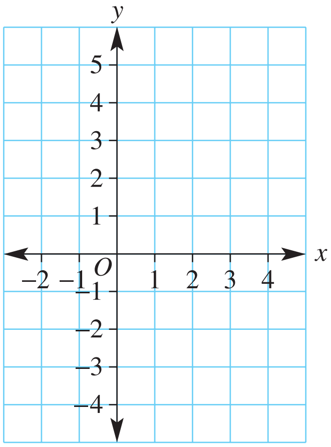
LESSON STARTER Finding features

A quadratic is given by the equation $y = x^2 - 2x - 3$. Complete these tasks to discover its graphical features.

- Use the rule to complete this table of values.

x	−2	−1	0	1	2	3	4
y							

- Plot your points on a copy of the axes shown at right and join them to form a smooth curve.
- Describe these features:
  - minimum turning point
  - axis of symmetry
  - coordinates of the y-intercept
  - coordinates of the x-intercepts.



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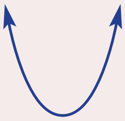
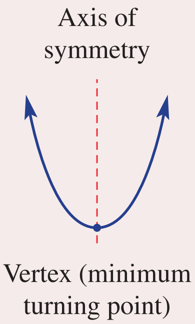
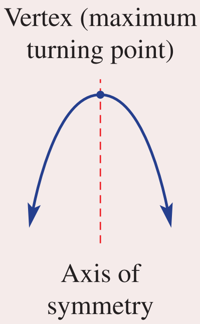
KEY IDEAS

- The graph of a quadratic relation is called a **parabola**. Its basic shape is shown here.
  - The basic quadratic rule is $y = x^2$.

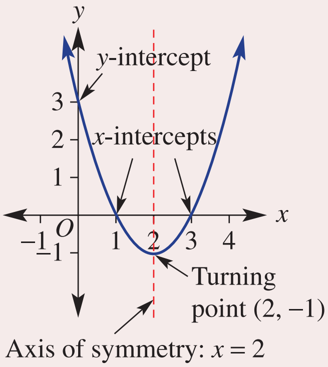


- The general equation of a quadratic is $y = ax^2 + bx + c$.
- A parabola is symmetrical about a line called the **axis of symmetry** and it has one **turning point** (a vertex), which may be a **maximum** or a **minimum**.





Here is an example of a parabola with equation $y = x^2 - 4x + 3$, showing all the key features.



BUILDING UNDERSTANDING

1 Choose a word from this list to complete each sentence.

lowest, parabola, vertex, highest, intercepts, zero

a A maximum turning point is the ____________ point on the graph.

b The graph of a quadratic is called a ____________.

c The x-____________ are the points where the graph cuts the x-axis.

d The axis of symmetry is a vertical line passing through the ____________.

e A minimum turning point is the ____________ point on the graph.

f The y-intercept is at x equals ____________.

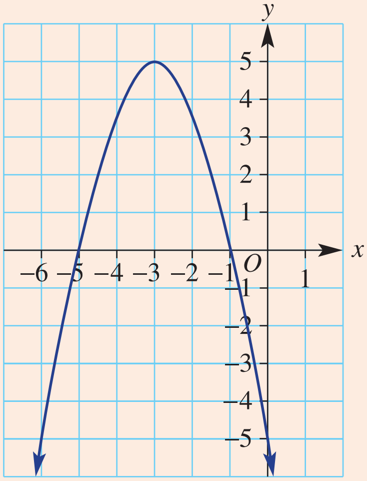
2 State the equation of a vertical line (e.g. $x = 2$) that passes through these points.

a (3, 0) **b** (1, 5) **c** (−2, 4)

Example 9 Identifying the features of a parabola

For this graph state the:

a equation of the axis of symmetry
b type of turning point
c coordinates of the turning point
d x-intercepts
e y-intercept.



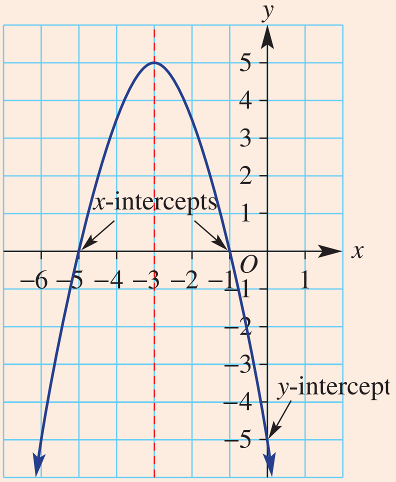
SOLUTION

a $x = -3$
b Maximum turning point
c Turning point is $(-3, 5)$
d x-intercepts: -5 and -1
e y-intercept: -5

EXPLANATION

Graph is symmetrical about the vertical line $x = -3$. Graph has its highest y-coordinate at the turning point, so it is a maximum point.

Highest point: turning point = $(-3, 5)$



Axis of symmetry: $x = -3$

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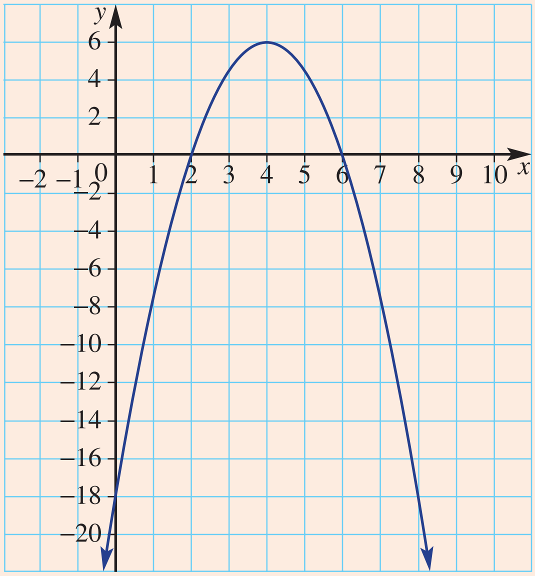
Now you try

For this graph state the:

a equation of the axis of symmetry
b type of turning point
c coordinates of the turning point
d x-intercepts
e y-intercept.



Example 10 Plotting a parabola

Use the quadratic rule $y = x^2 - 4$ to complete these tasks.

a Complete this table of values.

x	−3	−2	−1	0	1	2	3
y							

b Draw a set of axes using a scale that suits the numbers in your table. Then plot the points to form a parabola.

c State these features.
  i Type of turning point
  ii Axis of symmetry
  iii Coordinates of the turning point
  iv The y-intercept
  v The x-intercepts

SOLUTION

a

x	−3	−2	−1	0	1	2	3
y	5	0	−3	−4	−3	0	5

EXPLANATION

Substitute each x-value into the rule to find each y-value: e.g. $x = -3$,
$$\begin{aligned} y &= (-3)^2 - 4 \\ &= 9 - 4 \\ &= 5 \end{aligned}$$

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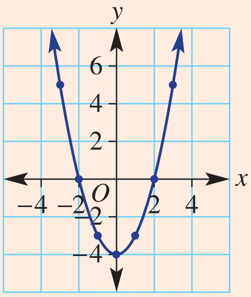
b



Plot each coordinate pair and join to form a smooth curve.

c **i** Minimum turning point
ii $x = 0$ is the axis of symmetry
iii Turning point $(0, -4)$
iv y-intercept: -4
v x-intercepts: -2 and 2

The turning point at $(0, -4)$ has the lowest y-coordinate for the entire graph.
The vertical line $x = 0$ divides the graph like a mirror line. The y-intercept is at $x = 0$.
The x-intercepts are at $y = 0$ on the x-axis.

Now you try

Use the quadratic rule $y = x^2 - 9$ to complete these tasks.

a Complete this table of values.

x	−3	−2	−1	0	1	2	3
y							

b Draw a set of axes using a scale that suits the numbers in your table. Then plot the points to form a parabola.

c State these features.

- **i** Type of turning point
- **ii** Axis of symmetry
- **iii** Coordinates of the turning point
- **iv** The y-intercept
- **v** The x-intercepts

The cross-section of a satellite dish is parabolic.

Exercise 10E

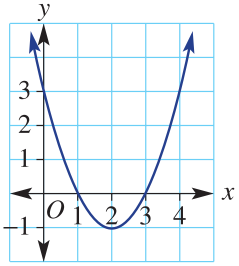
FLUENCY	1($^1/_2$), 2, 4	1($^1/_2$), 2–4	1($^1/_3$), 2, 4, 5

Example 9

1 For each of the following graphs, state:

- **i** the equation of the axis of symmetry
- **ii** the type of turning point (maximum or minimum)
- **iii** the coordinates of the turning point
- **iv** the x-intercepts
- **v** the y-intercept.

a



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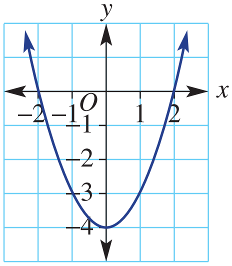

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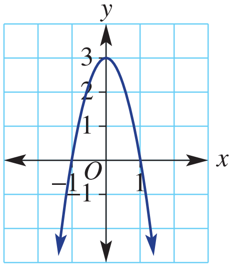

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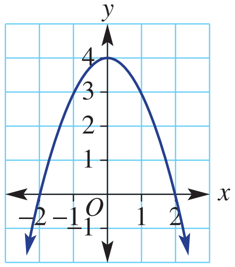

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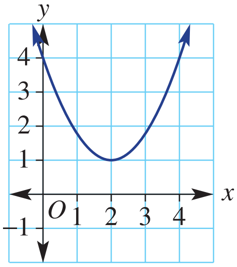

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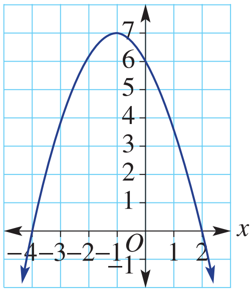

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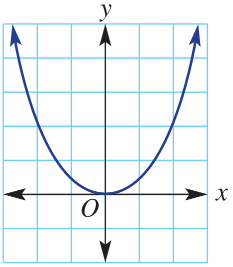

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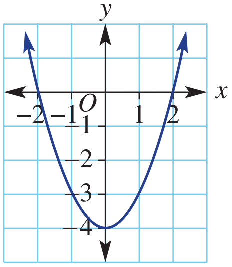

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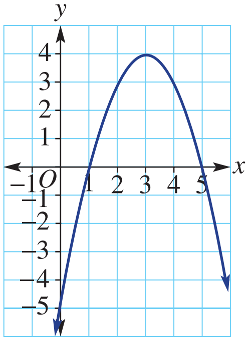

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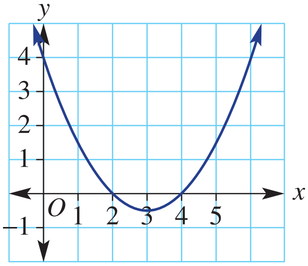

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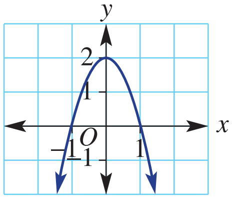

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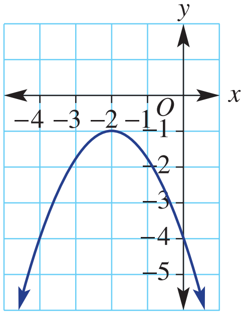


Example 10

2 Use the quadratic rule $y = x^2 - 1$ to complete these tasks.

a Complete the table of values.

x	–2	–1	0	1	2
y					

b Draw a set of axes using a scale that suits the numbers in your table. Then plot the points to form a parabola.

c State these features.

i Type of turning point
ii Axis of symmetry
iii Coordinates of the turning point
iv The y-intercept
v The x-intercepts

3 Use the quadratic rule $y = 9 - x^2$ to complete these tasks.

a Complete the table of values.

x	–3	–2	–1	0	1	2	3
y							

b Draw a set of axes using a scale that suits the numbers in your table. Then plot the points to form a parabola.

c State these features.

i Type of turning point
ii Axis of symmetry
iii Coordinates of the turning point
iv The y-intercept
v The x-intercepts

4 Use the quadratic rule $y = x^2 + 2x - 3$ to complete these tasks.

a Complete the table of values.

x	–4	–3	–2	–1	0	1	2
y							

b Draw a set of axes using a scale that suits the numbers in your table. Then plot the points to form a parabola.

c State these features.

i Type of turning point
ii Axis of symmetry
iii Coordinates of the turning point
iv The y-intercept
v The x-intercepts

5 Use the quadratic rule $y = -x^2 + x + 2$ to complete these tasks. Recall that $-x^2 = -1 \times x^2$.

a Complete the table of values.

x	–2	–1	0	1	2	3
y						

b Draw a set of axes using a scale that suits the numbers in your table. Then plot the points to form a parabola.

c State these features.

i Type of turning point
ii Axis of symmetry
iii Coordinates of the turning point
iv The y-intercept
v The x-intercepts

ISBN 978-1-108-77288-4

Updated September 2021

PROBLEM-SOLVING	6, 7	6, 8, 10	7, 9, 10

6 This graph shows the height of a cricket ball, y metres, as a function of time t seconds.

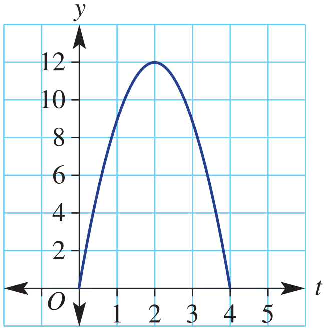
a **i** At what times is the ball at a height of 9 m?

ii Why are there two different times?

b **i** At what time is the ball at its greatest height?

ii What is the greatest height the ball reaches?

iii After how many seconds does it hit the ground?



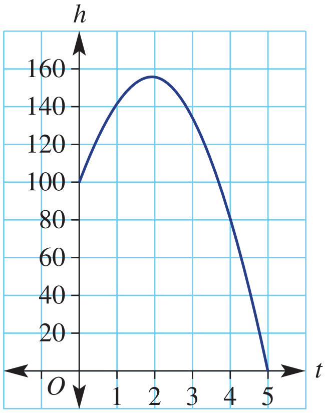
7 The graph gives the height, h m, at time t seconds, of a rocket which is fired up in the air.

a From what height is the rocket launched?

b What is the approximate maximum height that the rocket reaches?

c For how long is the rocket in the air?

d What is the difference in time for when the rocket is going up and when it is going down?



8 A parabola has x-intercepts at -2 and 4. The y-coordinate of the turning point is -3.

a What is the equation of its axis of symmetry?

b What are the coordinates of the turning point?

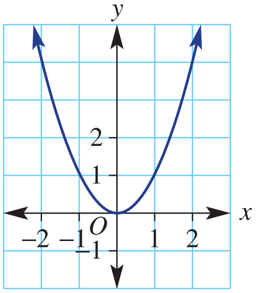
9 A parabola has a turning point at $(1, 3)$ and one of its x-intercepts at 0.

a What is the equation of its axis of symmetry?

b What is the other x-intercept?

10 Write the rule for these parabolas. Use trial and error to help and check your rule by substituting a known point.

a



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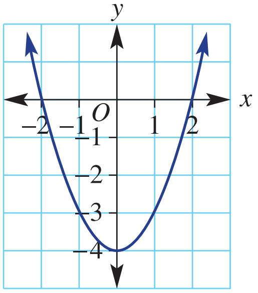

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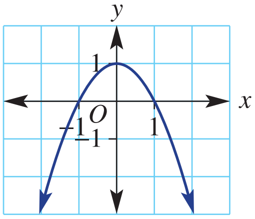

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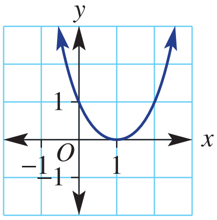

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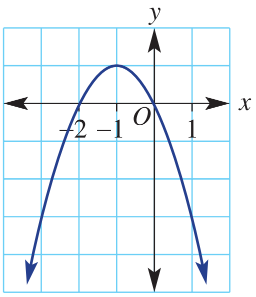

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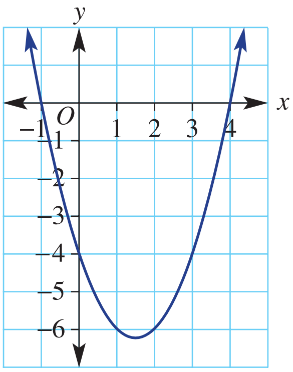


REASONING	11, 12	11–13	11, 13, 14

11 Is it possible to draw a parabola with the following properties? If yes, draw an example.

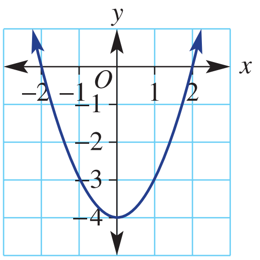
a Two x-intercepts **b** One x-intercept

c No x-intercepts **d** No y-intercept

12 **a** Mal calculates the y-value for $x = 2$ using $y = -x^2 + 2x$ and gets $y = 8$. Explain his error.

b Mai calculates the y-value for $x = -3$ using $y = x - x^2$ and gets $y = 6$. Explain her error.

13 This graph shows the parabola for $y = x^2 - 4$.



a For what values of x is $y = 0$?

b For what value of x is $y = -4$?

c How many values of x give a y-value which is:

- **i** greater than -4?
- **ii** equal to -4?
- **iii** less than -4?

14 This table corresponds to the rule $y = x^2 - 2x$.

x	−1	0	1	2	3	4
y	3	0	−1	0	3	8

a Use this table to solve these equations.

i $0 = x^2 - 2x$ **ii** $3 = x^2 - 2x$

b How many solutions would there be to the equation $8 = x^2 - 2x$? Why?

c How many solutions would there be to the equation $-1 = x^2 - 2x$? Why?

d How many solutions would there be to the equation $-2 = x^2 - 2x$? Why?

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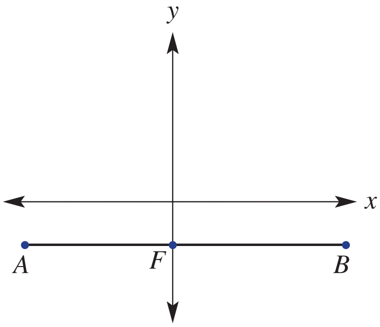
ENRICHMENT: Using software to construct a parabola	–	–	15

15 Follow the steps below to construct a parabola using an **interactive geometry package**.

Step 1. Show the coordinate axes system by selecting **Show Axes** from the **Draw** toolbox.

Step 2. Construct a line which is parallel to the x-axis and passes through a point F on the y-axis near the point $(0, -1)$.

Step 3. Construct a line segment AB on this line as shown in the diagram.

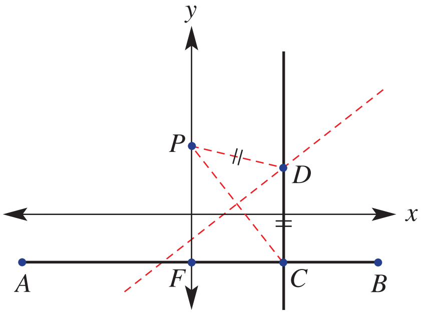


Step 4. Hide the line AB and then construct:

- a point C on the line segment AB
- a point P on the y-axis near the point $(0, 1)$.

Step 5. Construct a line which passes through the point C and is perpendicular to AB.

Step 6. Construct the point D which is equidistant from point P and segment AB. (*Hint*: Use the perpendicular bisector of PC.)

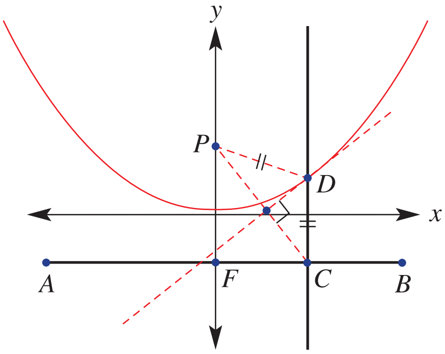


Step 7. Select **Trace** from the **Display** toolbox and click on the point D.

Step 8. Animate point C and observe what happens.

Step 9. Select **Locus** from the **Construct** toolbox and click at D and then at C.

Step 10. Drag point P and/or segment AB (by dragging F). (Clear the trace points by selecting **Refresh** drawing from the **Edit** menu.) What do you notice?



Using CAS and non-CAS calculators 10E: Sketching parabolas

This activity is in the Interactive Textbook in the form of a printable PDF.

10F Sketching $y = ax^2$ with dilations and reflections

Learning intentions

- To understand the effect of the value and sign of a in the rule $y = ax^2$ on the shape of the graph and key features
- To know which values of a make the parabola appear narrower or wider than $y = x^2$
- To know which values of a cause the parabola to be upright or inverted

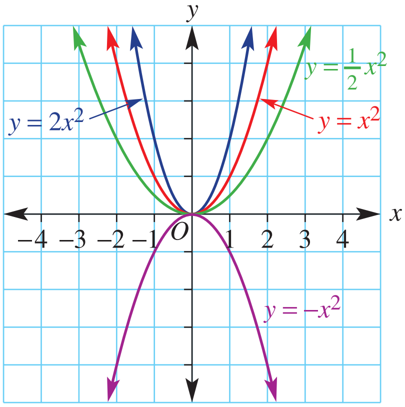
In geometry we know that shapes can be transformed by applying reflections, rotations, translations and dilations (enlargement). The same types of transformations can also be applied to graphs, including parabolas. Altering the value of a in $y = ax^2$ causes both dilations and reflections.

Suspension bridges have supporting cables that form parabolic curves, such as those on this New York bridge. Engineers model such parabolas with quadratic equations that are then used to calculate the vertical support heights.

LESSON STARTER What is the effect of a?

This table and graph show a number of examples of $y = ax^2$ with varying values of a. They could also be produced using technology.

x	-2	-1	0	1	2
$y = x^2$	4	1	0	1	4
$y = 2x^2$	8	2	0	2	8
$y = \frac{1}{2}x^2$	2	$\frac{1}{2}$	0	$\frac{1}{2}$	2
$y = -x^2$	-4	-1	0	-1	-4

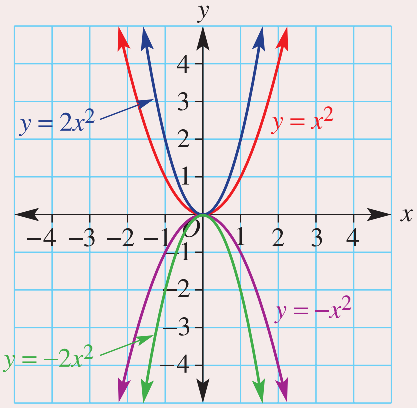


- Discuss how the different values of a affect the y-values in the table.
- Discuss how the different values of a affect the shape of the graph.
- How would the graphs of the following rules compare to the graphs shown above?

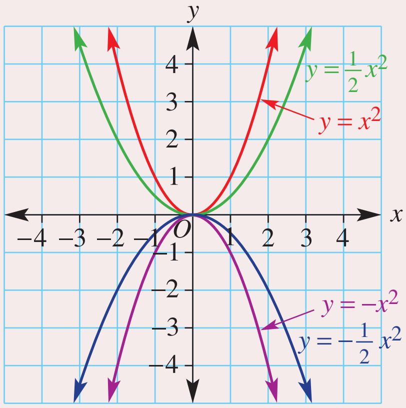
a $y = 3x^2$ **b** $y = \frac{1}{4}x^2$ **c** $y = -\frac{1}{2}x^2$

KEY IDEAS

- The equation $y = ax^2$ describes a family of parabolas including $y = x^2$, $y = -x^2$, $y = 2x^2$, $y = -2x^2$. They contain the following features:
  - the vertex (or turning point) is $(0, 0)$
  - the axis of symmetry is $x = 0$
  - if $a > 0$ the graph is **upright** (or **concave up**) and has a minimum turning point
  - if $a < 0$ the graph is **inverted** (or **concave down**) and has a maximum turning point.
- For $y = ax^2$, if $a > 1$ or $a < -1$:
  the graph appears narrower than the graph of either $y = x^2$ or $y = -x^2$. For example: $y = 2x^2$ or $y = -2x^2$



- For $y = ax^2$, if $-1 < a < 1$:
  the graph appears wider than the graph of either $y = x^2$ or $y = -x^2$. For example: $y = \frac{1}{2}x^2$ or $y = -\frac{1}{2}x^2$



- For $y = 2x^2$ we say that the graph of $y = x^2$ is **dilated** from the x-axis by a factor of 2.
- For $y = -x^2$ we say that the graph of $y = x^2$ is **reflected** in the x-axis.

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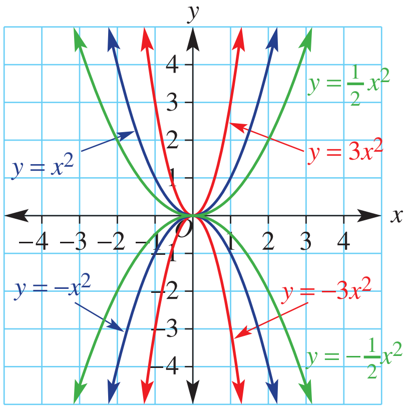
BUILDING UNDERSTANDING

1 Shown here are the graphs of $y = x^2$, $y = 3x^2$, $y = \frac{1}{2}x^2$, $y = -x^2$, $y = -3x^2$ and $y = -\frac{1}{2}x^2$.



- **a** State the rules of the three graphs which have a minimum turning point.
- **b** State the rules of the three graphs which have a maximum turning point.
- **c** What are the coordinates of the turning point for all the graphs?
- **d** What is the equation of the axis of symmetry for all the graphs?
- **e** State the rule of the graph which is:
  - **i** upright (concave up) and narrower than $y = x^2$
  - **ii** upright (concave up) and wider than $y = x^2$
  - **iii** inverted (concave down) and narrower than $y = -x^2$
  - **iv** inverted (concave down) and wider than $y = -x^2$.
- **f** State the rule of the graph which is:
  - **i** a reflection of $y = x^2$ in the x-axis
  - **ii** a reflection of $y = 3x^2$ in the x-axis
  - **iii** a reflection of $y = -\frac{1}{2}x^2$ in the x-axis.

2 Select the word *positive* or *negative* to suit each sentence.

- **a** The graph of $y = ax^2$ will be upright (concave up) with a minimum turning point if a is ___________.
- **b** The graph of $y = ax^2$ will be inverted (concave down) with a maximum turning point if a is ___________.

Example 11 Comparing graphs of $y = ax^2$, $a > 0$

Complete the following for $y = x^2$, $y = 2x^2$ and $y = \frac{1}{2}x^2$.

- **a** Draw up and complete a table of values for $-2 \leqslant x \leqslant 2$.
- **b** Plot their graphs on the same set of axes.
- **c** Write down the equation of the axis of symmetry and the coordinates of the turning point.
- **d** **i** Does the graph of $y = 2x^2$ appear wider or narrower than the graph of $y = x^2$?
  - **ii** Does the graph of $y = \frac{1}{2}x^2$ appear wider or narrower than the graph of $y = x^2$?

SOLUTION

a $y = x^2$

x	–2	–1	0	1	2
y	4	1	0	1	4

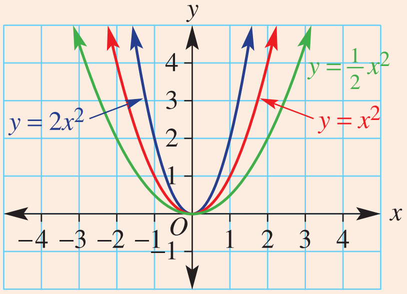
$y = 2x^2$

x	–2	–1	0	1	2
y	8	2	0	2	8

$y = \frac{1}{2}x^2$

x	–2	–1	0	1	2
y	2	$\frac{1}{2}$	0	$\frac{1}{2}$	2

b



c Axis of symmetry: y-axis ($x = 0$)
Turning point: minimum at (0, 0)

d **i** The graph of $y = 2x^2$ appears narrower than the graph of $y = x^2$.

ii The graph of $y = \frac{1}{2}x^2$ appears wider than the graph of $y = x^2$.

EXPLANATION

Substitute each x-value into $y = x^2$, $y = 2x^2$ and $y = \frac{1}{2}x^2$.

e.g. for $y = 2x^2$, if $x = 2$,

$$\begin{aligned} y &= 2(2)^2 \\ &= 2(4) \\ &= 8 \end{aligned}$$

$$\text{If } x = -1, \begin{aligned}[t] y &= 2(-1)^2 \\ &= 2(1) \\ &= 2 \end{aligned}$$

Plot the points for each graph using the coordinates from the tables and join them with a smooth curve.

Look at graphs to see symmetry about the y-axis and a minimum turning point at the origin.

For each value of x, $2x^2$ is twice that of x^2; hence, the graph (y-values) of $y = 2x^2$ rises more quickly.

For each value of x, $\frac{1}{2}x^2$ is half that of x^2; hence, the graph of $y = \frac{1}{2}x^2$ rises more slowly.

Now you try

Complete the following for $y = x^2$, $y = 4x^2$ and $y = \frac{1}{4}x^2$.

a Draw up and complete a table of values for $-2 \leqslant x \leqslant 2$.

b Plot their graphs on the same set of axes.

c Write down the equation of the axis of symmetry and the coordinates of the turning point.

d **i** Does the graph of $y = 4x^2$ appear wider or narrower than the graph of $y = x^2$?

ii Does the graph of $y = \frac{1}{4}x^2$ appear wider or narrower than the graph of $y = x^2$?

ISBN 978-1-108-77288-4

Updated September 2021

Example 12 Comparing graphs of $y = ax^2$, $a < 0$

Complete the following for $y = -x^2$, $y = -3x^2$ and $y = -\frac{1}{2}x^2$.

a Draw up and complete a table of values for $-2 \leqslant x \leqslant 2$.

b Plot their graphs on the same set of axes.

c Write down the the equation of the axis of symmetry and the coordinates of the turning point.

d **i** Does the graph of $y = -3x^2$ appear wider or narrower than the graph of $y = -x^2$?

ii Does the graph of $y = -\frac{1}{2}x^2$ appear wider or narrower than the graph of $y = -x^2$?

SOLUTION

a $y = -x^2$

x	−2	−1	0	1	2
y	−4	−1	0	−1	−4

$y = -3x^2$

x	−2	−1	0	1	2
y	−12	−3	0	−3	−12

$y = -\frac{1}{2}x^2$

x	−2	−1	0	1	2
y	−2	$-\frac{1}{2}$	0	$-\frac{1}{2}$	−2

EXPLANATION

Substitute each x-value into

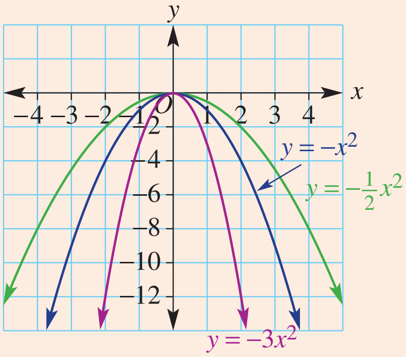
$y = -x^2$, $y = -3x^2$ and $y = -\frac{1}{2}x^2$.

e.g. for $y = -3x^2$, if $x = 2$,

$$\begin{aligned} y &= -3(2)^2 \\ &= -3(4) \\ &= -12 \end{aligned}$$

$$\begin{aligned} \text{If } x = -1,\ y &= -3(-1)^2 \\ &= -3(1) \\ &= -3 \end{aligned}$$

b



Plot the coordinates for each graph from the tables and join them with a smooth curve.

c Axis of symmetry: y-axis ($x = 0$)
Turning point: maximum at (0, 0)

Graphs are symmetrical about the y-axis with a maximum turning point at the origin.

d **i** The graph of $y = -3x^2$ appears narrower than the graph of $y = -x^2$.

ii The graph of $y = -\frac{1}{2}x^2$ appears wider than the graph of $y = -x^2$.

For each value of x, $-3x^2$ is three times that of $-x^2$; hence, the graph of $y = -3x^2$ gets larger in the negative direction more quickly. For each value of x, $-\frac{1}{2}x^2$ is half that of $-x^2$.

Now you try

Complete the following for $y = -x^2$, $y = -4x^2$ and $y = -\frac{1}{4}x^2$.

a Draw up and complete a table of values for $-2 \leqslant x \leqslant 2$.
b Plot their graphs on the same set of axes.
c Write down the equation of the axis of symmetry and the coordinates of the turning point.
d **i** Does the graph of $y = -4x^2$ appear wider or narrower than the graph of $y = -x^2$?
  ii Does the graph of $y = -\frac{1}{4}x^2$ appear wider or narrower than the graph of $y = -x^2$?

Exercise 10F

FLUENCY	1, 2–3(½)	2–3(½)	2–3(½)

Example 11

1 Complete the following for $y = x^2$, $y = 3x^2$ and $y = \frac{1}{3}x^2$.

a Draw up and complete a table of values for $-2 \leqslant x \leqslant 2$.
b Plot their graphs on the same set of axes.
c Write down the equation of the axis of symmetry and the coordinates of the turning point.
d **i** Does the graph of $y = 3x^2$ appear wider or narrower than the graph of $y = x^2$?
  ii Does the graph of $y = \frac{1}{3}x^2$ appear wider or narrower than the graph of $y = x^2$?

Example 11

2 For the equations given below, complete these tasks.

i Draw up and complete a table of values for $-2 \leqslant x \leqslant 2$.
ii Plot the graphs of the equations on the same set of axes.
iii Write down the coordinates of the turning point and the equation of the axis of symmetry.
iv Determine whether the graphs of the equations each appear wider or narrower than the graph of $y = x^2$.

a $y = 6x^2$ **b** $y = 5x^2$ **c** $y = \frac{1}{6}x^2$ **d** $y = \frac{1}{5}x^2$

Example 12

3 For the equations given below, complete these tasks.

i Draw up and complete a table of values for $-2 \leqslant x \leqslant 2$.
ii Plot the graphs of the equations on the same set of axes.
iii List the key features for each graph, such as the axis of symmetry, turning point, x-intercept and y-intercept.
iv Determine whether the graphs of the equations each appear wider or narrower than the graph of $y = -x^2$.

a $y = -2x^2$ **b** $y = -3x^2$ **c** $y = -\frac{1}{2}x^2$ **d** $y = -\frac{1}{3}x^2$

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PROBLEM-SOLVING	4	4, 5	5

4 Here are eight quadratics of the form $y = ax^2$.

A $y = 6x^2$ **B** $y = -7x^2$ **C** $y = 4x^2$ **D** $y = \frac{1}{9}x^2$

E $y = \frac{x^2}{7}$ **F** $y = 0.3x^2$ **G** $y = -4.8x^2$ **H** $y = -0.5x^2$

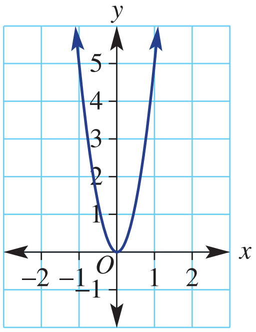
a Which rule would give a graph which is upright (concave up) and the narrowest?

b Which rule would give a graph which is inverted (concave down) and the widest?

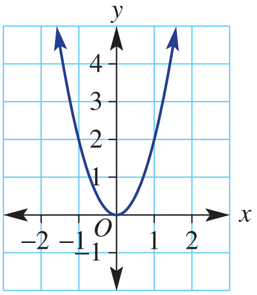
5 Match each of the following parabolas with the appropriate equation from the list below. Do a mental check by substituting the coordinates of a known point.

a $y = 3x^2$ **b** $y = -x^2$ **c** $y = 5x^2$ **d** $y = \frac{1}{2}x^2$ **e** $y = -5x^2$ **f** $y = 2x^2$

A



B



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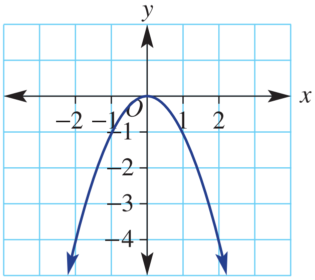

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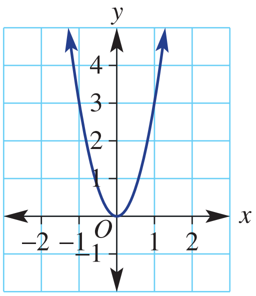
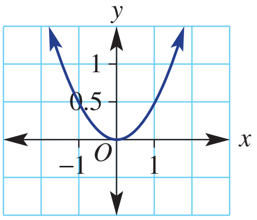
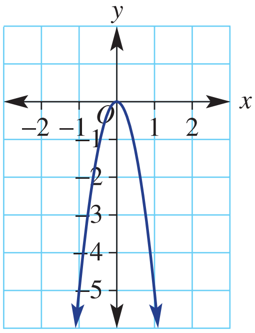


E



F



REASONING	6	6, 7	6(½), 7–9

6 The graph of $y = -2x^2$ can be obtained from $y = x^2$ by conducting these transformations:

- reflection in the x-axis
- dilation by a factor of 2 from the x-axis.

In the same way as above, describe the two transformations which take:

a $y = x^2$ to $y = -3x^2$

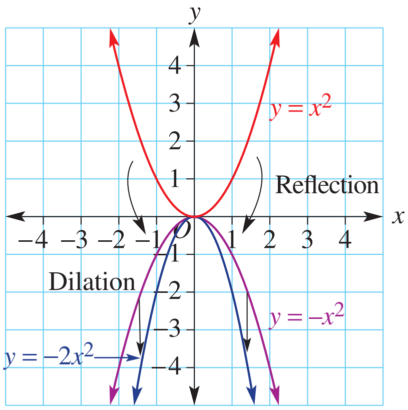
b $y = x^2$ to $y = -6x^2$

c $y = x^2$ to $y = -\frac{1}{2}x^2$

d $y = -x^2$ to $y = 2x^2$

e $y = -x^2$ to $y = 3x^2$

f $y = -x^2$ to $y = \frac{1}{3}x^2$



7 Write the rule for the graph after each set of transformations.

- **a** The graph of $y = x^2$ is reflected in the x-axis and then dilated by a factor of 4 from the x-axis.
- **b** The graph of $y = -x^2$ is reflected in the x-axis and then dilated by a factor of $\frac{1}{3}$ from the x-axis.
- **c** The graph of $y = 2x^2$ is reflected in the x-axis and then dilated by a factor of 2 from the x-axis.
- **d** The graph of $y = \frac{1}{3}x^2$ is reflected in the x-axis and then dilated by a factor of 4 from the x-axis.

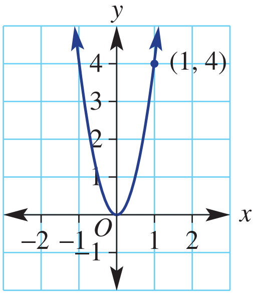
8 The graph of $y = ax^2$ is reflected in the x-axis and then dilated from the x-axis by a given factor. Does it matter which transformation is completed first? Explain.

9 The graph of the rule $y = ax^2$ is reflected in the y-axis. What is the new rule of the graph?

ENRICHMENT: Substitute to find the rule	–	–	10(1/2), 11

10 If a rule is of the form $y = ax^2$ and it passes through a point, say (1, 4), we can substitute this point to find the value of a.

$$y = ax^2$$
$$4 = a \times (1)^2$$
$$\therefore a = 4 \text{ and } y = 4x^2$$



Use this method to determine the equation of a quadratic relation that has an equation of the form $y = ax^2$ and passes through:

- **a** (1, 5)
- **b** (1, 7)
- **c** (−1, 1)
- **d** (−2, 7)
- **e** (−5, 4)
- **f** (3, 26)
- **g** (4, 80)
- **h** (−1, −52).

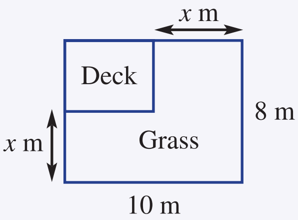
11 The rule of the form $y = ax^2$ models the shape of the parabolic cables of the Golden Gate Bridge. If the cable is centred at (0, 0) and the top of the right pylon has the coordinates (492, 67), find a possible equation that describes this shape. The numbers given are in metres.

Applications and problem-solving

The following problems will investigate practical situations drawing upon knowledge and skills developed throughout the chapter. In attempting to solve these problems, aim to identify the key information, use diagrams, formulate ideas, apply strategies, make calculations and check and communicate your solutions.

Backyard blitz

1 Jamie is planning to renovate his rectangular backyard. He wishes to have a rectangular deck that is positioned against the house and a fence. The remainder of the yard is to have synthetic grass of width x m along the house and fence near the deck. The backyard has dimensions 10 m by 8 m. The synthetic grass he has chosen costs \$35 per square metre, and the timber for the deck is \$200 per square metre. Jamie's budget is \$4000.



Jamie is investigating possible designs for his backyard by varying the width of the grassed area to suit his budget of \$4000 for the grass and timber materials.

- **a** Give expressions in metres in terms of x for:
  - **i** the length (longer side) of the deck
  - **ii** the width of the deck.
- **b** Give an expression for the area in expanded and simplified form of:
  - **i** the deck
  - **ii** the grassed area.
- **c** One design Jamie is considering is such that the area of the deck occupies one-tenth of the backyard area.
  - **i** By solving an appropriate equation, determine the value of x that satisfies this design.
  - **ii** What is the cost of the synthetic grass for this value of x?
  - **iii** Will Jamie come in under budget using this model?
- **d** Give a rule in terms of x for the cost, \$$C$, of renovating the backyard. Expand and simplify your rule and use it to verify your answer to part **c iii**.
- **e** If one quote for the cost of materials is \$5275, use your rule in part **d** to solve an equation to find the value of x and hence the deck area that gives this quote. (*Hint*: Remove the coefficient of x^2 as the common factor when factorising.)

Computer golf

2 A computer game designer is using quadratic rules to model the path of a golf ball for a variety of shot types in a new golf game. Based on the user's selection of club and other parameters, including distance from the hole, a quadratic rule is assigned to simulate their shot. The rule models the ball's path from when it is struck until when it first bounces.

The game designer is in the testing phase and is checking some of the rules for accuracy and to see how they can be manipulated to create other scenarios in the game.

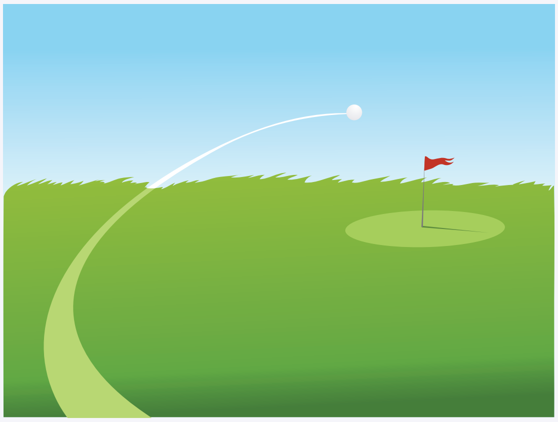
On the easy level of the game, the rules used give the ball a direct path in line with the hole. One such rule used to model certain pitch shots from the fairway onto the putting green is given by $y = -\frac{1}{75}x(x - 60)$, where x is the horizontal distance in metres from where the ball was hit and y is the vertical height of the ball above ground in metres.

A pitch shot using this model is taken 80 m from the hole.

- **a** Use the rule to find:
  - **i** the height above ground of the ball after it has travelled 15 m horizontally
  - **ii** the height above ground from which the ball is struck.
- **b** Use an equation to determine how far from the hole the ball first bounces.
- **c** Using some of the key points found in parts **a** and **b**:
  - **i** plot a graph showing the path of the ball until it first bounces
  - **ii** use the graph and symmetry of the flight path to determine how far the ball has travelled horizontally when it reaches its maximum height
  - **iii** hence use the rule to find the maximum height reached by the ball.

The game designer adds some trees to the golf course for added difficulty. From where the above shot is taken, one particular tree is placed in line with the hole 18 m from where the shot is taken. The tree is 10.8 m high.

- **d** Using the given rule for the shot simulation, determine if the ball will clear the tree.



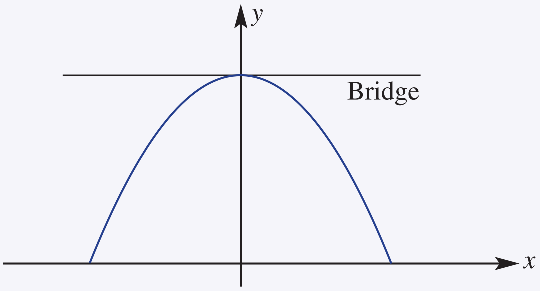
- **e** Different flight paths are obtained by adjusting the rule for the shot.
  - **i** Adjust the given rule so that the ball is still hit from ground level but first bounces 12 m from the hole.
  - **ii** Check algebraically whether or not this shot will clear the tree.
- **f** A water hazard is also placed on the course in line with the ball and the hole. The hazard covers a distance of between 48 m and 55 m from where the shot is being taken. Write a rule that the programmer could use such that the shot lands in the water before it bounces and reaches a maximum height of at least 13 m.

Bridges and barges

3 A pre-existing bridge across a river is to be replaced by a new bridge further downstream. The new bridge will be wider to ease traffic congestion.

Barges are moved by tugboats under this bridge to deliver cargo to the port. The design of new barges which are being built cannot fit under the current bridge. The new barge is rectangular and has a width of 6 m and a height, including cargo, of 5 m.

The current bridge is supported by a parabolic arch and the design is modelled on a Cartesian plane with rule $y = -\frac{1}{2}x^2 + 8$, as shown below. The y-axis indicates the middle of the bridge and the x-axis represents water level. The parabolic arch is at water level at the edges of the river.



The designer's aim is to keep the current features of the bridge and parabolic arch design, but to ensure that the new barges fit under the bridge.

a Use the model of the current bridge to determine the height of the bridge above water in the middle of the bridge.

b By solving an appropriate equation, determine the width of the river.

c Verify that the new barge design will not be able to pass under the arch of the bridge.

The specifications for the parabolic arch of the new bridge are a maximum height of 8 m and a width of 6 m at a height of 5 m above water level.

d **i** Draw a model of the new bridge on a set of axes with the y-axis indicating the centre of the bridge and the x-axis at water level.

ii Find a quadratic rule in the form $y = ax^2 + c$ that fits the arch specifications.

This bridge is placed further down the river where the river is wider.

iii Using your rule, find the width of this bridge. Answer correct to one decimal place.

e The bridge designers are looking to the future and decide they should leave more room for extra cargo to fit under the bridge. Find a rule for a parabolic arch under the bridge that will be 10 m high and that will allow the total height above water of the barge to be at least 8 m on a 6 m wide barge.

10G Sketching translations of $y = x^2$

Learning intentions

- To understand what it means for a graph to be translated
- To know what causes a graph to be translated vertically and horizontally
- To understand the turning point form of a quadratic equation
- To be able to sketch a graph of a quadratic equation expressed in turning point form, labelling the turning point and y-intercept

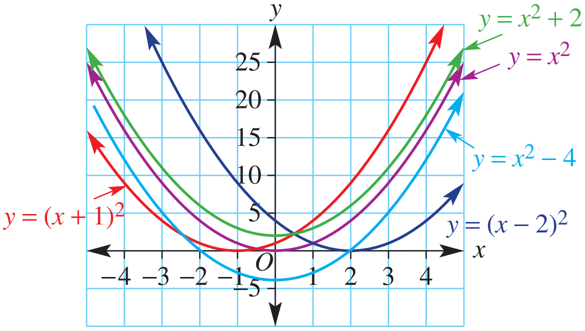
As well as reflection and dilation, there is a third type of transformation called translation. Translation involves a shift of every point on the graph horizontally and/or vertically. Unlike reflections and dilations, a translation alters the coordinates of the turning point. The shape of the curve is unchanged, but its position in relation to the origin is altered.

Manufacturers employ mathematicians to develop profit equations, which are quadratics when sales and profit per item are linear relations of the selling price. As the selling price increases, profit increases to the parabola's maximum point and then decreases due to declining sales.

LESSON STARTER Which way: Left, right, up or down?

This table and graph shows the quadratics $y = x^2$, $y = (x - 2)^2$, $y = (x + 1)^2$, $y = x^2 - 4$ and $y = x^2 + 2$. The table could also be produced using technology.

x	−3	−2	−1	0	1	2	3
$y = x^2$	9	4	1	0	1	4	9
$y = (x - 2)^2$	25	16	9	4	1	0	1
$y = (x + 1)^2$	4	1	0	1	4	9	16
$y = x^2 - 4$	5	0	−3	−4	−3	0	5
$y = x^2 + 2$	11	6	3	2	3	6	11



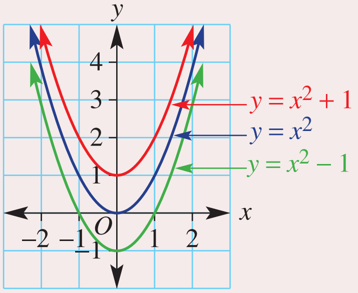
- Discuss what effect the different numbers in the rules had on the y-values in the table.
- Also discuss what effect the numbers in the rules have on each graph. How are the coordinates of the turning point changed?
- What conclusions could you draw on the effect of h in the rule $y = (x - h)^2$?
- What conclusions could you draw on the effect of k in the rule $y = x^2 + k$?
- What if the rule was $y = -x^2 + 2$ or $y = -(x + 1)^2$? Describe how the graphs would look.

ISBN 978-1-108-77288-4

Updated September 2021

KEY IDEAS

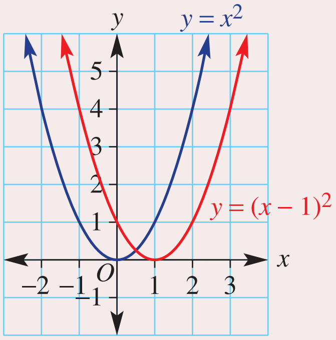
- A **translation** of a graph involves a shift of every point horizontally and/or vertically.
- Vertical translations: $y = x^2 + k$
  - If $k > 0$, the graph is translated k units up (red curve).
  - If $k < 0$, the graph is translated k units down (green curve).
  - The turning point is $(0, k)$ for all curves.
  - The axis of symmetry is the line $x = 0$ for all curves.
  - The y-intercept is k for all curves.

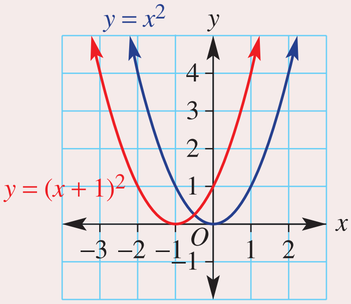


- Horizontal translations: $y = (x - h)^2$

  If $h > 0$, the graph is translated h units to the right.

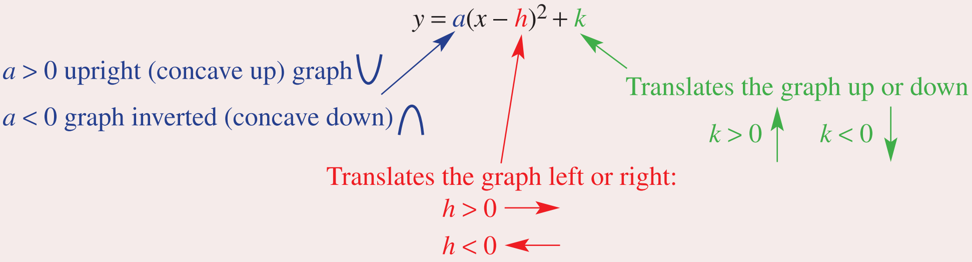
  If $h < 0$, the graph is translated h units to the left.



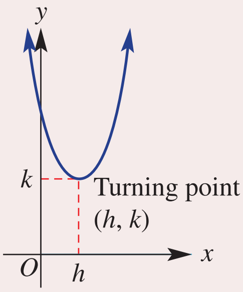


  - The turning point is $(h, 0)$ in both cases.
  - The axis of symmetry is the line $x = h$ in both cases.
  - The y-intercept is h^2 in both cases.

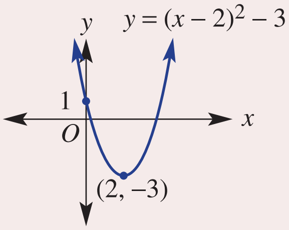
- The **turning point form** of a quadratic is given by:


y = a(x − h)² + k
a > 0 upright (concave up) graph
a < 0 graph inverted (concave down)
Translates the graph up or down
k > 0
k < 0
Translates the graph left or right:
h > 0
h < 0

Axis of symmetry: $x = h$
Turning point: (h, k)



- To sketch a graph of a quadratic equation in turning point form, follow these steps.
  - Draw and label a set of axes.
  - Identify important points including the turning point and y-intercept.
  - Sketch the curve connecting the key points and making the curve symmetrical.



BUILDING UNDERSTANDING

1 This diagram shows the graphs of $y = x^2$, $y = -x^2$, $y = (x + 2)^2$ and $y = -(x - 3)^2$.

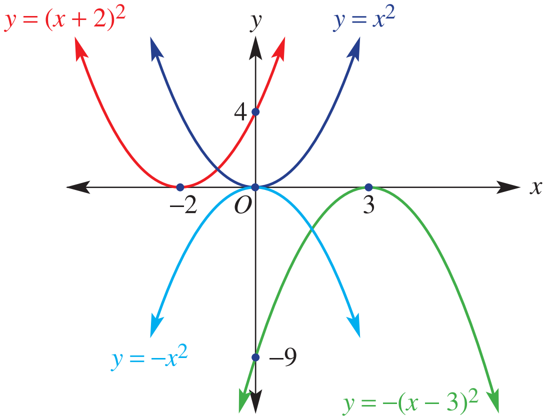
a State the coordinates of the turning point of the graph of:

i $y = (x + 2)^2$

ii $y = -(x - 3)^2$

b Compared to the graph of $y = x^2$, which way has the graph of $y = (x + 2)^2$ been translated (left or right)?

c Compared to the graph of $y = -x^2$ which way has the graph of $y = -(x - 3)^2$ been translated (left or right)?



2 This diagram shows the graphs of $y = x^2$, $y = -x^2$, $y = x^2 - 3$ and $y = -x^2 + 2$.

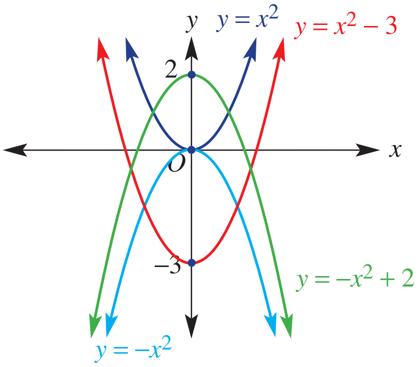
a State the coordinates of the turning point of the graph of:

i $y = x^2 - 3$

ii $y = -x^2 + 2$

b Compared to the graph of $y = x^2$, which way has the graph of $y = x^2 - 3$ been translated (up or down)?

c Compared to the graph of $y = -x^2$, which way has the graph of $y = -x^2 + 2$ been translated (up or down)?



3 Substitute $x = 0$ to find the y-intercept for these rules.

a $y = x^2 + 3$

b $y = -x^2 - 4$

c $y = -(x - 2)^2$

d $y = (x + 5)^2$

Example 13 Sketching with horizontal and vertical translations

Sketch the graphs of these rules showing the y-intercept and the coordinates of the turning point.

a $y = x^2 + 2$

b $y = -x^2 - 1$

c $y = (x - 3)^2$

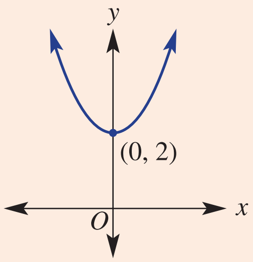
d $y = -(x + 2)^2$

SOLUTION

a $y = x^2 + 2$

Turning point is $(0, 2)$

y-intercept: $y = (0)^2 + 2 = 2$



EXPLANATION

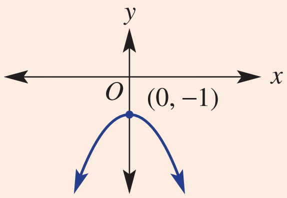
For $y = x^2 + k$, $k = 2$, so the graph of $y = x^2$ is translated 2 units up.

The point $(0, 0)$ shifts to $(0, 2)$.

b $y = -x^2 - 1$

Turning point is $(0, -1)$

y-intercept: $y = -(0)^2 - 1 = -1$



The graph of $y = -x^2$ is a reflection of the graph of $y = x^2$ in the x-axis.

For $y = -x^2 + k$, $k = -1$ so the graph of $y = -x^2$ is translated down 1 unit.

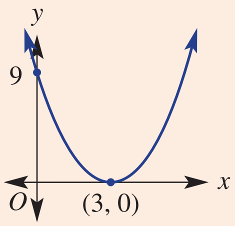
c $y = (x - 3)^2$

Turning point is $(3, 0)$

y-intercept: $y = (0 - 3)^2$

$= (-3)^2$

$= 9$



For $y = (x - h)^2$, $h = 3$, so the graph of $y = x^2$ is translated 3 units to the right.

The y-intercept is found by substituting $x = 0$ into the rule.

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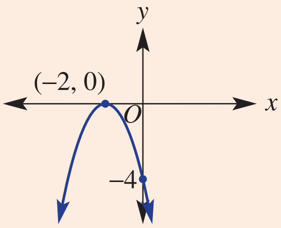
d $y = -(x + 2)^2$

Turning point is $(-2, 0)$

y-intercept: $y = -(0 + 2)^2$

$= -(2)^2$

$= -4$



For $y = -(x - h)^2$, $h = -2$ since $-(x - (-2))^2 = -(x + 2)^2$. So the graph of $y = -x^2$ is translated 2 units to the left.
The y-intercept is found by substituting $x = 0$.

The negative sign in front means the graph is inverted.

Now you try

Sketch the graphs of these rules showing the y-intercept and the coordinates of the turning point.

a $y = x^2 + 4$ **b** $y = -x^2 - 2$

c $y = (x - 1)^2$ **d** $y = -(x + 3)^2$

Example 14 Sketching with combined translations

Sketch these graphs, showing the y-intercept and the coordinates of the turning point.

a $y = (x - 2)^2 - 1$ **b** $y = -(x + 3)^2 + 2$

SOLUTION

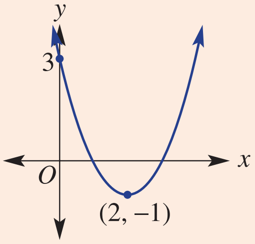
a $y = (x - 2)^2 - 1$

Turning point is $(2, -1)$

y-intercept: $y = (0 - 2)^2 - 1$

$= 4 - 1$

$= 3$



EXPLANATION

For $y = (x - h)^2 + k$, $h = 2$ and $k = -1$, so the graph of $y = x^2$ is shifted 2 to the right and 1 down.
Substitute $x = 0$ for the y-intercept.

First position the coordinates of the turning point and y-intercept, and then join to form the curve.

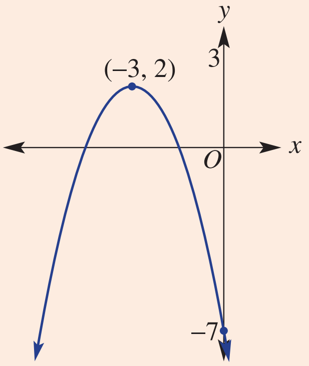
Continued on next page

b $y = -(x + 3)^2 + 2$

Turning point is $(-3, 2)$

$$\begin{aligned} y\text{-intercept: } y &= -(0 + 3)^2 + 2 \\ &= -9 + 2 \\ &= -7 \end{aligned}$$

For $y = -(x - h)^2 + k$, $h = -3$ and $k = 2$, so the graph of $y = x^2$ is shifted 3 to the left and 2 up.



First position the coordinates of the turning point and y-intercept and then join to form the curve.

Now you try

Sketch these graphs on the same set of axes showing the y-intercept and the coordinates of the turning point.

a $y = (x - 3)^2 - 2$ **b** $y = -(x + 2)^2 + 1$

Exercise 10G

FLUENCY	1–3, 4(1/2)	1–2(1/2), 3, 4(1/2)	1–2(1/2), 4(1/2)

1 Sketch the graphs of these rules showing the y-intercept and the coordinates of the turning point.

Example 13a **a** $y = x^2 + 1$ **b** $y = x^2 + 3$ **c** $y = x^2 - 2$

Example 13b **d** $y = -x^2 + 4$ **e** $y = -x^2 + 1$ **f** $y = -x^2 - 5$

2 Sketch the graphs of these rules showing the y-intercept and the coordinates of the turning point.

Example 13c **a** $y = (x - 2)^2$ **b** $y = (x - 4)^2$ **c** $y = (x + 3)^2$

Example 13d **d** $y = -(x - 3)^2$ **e** $y = -(x + 6)^2$ **f** $y = -(x + 2)^2$

3 The below diagram shows the graphs of $y = x^2$, $y = -x^2$, $y = -(x - 2)^2 + 3$ and $y = (x + 1)^2 - 1$.

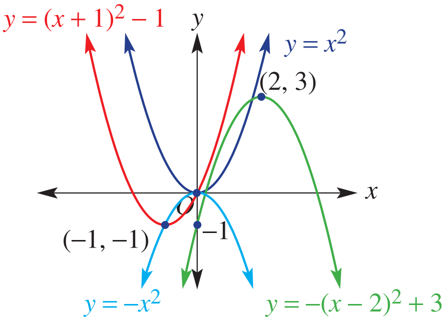
a State the coordinates of the turning point of the graph of:

i $y = -(x - 2)^2 + 3$ **ii** $y = (x + 1)^2 - 1$

b State the missing words and numbers.

i Compared to the graph of $y = x^2$, the graph of $y = (x + 1)^2 - 1$ has to be translated ______ unit to the ______ and ______ unit ______.

ii Compared to the graph of $y = -x^2$, the graph of $y = -(x - 2)^2 + 3$ has to be translated ______ units to the ______ and ______ units ______.



Example 14

4 Sketch each graph showing the y-intercept and the coordinates of the turning point.

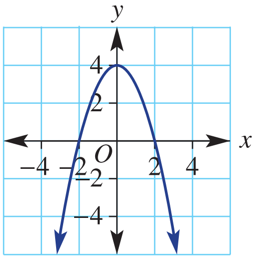
a $y = (x - 3)^2 + 2$ **b** $y = (x - 1)^2 - 1$ **c** $y = (x + 2)^2 - 3$
d $y = (x + 1)^2 + 7$ **e** $y = -(x - 2)^2 + 1$ **f** $y = -(x - 5)^2 + 3$
g $y = -(x + 3)^2 - 4$ **h** $y = -(x + 1)^2 - 5$ **i** $y = -(x - 3)^2 - 6$

PROBLEM-SOLVING 5 | 5, 6 | 5, 6(½), 7

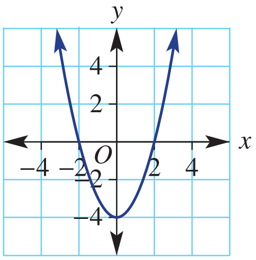
5 Match each parabola with the appropriate equation from the list below.

a $y = x^2$ **b** $y = (x + 2)^2$ **c** $y = x^2 - 4$ **d** $y = -x^2 - 3$
e $y = (x - 2)^2$ **f** $y = x^2 + 4$ **g** $y = 4 - x^2$ **h** $y = (x + 3)^2$
i $y = (x - 5)^2$ **j** $y = x^2 + 3$ **k** $y = -(x + 3)^2$ **l** $y = -x^2 + 3$

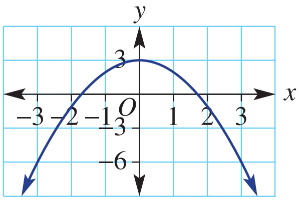
A



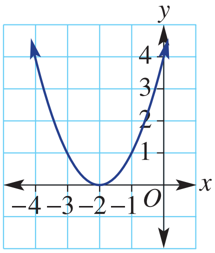
B



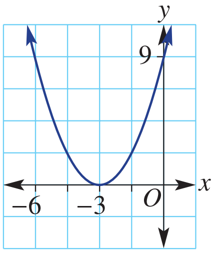
C



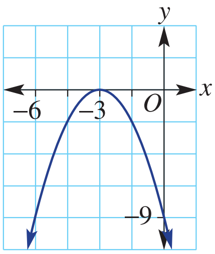
D



E



F



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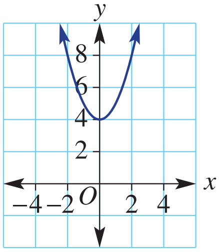

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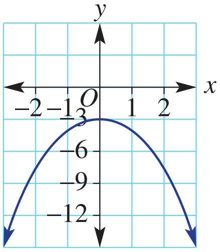

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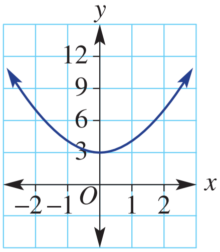

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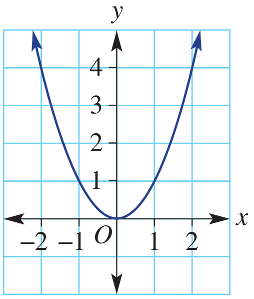
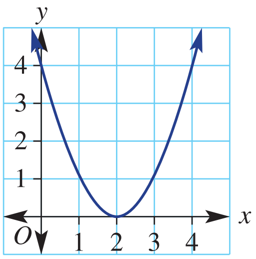
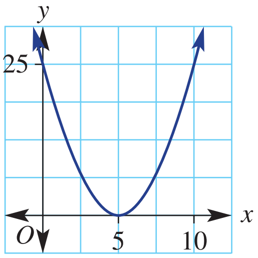


K

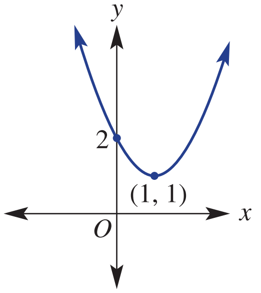


L

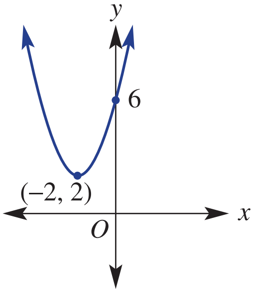


6 Write the rule for each graph in the turning point form: $y = (x - h)^2 + k$.

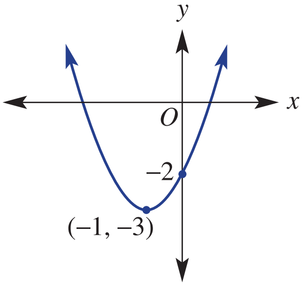
a



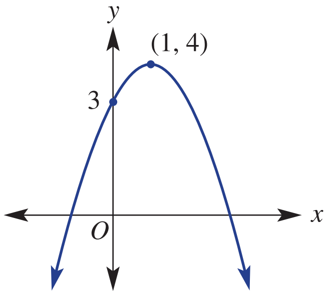
b



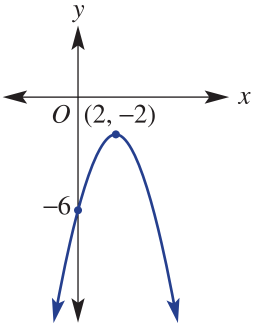
c



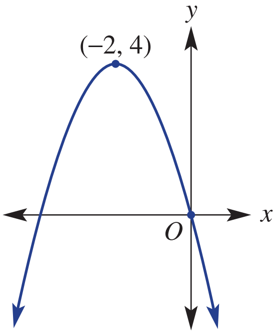
d



e



f



7 A bike track can be modelled approximately by combining two different quadratic equations. The first part of the bike track can be modelled by the equation $y = -(x - 2)^2 + 9$ for $-2 \leqslant x \leqslant 5$. The second part of the bike track can be modelled by the equation $y = (x - 7)^2 - 4$ for $5 \leqslant x \leqslant 10$.

a Find the turning point of the graph of each quadratic equation.

b Sketch each graph on the same set of axes. On your sketch of the bike track you need to show the coordinates of the start and finish of the track and where it crosses the x-axis.

REASONING	8	8, 9	9, 10

8 Written in the form $y = a(x - h)^2 + k$, the rule $y = 4 - (x + 2)^2$ could be rearranged to give $y = -(x + 2)^2 + 4$.

- **a** Rearrange these rules and write in the form $y = a(x - h)^2 + k$.
  - **i** $y = 3 - (x + 1)^2$
  - **ii** $y = 4 + (x + 3)^2$
  - **iii** $y = -3 + (x - 1)^2$
  - **iv** $y = -7 - (x - 5)^2$
  - **v** $y = -2 - x^2$
  - **vi** $y = -6 + x^2$
- **b** Write down the coordinates of the turning point for each of the above quadratics.

9 A quadratic has the rule $y = a(x - h)^2 + k$.

- **a** What are the coordinates of the turning point?
- **b** Write an expression for the y-intercept.

10 Investigate and explain how the graph of:

- **a** $y = (2 - x)^2$ compares to the graph of $y = (x - 2)^2$
- **b** $y = (1 - x)^2$ compares to the graph of $y = (x - 1)^2$.

ENRICHMENT: Finding rules	–	–	11–14($\frac{1}{2}$)

11 Find the equation of the quadratic relation which is of the form $y = x^2 + c$ and passes through:

a (1, 4) **b** (3, 5) **c** (2, 1) **d** (2, −1).

12 Find the equation of the quadratic relation which is of the form $y = -x^2 + c$ and passes through:

a (1, 3) **b** (−1, 3) **c** (3, 15) **d** (−2, 6).

13 Find the possible equations of each of the following quadratics if their equation is of the form $y = (x - h)^2$ and their graph passes through the point:

a (1, 16) **b** (3, 1) **c** (−1, 9) **d** (3, 9).

14 Find the rule for each of these graphs with the given turning point (TP) and y-intercept.

- **a** TP = (1, 1), y-intercept = 0
- **b** TP = (−2, 0), y-intercept = 4
- **c** TP = (3, 0), y-intercept = −9
- **d** TP = (−3, 2), y-intercept = −7
- **e** TP = (−1, 4), y-intercept = 5
- **f** TP = (3, −9), y-intercept = 0

10H Sketching parabolas using intercept form EXTENDING

Learning intentions

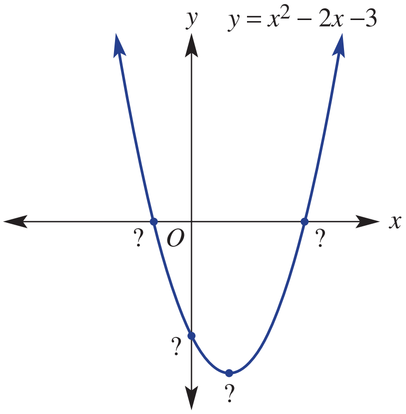
- To understand how a parabola can have 0, 1 or 2 x-intercepts
- To be able to find the x-intercepts of a quadratic graph by solving an equation using the Null Factor Law
- To know how to use symmetry and the x-intercepts to help locate the turning point
- To be able to sketch a quadratic graph in expanded form using factorisation

So far we have sketched parabolas using rules of the form $y = a(x - h)^2 + k$ where the coordinates of the turning point can be determined directly from the rule. An alternative method for sketching parabolas uses the factorised form of the quadratic rule and the Null Factor Law to find the x-intercepts. The turning point can be found by considering the axis of symmetry, which sits halfway between the two x-intercepts.

Parallel lines reflected from a parabola, and parallel light or radio waves reflected from a parabolic dish, meet at a focal point above the vertex. Scientists use CSIRO's six radio telescopes, with parabolic dishes, to research objects in outer space.

LESSON STARTER From x-intercepts to turning point

This graph has the rule $y = x^2 - 2x - 3$ but all its important features are not shown.
Find the features by discussing these questions.

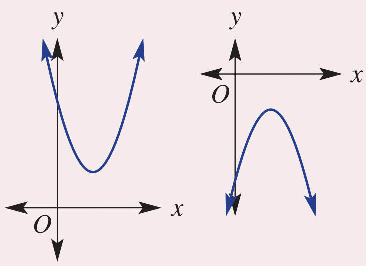


- Can the quadratic rule be factorised?
- What is the value of y at the x-intercepts?
- How can the factorised form of the rule help to find the x-intercepts?
- How does the x-coordinate of the turning point relate to the x-intercepts?
- Discuss how the y-coordinate of the turning point can be found.
- Finish by finding the y-intercept.

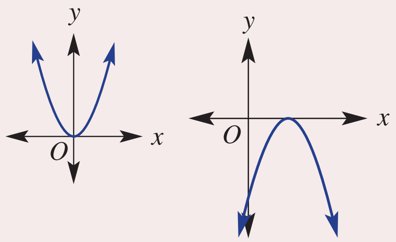
KEY IDEAS

- All parabolas have one y-intercept and can have zero, one or two x-intercepts.

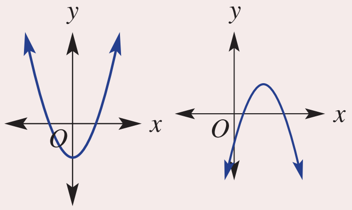
Zero x-intercepts



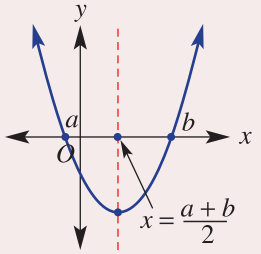
One x-intercept



Two x-intercepts



- x-intercepts can be found by substituting $y = 0$ and using the Null Factor Law.
- If the graph has two x-intercepts (a and b), the turning point can be found by:
  - calculating the x-coordinate of the turning point, which is the midpoint of a and b; that is, $x = \frac{a+b}{2}$
  - calculating the y-coordinate of the turning point by substituting the x-coordinate into the rule for the quadratic.



$$\begin{aligned} y &= (x-a)(x-b) \\ x\text{-intercepts: } 0 &= (x-a)(x-b) \\ x - a &= 0 \text{ or } x - b = 0 \\ x &= a \text{ or } x = b \end{aligned}$$

BUILDING UNDERSTANDING

① Factorise these quadratics.

a $x^2 + 2x$ **b** $x^2 - 3x$ **c** $x^2 - 9$

d $x^2 - 49$ **e** $x^2 - x - 12$ **f** $x^2 - 4x + 4$

② For these factorised quadratics, use the Null Factor Law to solve for x, and then find the x-value halfway between the two values.

a $0 = (x - 2)(x + 2)$ **b** $0 = (x + 1)(x + 5)$

c $0 = (x - 1)(x + 3)$ **d** $0 = (x - 2)(x + 3)$

③ Use substitution to find the y-coordinate of the turning point of these quadratics. The x-coordinate of the turning point is given.

a $y = x^2 - 4, x = 0$ **b** $y = x^2 - 6x, x = 3$ **c** $y = x^2 + 4x + 3, x = -2$

Example 15 Finding intercepts

For each of the following quadratic relations, find:

- **i** the x-intercepts
- **ii** the y-intercept.

a $y = x(x + 1)$

b $y = 2(x + 2)(x - 3)$

SOLUTION	EXPLANATION
a **i** x-intercepts (let $y = 0$): $x(x + 1) = 0$	Let $y = 0$ to find the x-intercepts.
$x = 0$ or $x + 1 = 0$ $x = 0$ or $x = -1$	Apply the Null Factor Law to set each factor equal to 0 and solve.
ii y-intercept (let $x = 0$): $y = 0(0 + 1)$ $y = 0$	Let $x = 0$ to find the y-intercept.
b **i** $y = 2(x + 2)(x - 3)$ has two x-intercepts.	There are two different factors.
x-intercepts (let $y = 0$): $2(x + 2)(x - 3) = 0$ $x + 2 = 0$ or $x - 3 = 0$ $x = -2$ or $x = 3$	Let $y = 0$ to find the x-intercepts. Set each factor equal to 0 and solve.
iii y-intercept (let $x = 0$): $y = 2(0 + 2)(0 - 3)$ $y = 2(2)(-3)$ $y = -12$	Let $x = 0$ to find the y-intercept.

Now you try

For each of the following quadratic relations, find:

- **i** the x-intercepts
- **ii** the y-intercept.

a $y = x(x + 5)$

b $y = 3(x + 1)(x - 4)$

Example 16 Sketching using intercept form

Consider the quadratic relation $y = x^2 - 2x$.

- **a** Factorise the relation.
- **b** Find the y-intercept.
- **c** Find the x-intercepts.
- **d** Find the axis of symmetry.
- **e** Find the turning point.
- **f** Sketch the graph clearly showing all the key features.

SOLUTION

a $y = x^2 - 2x$

$= x(x - 2)$

b y-intercept (let $x = 0$): $y = 0$

c x-intercepts (let $y = 0$):

$0 = x(x - 2)$

$x = 0$ or $x - 2 = 0$

$x = 0$ or $x = 2$

d Axis of symmetry: $x = \frac{0 + 2}{2}$

$\therefore x = 1$

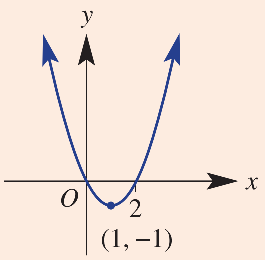
e Turning point occurs when $x = 1$.

When $x = 1$, $y = (1)^2 - 2(1)$

$y = -1$

$\therefore$ there is a minimum turning point at $(1, -1)$.

f



EXPLANATION

Take out the common factor of x.

Let $x = 0$ to find the y-intercept.

Let $y = 0$ to find the x-intercepts. Set each factor equal to 0 and solve.

The axis of symmetry is halfway between the x-intercepts.

Substitute $x = 1$ into $y = x^2 - 2x$ to find the y-coordinate.

$x = 1$ and $y = -1$

The coefficient of x^2 is positive, therefore the basic shape is .

Sketch the graph, labelling the key features found above.

Now you try

Consider the quadratic relation $y = x^2 - 4x$.

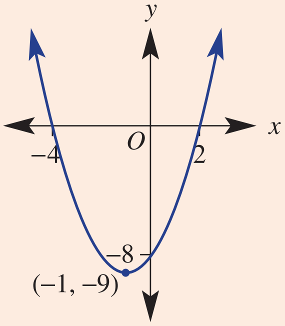
- **a** Factorise the relation.
- **b** Find the y-intercept.
- **c** Find the x-intercepts.
- **d** Find the axis of symmetry.
- **e** Find the turning point.
- **f** Sketch the graph clearly showing all the key features.

Example 17 Sketching a quadratic trinomial

Consider the quadratic relation $y = x^2 + 2x - 8$.

- **a** Factorise the relation.
- **b** Find the y-intercept.
- **c** Find the x-intercepts.
- **d** Find the axis of symmetry.
- **e** Find the turning point.
- **f** Sketch the graph clearly showing all the key features.

SOLUTION	EXPLANATION
a $y = x^2 + 2x - 8$ $= (x + 4)(x - 2)$	Factorise the quadratic trinomial: $4 \times (-2) = -8$ and $4 + (-2) = 2$.
b y-intercept (let $x = 0$): $y = -8$	Let $x = 0$ to find the y-intercept.
c x-intercepts (let $y = 0$): $0 = (x + 4)(x - 2)$ $x + 4 = 0$ or $x - 2 = 0$ $x = -4$ or $x = 2$	Let $y = 0$ to find the x-intercepts.
d Axis of symmetry: $x = \dfrac{-4 + 2}{2}$ $\therefore x = -1$	The axis of symmetry is halfway between the x-intercepts.
e Turning point occurs when $x = -1$. When $x = -1$, $y = (-1)^2 + 2(-1) - 8$ $y = -9$ $\therefore$ there is a minimum turning point at $(-1, -9)$.	Substitute $x = -1$ into $y = x^2 + 2x - 8$ to find the y-coordinate. $x = -1$ and $y = -9$ The coefficient of x^2 is positive, therefore the basic shape is ∪.
f	Sketch the graph showing the key features.



Now you try

Consider the quadratic relation $y = x^2 - 4x - 12$.

- **a** Factorise the relation.
- **b** Find the y-intercept.
- **c** Find the x-intercepts.
- **d** Find the axis of symmetry.
- **e** Find the turning point.
- **f** Sketch the graph clearly showing all the key features.

Exercise 10H

FLUENCY	1, 2–4(1/2)	2–5(1/2)	2–5(1/3)

1 For each of the following quadratic relations, find:

- **i** the x-intercepts
- **ii** the y-intercept.

Example 15a

a $y = x(x + 2)$

Example 15b

b $y = 4(x - 1)(x + 2)$

Example 15

2 For each of the following quadratic relations, find:

- **i** the x-intercepts
- **ii** the y-intercept.

a $y = x(x + 7)$ **b** $y = x(x + 3)$ **c** $y = x(x + 4)$

d $y = (x - 4)(x + 2)$ **e** $y = (x + 2)(x - 5)$ **f** $y = (x - 7)(x + 3)$

g $y = 2(x + 3)(x - 1)$ **h** $y = 3(x + 4)(x + 1)$ **i** $y = (2 - x)(3 - x)$

Example 16

3 Consider each of the following quadratic relations.

- **i** Factorise the relation.
- **ii** Find the y-intercept.
- **iii** Find the x-intercepts.
- **iv** Find the axis of symmetry.
- **v** Find the turning point.
- **vi** Sketch the graph clearly showing all the key features.

a $y = x^2 - 5x$ **b** $y = x^2 + x$ **c** $y = x^2 - 3x$

d $y = 2x + x^2$ **e** $y = 5x + x^2$ **f** $y = 3x - x^2$

g $y = -x^2 - 8x$ **h** $y = -2x - x^2$ **i** $y = -x^2 + x$

Example 17

4 Consider each of the following quadratic relations.

- **i** Factorise the relation.
- **ii** Find the y-intercept.
- **iii** Find the x-intercepts.
- **iv** Find the axis of symmetry.
- **v** Find the turning point.
- **vi** Sketch each graph clearly showing all the key features.

a $y = x^2 - 3x + 2$ **b** $y = x^2 - 4x + 3$ **c** $y = x^2 + 2x - 3$

d $y = x^2 + 4x + 4$ **e** $y = x^2 + 2x + 1$ **f** $y = x^2 + 2x - 8$

5 For each of the following relations, sketch the graph, clearly showing the x- and y-intercepts and the turning point.

a $y = x^2 - 1$ **b** $y = 9 - x^2$ **c** $y = x^2 + 5x$

d $y = 2x^2 - 6x$ **e** $y = 4x^2 - 8x$ **f** $y = x^2 - 3x - 4$

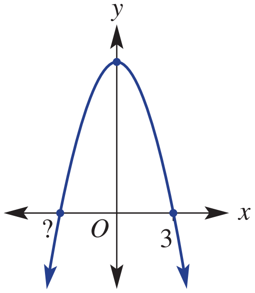
g $y = x^2 + x - 12$ **h** $y = x^2 + 2x + 1$ **i** $y = x^2 - 6x + 9$

Design structures, such as this building at LAX airport, can be modelled on a set of axes.

PROBLEM-SOLVING	6	6, 7	6($\frac{1}{2}$), 7, 8

6 State the missing number (?) in these quadratic graphs.

a



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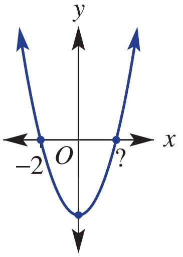

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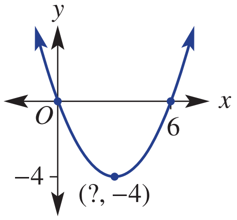
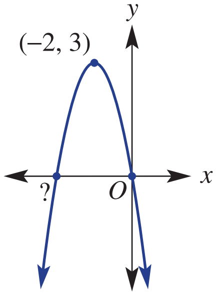
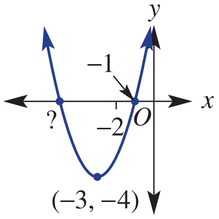


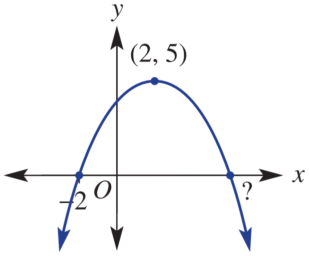
d



e



f



7 The path of a golf ball is given by the rule $y = 30x - x^2$, where y is the height in metres above the ground and x is the horizontal distance in metres. Find:

a how far the ball travels horizontally

b how high the ball reaches mid-flight.

8 A test rocket is fired and follows a path described by $y = 0.1x(200 - x)$. The height is y metres above ground and x is the horizontal distance in metres.

a How far does the rocket travel horizontally?

b How high does the rocket reach mid-flight?

REASONING	9	9, 10	10, 11

9 Explain why the coordinates of the x-intercept and the turning point for $y = (x - 2)^2$ are the same.

10 Write down an expression for the y-intercept for these quadratics.

a $y = ax^2 + bx + c$

b $y = (x - a)(x - b)$

11 $y = x^2 - 2x - 15$ can also be written in the form $y = (x - 1)^2 - 16$.

a Use the second rule to state the coordinates of the turning point.

b Use the first rule to find the x-intercepts and then the turning point. Check you get the same result.

c $y = x^2 - 4x - 45$ can be written in the form $y = (x - h)^2 + k$. Find the value of h and k.

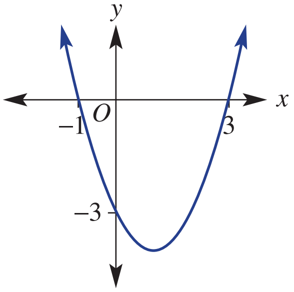
ISBN 978-1-108-77288-4

Updated September 2021

ENRICHMENT: Rule finding using x-intercepts	–	–	12(½)

12 Find the rule for these graphs using intercept form. The first one is done for you.

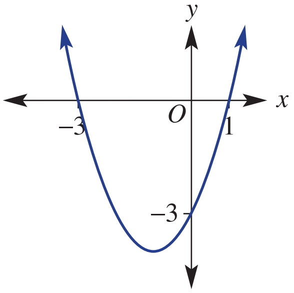
a



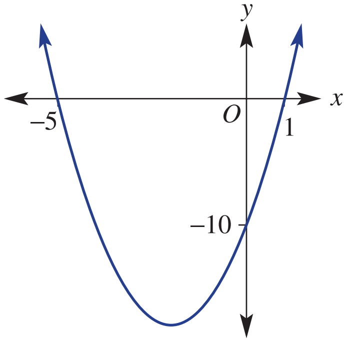
$$
\begin{aligned}
& & y &= a(x+1)(x-3)\\
&\text{Substitute }(0,-3) & -3 &= a(0+1)(0-3)\\
& & -3 &= a(1)\times(-3)\\
& & -3 &= -3a\\
& & a &= 1
\end{aligned}
$$

$\therefore\ y = (x+1)(x-3)$

b



c



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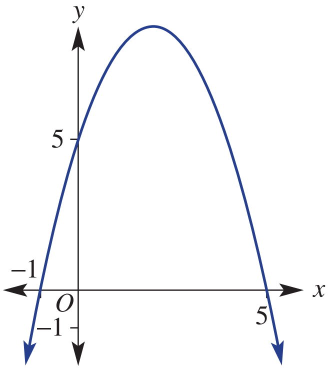

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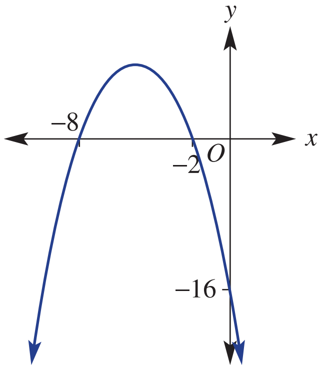

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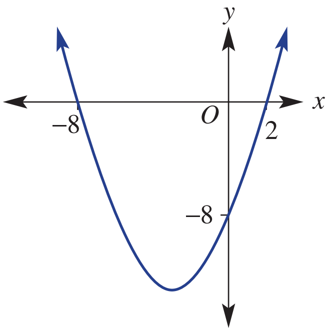
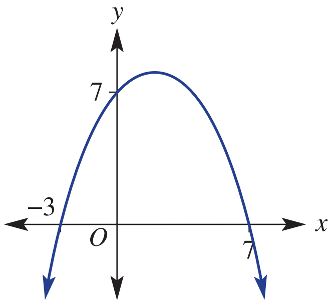


g



Modelling

Bridge support arch

Parabolic shapes are often used as bridge arches due to their self-supporting properties. Using the coordinates (0, 0) for one side of a bridge arch, a model for a bridge parabolic curve is given by $y = kx(a - x)$ where:

- y metres is the vertical distance above water level
- x metres is the horizontal distance from the left-hand side
- a and k are positive numbers.

Present a report for the following tasks and ensure that you show clear mathematical workings and explanations where appropriate.

Preliminary task

a If $k = 0.5$ and $a = 30$:
  - **i** construct the rule connecting y and x
  - **ii** sketch a graph representing the bridge arch by firstly finding the x-intercepts
  - **iii** determine the maximum height of the arch.

b Repeat part **a** for $k = 0.2$ and $a = 50$.

Modelling task

Formulate

a The problem is to determine a rule of a new bridge design which clears a river of a certain width and has a maximum arch height of less than or equal to 40 metres. Write down all the relevant information that will help solve this problem with the aid of a rough sketch.

b Construct the rule connecting y and x using the following values for a and k.
  - **i** $a = 50$ and $k = 0.1$
  - **ii** $a = 70$ and $k = 0.02$

Solve

c If $a = 50$ and $k = 0.1$, sketch a graph of the bridge arch by locating and labelling the following:
  - **i** x-intercepts
  - **ii** turning point.

d Decide if an arch that is designed using $a = 50$ and $k = 0.1$ meets the requirement that the maximum height is to be less than or equal to 40 metres.

e If $a = 70$ and $k = 0.02$, sketch a graph of the bridge arch by locating and labelling the following:
  - **i** x-intercepts
  - **ii** turning point.

f Decide if an arch that is designed using $a = 70$ and $k = 0.02$ meets the requirement that the maximum height is to be less than or equal to 40 metres.

Evaluate and verify

g Choose your own values of a and k so that a bridge that spans a 60 metre-wide river has a maximum arch height of less than 40 metres. Justify your choice by showing your calculations and a graph.

h Choose your own values of a and k so that a bridge that spans a 60 metre-wide river has a maximum arch height of exactly 40 metres. Justify your choice by showing your calculations and a graph.

Communicate

i Summarise your results and describe any key findings.

Extension questions

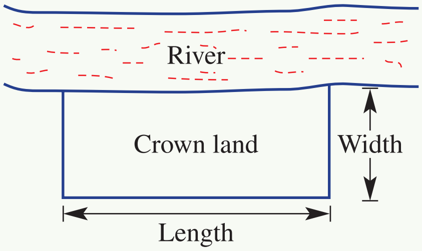
a Show that the rule for k in terms of a is $k = \frac{160}{a^2}$ if the maximum height of the arch is to be 40 metres.

b Use the rule for k in part **a** to determine the value of k if the width of the river is 100 m.

c Use the rule for k in part **a** to determine the width of the river, correct to one decimal place, if $k = 0.3$.

Grazing Crown land

Land along the side of rivers is usually owned by the government and is sometimes called Crown land. Farmers can often lease this land to graze their sheep or cattle.



A farmer has a permit to fence off a rectangular area of land alongside the river. She has 400 m of fencing available and does not need to fence along the river.

A given width

a Find the length of the rectangular area when the width is:
i 50 m **ii** 120 m **iii** 180 m.

b Find the area of the land when the width is:
i 50 m **ii** 120 m **iii** 180 m.

c Which width from part **b** above gave the largest area? Explain why the area decreases for small and large values for the width.

The variable width

a Using x metres to represent the width, write an expression for the length showing working.

b Write an expression for the area of the land in terms of x.

c Use your area expression from part **b** to find the area of the land when x is:
i 20 **ii** 80 **iii** 160.

The graph

a Using your expression from part **b** above, sketch a graph of Area (A) versus x. You should find the following to help complete the graph.
i y-intercept **ii** x-intercepts **iii** axis of symmetry **iv** turning point

b What value of x gives the maximum area of land for grazing? Explain your choice and give the dimensions of the rectangular area of land.

General observations

a What do you notice about the width and the length when the area is a maximum?

b See if the same is true if the farmer had 600 m of fencing instead. Show your expressions and graph.

c Prove your observation to parts **a** and **b** above by finding the x-value that gives a maximum area using k metres of fencing. (*Hint*: Use $A = x(k - 2x)$.)

Problems and challenges

Up for a challenge? If you get stuck on a question, check out the 'Working with unfamiliar problems' poster at the end of the book to help you.

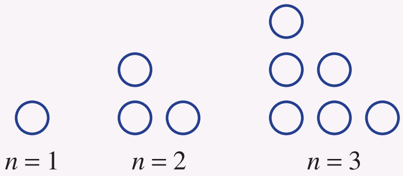
1 a What do you notice about the sum of these numbers?
   - i $1 + 3$
   - ii $1 + 3 + 5$
   - iii $1 + 3 + 5 + 7$
   - iv $1 + 3 + 5 + 7 + 9$

  b Find the sum of the first 100 odd integers.

2 Solve these equations.
  - a $6x^2 = 35 - 11x$
  - b $\frac{2}{x^2} = 1 - \frac{1}{x}$
  - c $\sqrt{x - 1} = \frac{2}{x - 1}$

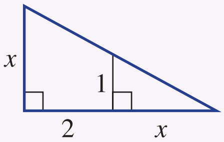
3 The height of a projectile (in metres) above the ground is given by the expression $t(14 - t)$ where time t is in seconds. For how long is the projectile at a height of above 40 m?

4 Find the quadratic rule that relates the number of balls to the term number (n) in the pattern below.



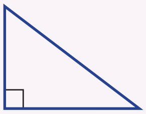
If there are 66 balls in the pattern, what term number is it?

5 Find the value of x in this diagram.



6 a A right-angled triangle's shortest side is of length x, the hypotenuse is 9 units longer than the shortest side and the other side is 1 unit longer than the shortest side. Find the side lengths of the triangle.

  b The area of a right-angled triangle is 60 square units and the lengths of the two shorter sides differ by 7 units. Find the length of the hypotenuse.



7 Given $a > 0$, for what values of k does $y = a(x - h)^2 + k$ have:
  - a two x-intercepts?
  - b one x-intercept?
  - c no x-intercepts?

8 For the following equations, list the possible values of a that will give integer solutions for x.
  - a $x^2 + ax + 24 = 0$
  - b $x^2 + ax - 24 = 0$

9 Solve the equation $(x^2 - 3x)^2 - 16(x^2 - 3x) - 36 = 0$ for all values of x. (*Hint*: Let $a = x^2 - 3x$.)

10 Solve the equation $x^4 - 10x^2 + 9 = 0$ for all values of x. (*Hint*: Let $a = x^2$.)

ISBN 978-1-108-77288-4

Introduction to quadratic equations and graphs

Turning point form

$y = a(x - h)^2 + k$
turning point (h, k)
e.g. $y = 2(x + 1)^2 + 3$
TP: $(-1, 3)$
y-int: $x = 0$, $y = 2(0 + 1)^2 + 3$
$= 2(1)^2 + 3$
$= 5$

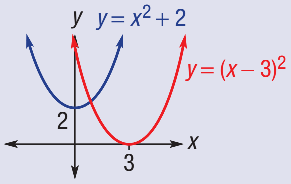
Standard form

A quadratic is of the form
$y = ax^2 + bx + c$, $a \neq 0$

Translation

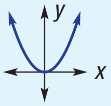
$y = x^2 + k$ $\quad$ $k > 0$, graph shifts up k units
$k < 0$, graph shifts down k units
$y = (x - h)^2$ $\quad$ $h > 0$, graph shifts h units right
$h < 0$, graph shift h units left

e.g.

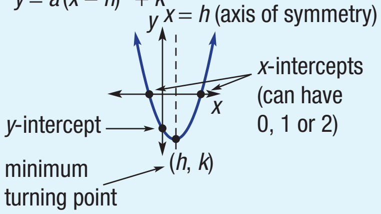


Parabola

Basic form $y = x^2$

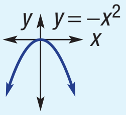


Turning point form
$y = a(x - h)^2 + k$



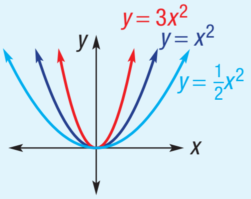
Dilation and reflection

$y = ax^2$
$a < 0$, graph is reflected in x-axis



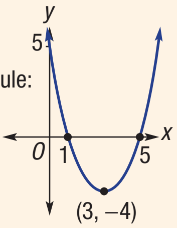
$-1 < a < 1$:
graph appears wider than $y = x^2$ or $y = -x^2$

$a > 1$ or $a < -1$:
graph appears narrower than $y = x^2$ or $y = -x^2$
e.g.



Sketching $y = ax^2 + bx + c$ (Ext)

Find x- and y-intercepts and turning point and then sketch.
e.g. $y = x^2 - 6x + 5$
y-int: $x = 0$, $y = 0^2 - 6(0) + 5$
$= 5$
x-int: $y = 0$, $0 = x^2 - 6x + 5$
$0 = (x - 1)(x - 5)$
$x - 1 = 0$ or $x - 5 = 0$
$x = 1$ or $x = 5$
Turning point: halfway between the x-intercepts:
$x = \frac{1 + 5}{2} = 3$
Substitute $x = 3$ into rule:
$y = 3^2 - 6(3) + 5$
$= -4$
$(3, -4)$ minimum turning point



Applications (Ext)

Define variable.
Set up equation.
Solve using Null Factor Law.
Check answer makes sense.
Answer the question in words.

Solving quadratic equations (Ext)

- Rewrite equation in standard form: i.e. $ax^2 + bx + c = 0$
- Factorise
- Apply Null Factor Law: If $a \times b = 0$, $a = 0$ or $b = 0$.
- Solve remaining equation:
  e.g. $x^2 + 5x = 0$
  $x(x + 5) = 0$
  $x = 0$ or $x + 5 = 0$
  $x = 0$ or $x = -5$
  e.g. $x^2 - 12 = 4x$
  $x^2 - 4x - 12 = 0$
  $(x - 6)(x + 2) = 0$
  $x - 6 = 0$ or $x + 2 = 0$
  $x = 6$ or $x = -2$

ISBN 978-1-108-77288-4

Updated September 2021

Chapter checklist: Success criteria

✔

10A **1. I can write a quadratic equation in standard form.**
e.g. Write $x(x - 3) = 6$ in standard form. Ext ☐

10A **2. I can determine if a value is a solution of an equation.**
e.g. Use substitution to determine if $x = -2$ is a solution of $2x^2 + 3x - 2 = 0$. Ext ☐

10A **3. I can use the Null Factor Law to solve an equation.**
e.g. Use the Null Factor Law to solve $(x + 2)(x - 5) = 0$. Ext ☐

10B **4. I can solve a quadratic equation with a common factor.**
e.g. Solve $2x^2 - 10x = 0$. Ext ☐

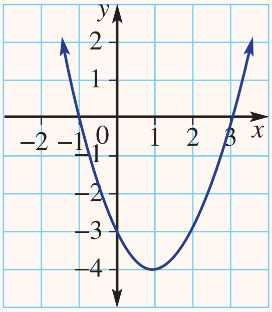
10B **5. I can solve a quadratic equation of the form $ax^2 = d$.**
e.g. Solve $6x^2 = 24$. Ext ☐

10C **6. I can solve equations involving quadratic trinomials.**
e.g. Solve $x^2 - 3x - 70 = 0$. Ext ☐

10C **7. I can solve quadratic equations not in standard form**
e.g. Solve $x^2 = 9x - 20$. Ext ☐

10D **8. I can set up and solve a quadratic equation from a word problem.**
e.g. The length of a postcard is 5 cm more than its width and the area of the postcard is 84 cm^2. Find the dimensions of the postcard. Ext ☐

10E **9. I can identify the key features of a parabola.**
e.g. Consider this graph. ☐



State:
- **i.** the equation of the axis of symmetry
- **ii.** the type and coordinates of the turning point
- **iii.** the x- and y-intercepts.

10E **10. I can plot a parabola from a table of values.**
e.g. For $y = x^2 - 4x + 4$, fill in the table of values and plot the points on a set of axes to form a parabola. ☐

x	−3	−2	−1	0	1	2	3
y							

ISBN 978-1-108-77288-4

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✔

10E

11. I can compare graphs of $y = ax^2$, $a > 0$.

e.g. Sketch $y = x^2$, $y = 3x^2$ and $y = \frac{1}{3}x^2$ by first completing the table of values below.

x	-2	-1	0	1	2
$y = x^2$					
$y = 3x^2$					
$y = \frac{1}{3}x^2$					

State the turning point of each graph and whether the graphs of $y = 3x^2$ and $y = \frac{1}{3}x^2$ are wider or narrower than the graph of $y = x^2$.

10F

12. I can compare graphs of $y = ax^2$, $a < 0$.

e.g. Sketch $y = -x^2$, $y = -4x^2$ and $y = -\frac{1}{4}x^2$ by first completing the table of values below.

x	-2	-1	0	1	2
$y = -x^2$					
$y = -4x^2$					
$y = -\frac{1}{4}x^2$					

State the turning point of each graph and whether the graphs of $y = -4x^2$ and $y = -\frac{1}{4}x^2$ are wider or narrower than the graph of $y = -x^2$.

10G

13. I can sketch with horizontal and vertical translations.

e.g. Sketch $y = x^2 + 3$ and $y = -(x + 1)^2$ showing the y-intercept and turning point.

10G

14. I can sketch with combined translations.

e.g. Sketch $y = (x - 1)^2 - 3$ showing the y-intercept and turning point.

10H

15. I can sketch using intercept form.

e.g. For the quadratic relation $y = x^2 - 6x$, find the x- and y-intercepts and the turning point and sketch its graph, labelling key features.

Ext

10H

16. I can sketch a quadratic trinomial.

e.g. For the quadratic relation $y = x^2 - 4x - 12$, find the x- and y-intercepts and the turning point and sketch its graph, labelling key features.

Ext

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Short-answer questions

10E

1 Consider the quadratic $y = x^2 - 2x - 3$.

a Complete this table of values for the equation.

x	-3	-2	-1	0	1	2	3
y							

b Plot the points in part **a** on a Cartesian plane and join in a smooth curve.

10A Ext

2 Use the Null Factor Law to solve the following equations.

a $x(x + 2) = 0$

b $3x(x - 4) = 0$

c $(x + 3)(x - 7) = 0$

d $(x - 2)(2x + 4) = 0$

e $(x + 1)(5x - 2) = 0$

f $(2x - 1)(3x - 4) = 0$

10B/C Ext

3 Solve the following quadratic equations by first factorising.

a $x^2 + 3x = 0$

b $2x^2 - 8x = 0$

c $x^2 = 25$

d $x^2 = 81$

e $5x^2 = 20$

f $3x^2 - 30 = 0$

g $x^2 + 10x + 21 = 0$

h $x^2 - 3x - 40 = 0$

i $x^2 - 8x + 16 = 0$

10C Ext

4 Write the following quadratic equations in standard form and solve for x.

a $x^2 = 5x$

b $3x^2 = 18x$

c $x^2 + 12 = -8x$

d $2x + 15 = x^2$

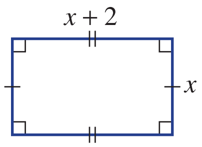
e $x^2 + 15 = 8x$

f $4 - x^2 = 3x$

10D Ext

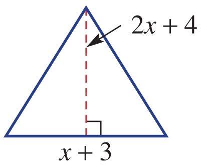
5 Set up and solve a quadratic equation to determine the value of x that gives the specified area of the shapes below.

a



$A = 80 \text{ units}^2$

b



$A = 30 \text{ units}^2$

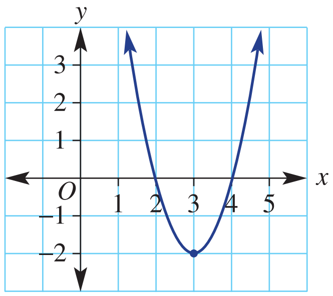
10E

6 For the following graphs state:

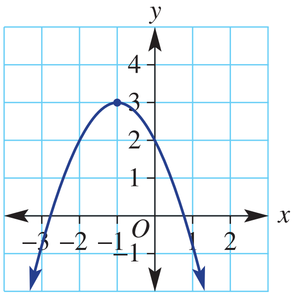
i the axis of symmetry

ii the turning point and its type.

a



b



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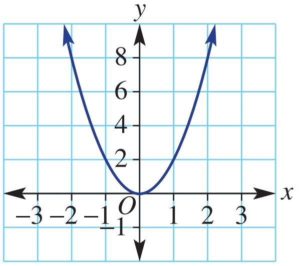
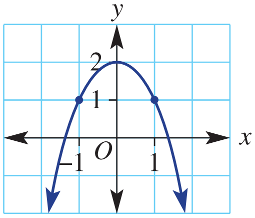
10F/G **7** Match the following quadratic rules with their graphs.

$y = x^2 - 4$, $y = 2x^2$, $y = (x + 1)^2$, $y = (x - 3)^2$, $y = -x^2 + 2$

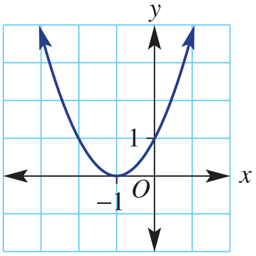
a



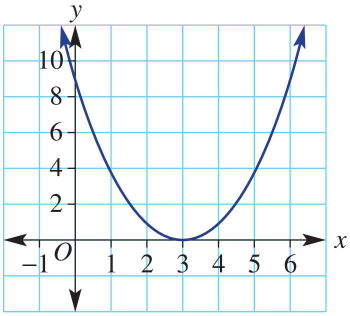
b



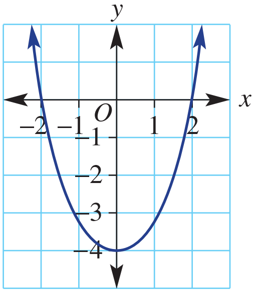
c



d



e



10G **8** Sketch the following graphs labelling the turning point and y-intercept.

a $y = x^2 + 2$

b $y = -x^2 - 5$

c $y = (x + 2)^2$

d $y = -(x - 3)^2$

e $y = (x - 1)^2 + 3$

f $y = 2(x + 2)^2 - 4$

10G **9** State the transformations that take $y = x^2$ to each of the graphs in Question **8**.

10H Ext **10** Sketch the following graphs labelling the y-intercept, turning point and x-intercepts.

a $y = x^2 - 8x + 12$

b $y = x^2 + 10x + 16$

c $y = x^2 + 2x - 15$

d $y = x^2 + 4x - 5$

Multiple-choice questions

10A Ext **1** (1, 3) is a point on a curve with which equation?

A $y = x^2$

B $y = (x - 3)^2$

C $y = x^2 + 2x - 3$

D $y = x^2 + 2$

E $y = 6 - x^2$

10A Ext **2** The solution(s) to $2x(x - 3) = 0$ is/are:

A $x = -3$

B $x = 0$ or $x = 3$

C $x = 2$ or $x = 3$

D $x = 0$ or $x = -3$

E $x = 0$

10A Ext **3** The quadratic equation $x^2 = 7x - 12$ in standard form is:

A $x^2 - 7x - 12 = 0$

B $-x^2 + 7x + 12 = 0$

C $x^2 - 7x + 12 = 0$

D $x^2 + 7x - 12 = 0$

E $x^2 + 7x + 12 = 0$

10B Ext **4** The solution(s) to $x(x + 2) = 2x + 9$ is/are:

A $x = -3$ or $x = 3$

B $x = 0$ or $x = -2$

C $x = 3$

D $x = 9$ or $x = -1$

E $x = 9$ or $x = 0$

The following applies to Questions **5** and **6**.
The height, h metres, of a toy rocket above the ground t seconds after launch is given by $h = 6t - t^2$.

10D Ext

5 The rocket returns to ground level after:

A 5 seconds **B** 3 seconds **C** 12 seconds
D 6 seconds **E** 8 seconds

10H Ext

6 The rocket reaches its maximum height after:

A 6 seconds **B** 3 seconds **C** 10 seconds
D 4 seconds **E** 9 seconds

10G

7 The turning point of $y = (x - 2)^2 - 4$ is:

A a maximum at $(2, 4)$
B a minimum at $(-2, 4)$
C a maximum at $(-2, 4)$
D a minimum at $(-2, -4)$
E a minimum at $(2, -4)$

10G

8 The transformation of the graph of $y = x^2$ to $y = x^2 - 2$ is described by:

A a translation of 2 units to the left
B a translation of 2 units to the right
C a translation of 2 units down
D a translation of 2 units up
E a translation of 2 units right and 2 units down

10F

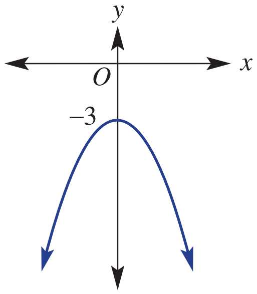
9 Compared to $y = x^2$, the narrowest graph is:

A $y = 5x^2$ **B** $y = 0.2x^2$ **C** $y = 2x^2$ **D** $y = \frac{1}{2}x^2$ **E** $y = 3.5x^2$

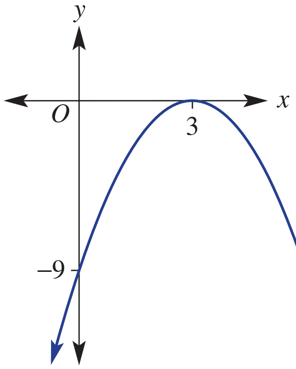
10G

10 The graph of $y = -(x - 3)^2$ is:

A



B



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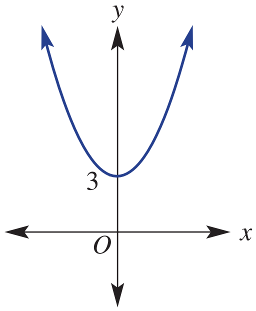

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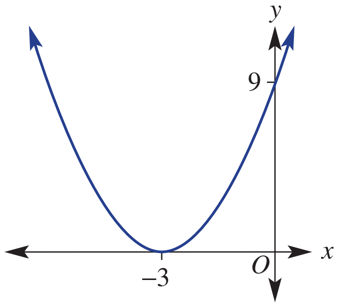
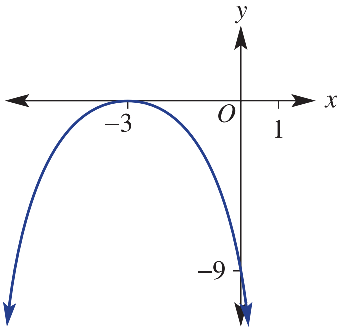


E



Extended-response questions

Ext **1** A sail of a yacht is in the shape of a right-angled triangle. It has a base length of $2x$ metres and its height is 5 metres more than half its base.

a Write an expression for the height of the sail.

b Give an expression for the area of the sail in expanded form.

c If the area of the sail is $14\,\text{m}^2$, find the value of x.

d Hence, state the dimensions of the sail.

Ext **2** Connor and Sam are playing in the park with toy rockets they have made. They launch their rockets at the same time to see whose is better.

a The path of Sam's rocket is modelled by the equation $h = 12t - 2t^2$, where h is the height of the rocket in metres after t seconds.

- **i** Find the axis intercepts.
- **ii** Find the turning point.
- **iii** Sketch a graph of the height of Sam's rocket over time.

b The path of Connor's rocket is modelled by the equation $h = -(t - 4)^2 + 16$, where h is the height of the rocket in metres after t seconds.

- **i** Find the h-intercept (i.e. $t = 0$).
- **ii** State the turning point.
- **iii** Use the answers from parts **b i** and **ii** to state the two t-intercepts.
- **iv** Sketch a graph of the height of Connor's rocket over time.

c
- **i** Whose rocket was in the air for longer?
- **ii** Whose rocket reached the greater height and by how much?
- **iii** How high was Sam's rocket when Connor's rocket was at its maximum height?

Indices and surds

Short-answer questions

1 Use index laws to simplify the following.

a $\dfrac{9a^6b^3}{18a^4b^2}$ **b** $\dfrac{(-3x^4y^2)^2 \times 6xy^2}{27x^6y}$ **c** $(2x^2)^3 - 3x^0 + (5x)^0$

2 Write each of the following using positive indices and simplify.

a $\dfrac{5}{m^{-2}}$ **b** $\dfrac{4a^6b^{-4}}{6a^{-2}b}$ Ext **c** $3(x^{\frac{1}{2}})^3y^{-\frac{1}{3}} \times x^{\frac{1}{2}}y^{-\frac{2}{3}}$

3 Convert these numbers to the units given in brackets. Write your answer using scientific notation with three significant figures.

a 30.71 g (kg)
b 4236 tonnes (kg)
c 3.4 hours (seconds)
d 235 nanoseconds (seconds)

Ext **4** Simplify the following.

a $144^{\frac{1}{2}}$ **b** $27^{\frac{1}{3}}$ **c** $2\sqrt{3} + 3\sqrt{5} + 4\sqrt{5}$ **d** $\sqrt{35} \div \sqrt{5}$

Multiple-choice questions

1 $3a^2b^3 \times 4ab^2$ is equivalent to:

A $12a^2b^6$ **B** $7a^3b^5$ **C** $12a^3b^5$ **D** $12a^4b^5$ **E** $7a^2b^6$

2 $\left(\dfrac{2x}{5}\right)^3$ is equivalent to:

A $\dfrac{6x^3}{5}$ **B** $\dfrac{8x^3}{125}$ **C** $\dfrac{2x^3}{5}$ **D** $\dfrac{2x^4}{15}$ **E** $\dfrac{2x^3}{125}$

3 4^{-2} can be expressed as:

A $\dfrac{1}{4^{-2}}$ **B** $\dfrac{1}{8}$ **C** -16 **D** $\dfrac{1}{16}$ **E** -8

4 $3x^{-4}$ written with positive indices is:

A $-3x^4$ **B** $\dfrac{1}{3x^4}$ **C** $-\dfrac{3}{x^4}$ **D** $\dfrac{1}{3x^{-4}}$ **E** $\dfrac{3}{x^4}$

5 0.00371 in scientific notation is:

A 0.371×10^{-3} **B** 3.7×10^{-2} **C** 3.71×10^{-3}
D 3.71×10^3 **E** 371×10^3

Extended-response question

The average human body contains about 74 billion cells.

a Write this number of cells:

i in decimal form **ii** using scientific notation.

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b If the population of a particular city is 2.521×10^6, how many human cells are there in the city? Give your answer using scientific notation correct to three significant figures.

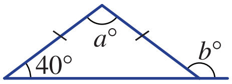
c If the average human weighs 64.5 kilograms, what is the average mass of one cell in grams? Give your answer using scientific notation correct to three significant figures.

Geometry

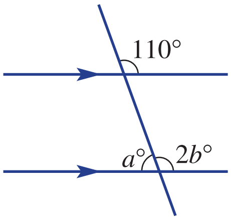
Short-answer questions

1 Find the value of each pronumeral in the following.

a



b



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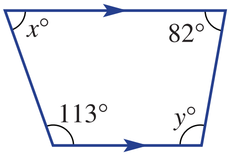
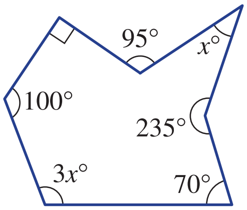
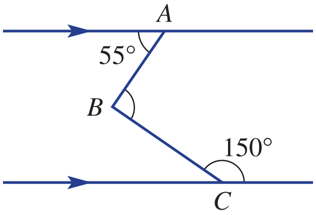


d

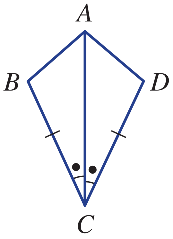


2 Find the value of $\angle ABC$ by adding a third parallel line.



Ext

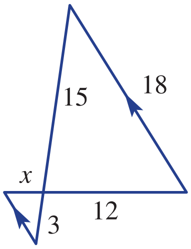
3 Prove $\Delta ABC \equiv \Delta ADC$.



4 Consider the pair of triangles shown at right.

a Give a reason (SSS, SAS, AAA or RHS) why the two triangles are similar.

b Find the value of x.



Multiple-choice questions

1 The supplementary angle to 55° is:

A 55° **B** 35° **C** 125° **D** 135° **E** 70°

2 A quadrilateral with all four sides equal and opposite sides parallel is best described by a:

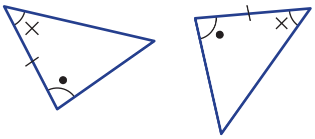
A parallelogram **B** rhombus **C** rectangle
D trapezium **E** kite

3 The size of the interior angle in a regular pentagon is:

A 108° **B** 120° **C** 96° **D** 28° **E** 115°

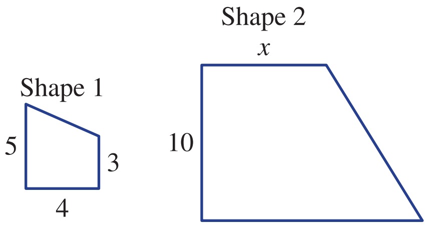
4 The test that proves congruence in these two triangles is:

A SAS **B** RHS **C** AAA
D SSS **E** AAS



5 What is the scale factor that enlarges shape 1 to shape 2 in these similar figures, and what is the value of x?

A 2 and $x = 8$
B 2.5 and $x = 7.5$
C 3.33 and $x = 13.33$
D 2.5 and $x = 12.5$
E 2 and $x = 6$



Extended-response question

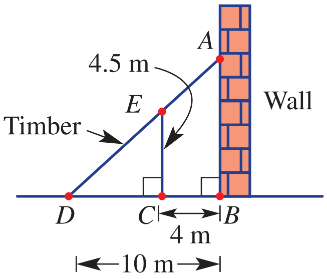
A vertical wall is being supported by a piece of timber that touches the ground 10 m from the base of the wall. A vertical metal support 4.5 m high is placed under the timber support 4 m from the wall.

a Prove $\Delta ABD \,|||\, \Delta ECD$.

b Find how far up the wall the timber reaches.

c How far above the ground is the point halfway along the timber support?

d The vertical metal support is moved so that the timber support is able to reach one metre higher up the wall. If the piece of timber now touches the ground 9.2 m from the wall, find how far the metal support is from the wall. Give your answer correct to one decimal place.

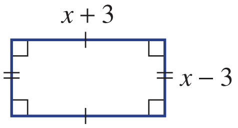


Algebraic techniques

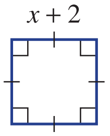
Short-answer questions

1 Find the area of the following shapes in expanded form.

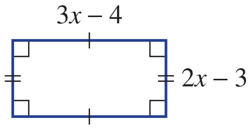
a



b



c



2 Factorise each of the following fully.

a $8ab + 2a^2b$ **b** $9m^2 - 25$ **c** $3b^2 - 48$

d $(a + 7)^2 - 9$ Ext **e** $x^2 + 6x + 9$ Ext **f** $x^2 + 8x - 20$

Ext **g** $2x^2 - 16x + 30$ Ext **h** $2x^2 - 11x + 12$ Ext **i** $6x^2 + 5x - 4$

3 Factorise the following by grouping.

a $x^2 - 3x - x + 3$ **b** $2x^2 - 10 - 5x + 4x$

4 **a** Simplify these algebraic fractions.

i $\dfrac{3x + 24}{2x + 16}$ **ii** $\dfrac{12}{x^2 - 9} \div \dfrac{3}{x - 3}$ **iii** $\dfrac{x + 3}{2} + \dfrac{3x}{7}$

iv $\dfrac{2}{x} - \dfrac{5}{3x}$ **v** $\dfrac{7}{x + 1} + \dfrac{4}{x - 2}$ Ext **vi** $\dfrac{5}{x - 5} - \dfrac{3}{x + 2}$

Ext **b** Solve these equations involving algebraic fractions.

i $\dfrac{5}{2x} + \dfrac{1}{3x} = 2$ **ii** $\dfrac{4}{x - 5} = \dfrac{2}{x + 3}$

Multiple-choice questions

1 The expanded form of $(x + 4)(3x - 2)$ is:

A $3x^2 + 12x - 8$ **B** $4x^2 + 14x - 8$ **C** $3x^2 + 12x - 10$

D $3x^2 - 8$ **E** $3x^2 + 10x - 8$

2 $2(x + 2y) - x(x + 2y)$ factorises to:

A $2x(x + 2y)$ **B** $(x + 2y)^2(2 - x)$ **C** $(2 + x)(x + 2y)$

D $(x + 2y)(2 - x)$ **E** $(x + 2y)(2 - x^2 - 2xy)$

Ext **3** $x^2 - 2x - 24$ in factorised form is:

A $(x - 2)(x + 12)$ **B** $(x - 6)(x + 4)$ **C** $(x - 8)(x + 3)$

D $(x + 6)(x - 4)$ **E** $(x - 6)(x - 4)$

4 $\dfrac{3x + 6}{(x - 2)(x - 4)} \times \dfrac{x^2 - 4}{(x + 2)^2}$ is equivalent to:

A $\dfrac{3}{x - 4}$ **B** $\dfrac{3(x - 2)}{(x - 4)(x + 2)}$ **C** $\dfrac{3x - 2}{(x + 1)^2}$

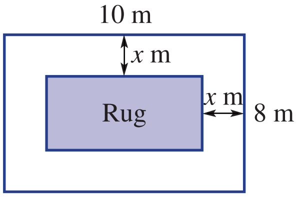
D $\dfrac{3}{(x + 2)(x - 2)}$ **E** $\dfrac{3x^2}{(x - 2)(x + 2)}$

Ext **5** $\dfrac{7}{(x+1)^2} - \dfrac{4}{x+1}$ simplifies to:

A $\dfrac{3x+1}{(x+1)^2}$ **B** $\dfrac{6-4x}{(x+1)^2}$ **C** $\dfrac{-9}{(x+1)^2}$ **D** $\dfrac{3-4x}{(x+1)^2}$ **E** $\dfrac{11-4x}{(x+1)^2}$

Extended-response question

A rectangular room 10 metres long and 8 metres wide has a rectangular rug in the middle of it that leaves a border x metres wide all the way around it as shown.



a Write expressions for the length and the width of the rug.

b Write an expression for the area of the rug in expanded form.

c What is the area of the rug when $x = 1$?

d Fully factorise your expression in part **b** by first removing the common factor.

Ext **e** What happens when $x = 4$?

Probability and statistics

Short-answer questions

1 In a survey of 30 people, 18 people drink coffee, 14 people drink tea and 8 people drink both. Let C be the set of people who drink coffee and T the set of people who drink tea.
- **a** Construct a Venn diagram for the survey results.
- **b** Find:
  - **i** $n(C \cup T)$
  - **ii** $n(T')$.
- **c** If one of the 30 people was randomly selected, find:
  - **i** Pr(drinks neither coffee nor tea)
  - **ii** Pr(drinks coffee only).

2 Two ice-creams are randomly selected without replacement from a box containing one vanilla (V), two strawberry (S) and one chocolate (C) flavoured ice-creams.
- **a** Draw a tree diagram to show each of the possible outcomes.
- **b** What is the probability of selecting:
  - **i** a vanilla and a strawberry flavoured ice-cream?
  - **ii** two strawberry flavoured ice-creams?
  - **iii** no vanilla flavoured ice-cream?

3 The data below shows the number of aces served by a player in each of their grand slam tennis matches for the year.

15 22 11 17 25 25 12 31 26 18 32 11 25 32 13 10

- **a** Construct a stem-and-leaf plot for the data.
- **b** From the stem-and-leaf plot, find the mode and median number of aces.
- **c** Is the data symmetrical or skewed?

4 The frequency table shows the number of visitors, in intervals of fifty, to a theme park each day in April.

a Complete the frequency table shown. Round to one decimal place where necessary.

b Construct a frequency histogram.

c **i** On how many days were there fewer than 100 visitors?

ii On what percentage of days were there between 50 and 200 visitors?

Class interval	Frequency	Percentage frequency
0–	2	
50–	4	
100–	5	
150–	9	
200–		
250–	3	
Total	30	

Multiple-choice questions

1 The probability of not rolling a number less than 3 on a normal six-sided die is:

A $\frac{1}{3}$ **B** 4 **C** $\frac{1}{2}$ **D** $\frac{2}{3}$ **E** 3

2 From the two-way table, $\Pr(A \cap B)$ is:

A $\frac{1}{5}$ **B** 4 **C** $\frac{9}{20}$ **D** $\frac{1}{4}$ **E** 16

	A	A′	Total
B		7	
B′	5		
Total		11	20

3 In the selection of 40 marbles, 28 were blue. The experimental probability of the next one selected being blue is:

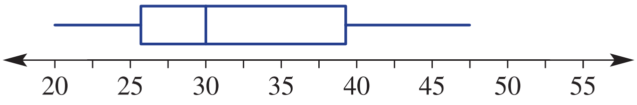
A 0.28 **B** 0.4 **C** 0.7 **D** 0.54 **E** 0.75

4 The median, mean and range of the data set 12 3 1 6 10 1 5 18 11 15 are, respectively:

A 5.5, 8.2, 1–18 **B** 8, 8.2, 17 **C** 5.5, 8, 17

D 8, 8.2, 1–18 **E** 8, 74.5, 18

Ext **5** The interquartile range of the data in the box plot shown is:



A 10 **B** 27 **C** 13 **D** 30 **E** 17

Extended-response question

A game at the school fair involves randomly selecting a green ball and a red ball each numbered 1, 2 or 3.

a List the outcomes in a table.

b What is the probability of getting an odd and an even number?

c Participants win $1 when they draw two balls showing the same number.

i What is the probability of winning $1?

ii If someone wins six times, how many games are they likely to have played?

Ext **d** The ages of those playing the game in the first hour are recorded and are shown below.

12 16 7 24 28 9 11 17 18 18 37 9 40 16 32 42 14

i Draw a box plot to represent the data.

ii Twenty-five per cent of the participants are below what age?

iii If this data is used as a model for the 120 participants throughout the day, how many would be expected to be aged less than 30?

ISBN 978-1-108-77288-4

Introduction to quadratic equations and graphs

Short-answer questions

Ext

1 Solve the following quadratic equations.

a $(x + 5)(x - 3) = 0$
b $(2x - 1)(3x + 5) = 0$
c $4x^2 + 8x = 0$
d $5x^2 = 45$
e $x^2 - 9x + 14 = 0$
f $8x = -x^2 - 16$

Ext

2 The length of a rectangular swimming pool is x m and its width is 7 m less than its length. If the area occupied by the pool is 120 m^2, solve an appropriate equation to find the dimensions of the pool.

3 Sketch the following graphs showing the y-intercept and turning point, and state the transformations that have taken place from the graph of $y = x^2$.

a $y = x^2 + 3$
b $y = -(x + 4)^2$
c $y = (x - 2)^2 + 5$

Ext

4 Consider the quadratic relation $y = x^2 - 4x - 12$.

a Find the y-intercept.
b Factorise the relation and find the x-intercepts.
c Find the coordinates of the turning point.
d Sketch the graph.

Multiple-choice questions

Ext

1 The solution(s) to $3x(x + 5) = 0$ is/are:

A $x = -5$
B $x = 0$ or $x = -5$
C $x = 3$ or $x = 5$
D $x = 5$
E $x = 0$ or $x = 5$

Ext

2 $x^2 = 3x - 2$ is the same as the equation:

A $x^2 + 3x - 2 = 0$
B $x^2 - 3x + 2 = 0$
C $x^2 - 3x - 2 = 0$
D $-x^2 + 3x + 2 = 0$
E $x^2 + 3x + 2 = 0$

3 The *incorrect* statement about the graph of $y = 2x^2$ is:

A the graph is the shape of a parabola
B the point $(-2, 8)$ is on the graph
C its turning point is at $(0, 0)$
D it has a minimum turning point
E the graph is wider than the graph of $y = x^2$

4 The type and coordinates of the turning point of the graph of $y = -(x + 3)^2 + 2$ are:

A a minimum at $(3, 2)$
B a maximum at $(3, 2)$
C a maximum at $(-3, 2)$
D a minimum at $(-3, 2)$
E a minimum at $(3, \ -2)$

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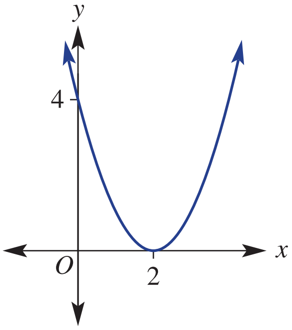
5 The graph shown has the equation:

A $y = 4x^2$
B $y = x^2 + 4$
C $y = (x + 2)^2$
D $y = x^2 + 2$
E $y = (x - 2)^2$



Extended-response question

Ext

The flight path of a soccer ball kicked upwards from the ground is given by the equation $y = 120x - 20x^2$, where y is the height of the ball above the ground in centimetres at any time x seconds.

a Find the x-intercepts to determine when the ball lands on the ground.

b Find the coordinates of the turning point and state:
- **i** the maximum height reached by the ball
- **ii** after how many seconds the ball reaches this maximum height.

c At what times was the ball at a height of 160 cm?

d Sketch a graph of the path of the ball until it returns to the ground.

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Working with unfamiliar problems: Part 1

1 **a** 999 999 998 000 000 001
  b 99 999 999 990 000 000 001
2 31 lockers open
1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441, 484, 529, 576, 625, 676, 729, 784, 841, 900, 961
3 61
4 12, 23, 34, 45, 56, 67, 78 and 89
5 85 km; Adina: 20 km, 53.13°; Birubi: 65 km, 306.87°
6 $\frac{1}{3}$
7 approximately 70 days
8 Answers may vary.
9 20% **10** $x = 15$ **11** 48 **12** 72
13 6 m, 11 m
14 **a** one revolution of a spiral
  b 6.18 m **c** 75.82 m
15 49 **16** 254 **17** $\frac{x}{y}$

Working with unfamiliar problems: Part 2

1 $x = 1$ and $y = 3$
2 any prism with a quadrilateral cross-section (e.g. cube, rectangular prism, trapezoidal prism), heptagonal pyramid, truncated rectangular pyramid
3 $x = 20$
4 $2n - 1 + 2m - 1 = 2(n + m - 1)$, which is a multiple of two and is therefore even where m and n are integers
5 $xy = 24$ **6** 98° **7** $n + 2\sqrt{n} + 1$
8 **a** 64 cm^2, 32 cm^2, 16 cm^2, 8 cm^2, 4 cm^2
  $2^6, 2^5, 2^4, 2^3, 2^2$
  b $2^1, 2^0, 2^{-1}, 2^{-2}, 2^{-3}$; 1 cm^2, $\frac{1}{8}$ cm^2
  c $A = 2^{7-n}$; $\frac{1}{256}$
9 7 : 5 : 3 **10** 2
11 Sam 36 years, Noah 12 years
12 20 cm^2
13 **a** 3^{x-1} **b** 0
14 **a** 96 cm^3 **b** $24\frac{1}{40}$ unit^2
15 7 or –4
16 $2\frac{1}{2}$ hours, $\frac{3}{4}$ hour **17** 5 : 2

Chapter 1

1A

Building understanding

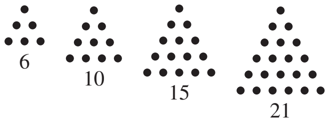
1 **a** 1, 2, 4, 7, 8, 14, 28, 56 **b** HCF = 8
  c 3, 6, 9, 12, 15, 18, 21 **d** LCM = 15
  e 2, 3, 5, 7, 11, 13, 17, 19, 23, 29
2 **a** 121 **b** 12 **c** 27 **d** 2
3 **a** –1 **b** –1 **c** 15
  d –6 **e** 22 **f** –9

Now you try

Example 1
a 5 **b** 4 **c** –65

Exercise 1A

1 **a** **i** 5 **ii** –3
  b **i** –12 **ii** 21
  c **i** 5 **ii** 28
2 **a** 2 **b** 2 **c** 10 **d** 16
  e –9 **f** –3 **g** –4 **h** 10
  i –11 **j** 2 **k** –3 **l** 4
  m –23 **n** 10 **o** 0 **p** 3
  q –9 **r** 7 **s** –1 **t** 4
3 **a** 28 **b** 24 **c** 187 **d** 30
4 **a** 4 **b** 5 **c** 1 **d** 23
5 **a** 4 **b** 23 **c** –3
  d 2 **e** –2 **f** –18
  g –6 **h** –1 **i** 1
6 **a** –2 **b** –38 **c** –8
  d 27 **e** 1 **f** 24
  g 21 **h** 0 **i** 24
7 **a** $-2 \times [11 + (-2)] = -18$ **b** $[-6 + (-4)] \div 2 = -5$
  c $[2 - 5] \times (-2) = 6$ **d** $-10 \div [3 + (-5)] = 5$
  e $3 - [(-2) + 4] \times 3 = -3$ **f** $[(-2)^2 + 4] \div (-2) = -2^2$
8 4 **9** 252 days
10 **a** 7 and –2 **b** –5 and 2
11 8
12 **a** **i** 16 **ii** 16
  b $a = \pm 4$ **c** $a = 3$
  d The square of a negative number has negative signs occurring in pairs and will create a positive answer.
  e –3
  f The squaring of any number produces a positive answer.
  g **i** –4 **ii** –125 **iii** –9 **iv** –16
  h no **i** yes
  j A prime number has only two factors – itself and 1, therefore the only common factor for any pair of prime numbers is 1.
  k Again as there are only two factors of any prime, the LCM must be the multiple of primes.
13 **a** False **b** False **c** True
  d True **e** False **f** True
14 **a** **i** $1 + 2 + 3 = 6$
  ii 28 $(1 + 2 + 4 + 7 + 14 = 28)$
  iii $1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248 = 496$
  b **i**



  ii 28, 36
  c **i** 0, 1, 1, 2, 3, 5, 8, 13, 21, 34
  ii –21, 13, –8, 5, –3, 2, –1, 1, 0, 1 …
  $\therefore$ –21, –8, –3, –1

1B

Building understanding

1 a 40 b 270 c 7.9 d 0.04
2 a 32 100 b 432 c 5.89 d 0.443
3 a 60 b 57.375 c 2.625

Now you try

Example 2
a 138.47 b 0.03 c 2.80

Example 3
a 3600 b 49 000 c 0.029

Example 4
26

Exercise 1B

1 a i 124.27 ii 830.44
b i 0.02 ii 0.07
c i 7.69 ii 13.50
2 a 17.96 b 11.08
c 72.99 d 47.86
e 63.93 f 23.81
g 804.53 h 500.57
i 821.27 j 5810.25
k 1005.00 l 2650.00
3 a 7 b 73 c 130 d 36 200
4 a 0.333 b 0.286 c 1.182 d 13.793
5 a 2400 b 35 000 c 0.060 d 34
e 110 000 f 0.0025 g 2.1 h 0.71
6 a 30 000 b 200 c 0.05 d 0.0006
7 a 3600, 3693 b 760, 759.4
c 4000, 4127.16 d 3000, 3523.78
e 0, 0.72216 f 4, 0.716245
g 0.12, 0.1186 h 0.02, 0.02254
i 10, 8.4375 j 1600, 1683.789156
k 0.08, 0.074957... l 11, 10.25538...
8 a A: 54.3, B: 53.8, 0.5 b A: 54.28, B: 53.79, 0.49
c A: 54, B: 54, 0 d A: 50, B: 50, 0
9 a each to 1 significant figure
b each to 2 significant figures
c each to the nearest integer
10 8.33 m 11 0.143 tonnes
12 2.14999 is closer to 2.1 correct to 1 decimal place
$\therefore$ round down
13 as magnesium in this case would be zero if rounded to two decimal places rather than 2 significant figures
14 a i 50 ii 624
b 50 c 600
d The addition is the same as the original but the multiplication is lower ($20 \times 30 < 24 \times 26$).
15 a 0.18181818
b i 8 ii 1 iii 8
c 0.1428571428571
d i 4 ii 2 iii 8
e not possible

1C

Building understanding

1 a $1\frac{2}{5}$ b $4\frac{1}{3}$ c $4\frac{4}{11}$
2 a $\frac{11}{7}$ b $\frac{16}{3}$ c $\frac{19}{2}$
3 a $\frac{2}{5}$ b $\frac{4}{29}$ c $4\frac{1}{6}$
4 a 9 b 24 c 3 d 10

Now you try

Example 5
a 4.625 b $0.\overline{18}$

Example 6
a $\frac{9}{20}$ b $1\frac{17}{40}$

Example 7
$\frac{7}{11}$

Exercise 1C

1 a i 2.375 ii 6.875
b i $0.\overline{230769}$ ii $0.4\dot{6}$
2 a 2.75 b 0.35 c 3.4
d 1.875 e 2.625 f 3.8
g 2.3125 h 0.21875
3 a $0.\overline{27}$ b $0.\dot{7}$ c $1.\overline{285714}$
d $0.41\dot{6}$ e $1.\dot{1}$ f $3.8\dot{3}$
g $7.2\dot{6}$ h $2.\overline{63}$
4 a $\frac{7}{20}$ b $\frac{3}{50}$ c $3\frac{7}{10}$
d $\frac{14}{25}$ e $1\frac{7}{100}$ f $\frac{3}{40}$
g $3\frac{8}{25}$ h $7\frac{3}{8}$ i $2\frac{1}{200}$
j $10\frac{11}{250}$ k $6\frac{9}{20}$ l $2\frac{101}{1000}$
5 a $\frac{5}{6}$ b $\frac{13}{20}$ c $\frac{7}{10}$
d $\frac{5}{12}$ e $\frac{7}{16}$ f $\frac{11}{14}$
g $\frac{19}{30}$ h $\frac{11}{27}$
6 a $\frac{5}{12}, \frac{7}{18}, \frac{3}{8}$ b $\frac{5}{24}, \frac{3}{16}, \frac{1}{6}$ c $\frac{7}{12}, \frac{23}{40}, \frac{8}{15}$
7 a $\frac{9}{20}$ b $\frac{3}{20}$ c $\frac{32}{45}$ d $\frac{23}{75}$
8 a $\frac{11}{6}, \frac{7}{3}$ b $\frac{2}{5}, \frac{2}{15}$ c $\frac{11}{12}, \frac{12}{12}$ d $\frac{5}{7}, \frac{11}{14}$
9 Weather forecast
10 a $\frac{3}{5}$ b $\frac{5}{9}$ c $\frac{8}{13}$ d $\frac{23}{31}$
11 a 31, 32 b 36, 37, 38, …, 55
c 4, 5 d 2, 3
e 43, 44, 45, …, 55 f 4, 5, 6

ISBN 978-1-108-77288-4

Updated September 2021

12 $\frac{ac+b}{c}$

13 a yes, e.g. $\frac{7}{14}=\frac{1}{2}$ and 7 is prime

b no as *a* and *b* will have no common factors other than one

c no as then a factor of 2 can be used to cancel

d yes, e.g. $\frac{5}{7}$

14 a $\frac{8}{9}$ b $1\frac{2}{9}$ c $\frac{9}{11}$ d $3\frac{43}{99}$

e $9\frac{25}{33}$ f $\frac{44}{333}$ g $2\frac{917}{999}$ h $13\frac{8125}{9999}$

1D

Building understanding

1 a 6 b 63 c 4 d 60

2 a $\frac{7}{3}$ b $\frac{39}{5}$ c $\frac{41}{4}$

3 a $\frac{9}{6}+\frac{8}{6}=\frac{17}{6}$ b $\frac{4}{3}-\frac{2}{5}=\frac{20}{15}-\frac{6}{15}=\frac{14}{15}$

c $\frac{5}{3}\times\frac{7}{2}=\frac{35}{6}$

Now you try

Example 8

a $\frac{11}{15}$ b $\frac{41}{8}$ or $5\frac{1}{8}$ c $\frac{37}{20}$ or $1\frac{17}{20}$

Example 9

a $\frac{12}{35}$ b $\frac{23}{3}$ or $7\frac{2}{3}$

Example 10

a $\frac{5}{6}$ b $\frac{46}{33}$ or $1\frac{13}{33}$

Exercise 1D

1 a i $\frac{7}{6}$ or $1\frac{1}{6}$ ii $\frac{17}{30}$

b i $\frac{25}{6}$ or $4\frac{1}{6}$ ii $\frac{59}{12}$ or $4\frac{11}{12}$

c i $\frac{7}{10}$ ii $\frac{24}{35}$

2 a $\frac{3}{5}$ b $\frac{4}{9}$ c $1\frac{2}{7}$ d $\frac{19}{20}$

e $\frac{19}{21}$ f $1\frac{7}{40}$ g $\frac{7}{10}$ h $\frac{17}{27}$

3 a 5 b $3\frac{2}{5}$ c $5\frac{1}{7}$

d $6\frac{11}{15}$ e $7\frac{17}{63}$ f $17\frac{13}{16}$

4 a $\frac{2}{5}$ b $\frac{1}{45}$ c $\frac{11}{20}$ d $\frac{1}{10}$

e $\frac{1}{18}$ f $\frac{1}{8}$ g $\frac{13}{72}$ h $\frac{5}{48}$

5 a $1\frac{1}{2}$ b $\frac{3}{4}$ c $\frac{13}{20}$

d $\frac{29}{40}$ e $\frac{5}{6}$ f $1\frac{16}{77}$

6 a $\frac{6}{35}$ b $\frac{1}{2}$ c $\frac{1}{12}$ d $\frac{4}{9}$

e $4\frac{1}{2}$ f $5\frac{1}{3}$ g $7\frac{1}{2}$ h 5

i 15 j 26 k $\frac{2}{3}$ l $\frac{5}{6}$

m $2\frac{1}{4}$ n $3\frac{1}{2}$ o 6 p $1\frac{1}{3}$

7 a $\frac{1}{3}$ b $\frac{7}{5}$ c 8 d $\frac{9}{13}$

8 a $\frac{20}{21}$ b $1\frac{1}{8}$ c $\frac{45}{56}$ d $\frac{27}{28}$

e $1\frac{1}{3}$ f $1\frac{1}{2}$ g 6 h $\frac{7}{8}$

i 18 j 9 k 16 l 64

m $\frac{1}{10}$ n $\frac{1}{12}$ o $\frac{4}{27}$ p $3\frac{1}{3}$

q 4 r $\frac{1}{6}$ s $1\frac{1}{2}$ t $\frac{7}{8}$

9 a $\frac{2}{7}$ b $\frac{8}{15}$ c 16

d $\frac{66}{85}$ e $2\frac{10}{21}$ f $\frac{3}{13}$

10 $\frac{7}{8}$ tonnes

11 $5\frac{29}{56}$ tonnes

12 $\frac{5}{12}$ hours (25 min)

13 7 truckloads

14 3 hours

15 $\frac{5}{6}$, problem is the use of negatives in the method since $\frac{1}{3}<\frac{1}{2}$

16 a $\frac{b}{a}$ b $\frac{c}{ac+b}$

17 a 1 b $\frac{a^2}{b^2}$ c 1

d $\frac{c}{a}$ e a f $\frac{c}{b}$

18 a $-\frac{2}{3}$ b $\frac{5}{4}$ c $\frac{83}{10}$ d 1

e $\frac{50}{31}$ f $\frac{81}{400}$ g $\frac{329}{144}$ h $\frac{969}{100}$

i $\frac{583}{144}$ j $\frac{5}{11}$

1E

Building understanding

1 a 4 b 12 c 11 d 3

2 a 9 b $\frac{4}{9}$ c $\frac{5}{9}$ d 8 e 10

3 **a** **i** 240 km **ii** 40 km **iii** 520 km
b **i** 5 h **ii** $4\frac{1}{2}$ h **iii** 15 min
4 **a** $4 **b** $3 **c** $2.50

Now you try

Example 11
a 3 : 2 **b** 16 : 9 **c** 5 : 3

Example 12
$200

Example 13
a 35 km/h **b** 400 revs/min

Example 14
7 kg bag

Exercise 1E

1 **a** 11 : 2 **b** 32 : 15 **c** 5 : 2
2 **a** 1 : 5 **b** 2 : 5 **c** 3 : 2 **d** 4 : 3
e 9 : 20 **f** 45 : 28 **g** 3 : 14 **h** 22 : 39
i 1 : 3 **j** 1 : 5 **k** 20 : 7 **l** 10 : 3
3 **a** 1 : 10 **b** 1 : 5 **c** 2 : 3 **d** 7 : 8
e 25 : 4 **f** 1 : 4 **g** 1 : 4 **h** 24 : 5
i 4 : 20 : 5 **j** 4 : 3 : 10 **k** 5 : 72 **l** 3 : 10 : 40
4 **a** $200, $300 **b** $150, $350
c $250, $250 **d** $175, $325
5 126 g
6 **a** $10, $20, $40 **b** $14, $49, $7 **c** $40, $25, $5
7 **a** 15 km/h **b** 2000 rev/min **c** 45 strokes/min
d 14 m/s **e** 8 mL/h **f** 92 beats/min
8 **a** 55 km **b** $16\frac{1}{2}$ km **c** $5\frac{1}{2}$ km
9 **a** 3 kg deal **b** Red delicious
c 2.4 L **d** 0.7 GB
10 **a** coffee A: $3.60, coffee B: $3.90
Therefore, coffee A is the best buy.
b pasta A: $1.25, pasta B: $0.94
Therefore, pasta B is the best buy.
c cereal A: $0.37, cereal B: $0.40
Therefore, cereal A is the best buy.
11 120
12 $3000, $1200, $1800 respectively
13 $15.90 14 108 L
15 $3600, $1200, $4800 respectively
16 $1.62 17 1 : 4
18 36°, 72°, 108°, 144°
19 Find cost per kilogram or number of grams per dollar.
Cereal A is the best buy.
20 **a** False **b** False **c** True **d** True
21 **a** $a + b$ **b** $\frac{a}{a+b}$ **c** $\frac{b}{a+b}$
22 **a** **i** 100 mL **ii** 200 mL
b **i** 250 mL **ii** 270 mL
c **i** 300 mL **ii** 1 : 4
d **i** 1 : 3 **ii** 7 : 19 **iii** 26 : 97 **iv** 21 : 52
e jugs 3 and 4

1F

Building understanding

1 **a** $\frac{3}{100}$ **b** $\frac{11}{100}$ **c** $\frac{7}{20}$ **d** $\frac{2}{25}$
2 **a** 0.04 **b** 0.23 **c** 0.86 **d** 0.463
3 **a** 50% **b** 25% **c** 75% **d** 20%

Now you try

Example 15
a 75% **b** 0.6 **c** $\frac{11}{400}$

Example 16
12.5%

Example 17
$8.40

Example 18
$300

Exercise 1F

1 **a** **i** 55% **ii** 91%
b **i** 0.75 **ii** 0.61
c **i** $\frac{1}{40}$ **ii** $\frac{4}{75}$
2 **a** 34% **b** 40% **c** 6% **d** 70%
e 100% **f** 132% **g** 109% **h** 310%
3 **a** 0.67 **b** 0.3 **c** 2.5 **d** 0.08
e 0.0475 **f** 0.10625 **g** 0.304 **h** 0.4425
4 **a** $\frac{67}{100}$ **b** $\frac{3}{10}$ **c** $2\frac{1}{2}$ **d** $\frac{2}{25}$
e $\frac{19}{400}$ **f** $\frac{17}{160}$ **g** $\frac{38}{125}$ **h** $\frac{177}{400}$

5

Percentage	Fraction	Decimal
10%	$\frac{1}{10}$	0.1
50%	$\frac{1}{2}$	0.5
5%	$\frac{1}{20}$	0.05
25%	$\frac{1}{4}$	0.25
20%	$\frac{1}{5}$	0.2
12.5%	$\frac{1}{8}$	0.125
1%	$\frac{1}{100}$	0.01
$11.\dot{1}\%$	$\frac{1}{9}$	$0.\dot{1}$
$22.\dot{2}\%$	$\frac{2}{9}$	$0.\dot{2}$
75%	$\frac{3}{4}$	0.75
15%	$\frac{3}{20}$	0.15
90%	$\frac{9}{10}$	0.9
37.5%	$\frac{3}{8}$	0.375

Percentage	Fraction	Decimal
$33\frac{1}{3}\%$	$\frac{1}{3}$	$0.\dot{3}$
$66\frac{2}{3}\%$	$\frac{2}{3}$	$0.\dot{6}$
62.5%	$\frac{5}{8}$	0.625
$16.\dot{6}\%$	$\frac{1}{6}$	$0.1\dot{6}$

6 **a** 25% **b** $33\frac{1}{3}\%$ **c** 16%
d 200% **e** 2800% **f** 25%
7 **a** $36 **b** $210 **c** 48 kg
d 30 km **e** 15 apples **f** 350 m
g 250 people **h** 200 cars **i** $49
8 **a** $120 **b** $700 **c** $300
d $7 **e** $0.20 **f** $400
9 **a** $540 **b** $600 **c** $508
d $1250 **e** $120 **f** $40
10 $16\frac{2}{3}\%$ **11** $6\frac{1}{4}\%$ **12** 48 kg
13 15 students **14** 9 students **15** $1150
16 **a** $P = 100$ **b** $P < 100$ **c** $P > 100$
17 **a** $x = 2y$ **b** $x = 5y$
c $x = \frac{3}{5}y$ (or $5x = 3y$) **d** $14x = 5y$
18 **a** 72 **b** $\frac{10}{11}$ **c** 280% **d** $3\frac{1}{4}$ **e** 150%

1G

Building understanding

1 **a** 1.4 **b** 21% **c** 0.27 **d** 6%
2 **a** $30 **b** 25%
3 **a** 12 kg **b** 11.1%

Now you try

Example 19
$50

Example 20
$4.76

Example 21
a 25% **b** 12%

Example 22
227 m^2

Exercise 1G

1 **a** $60 **b** $153.76
2 **a** $52.50 **b** 37.8 min (37 min 48 s)
c 375 mL **d** 1.84 m
e 27.44 kg **f** 36 watts
g $13 585 **h** $1322.40
3 **a** 19.2 cm **b** 24.5 cm
c 39.06 kg **d** 48.4 min (49 m in 24 s)
e $78.48 **f** 202.4 mL
g 18°C **h** $402.36
4 50% **5** 44% **6** 28% **7** 4%
8 **a** 22.7% **b** 26.7% **c** 30.9% **d** 38.4%
9 $21.50 **10** 30 068 **11** $14 895
12 **a** 10% **b** 12.5% **c** 16% **d** 9.375%
13 $10.91 **14** $545.45
15 **a** $900 **b** $990
c As 10% of 1000 = 100 but 10% of 900 = 90
16 25% **17** 100% **18** 42.86%
19 **a** $635.58 **b** $3365.08 **c** $151.20 **d** $213.54
20 **a** 79.86 g **b** $97 240.50
c $336 199.68 **d** 7.10 cm

Progress quiz

1 **a** –54 **b** 8
2 **a** 3.46 **b** 45.9 **c** 0.0079 **d** 46 800 000
3 **a** 0.75 **b** 0.8 **c** 0.35 **d** $0.\dot{3}$
4 **a** $\frac{9}{10}$ **b** $\frac{17}{20}$ **c** $\frac{1}{8}$
5 **a** $1\frac{1}{3}$ **b** $\frac{4}{21}$ **c** $1\frac{1}{5}$ **d** $2\frac{1}{12}$
6 **a** 30 : 1 **b** 5 : 9 **c** 20 : 3 **d** 3 : 32
7 **a** $250, $150 **b** 1.8 kg, 4.2 kg
c $266\frac{2}{3}$ cm, $333\frac{1}{3}$ cm, 400 cm
8 **a** $70/h **b** 80 km/h
9 B
10 **a** 80% **b** 96% **c** 375% **d** 8%
11 2040 cm **12** $400
13 **a** $504 **b** 475 kg
14 40% **15** 850 L

1H

Building understanding

1 $3 profit, $2.50 loss, $7.30, $2070
2 **a** 90% **b** 85%

Now you try

Example 23
a $144 **b** 45%

Example 24
$16.80

Example 25
$171.36

Example 26
$40 000

Exercise 1H

1 **a** $75 **b** 15%
2 **a** **i** $2 **ii** 20%
b **i** $5 **ii** 25%
c **i** $16.80 **ii** 14%
d **i** $2450 **ii** 175%

3 166.67% **4** 40% **5** 92.5%
6 $37.50 **7** $1001.25 **8** 42.3%
9 $148.75 **10** $760.50 **11** $613.33
12 increased by 4% **13** 25%
14 a $54.75 **b** 128%
15 No, both methods give the same price.
16 $2100
17 a i $54 187.50 **ii** $33 277.90
b 10 years
18 a $34 440 **b** $44 000 **c** $27 693.75
d $32 951.10 **e** $62 040 **f** $71 627.10

1I

Building understanding

1 a $3952 **b** $912 **c** $24
2 a $79.80 **b** $62.70 **c** $91.20 **d** $102.60
3 a $200 **b** $900
4 $46 166

Now you try

Example 27
a $2394.23 **b** Gary **c** $67 080

Example 28
a $418 **b** $16.50

Example 29
$1299.50

Example 30
a $6791.67 **b** 22.7% **c** $79 037.50

Exercise 1I

1 a $1826.92 **b** Georgia **c** $37 606.40
2 a i $19.50 **ii** $27 **iii** $42.50
b i $30 201.60 **ii** $44 044 **iii** $20 134.40
3 a $165.60 **b** $239.20 **c** $368
d $552 **e** $515.20 **f** $809.60
4 a 7 **b** 18 **c** 33
d 25 **e** 37 **f** 40
5 $14.50 per hour **6** $12.20 per hour **7** $490
8 $4010
9 a i $40 035 **ii** 17.0%
b i $53 905.80 **ii** 20.1%
c i $41 218.20 **ii** 15%
d i $30 052.56 **ii** 22.2%
10 a $4547 **b** $9747 **c** $11697 **d** $19 659.50
11 Cate, Adam, Ed, Diana, Bill
12 $839.05
13 $1239.75
14 4.58%
15 $67 400
16 a 12 hours **b** 9 and 2, 6 and 4, 3 and 6
17 a i $920 **ii** $1500
b i $A = 0.02x$
ii $A = 1200 + 0.025(x - 60\,000)$ or $0.025x - 300$
18 a $5500
b Choose plan A if you expect that you will sell less than $5500 worth of jewellery in a week and plan B if you expect to sell more than $5500.
19 a i $2000 **ii** $11 500 **iii** $35 000
b

Income	Rate	Tax payable
$40 001–$90 000	25%	$3750 + 25% of (income – $40 000)
$90 001–	33%	$16 250 + 33% of (income – $90 000)

c i $2000 **ii** $6300
iii $24 000 **iv** $40 000.50
d An extra dollar of income can push you into a higher tax bracket where you don't just pay the higher tax rate on the dollar but on your entire income. No incentive to earn more.

1J

Building understanding

1 a $12 000 **b** 6% p.a. **c** 3.5 years
d $720 **e** $1440 **f** $2520
2 a i 3 **ii** 1.5 **iii** 5.5
b i 48 **ii** 30 **iii** 87

Now you try

Example 31
$420

Example 32
$4550, $24 550

Example 33
9 months

Exercise 1J

1 a $3600 **b** $416
2 $600 **3** $2700, $17 700 **4** $1980, $23 980
5 $2560 **6** 9 months **7** 16 months
8 $2083.33 **9** Choice 2
10 a $14 400 **b** $240
11 10%
12 a $\$P$ **b** 12.5%
c i 20 years **ii** 40 years **iii** double
13 a $51 000 **b** 4 years
14 a $P = \frac{100I}{rt}$ **b** $t = \frac{100I}{Pr}$ **c** $r = \frac{100I}{Pt}$
15 a $1750 a month **b** $18 000
c $6000 **d** 2%

1K

Building understanding

1 a $200 b $2200 c $220
d $2420 e $242 f $2662
2 a 2731.82 b 930.44
c 2731.82 d 930.44
3 a $\$4000 \times (1.2)^3$
b $\$15000 \times (1.07)^6$
c $\$825 \times (1.11)^4$

Now you try

Example 34
$6749.18

Example 35
$25 207

Exercise 1K

1 $4282.32
2 a $6515.58 b $10 314.68
c $34 190.78 d $5610.21
3 $293 865.62
4 a 21.7% b 19.1% c 136.7% d 33.5%
5 $33 776
6 a $23 558 b $33 268
c $28 879 d $25 725
7 $543 651 8 6142 people 9 6.54 kg
10 Trial and error gives 12 years
11 Trial and error gives 5 years
12 a 35% b 40.26%
c It calculates each years' interest on the original $400 not the accumulated total that compound interest uses.
13 a 15.76% b 25.44%
c 24.02% d 86.96%
e $\left(\left(1 + \frac{r}{100}\right)^t - 1\right) \times 100\%$
14 a $7509.25 b 9.39% p.a.
15 a 70.81% b 7.08%
16 a 5.39% p.a. b 19.28% p.a.

Problems and challenges

1 Discuss the answer with classmates as more than one answer for each may be possible. Some suggestions are given below (be creative).
$(4 - 4) \times (4 + 4) = 0$ $(4 - 4) + (4 \div 4) = 1$
$4 \div 4 + 4 \div 4 = 2$ $\sqrt{4 \times 4} - (4 \div 4) = 3$
$4 + (4 - 4) \times 4 = 4$ $\sqrt{4 \times 4} + 4 \div 4 = 5$
$(4 + 4 + 4) \div \sqrt{4} = 6$ $4 + 4 - 4 \div 4 = 7$
$\sqrt{4} + \sqrt{4} + \sqrt{4} + \sqrt{4} = 8$ $4 + 4 + 4 \div 4 = 9$
$\sqrt{4} \times \sqrt{4} \times \sqrt{4} + \sqrt{4} = 10$
2 12
3 a $\frac{4}{7}$ b $\frac{14}{17}$
4 125 mL
5 40.95% reduction
6 7.91% p.a.
7 a 44% b 10%
8 200 000 cm^2 (20 m^2)
9 9, 7, 2, 14, 11, 5, 4, 12, 13, 3, 6, 10, 15, 1, 8

Short-answer questions

1 a –16 b 2 c 0
d 10 e –23 f 1
2 a 21.5 b 29 100 c 0.153 d 0.00241
3 a 200 b 60 c 2
4 a 2.125 b $0.8\dot{3}$ c $1.\overline{857142}$
5 a $\frac{3}{4}$ b $1\frac{3}{5}$ c $2\frac{11}{20}$
6 a $\frac{1}{2}$ b $2\frac{1}{6}$ c $\frac{7}{24}$
d 2 e $3\frac{3}{4}$ f $2\frac{19}{28}$
7 a 5 : 2 b 16 : 9 c 75 : 14
8 a 50, 30 b 25, 55 c 10, 20, 50
9 a store A: $2.25/kg; store B: $2.58/kg ∴ A is best buy
b store A: 444 g/$; store B: 388 g/$
10

Decimal	Fraction	Percentage
0.6	$\frac{3}{5}$	60%
$0.\dot{3}$	$\frac{1}{3}$	$33\frac{1}{3}\%$
0.0325	$\frac{13}{400}$	$3\frac{1}{4}\%$
0.75	$\frac{3}{4}$	75%
1.2	$1\frac{1}{5}$	120%
2	2	200%

11 a $77.50 b 1.65
12 a 150 b 25
13 a 72 b 1.17 c 20%
14 12.5 kg 15 $1800
16 a $25 b $16\frac{2}{3}\%$
17 a $18.25 b $14.30
18 $50 592 19 $525 20 $4\frac{1}{2}$ years
21 $63 265.95 22 $39 160

Multiple-choice questions

1 D 2 B 3 C 4 A 5 E 6 E
7 D 8 E 9 C 10 A 11 C 12 B

Extended-response questions

1 a $231 b $651
c i $63 ii $34.65
2 a i $26 625 ii $46 928.44
b 87.71% c $82 420
d 26.26% e 7.4% p.a.

Chapter 2

2A

Building understanding

1 a 2 b 2 c 3 d 1
2 a C b D c B d A e F f E
3 a 5 b –2 c $\frac{1}{3}$ d $-\frac{2}{5}$

Now you try

Example 1
a $a + 7$ b $1.2n$ c $\frac{60}{p}$

Example 2
a $x + 6$ b $3x - 1$ c $\frac{a - 2}{3}$ d $\sqrt{a + b}$

Example 3
a 8 b 13

Exercise 2A

1 a $5 + c$ b $2n$ c $\frac{10}{p}$
2 a i $4 + r$ ii $t + 2$ iii $b + g$ iv $x + y + z$
b i $6P$ ii $10n$ iii $2D$ iv $5P + 2D$
c $\frac{500}{C}$
3 a $2 + x$ b $ab + y$ c $x - 5$ d $3x$
e $3x - 2y$ or $2y - 3x$ f $3p$ g $2x + 4$
h $\frac{x + y}{5}$ i $4x - 10$ j $(m + n)^2$ k $m^2 + n^2$
l $\sqrt{x + y}$ m $a + \frac{1}{a}$ n $(\sqrt{x})^3$
4 a –31 b –25 c –33 d –19
e $\frac{1}{2}$ f 4 g 1 h 85
5 a $1\frac{1}{6}$ b $4\frac{1}{4}$ c $\frac{1}{6}$ d $-1\frac{1}{3}$
6 a 60 m^2 b length = $12 + x$, width = $5 - y$
c $A = (12 + x)(5 - y)$
7 a 18 square units b 1, 2, 3, 4, 5
8 a $\frac{P}{10}$ b $\frac{nP}{10}$
9 a i $P = 2x + 2y$ ii $A = xy$
b i $P = 4p$ ii $A = p^2$
c i $P = x + y + 5$ ii $A = \frac{5x}{2}$
10 a A: $2(x + y)$; B: $2x + y$, different b same
11 a B b A: $c^2 = (a + b)^2$
12 a $\frac{n(n + 1)}{2}$
b i 10 ii 55
c $\frac{n^2}{2} + \frac{n}{2}$ d 10, 55
e Half the sum of n and the square of n.

2B

Building understanding

1 a variable (pronumeral) b $5x$ c unlike
2 a $\frac{5}{2}$ b 3 c $\frac{1}{5}$ d $-\frac{1}{20}$
3 a like b unlike c unlike
d unlike e like f unlike

Now you try

Example 4
a $35n$ b $-18x^2y$

Example 5
a $\frac{y}{2}$ b $-2b$

Example 6
a $5a + 2$ b $6m + 7n$ c $3a^2b - 2ab$

Exercise 2B

1 a i $10m$ ii $12b$ iii $15p$
b i $24n^2$ ii $-3q^2$ iii $8b^2c$
2 a $18r$ b $16mn$ c $-14xy$ d $-15mn$
e $-12cd$ f $30ab$ g $-24rs$ h $-40jk$
3 a $21a^2b$ b $-15mn^2$ c $18gh^2$
d $12x^2y^2$ e $8a^2b^2$ f $-6m^2n^2$
4 a $4b$ b $-\frac{a}{3}$ c $\frac{2ab}{3}$
d $\frac{m}{2}$ e $-\frac{x}{4}$ f $\frac{5s}{3}$
g uv h $\frac{5rs}{8}$ i $\frac{5ab}{9}$
5 a $\frac{2x}{5}$ b $\frac{4}{3a}$ c $\frac{11mn}{3}$ d $6ab$
e $-\frac{5}{gh}$ f 8 g –3 h $\frac{7n}{3}$
i $-\frac{9q}{2}$ j $3b$ k $-5x$ l $\frac{m}{2}$
6 a $\frac{4x}{y}$ b $\frac{5p}{2}$ c $-6ab$ d $-\frac{3a}{2b}$
e $-\frac{7n}{5m}$ f $\frac{10s}{t}$ g $8n$ h $3y$
i $4b$ j $6xy$ k $5m$ l $3pq^2$
7 a $10a$ b $7n$ c $8y$ d $11x$
e $3ab$ f $7mn$ g $y + 8$ h $3x + 5$
i $7xy + 4y$ j $12ab + 3$ k $2 - 6m$ l $4 - x$
8 a $5a + 9b$ b $6x + 5y$ c $4t + 6$ d $11x + 4$
e $5xy + 4x$ f $7mn - 9$ g $5ab - a$ h 0
9 a xy^2 b $7a^2b$ c $3m^2n$ d p^2q^2
e $7x^2y - 4xy^2$ f $13rs^2 - 6r^2s$ g $-7x - 2x^2$
h $4a^2b - 3ab^2$ i $7pq^2 - 8pq$ j $8m^2n^2 - mn^2$
10 a $x + y$ b $4x + 2y$
11 a $8x$ b $3x^2$

12 **a** $30x$ cm **b** $30x^2$ cm^2

13 $\frac{20x + 75}{21}$

14 **a** True **b** False **c** True
d False **e** False **f** True

15 **a** $(4x + 4y)$ km **b** $(8x + 4)$ km **c** $(2x - 2)$ km

16 **a** a^3 **b** $\frac{b^2}{3}$ **c** $\frac{b^2}{3}$ **d** $\frac{3ab}{8}$
e $-\frac{2a^3}{3}$ **f** $-\frac{2}{5a^2}$ **g** $\frac{2}{5a^5}$ **h** $\frac{a^2}{4b^3}$
i $3a^3b$ **j** $\frac{4b^3}{a}$ **k** $\frac{a^3}{3b}$ **l** $-\frac{a}{2b^3}$

2C

Building understanding

1 **a** **i** $5x$ **ii** 10
b $5x + 10$ **c** $x + 2$ **d** $5(x + 2)$ **e** $5x + 10$

2 **a** **i** -16 **ii** -4
b no, $-2x - 6$
c **i** -27 **ii** -33
d no, $-3x + 3$

Now you try

Example 7
a $5x + 10$ **b** $3x - 21$ **c** $-3x + 6$

Example 8
a $3x + 21y$ **b** $-6a^2 + 12a$

Example 9
a $6 - 2y$ **b** $x - 9y$

Exercise 2C

1 **a** **i** $2x + 2$ **ii** $5x + 10$
b **i** $7x - 28$ **ii** $6x - 18$
c **i** $-4x + 8$ **ii** $-2x + 8$

2 **a** $2x + 6$ **b** $5x + 60$ **c** $2x - 14$ **d** $7x - 63$
e $6 + 3x$ **f** $21 - 7x$ **g** $28 - 4x$ **h** $2x - 12$

3 **a** $-3x - 6$ **b** $-2x - 22$ **c** $-5x + 15$
d $-6x + 36$ **e** $-8 + 4x$ **f** $-65 - 13x$
g $-180 - 20x$ **h** $-300 + 300x$

4 **a** $2a + 4b$ **b** $15a - 10$ **c** $12m - 15$
d $-16x - 40$ **e** $-12x - 15$ **f** $-4x^2 + 8xy$
g $-18ty + 27t$ **h** $3a^2 + 4a$ **i** $2d^2 - 5d$
j $-6b^2 + 10b$ **k** $8x^2 + 2x$ **l** $5y - 15y^2$

5 **a** $2x + 11$ **b** $6x - 14$ **c** $15x - 3$ **d** $3x + 1$
e $4x - 5$ **f** $2x + 1$ **g** $-3x - 4$ **h** $-5x - 19$
i $11 - x$ **j** $12 - x$ **k** $2 - 2x$ **l** $6 - 3x$

6 **a** $5x + 12$ **b** $4x - 8$ **c** $11x - 2$ **d** $17x - 7$
e $-4x - 6$ **f** $x - 7$ **g** $2x - 4$ **h** $-27x - 4$
i $-4x - 18$ **j** $-13x - 7$ **k** $11x - 7$ **l** $2x - 15$

7 $A = x^2 + 4x$

8 **a** $2x + 2$ **b** $2x^2 - 3x$ **c** $6x^2 - 2x$
d $2x^2 + 4x$ **e** $x^2 + 2x + 6$ **f** $20 + 8x - x^2$

9 $20n - 200$

10 $0.2x - 2000$

11 **a** $2x + 12$ **b** $x^2 - 4x$ **c** $-3x - 12$
d $-7x + 49$ **e** $19 - 2x$ **f** $x - 14$

12 **a** $6(50 + 2) = 312$ **b** $9(100 + 2) = 918$
c $5(90 + 1) = 455$ **d** $4(300 + 26) = 1304$
e $3(100 - 1) = 297$ **f** $7(400 - 5) = 2765$
g $9(1000 - 10) = 8910$ **h** $6(900 - 21) = 5274$

13 **a** $a = \$6000$, $b = \$21\,000$
b **i** $\$3000$ **ii** $\$12\,600$ **iii** $\$51\,000$
c **i** 0 **ii** $0.2x - 4000$ **iii** $0.3x - 9000$
iv $0.5x - 29\,000$
d Answers may vary.
e Answers may vary.

2D

Building understanding

1 **a** 3 **b** 40 **c** 5 **d** 6

2 **a** 3 **b** 2 **c** 12 **d** 6

3 B, C

Now you try

Example 10
a $x = 3$ **b** $x = 22$ **c** $x = -1$
d $x = \frac{6}{5}$ **e** $x = \frac{13}{4}$

Exercise 2D

1 **a** **i** 2 **ii** 1 **iii** 4
b **i** 8 **ii** 2 **iii** 12

2 **a** -1 **b** -3 **c** -6 **d** -4
e $-3\frac{2}{3}$ **f** $1\frac{2}{3}$ **g** $4\frac{1}{2}$ **h** $\frac{1}{2}$
i $1\frac{1}{3}$ **j** $-\frac{5}{7}$ **k** $-\frac{1}{8}$ **l** $-\frac{3}{20}$

3 **a** -6 **b** -6 **c** 30 **d** -15
e -8 **f** 12 **g** 20 **h** -5

4 **a** -3 **b** -1 **c** 2 **d** 8
e $-1\frac{2}{5}$ **f** $-2\frac{5}{7}$ **g** $\frac{3}{8}$ **h** $1\frac{3}{4}$

5 **a** 9 **b** 6 **c** $-6\frac{3}{4}$ **d** $-7\frac{1}{2}$
e $\frac{2}{3}$ **f** $-\frac{4}{25}$ **g** 12 **h** 12
i -9 **j** -8 **k** 6 **l** $-1\frac{1}{2}$

6 **a** 11 **b** 6 **c** -10 **d** -12
e -5 **f** -1 **g** 7 **h** 5
i $4\frac{2}{7}$ **j** $5\frac{1}{3}$ **k** -7 **l** 3

7 **a** 26 **b** 28 **c** 3
d 28 **e** 8 **f** $3\frac{3}{7}$

8 7

9 $\$900$

ISBN 978-1-108-77288-4

10 **a** should have +1 before ÷ 2
b should have × 3 before −2
c need to ÷ −1 as $-x = 7$
d should have +4 before × 3

11 **a** **i** 5 **ii** −3 **iii** $\frac{1}{5}$
iv 3 **v** $-\frac{5}{6}$ **vi** $\frac{3}{10}$
b when the common factor divides evenly into the RHS

12 **a** $a = b + c$ **b** $a = \frac{c - b}{2}$
c $a = \frac{c - 2d}{b}$ **d** $a = c(b + d)$
e $a = -\frac{cd}{b}$ **f** $a = \frac{b}{2c}$
g $a = \frac{c(3 - d)}{2b}$ **h** $a = \frac{2d(b - 3)}{c}$
i $a = cd - b$ **j** $a = b + cd$
k $a = \frac{2be + 6c}{d}$ **l** $a = \frac{d - 3ef}{4c}$

2E

Building understanding

1 **a** $4x - 12$ **b** 2 **c** $5x - 9$
2 **a** $2x + 6 = 5$ **b** $5 + 2x - 2 = 7$
c $2x + 1 = -6$ **d** $2x - 3 = 1$

Now you try

Example 11

a $x = \frac{7}{6}$ **b** $x = 3$ **c** $x = -3$ **d** $x = 14$

Exercise 2E

1 **a** **i** $2\frac{1}{2}$ **ii** $-1\frac{2}{5}$
b **i** 2 **ii** −10

2 **a** $6\frac{1}{3}$ **b** $4\frac{3}{5}$ **c** $-3\frac{3}{4}$ **d** $9\frac{1}{2}$
e $\frac{1}{2}$ **f** $-5\frac{1}{2}$ **g** $-\frac{4}{5}$ **h** $\frac{1}{14}$
i $3\frac{1}{6}$ **j** $\frac{2}{3}$ **k** $\frac{9}{10}$ **l** $1\frac{1}{6}$

3 **a** 1 **b** −2 **c** 5 **d** 0
e 1 **f** 5 **g** 3 **h** 3

4 **a** 1 **b** −4 **c** 2
d 8 **e** 8 **f** 3
g 4 **h** −1 **i** $\frac{5}{11}$

5 **a** −1 **b** $3\frac{1}{2}$ **c** −9 **d** −14
e −16 **f** 3 **g** 19 **h** −3
i −13 **j** −26 **k** $-2\frac{2}{3}$ **l** $-1\frac{1}{10}$

6 **a** $\frac{1}{2}$ **b** $2\frac{2}{3}$ **c** $1\frac{1}{2}$ **d** $-\frac{5}{6}$
e $\frac{2}{5}$ **f** $1\frac{1}{2}$ **g** −6

7 Let \$$w$ be the wage, $2(3w - 6) = 18$, \$5

8 11 marbles

9 **a** $x = 5$ **b** $x = 5$
c Dividing both sides by 3 is faster because 9 ÷ 3 is an integer.

10 **a** $x = 4\frac{1}{3}$ **b** $x = 4\frac{1}{3}$
c Expanding the brackets is faster because 7 ÷ 3 gives a fraction answer.

11 **a** $x = 4$ **b** $x = 4$
c Method a is preferable as you don't have to deal with negatives.
Final step in method a: divide both sides by a positive number.
Final step in method b: divide both sides by a negative number.

12 **a** $x = \frac{d}{a - b}$ **b** $x = \frac{2}{a - b}$
c $x = \frac{c}{5a - b}$ **d** $x = -\frac{6}{3a - 4b}$ or $\frac{6}{4b - 3a}$
e $x = -c$ **f** $x = b$
g $x = \frac{d + bd + c}{a - b}$ **h** $x = -\frac{ab + bc}{a - b + 1}$ or $\frac{ab + bc}{b - a - 1}$

2F

Building understanding

1 **a** $2x - 1 = 11$ **b** $2x + 100 = 2200$
c $m + (m + 12) = 60$ **d** $80n + 100 = 480$

Now you try

Example 12
5

Example 13
\$187 and \$239

Exercise 2F

1 **a** $x - 3 = 4x - 9$ **b** $3(x + 7) = 9$

2 **a** $4(x - 9) = 12, x = 12$ **b** $2x = x + 5, x = 5$
c $x - 8 = 3x + 2, x = -5$

3 **a** Let e be the number of goals for Emma.
b $e + 8$ **c** $e + e + 8 = 28$ **d** $e = 10$
e Emma scored 10 goals, Leonie scored 18 goals.

4 **a** Let w be the width in centimetres.
b length = $4w$ **c** $2w + 2(4w) = 560$ **d** $w = 56$
e length = 224 cm, width = 56 cm

5 $40x + 50 = 290$, 6 days.

6 $x + 2x + 2 = 32$, 10 km, 20 km

7 $x + x + 280 = 1000$, \$360, \$640.

8 15, 45

9 19 km

10 Andrew \$102.50, Brenda \$175, Cammi \$122.50
11 4 fiction, 8 non-fiction books
12 I am 10 years old.
13 Eric is 18 years old now.
14 first leg = 54 km, second leg = 27 km, third leg = 18 km, fourth leg = 54 km
15 2 hours
16 8 pm
17 rectangle $L = 55$ m, $W = 50$ m; triangle side = 70 m
18 a 27, 28, 29
  b i $x, x+2, x+4$ ii 4, 6, 8
  c i $x, x+2, x+4$ ii 15, 17, 19
  d i $x, x+3, x+6$ ii 24, 27, 30
19 a $T = 8x + 7200$ b 300 c $R = 24x$
  d $x = 350$ e 3825
20 \$15 200 21 438 km
22 Anna 6; Henry 4; Chloe 12; twins 11

Progress quiz

1 a $5 + x + y$ b $4y - 15$ c $k^2 + p^2$
  d $(m-n)^2$ or $(n-m)^2$ e $\sqrt{x+16}$
2 a 12 b −28 c $\frac{1}{2}$ d −12
3 a $14ab$ b $-40p^2q$ c $\frac{4k}{m}$ d $-6y^2$
4 a $2xy + 7$ b $11 + km$
  c $5xy - 3x + 2y$ d $a^2b + ab^2 + b$
5 a $4a + 28$ b $-6x^2 + 15x$
6 a $16 - 3a$ b $5x - 2y$
7 a $x = 4$ b $a = 21$ c $p = -3\frac{1}{2}$
  d $x = -6$ e $t = 20$ f $k = \frac{5}{12}$
  g $x = -9$ h $k = 7$
8 a $x = 6\frac{1}{3}$ b $a = 3$ c $m = -2$ d $y = 9\frac{1}{2}$
9 a $2(x + 5) = 14$ b $x - 9 = 4x + 6$
10 Let g be Jed's team's goals: $g + g + 14 = 30$; 22 goals
11 Let a be my age, $4(a + 5) = 88$, 17 years old

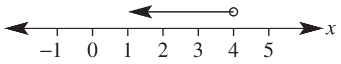
2G

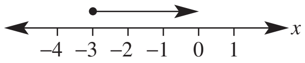
Building understanding

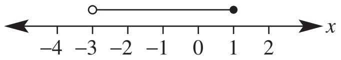
1 a $3 > 2$ b $-1 < 4$ c $-7 < -3$ d $5 > -50$
2 a C b A c B

Now you try

Example 14

a


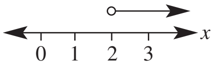
b


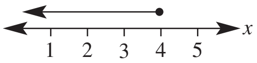
c


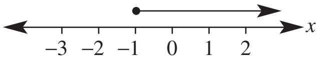
Example 15

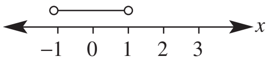
a $x < 11$ b $x < -1$ c $a \geqslant -9$ d $a \geqslant -\frac{7}{2}$

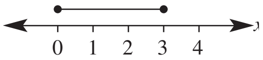
Exercise 2G

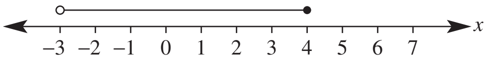
1 a


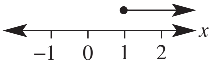
b


c


d


e


f


2 a


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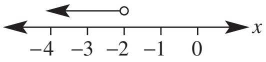

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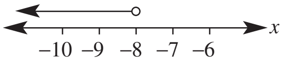

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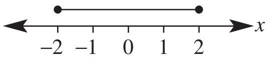

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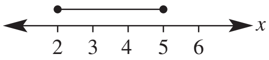
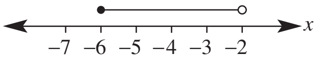


f


3 a $x < 3$ b $b > 5$ c $y > 6$
  d $m < 5$ e $x \geqslant 3$ f $t > -5$
  g $x \geqslant 12$ h $y \geqslant -10$ i $m < 6$
  j $a \geqslant 1\frac{1}{2}$ k $x < 1$ l $x > 8$
4 a $x < 4$ b $n \leqslant -1$ c $x \geqslant \frac{3}{5}$
  d $a \geqslant 4$ e $x \geqslant -6$ f $x \geqslant 10$
  g $x < -6$ h $t \leqslant -2$ i $m > -4\frac{5}{6}$
5 a $x \leqslant 16$ b $x \leqslant -9$ c $x \leqslant 20$
  d $x < 11$ e $x > -2\frac{2}{3}$ f $x \geqslant -2$
6 a $x < 1$ b $a < -8$ c $x \leqslant -2$
  d $x < 2\frac{1}{2}$ e $y < -3\frac{1}{5}$ f $x < -\frac{4}{7}$
7 a $x \geqslant 2\frac{1}{2}$ b $t > -\frac{3}{5}$ c $y \leqslant \frac{3}{8}$
  d $a < 1\frac{1}{5}$ e $m \geqslant 6$ f $b < 5\frac{1}{2}$
8 less than 18
9 less than 13 cm
10 a 3 b 2 c 4 d 0

11 $x = 3$ or $x = 4$ or $x = 5$

12 399 km

13 **a** **i** −0.9, 0, 0.5, 1, 1.8 etc
ii Numbers must be less than 2.
b **i** −4, −2, 0, 1, 5 etc
ii Numbers must be greater than −5.
c **i** $x < a$ **ii** $x > -a$

14 **a** $x < 3$ **b** $x < 3$
c Reverse the inequality sign when dividing by a negative number.

15 **a** $x < -13$ **b** $x \geqslant -3$ **c** $x > \frac{4}{7}$
d $x \leqslant \frac{13}{5}$ **e** $x > \frac{10}{17}$ **f** $x \geqslant \frac{3}{4}$

16 **a** $x > \frac{b - c}{a}$ **b** $x \geqslant b - a$ **c** $x \leqslant a(b + c)$
d $x \leqslant \frac{ac}{b}$ **e** $x < \frac{cd - b}{a}$ **f** $x \geqslant \frac{b - cd}{2}$
g $x < \frac{c}{a} - b$ **h** $x > \frac{b - cd}{a}$ **i** $x < b - \frac{c}{a}$
j $x \leqslant \frac{b + c}{1 - a}$ **k** $x < \frac{b + 1}{b - a}$ **l** $x \leqslant \frac{c - b}{b - a}$

2H

Building understanding

1 **a** A **b** D **c** M **d** A

2 **a** subtract 3 **b** multiply by 4
c square both sides **d** subtract c

3 **a** **i** subtract 5, divide by 3
ii subtract b, divide by a
b **i** divide by 3, take square root of both sides
ii divide by a, take square root of both sides

Now you try

Example 16
a 10 **b** 180

Example 17
21

Example 18
a $a = \frac{b}{2} + c$ or $a = \frac{b + 2c}{2}$ **b** $a = \sqrt{c + b}$

Exercise 2H

1 **a** 21 **b** 24 **c** 2 **d** 6

2 **a** 452.39 **b** 33.51 **c** 25.06
d 14.95 **e** 249.86 **f** 80

3 **a** 36 **b** 5 **c** 20 **d** 4.14
e 3.39 **f** 18.67 **g** 0.06

4 **a** $r = \frac{A}{2\pi h}$ **b** $r = \frac{100I}{Pt}$ **c** $n = \frac{p}{m} - x$
d $x = \frac{cd - a}{b}$ **e** $r = \sqrt{\frac{V}{\pi h}}$ **f** $v = \sqrt{PR}$
g $h = \frac{S - 2\pi r^2}{2\pi r}$ **h** $p = -q \pm \sqrt{A}$
i $g = \frac{4\pi^2 l}{T^2}$ **j** $A = (4C - B)^2$

5 **a** 88.89 km/h
b **i** $d = st$ **ii** 285 kmkm

6 **a** **i** 212°F **ii** 100.4°F
b $C = \frac{5}{9}(F - 32)$
c **i** −10°C **ii** 36.7°C

7 **a** 35 m/s **b** 2 s

8 **a** decrease **b** 3988 L
c 6 hours 57 minutes **d** 11 hours 7 minutes

9 **a** $D = \frac{c}{100}$ **b** $d = 100e$ **c** $D = 0.7M$
d $V = 1.15P$ **e** $C = 50 + 18t$ **f** $d = 42 - 14t$
g $C = \frac{c}{b}$

10 **a** $a = \frac{P}{4}$ **b** $a = 180 - b$ **c** $a = 90 - b$
d $a = \frac{180 - b}{2}$ **e** $a = \sqrt{c^2 - b^2}$ **f** $a = \sqrt{\frac{4A}{\pi}}$

11 **a** 73 **b** 7 **c** 476.3

2I

Building understanding

1 **a** $y = 5$ **b** $x = -3$ **c** $x = 10$

2 **a** A **b** C

3 **a** no **b** yes

Now you try

Example 19
a $x = 2$, $y = 6$ **b** $x = 2$, $y = -3$ **c** $x = -1$, $y = 4$

Exercise 2I

1 **a** $x = 1, y = 2$ **b** $x = 5, y = 1$ **c** $x = \frac{1}{2}, y = \frac{3}{2}$

2 **a** $x = -2, y = -1$ **b** $x = 2, y = 6$
c $x = -3, y = 9$

3 **a** $x = 3, y = 9$ **b** $x = -1, y = 3$
c $x = 1, y = 0$ **d** $x = 6, y = 11$
e $x = 4, y = 3$ **f** $x = 12, y = -3$
g $x = -18, y = -4$ **h** $x = -2, y = 10$
i $x = 2, y = -4$

4 **a** $x = 2, y = 1$ **b** $x = 1, y = 3$
c $x = 0, y = 4$ **d** $x = 3, y = -2$
e $x = 4, y = 1$ **f** $x = -1, y = 4$

5 17, 31

6 10 tonnes, 19 tonnes

7 width $= 1\frac{2}{3}$ cm, length $= 3\frac{5}{6}$ cm

8 $x - (3x - 1) = x - 3x + 1$; to avoid sign error use brackets when substituting

9 **a** $x = \frac{1}{3}, y = 2\frac{1}{3}$ **b** $x = -\frac{5}{6}, y = 6\frac{1}{3}$
c $x = -22, y = -7$

10 a $x = \frac{b}{a+b}, y = \frac{b^2}{a+b}$
b $x = \frac{b}{a+1}, y = \frac{1}{a+1}$
c $x = \frac{a-b}{2}, y = \frac{a+b}{2}$
d $x = \frac{a-ab}{a-b}, y = \frac{a-ab}{a-b} - a$
e $x = \frac{2a}{a-b}, y = \frac{2ab}{a-b} + a$
f $x = \frac{2a+ab}{a-b} - b, y = \frac{2a+ab}{a-b}$

2J

Building understanding

1 a – b + c + d –
2 a i subtraction ii addition iii subtraction
b i addition ii subtraction iii addition

Now you try

Example 20
a $x = 1$, $y = 2$ b $x = -1$, $y = 4$
Example 21
a $x = 2$, $y = -1$ b $x = 3$, $y = -2$

Exercise 2J

1 a $x = 1, y = 1$ b $x = 10, y = 2$
c $x = 1, y = 3$
2 a $x = 1, y = 1$ b $x = 2, y = 2$
c $x = 2, y = -1$
3 a $x = 2, y = 4$ b $x = 8, y = -1$
c $x = -2, y = 6$ d $x = -3, y = -13$
e $x = 2, y = 3$ f $x = 1, y = 3$
4 a $x = -3, y = 4$ b $x = 2, y = 1$
c $x = 2, y = 1$ d $x = -2, y = 3$
e $x = 7, y = -5$ f $x = 5, y = 4$
g $x = 5, y = -5$ h $x = -3, y = -2$
i $x = -1, y = -3$
5 a $x = 3, y = -5$ b $x = 2, y = -3$
c $x = 2, y = 4$ d $x = 3, y = 1$
e $x = -1, y = 4$ f $x = 3, y = -2$
g $x = 5, y = -3$ h $x = -2, y = -2$
i $x = 2, y = 1$ j $x = -5, y = -3$
k $x = -3, y = 9$ l $x = -1, y = -2$
6 21 and 9
7 102, 78
8 $L = 261.5$ m, $W = 138.5$ m
9 11 mobile phones, 6 iPads
10 a $x = 2, y = 1$ b $x = 2, y = 1$
c Method b is preferable as it avoids the use of a negative coefficient.
11 Rearrange one equation to make x or y the subject.
a $x = 4, y = 1$ b $x = -1, y = -1$
12 a no solution b no solution
13 a $x = \frac{a+b}{2}, y = \frac{a-b}{2}$ b $x = \frac{b}{2a}, y = -\frac{b}{2}$
c $x = -\frac{a}{2}, y = -\frac{3a}{2b}$ d $x = 0, y = b$
e $x = \frac{1}{3}, y = \frac{b}{3a}$ f $x = -\frac{4}{a}, y = 6$
g $x = \frac{3b}{7a}, y = \frac{b}{7}$ h $x = \frac{3b}{7a}, y = -\frac{b}{7}$
i $x = 0, y = \frac{b}{a}$ j $x = -\frac{c}{a}, y = c$
k $x = \frac{b-4}{ab-4}, y = \frac{1-a}{ab-4}$ l $x = 1, y = 0$
m $x = \frac{c-bd}{a(b+1)}, y = \frac{c+d}{b+1}$ n $x = \frac{a+2b}{a-b}, y = \frac{3a}{a-b}$
o $x = \frac{3b}{a+3b}, y = \frac{3b-2a}{a+3b}$ p $x = -\frac{b}{a-bc}, y = \frac{bc-2a}{a-bc}$
q $x = \frac{c+f}{a+d}, y = \frac{cd-af}{b(a+d)}$ r $x = \frac{c-f}{a-d}, y = \frac{af-cd}{b(a-d)}$

2K

Building understanding

1 a $x + y = 42, x - y = 6$
b elimination
2 a $l = w + 5, 2l + 2w = 84$
b substitution
3 a $l = 3w, 2l + 2w = 120$
b substitution

Now you try

Example 22
hat \$8.40, scarf \$11.20

Exercise 2K

1 a Let \$$m$ be the cost of milk.
Let \$$c$ be the cost of chips.
b $3m + 4c = 17, m + 5c = 13$
c $m = 3, c = 2$
d A bottle of milk costs \$3 and a bag of chips \$2.
2 a Let \$$g$ be the cost of lip gloss.
Let \$$e$ be the cost of eye shadow.
b $7g + 2e = 69, 4g + 3e = 45$
c $g = 9, e = 3$
d lip gloss \$9; eye shadow \$3
3 cricket ball \$12; tennis ball \$5
4 4 buckets of chips, 16 hot dogs
5 300 adults, 120 children
6 potatoes 480 ha; corn 340 ha
7 11 five-cent and 16 twenty-cent coins
8 Michael is 35 years old now.
9 Jenny \$100, Kristy \$50
10 160 adult tickets and 80 child tickets

11 5 hours

12 jogging 3 km/h, cycling 9 km/h

13 **a** Malcolm is 14 years old.
  b The second digit of Malcolm's age is 3 more than the first digit.
  c 6: 14, 25, 36, 47, 58 or 69

14 original number is 37

15 any two-digit number that has the first digit 2 more than the second (e.g. 42 or 64 etc.)

Problems and challenges

1 **a** $x = \frac{k+4}{w-a}$ **b** $K = \frac{a-b}{y}$ **c** $a = \frac{w}{1-km^2}$

2 39

3 \$140

4 8

5 **a** **i** $>$ **ii** $<$ **iii** $>$ **iv** $<$
  b c, b, a, d

6 $x = 1, y = -2, z = 5$

7 **a** $x = \frac{ab}{a-b}$ **b** $x = \frac{10}{3}$ or $3\frac{1}{3}$
  c $x = -\frac{7}{11}$ **d** $x = \frac{29}{4}$ or $7\frac{1}{4}$

Short-answer questions

1 **a** $7m$ **b** $2(x+y)$ **c** $3m$ **d** $\frac{n}{4} - 3$

2 **a** -7 **b** 7 **c** 24 **d** 8

3 **a** $8mn$ **b** $\frac{xy}{3}$ **c** $6b^2$
  d $4 - 3b$ **e** $2mn + 2m - 1$ **f** $2p + 4q$

4 **a** $2x + 14$ **b** $-6x - 15$ **c** $6x^2 - 8x$
  d $-10a + 8a^2$ **e** $13 - 4x$ **f** $27x - 10$

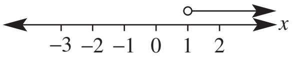
5 **a** $x = 9$ **b** $x = 26$ **c** $x = 15$ **d** $x = 1$
  e $x = 2$ **f** $x = -9$ **g** $x = -10$ **h** $x = \frac{5}{7}$

6 **a** $2n + 3 = 21, n = 9$ **b** $\frac{l-5}{3} = 7, l = 26$
  c $\frac{x}{4} - 5 = 0, x = 20$

7 **a** $x = 5$ **b** $x = 1\frac{5}{6}$ **c** $x = 4$
  d $x = -5$ **e** $x = 1$ **f** $x = 4$

8 \$260

9 **a**



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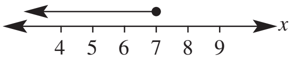

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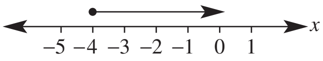

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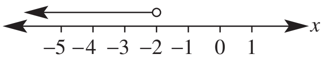

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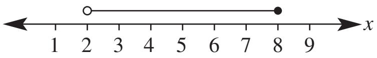

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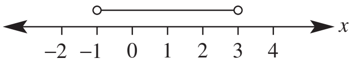


10 **a** $x < 12$ **b** $m > -1$ **c** $y \geqslant -6$
  d $x < 17$ **e** $a > 4$ **f** $x \geqslant 2$

11 n (sales) $\geqslant$ \$24 000

12 **a** $E = 60$ **b** $a = 5$ **c** $h = 10$

13 **a** $x = \frac{v^2 - u^2}{2a}$ **b** $\theta = \frac{2A}{r^2}$
  c $I = \sqrt{\frac{P}{R}}$ **d** $a = \frac{2S}{n} - l = \frac{2S - nl}{n}$

14 **a** $x = 8, y = 2$ **b** $x = -\frac{3}{5}, y = -1\frac{3}{5}$
  c $x = 4, y = 11$ **d** $x = 11, y = 4$
  e $x = -3, y = -5$ **f** $x = 4, y = 1$

15 3 show bags, 6 rides

Multiple-choice questions

1 C **2** D **3** D **4** C **5** B **6** A
7 A **8** E **9** B **10** A **11** B **12** D

Extended-response questions

1 **a** $0 < b \leqslant 10$ **b** $b = \frac{2A}{h} - a$ or $b = \frac{2A - ah}{h}$
  c 8 m **d** $h = \frac{2A}{a+b}$ **e** 8 m

2 **a** \$5 per ride
  b **i** Let \$$c$ be the cost of a bucket of chips.
    Let \$$d$ be the cost of a drink.
   ii $2d + c = 11, 3d + 2c = 19$
   iii $d = 3, c = 5$
   iv Chips cost \$5 per bucket and drinks cost \$3 each.

Chapter 3

3A

Building understanding

1 **a** 17 **b** 50 **c** $\sqrt{8}$

2 **a** $c^2 = a^2 + b^2$ **b** $x^2 = y^2 + z^2$ **c** $j^2 = k^2 + l^2$

3 **a** 10.24 **b** 13 **c** 10 **d** 4.90

Now you try

Example 1
a 5 **b** 5.59
Example 2
$\sqrt{58}$

Exercise 3A

1 **a** $c = 10$ **b** $c = 6.62$
2 **a** $c = 26$ **b** $c = 17$ **c** $c = 15$
d $c = 25$ **e** $c = 41$ **f** $c = 50$
3 **a** 4.47 **b** 3.16 **c** 15.62
d 11.35 **e** 7.07 **f** 0.15
4 **a** $\sqrt{5}$ **b** $\sqrt{58}$ **c** $\sqrt{34}$
d $\sqrt{37}$ **e** $\sqrt{109}$ **f** $\sqrt{353}$
5 **a** 21.63 mm **b** 150 mm **c** 50.99 mm
d 155.32 cm **e** 1105.71 m **f** 0.02 m
6 **a** 8.61 m **b** 5.24 m **c** 13.21 cm
d 0.19 m **e** 17.07 mm **f** 10.93 cm
7 42 m **8** 4.4 m
9 250 m **10** 495 m
11 2.4 m
12 **a** 5 **b** $\frac{25}{3}$ or $8\frac{1}{3}$
13 5.83 m
14 **a** no **b** no **c** yes
d no **e** yes **f** yes
15 **a** 5.66 cm **b** 5.66 cm **c** yes
16 **a** 77.78 cm **b** 1.39 m
c reduce is by 7.47 cm **d** 43.73 cm
17 42.43 cm, 66.49 cm

3B

Building understanding

1 **a** 14 **b** 12 **c** 4 **d** 8
2 **a** True **b** False **c** False
d True **e** True **f** False

Now you try

Example 3
a $a = 7$ **b** $b = 10.27$
c $x = \sqrt{\frac{49}{2}} \left(= \frac{7}{\sqrt{2}} = \frac{7\sqrt{2}}{2}\right)$

Exercise 3B

1 **a** 40 **b** 2.26 **c** $\sqrt{32}$ or $4\sqrt{2}$
2 **a** 16 **b** 24 **c** 6
d 21 **e** 60 **f** 27
3 **a** 8.66 **b** 11.31 **c** 5.11
d 17.55 **e** 7.19 **f** 0.74
4 **a** 30 cm **b** 149.67 cm **c** 1.65 cm
d 2.24 km **e** 12 cm **f** 52.92 mm
5 **a** $\sqrt{\frac{25}{2}} \left(= \frac{5\sqrt{2}}{2}\right)$ **b** $\sqrt{8} \left(= 2\sqrt{2}\right)$
c $\sqrt{\frac{1521}{200}} \left(= \frac{39\sqrt{2}}{20}\right)$
6 **a** $\sqrt{187}$ **b** $\sqrt{567}$ **c** 40
7 14.2 m **8** 5.3 m
9 49 cm **10** 1.86 m
11 **a** $\sqrt{\frac{81}{5}} \left(= \frac{9\sqrt{5}}{5}\right)$ **b** $\sqrt{5}$ **c** $\sqrt{\frac{5}{2}}$
12 **a** $\sqrt{\frac{5}{2}}, 3\sqrt{\frac{5}{2}}$ **b** $\frac{10}{\sqrt{13}}, \frac{15}{\sqrt{13}}$ **c** $\frac{25}{\sqrt{74}}, \frac{35}{\sqrt{74}}$
13 **a** the side c **b** $\sqrt{6}$
c no, c^2 is enough
d error due to rounding, x would not be exact
14 **a** **i** $\sqrt{8}$ **ii** $\sqrt{7}$ **iii** $\sqrt{6}$
b *OB*: 2.6, *OC*: 2.79, *OD*: 2.96
c differ by 0.04; the small difference is the result of rounding errors
d Answers may vary.

3C

Building understanding

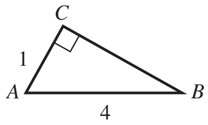
1 **a** C, II **b** A, III **c** B, I

Now you try

Example 4
40.45 m

Exercise 3C

1 3.0 m **2** 142.9 m
3 3823 mm **4** 1060 m
5 **a** 6.4 m **b** 5.7 cm **c** 5.4 m **d** 6.0 m
6 466.18 m
7 **a**



b 3.9 cm
8 **a** 47 m **b** 89.8 m
9 171 cm
10 **a** $\frac{5}{\sqrt{2}}$ **b** $6 + \frac{5}{\sqrt{2}}$ **c** 10.2
11 **a** 1.1 km **b** 1.1 km **c** 3.8 km
12 **a** 28.28 cm
b **i** 80 cm **ii** 68.3 cm **iii** 48.3 cm
c Answers may vary.
d Answers may vary.

3D

Building understanding

1 **a** yes **b** yes **c** no
d yes **e** yes **f** no
g yes **h** no **i** yes

Now you try

Example 5
7.60 cm

Exercise 3D

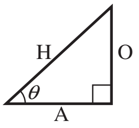
1 11.18 mm
2 a 0.34 b 19.75
3 a 3.6 b 2.5 c 3.1
4 a 18.0 cm b 7 mm c 0.037 m
5 a $\sqrt{2}$ cm b 1.7 cm
6 a $\sqrt{208}$ cm b 15.0 cm
7 84 m
8 2.86 cm
9 $\sqrt{3} : 2$
10 a i $\sqrt{8}$ cm ii 3.74 cm
b i $\sqrt{50}$ cm ii 5.45 cm
11 a $\sqrt{65}$ b 8.06 c $\sqrt{69}$, 8.31 d 8.30
e Rounding errors have accumulated.
12 a i 11.82 m ii 12.15 m
iii 11.56 m iv 11.56 m
b Shortest distance is 10.44 m.

3E

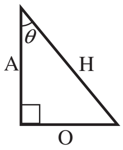
Building understanding

1 a hypotenuse b opposite c adjacent
d opposite e hypotenuse f adjacent

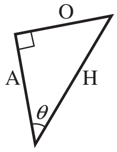
2 a



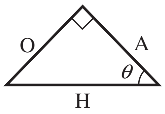
b



c



d



3 a 5 b 4 c 3 d 3 e 4

Now you try

Example 6
a $\tan\theta = \frac{6}{5}$ b $\cos\theta = \frac{3}{8}$ c $\sin\theta = \frac{2}{3}$

Exercise 3E

1 a $\sin\theta = \frac{4}{7}$ b $\tan\theta = \frac{5}{4}$ c $\cos\theta = \frac{3}{5}$

2 a $\sin\theta = \frac{2}{3}$ b $\tan\theta = 1$ c $\sin\theta = \frac{x}{y}$
d $\tan\theta = \frac{4}{5}$ e $\cos\theta = \frac{a}{2b}$ f $\sin\theta = \frac{5y}{3x}$

3 a i $\frac{5}{13}$ ii $\frac{5}{13}$ iii the same
b i $\frac{12}{13}$ ii $\frac{12}{13}$ iii the same
c i $\frac{5}{12}$ ii $\frac{5}{12}$ iii the same

4 a $\sin\theta = \frac{3}{5}, \cos\theta = \frac{4}{5}, \tan\theta = \frac{3}{4}$
b $\sin\theta = \frac{5}{13}, \cos\theta = \frac{12}{13}, \tan\theta = \frac{5}{12}$
c $\sin\theta = \frac{12}{13}, \cos\theta = \frac{5}{13}, \tan\theta = \frac{12}{5}$

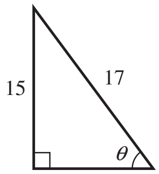
5 a $\frac{4}{5}$ b $\frac{3}{5}$ c $\frac{3}{5}$
d $\frac{3}{4}$ e $\frac{4}{5}$ f $\frac{4}{3}$

6 $\frac{3}{4}$

7 a $\frac{7}{25}$ b $\frac{24}{25}$ c $\frac{7}{24}$

8 a i 5 ii $\sin\theta = \frac{3}{5}, \cos\theta = \frac{4}{5}, \tan\theta = \frac{3}{4}$
b i 25 ii $\sin\theta = \frac{7}{25}, \cos\theta = \frac{24}{25}, \tan\theta = \frac{7}{24}$
c i 15 ii $\sin\theta = \frac{9}{15} = \frac{3}{5}, \cos\theta = \frac{12}{15} = \frac{4}{5}, \tan\theta = \frac{9}{12} = \frac{3}{4}$
d i 10 ii $\sin\theta = \frac{8}{10} = \frac{4}{5}, \cos\theta = \frac{6}{10} = \frac{3}{5}, \tan\theta = \frac{8}{6} = \frac{4}{3}$

9 a



b 8 c $\sin\theta = \frac{15}{17}, \cos\theta = \frac{8}{17}, \tan\theta = \frac{15}{8}$

10 a i $\frac{1}{2}$ ii $\frac{\sqrt{3}}{2}$ iii $\frac{1}{\sqrt{3}}$
iv $\frac{\sqrt{3}}{2}$ v $\frac{1}{2}$ vi $\sqrt{3}$
b i They are equal. ii They are equal.

11 a Answers may vary.
b i 0.766 ii 0.643 iii 0.839
iv 0.766 v 1.192 vi 0.643
c $\sin 40° = \cos 50°, \sin 50° = \cos 40°,$
$\tan 50° = \frac{1}{\tan 40°}, \tan 40° = \frac{1}{\tan 50°}$

12 a yes, any isosceles right-angled triangle
b no, as it would require the hypotenuse (the longest side) to equal the opposite

c no, adjacent side can't be zero
d no; the numerator < denominator for sin and cos as the hypotenuse is the longest side

13 a $\sin\theta = \frac{3}{5}$, $\tan\theta = \frac{3}{4}$
b i $\cos\theta = \frac{\sqrt{3}}{2}$, $\tan\theta = \frac{1}{\sqrt{3}}$
ii $\tan\theta = \sqrt{3}$, $\sin\theta = \frac{\sqrt{3}}{2}$
iii $\sin\theta = \frac{1}{\sqrt{2}}$, $\cos\theta = \frac{1}{\sqrt{2}}$
c equals one
d $(\sin\theta)^2 + (\cos\theta)^2 = 1$ (the Pythagorean identity)

3F

Building understanding

1 a A b O c H
2 a sin b tan c cos

Now you try

Example 7
a 0.94 b 0.87 c 1.38
Example 8
3.21
Example 9
a 2.12 b 4.49 c 2.02

Exercise 3F

1 a 0.34 b 0.80 c 2.05
2 a 0.73 b 0.10 c 0.25 d 0.46
3 a 3.06 b 18.94 c 5.03
d 0.91 e 1.71 f 9.00
g 2.36 h 4.79 i 7.60
4 a 5.95 b 0.39 c 12.59 d 3.83
e 8.40 f 1.36 g 29.00 h 1.62
i 40.10 j 4.23 k 14.72 l 13.42
m 17.62 n 5.48 o 9.75 p 1.01
5 1.12 m
6 44.99 m
7 10.11 m
8 a 20.95 m b 10 cm
9 a 65° b 1.69 c 1.69
d They are the same as both are suitable methods for finding x.
10 a i 80° ii 62° iii 36° iv 9°
b i both 0.173... ii both 0.469...
iii both 0.587... iv both 0.156...
c $\sin\theta° = \cos(90° - \theta°)$
d i 70° ii 31° iii 54° iv 17°

11 a $\sqrt{2}$
b i $\frac{1}{\sqrt{2}}$ ii $\frac{1}{\sqrt{2}}$ iii 1
c $\sqrt{3}$
d i $\frac{1}{2}$ ii $\frac{\sqrt{3}}{2}$ iii $\frac{1}{\sqrt{3}}$
iv $\frac{\sqrt{3}}{2}$ v $\frac{1}{2}$ vi $\sqrt{3}$

Progress quiz

1 a 6.66 b 7.88
2 a $\sqrt{41}$ b $\sqrt{50}\ (= 5\sqrt{2})$
3 224.5 cm
4 a 10.4 cm b 10.8 cm
5 a AC b BC c $\frac{3}{5}$ d $\frac{4}{3}$
6 a 7.46 b 6.93
7 a 13.80 b 25.27 c 2.50
8 a 3.464 cm b 6.9 cm^2

3G

Building understanding

1 a $x = 2$ b $x = 5$ c $x = 7$ d $x = 4$
2 a N b D c D

Now you try

Example 10
3.40
Example 11
a 8.00 b $x = 7.58$, $y = 23.27$

Exercise 3G

1 a 4.10 b 6.81 c 37.88
2 a 0.98 b 12.80 c 14.43
d 9.52 e 114.83 f 22.05
3 a 13.45 b 16.50 c 57.90 d 26.33
e 15.53 f 38.12 g 9.15 h 32.56
i 21.75 j 49.81 k 47.02 l 28.70
4 a $x = 7.5$, $y = 6.4$ b $a = 7.5$, $b = 10.3$
c $a = 6.7$, $b = 7.8$ d $x = 9.5$, $y = 12.4$
e $x = 12.4$, $y = 9.2$ f $x = 21.1$, $y = 18.8$
g $m = 56.9$, $n = 58.2$ h $x = 15.4$, $y = 6.0$
5 40 m
6 3848 m
7 a 23.7 m b 124.9 m
8 a B, as student B did not use an approximation in their working out
b Use your calculator and do not round sin 31° during working.
c i difference of 0.42 ii difference of 0.03
9 a i 10.990 ii 11.695

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b **i** 0.34 **ii** 0.94 **iii** 0.36
c equal to tan 20°
d **i** $\frac{b}{c}$ **ii** $\frac{a}{c}$ **iii** $\frac{b}{a}$ **iv** $\frac{b}{a}$
e same as tan θ

3H

Building understanding

1 **a** 11.54 **b** 41.41 **c** 26.57
2 **a** $\frac{1}{2}$ **b** 50° **c** $45° = \tan^{-1}(1)$
3 **a** sine **b** cosine **c** tan

Now you try

Example 12
a 65° **b** 59.0°
Example 13
34°

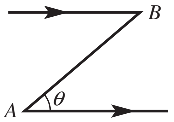
Exercise 3H

1 **a** 30° **b** 60° **c** 45°
2 **a** 30° **b** 45° **c** 30°
d 90° **e** 50° **f** 90°
3 **a** 34.85° **b** 19.47° **c** 64.16°
d 75.52° **e** 36.87° **f** 38.94°
g 30.96° **h** 57.99° **i** 85.24°
4 **a** 43° **b** 31° **c** 41° **d** 16°
e 55° **f** 50° **g** 49° **h** 41°
5 17° **6** 23.13°
7 25.4° **8** 26.6°
9 **a** 128.7° **b** 72.5° **c** 27.3°
10 **a** 90°, 37°, 53° **b** 90°, 23°, 67° **c** 90°, 16°, 74°
11 45°
12 $\angle ACM = 18.4° \angle ACB = 33.7°$, no it is not half
13 **a** 18° **b** 27° **c** 45°
d 5.67 m **e** up to 90°

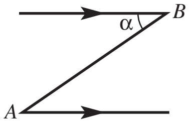
3I

Building understanding

1 $a = 65$, $b = 25$
2 **a** 22° **b** 22°
3 **a** **i**



ii



b yes, $\theta = \alpha$, alternate angles are equal on parallel lines

Now you try

Example 14
31 m
Example 15
22°
Example 16
Yes, 7 m above

Exercise 3I

1 29 m
2 16 m
3 157 m
4 38 m
5 90 m
6 37°
7 6°
8 10°
9 yes by 244.8 m
10 **a** 6° **b** 209 m
11 4634 mm
12 15°, 4319 mm
13 1.25 m
14 **a** 6.86 m **b** 26.6°
c **i** $m = h + y$ **ii** $y = x \tan\theta$ **iii** $m = h + x \tan\theta$
15 $A = \frac{1}{2}a^2 \tan\theta$
16 **a** **i** 47.5 km **ii** 16.25 km
b no, $\sin(2 \times \theta) < 2 \times \sin\theta$
c yes, $\sin\left(\frac{1}{2}\theta\right) > \frac{1}{2}\sin\theta$
17 **a** yes **b** 12.42 km
c no, after 10 min the plane will be above 4 km
d 93 km/h or more
18 **a** 1312 m **b** 236 km/h

3J

Building understanding

1 **a** 0° **b** 045° **c** 090° **d** 135°
e 180° **f** 225° **g** 270° **h** 315°
2 **a** 070° **b** 130° **c** 255° **d** 332°

Now you try

Example 17
a 230°T **b** 050°T
Example 18
5.7 km
Example 19
a 1.7 km
b 300°T

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Exercise 3J

1 a i 060° T ii 240° T
 b i 230° T ii 050° T
2 a i 040° T ii 220° T
 b i 142° T ii 322° T
 c i 125° T ii 305° T
 d i 067° T ii 247° T
 e i 330° T ii 150° T
 f i 228° T ii 048° T
3 3.28 km
4 59.45 km
5 3.4 km
6 11 km
7 a 39 km b 320° T
8 a 11.3 km b 070° T
9 310 km
10 3.6 km
11 6.743 km
12 a $180^\circ + a^\circ$ b $a^\circ - 180^\circ$
13 a 320° T b 245° T c 065° T
 d 238° T e 278° T
14 a 620 km b 606 km c 129 km
15 a i 115 km ii 96 km
 b 158 km
 c i 68 km ii 39.5 min

Problems and challenges

1 $P = 4 + 2\sqrt{8}$
2 $10\ \text{m}^2$
3 15 cm
4 010° T
5 round peg square hole
6 122° T
7 a i 3, 4, 5 ii 5, 12, 13
 b $m^2 + n^2$

Short-answer questions

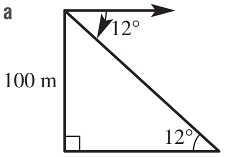
1 a 37 b $\sqrt{12}$ c $\sqrt{50}$
2 4.49 m
3 a 13 cm b 13.93 cm
4 19 m
5 a 4.91 m b $x = 2.83, h = 2.65$
6 a 0.64 b 2.25 c 0.72
7 a 7.83 b 48.02 c 50.71
8 28.01 m
9 25 m
10 a 59.45 km b 53.53 km
11 177.91 m
12 053.13°
13 63.2 m
14 5.3 m
15 a 52.5 km b 13.59 km

Multiple-choice questions

1 A 2 B 3 A 4 C 5 B
6 C 7 D 8 C 9 A 10 B

Extended-response questions

1 a 2.15 m
 b 0.95 m
 c i 3.05 m ii yes
2 a



 b 470 m
 c 3° d 1530 m
3 a 35 km b 399 km

Chapter 4

4A

Building understanding

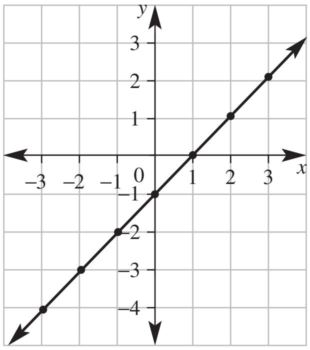
1 a *A* (4, 1) *B* (2, 3) *C* (0, 3)
 D (–2, 2) *E* (–3, 1) *F* (–1, 0)
 G (–3, –2) *H* (–2, –4) *I* (0, –3)
 J (2, –2) *K* (3, –4) *L* (2, 0)
 b i *F*, *L* ii *C*, *I*
 c i *D*, *E* ii *J*, *K*
2 a 2 b –7
3 a 1 b 21
4 Answers may vary; e.g. (1, 4), (–1, 2) etc.

Now you try

Example 1

a

x	–3	–2	–1	0	1	2	3
y	–4	–3	–2	–1	0	1	2



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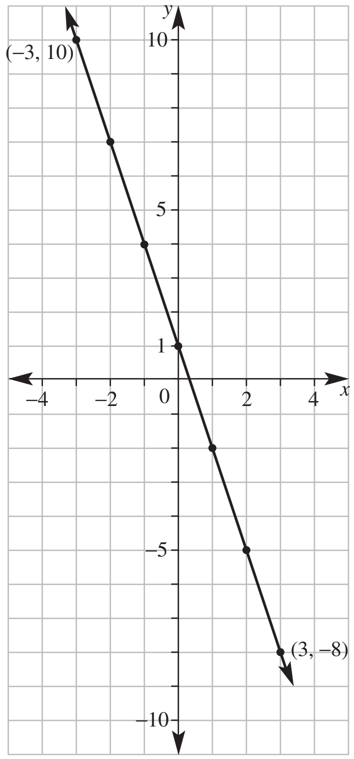
b

x	–3	–2	–1	0	1	2	3
y	10	7	4	1	–2	–5	–8



Example 2

a x-intercept is 1, y-intercept is 1

b x-intercept is 5, y-intercept is –3

Example 3

a yes

b no

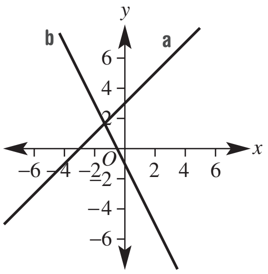
Exercise 4A

1 a $y = x + 3$

x	–3	–2	–1	0	1	2	3
y	0	1	2	3	4	5	6

b $y = -2x - 1$

x	–3	–2	–1	0	1	2	3
y	5	3	1	–1	–3	–5	–7



2 a $y = x - 1$

x	–3	–2	–1	0	1	2	3
y	–4	–3	–2	–1	0	1	2

b $y = 2x - 3$

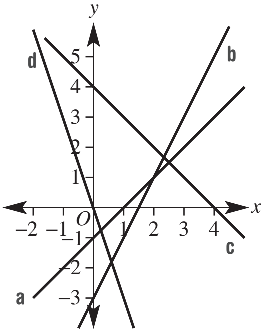
x	–3	–2	–1	0	1	2	3
y	–9	–7	–5	–3	–1	1	3

c $y = -x + 4$

x	–3	–2	–1	0	1	2	3
y	7	6	5	4	3	2	1

d $y = -3x$

x	–3	–2	–1	0	1	2	3
y	9	6	3	0	–3	–6	–9



3 a 1, 1 **b** –2, 2 **c** 4, 8 **d** –5, 10
e 2, 3 **f** 7, –3 **g** –11, 5 **h** –2, –5

4 a yes **b** no **c** yes
d yes **e** no **f** no

5 a no **b** yes **c** no
d no **e** yes **f** no

6 a $y = x + 2$ **b** $y = 2x$
c $y = 2x + 1$ **d** $y = -x + 2$

7 a C **b** D **c** B **d** A

8 a False **b** True **c** True **d** False

9 a For graph to move 1 down from (1, 1) it moves $\frac{1}{2}$ across.

b For graph to move 1 up from (0, –1) it moves $1\frac{1}{2}$ across.

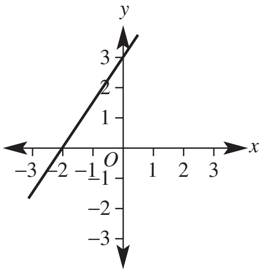
10 a yes **b** no **c** no **d** yes

11 a $y = 2x + 7$ **b** $y = -x + 20$
c $y = -3x - 10$ **d** $y = -5x + 4$
e $y = \frac{1}{2}x + \frac{1}{2}$ **f** $y = -\frac{1}{2}x - \frac{3}{2}$

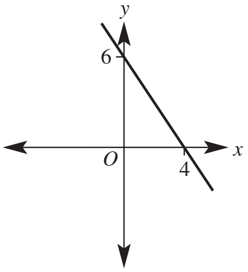
4B

Building understanding

1 a



b

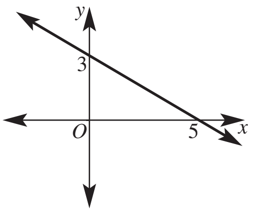


2 a i 3 ii 4 iii −6
b i 10 ii 1 iii 6
3 a 4 **b** −5 **c** 3
4 a −2 **b** 3 **c** 2

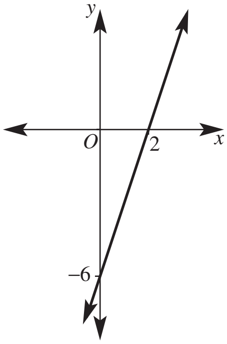
Now you try

Example 4

a

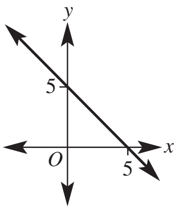


b

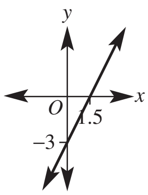


Exercise 4B

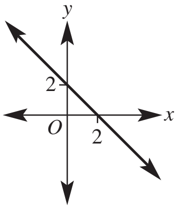
1 a



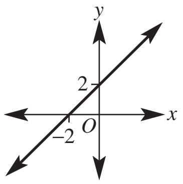
b



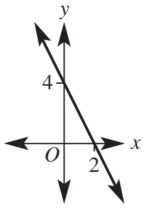
2 a



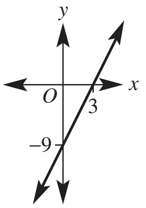
b



c



d



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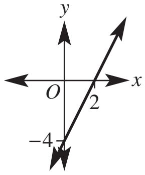
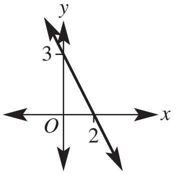
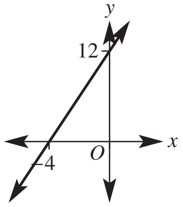


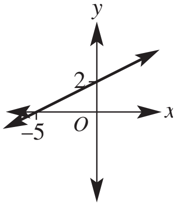
f



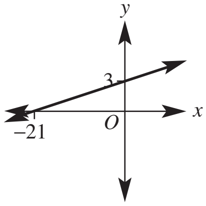
g



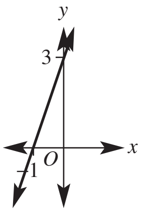
h



i



3 a



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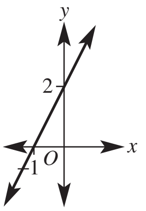

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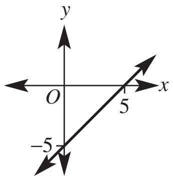

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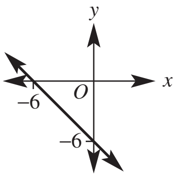

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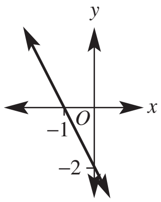

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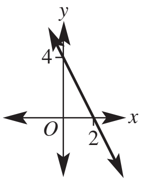
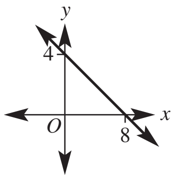


4 a



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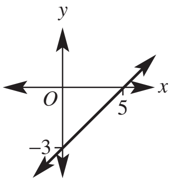

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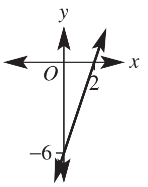
c



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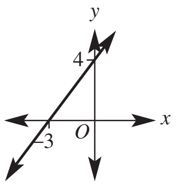
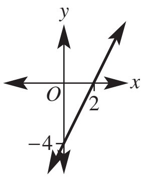


e



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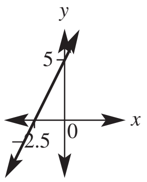
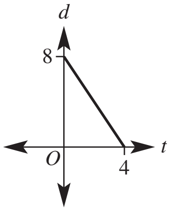


5 **a** 8 m **b** 4 seconds

c

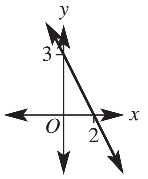


6 **a** 100 m **b** 12.5 s

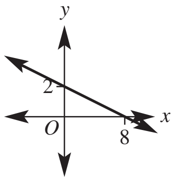
7 **a** $1\frac{2}{3}, -2\frac{1}{2}$ **b** $-7, -1\frac{2}{5}$ **c** $6\frac{1}{2}, -13$

d $-\frac{1}{2}, -1$ **e** $3, -1\frac{1}{2}$ **f** $\frac{1}{3}, -\frac{1}{7}$

8 **a**



b



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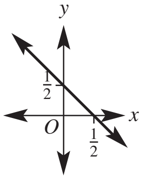


9 (0, 0) is the x- and y-intercept for all values of a and b

10 **a** $x + y = 4$ **b** $x + y = 2$

c $x - y = 3$ **d** $x - y = -1$

e $x + y = k$ **f** $x + y = -k$

11 **a** $\frac{c}{a}, \frac{c}{b}$ **b** $-\frac{cb}{a}, c$ **c** $\frac{c}{a}, -\frac{c}{b}$

d $-\frac{c}{b}, \frac{c}{a}$ **e** $-\frac{c}{b}, \frac{c}{a}$ **f** $\frac{bc}{a}, \frac{bc}{a}$

4C

Building understanding

1 **a** 2 **b** −3 **c** 0

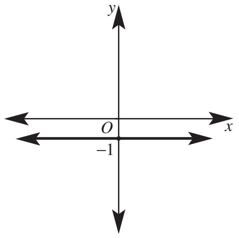
2 **a** 3 **b** −2 **c** 0

3 **a** 5 **b** $\frac{1}{3}$ **c** −4 **d** −0.1

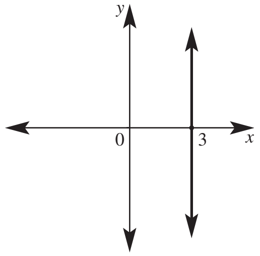
Now you try

Example 5

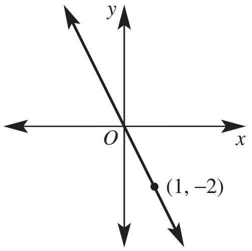
a



b

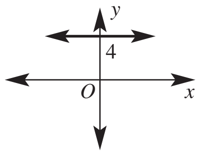


Example 6

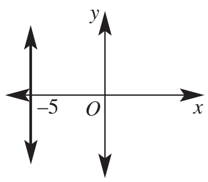


Exercise 4C

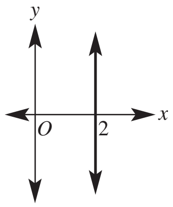
1 **a**



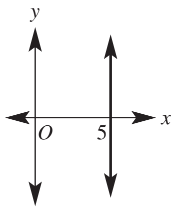
b



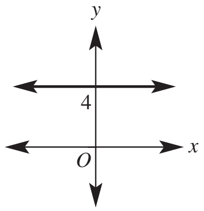
2 **a**



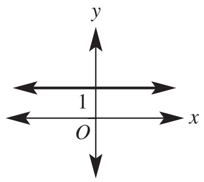
b



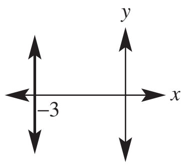
c



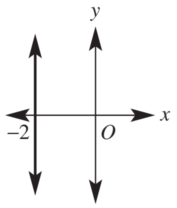
d



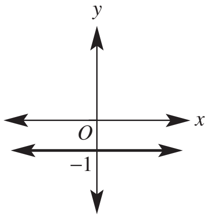
e



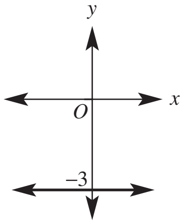
f



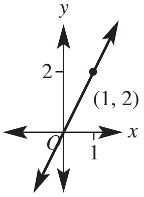
g



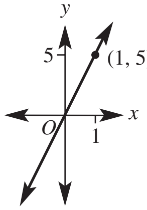
h



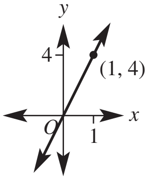
3 a



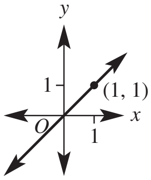
b



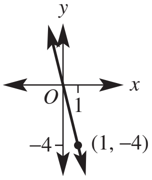
c



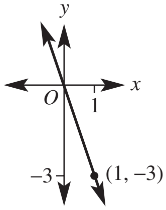
d



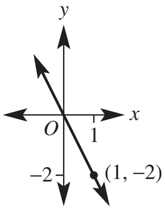
e



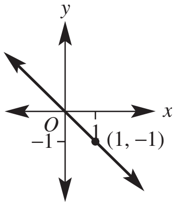
f



g



h



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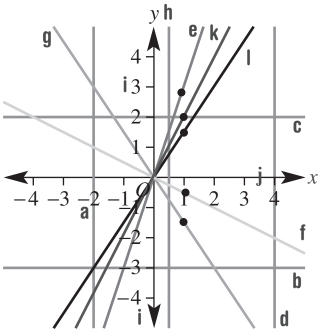


5 **a** $y = -2$ **b** $y = 4$ **c** $x = -2$
d $x = 5$ **e** $y = 1.5$ **f** $x = -6.7$

6 **a** $y = 3$ **b** $x = 5$ **c** $x = -2$ **d** $y = 0$

7 **a** $y = 250$ **b** $y = -45$

8 **a** (1, 2) **b** (−3, 5) **c** (0, −4)
d (4, 0) **e** (0, 0) **f** (1, 3)
g (3, −27) **h** (5, 40)

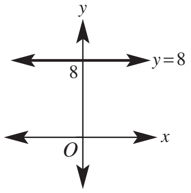
9 **a** $A = 15$ square units **b** $A = 68$ square units

10 **a** **i** $y = 1$ or $y = -5$
ii $y = 0$ or $y = -4$
iii $y = 3\frac{1}{2}$ or $y = -7\frac{1}{2}$
b **i** $y = 1$ or $y = -5$
ii $y = 7$ or $y = -11$
iii $y = 9\frac{1}{2}$ or $y = -13\frac{1}{2}$

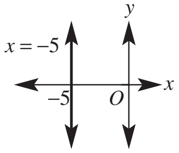
11 **a** $y = 2x$ **b** $y = -x$ **c** $y = 3x$ **d** $y = -3x$

12 **a** $y = 3x$ **b** $y = 4x$ **c** $y = -5x$ **d** $y = -2x$

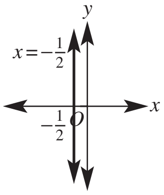
13 a



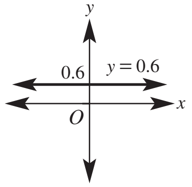
b



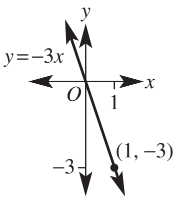
c



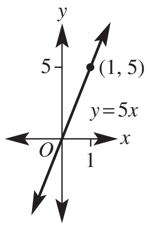
d



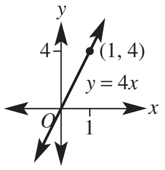
e



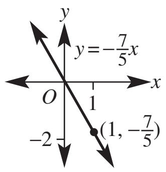
f



g



h



14 $x = -1$, $y = -5$, $y = 5x$

15 **a** (b, c) **b** $m = \frac{c}{b}$

16 $m = \frac{1}{2}$

4D

Building understanding

1 **a** 2 **b** $\frac{1}{2}$ **c** 3
d −1 **e** −3 **f** $-\frac{1}{4}$

2 **a** zero **b** negative
c positive **d** undefined

3 **a** 2 **b** 0.5 **c** 6 **d** −6

Now you try

Example 7

a negative, gradient = $-\frac{2}{3}$ b zero, gradient = 0

c undefined d positive, gradient = $\frac{3}{5}$

Example 8

a $m = 2$ b $m = -\frac{4}{7}$

Exercise 4D

1 a positive, 1 b zero
c negative, $-\frac{2}{3}$ d undefined

2 a positive, 2 b zero
c negative, $-\frac{3}{4}$ d undefined
e positive, $\frac{1}{2}$ f positive, 3
g positive, 2 h negative, −4

3 a 2 b 1 c $-\frac{1}{4}$ d $\frac{4}{3}$
e 2 f −2 g −1 h $\frac{1}{2}$
i $\frac{3}{2}$ j $-\frac{5}{2}$ k $\frac{1}{3}$ l $\frac{5}{2}$

4 A 1 B −2 C 1 D $\frac{1}{2}$ E −2 F 3

5 a $-\frac{1}{20}$ b $\frac{35}{2}$ c $-\frac{1}{5}$ d $\frac{15}{2}$

6 a 6 b −1 c 4 d $-\frac{3}{2}$

7 300 m

8 A 9 B −4 C −1 D 4 E −0.4 F 2.4

9 $\frac{7}{11} = \frac{35}{55}, \frac{3}{5} = \frac{33}{55}$ hence $\frac{7}{11} > \frac{3}{5}$ so $\frac{7}{11}$ is steeper

10 a run = $x_2 - x_1$ b rise = $y_2 - y_1$ c $m = \frac{y_2 - y_1}{x_2 - x_1}$
d i $m = \frac{3}{2}$ ii $m = \frac{5}{4}$ iii $m = -\frac{5}{3}$ iv $m = \frac{4}{3}$
e yes, the rise and the run work out regardless

11 a i −1 ii −3
b 4.5
c i 6 ii 2.4 iii $\frac{9}{7}$ iv 7.5

4E

Building understanding

1 a i 10 km ii 20 km iii 30 km
b 10 km/h c 10
d They are the same.

2 a 15 mm
b i 6 days ii 20 days
c

t	0	1	2	3	4
h	0	5	10	15	20

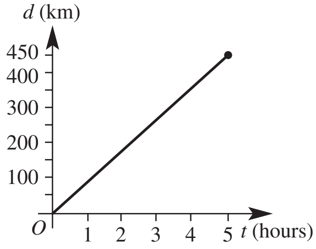
d B e 5

Now you try

Example 9

a 90 km/h

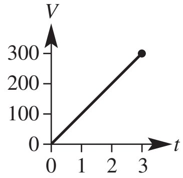
b



c i 90 ii $d = 90t$
d i 315 km ii 1.5 hours

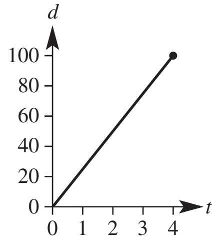
Exercise 4E

1 a 100 L/hour
b



c i 100 ii $V = 100t$
d i $V = 150$ L ii $t = 2$ hours

2 a 25 km/h
b



c i 25 ii $d = 25t$
d i 62.5 km ii 1.6 hours or 1 h 36 min

3 a $d = 50t$ b $g = 2t$ c $C = 1.25n$ d $P = 20t$

4 a 100 km/h b 7 cm/s
c 2.5 cm/min d 49 mm/min

5 a 10.2 L b 72.25 L c 800 km

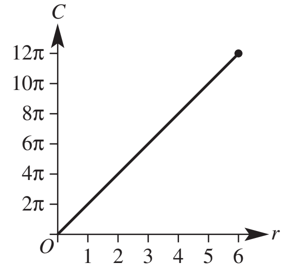
6 Sally

7 Leopard

8 25 months

9 a i 0 ii 4π iii 12π

b



c gradient is 2π, the coefficient of r

10 No, $A = \pi r^2$ so area (A) is proportional to square of radius (r^2).

11 a $A = 2h$

b 2 cm^2 increase for each 1 cm increase in height

12 yes, for any fixed time, e.g. $t = 5$, $s = \frac{d}{5}$

13 1.2 minutes or 72 seconds

14 $\frac{5}{12}$ km

4F

Building understanding

1 a $y = 2x + 5$ b $y = -2x + 3$

c $y = -x - 2$ d $y = -\frac{1}{2}x - 10$

2 a 1 b -5 c 9

3 a $y = x + 7$ b $y = -x + 3$ c $y = 2x + 5$

Now you try

Example 10

a gradient = 5, y-intercept = -2

b gradient = 6, y-intercept = 0

Example 11

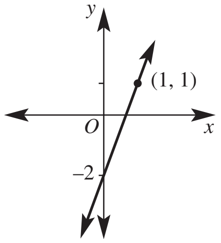
a $y = -5x + 10$

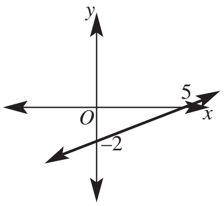
b $2x + y = 3$

Example 12

a gradient = 3, y-intercept = -2

b gradient = $\frac{2}{5}$, y-intercept = -2





Exercise 4F

1 a gradient = 7, y-intercept = 3

b gradient = -2, y-intercept = 0

2 a gradient = 3, y-intercept = -4

b gradient = -5, y-intercept = -2

c gradient = -2, y-intercept = 3

d gradient = $\frac{1}{3}$, y-intercept = 4

e gradient = -4, y-intercept = 0

f gradient = 2, y-intercept = 0

g gradient = 1, y-intercept = 0

h gradient = -0.7, y-intercept = 0

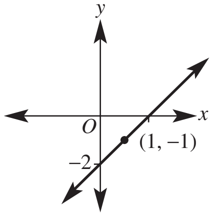
3 a $y = -2x + 3$ b $y = 3x - 1$

c $y = -3x + 2$ d $y = x + 2$

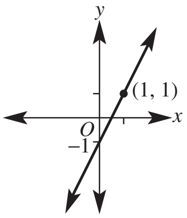
e $2x - y = 1$ f $3x + y = 4$

g $x - 3y = 1$ h $2x - 7y = -2$

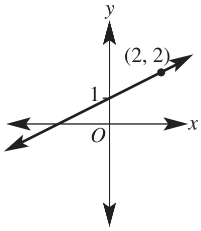
4 a 1, -2



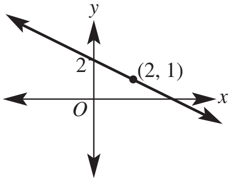
b 2, -1



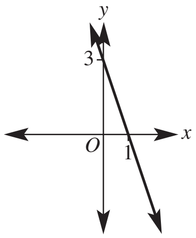
c $\frac{1}{2}$, 1



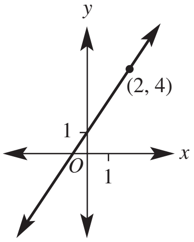
d $-\frac{1}{2}$, 2



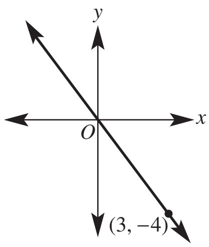
e -3, 3



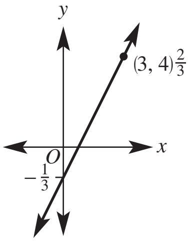
f $\frac{3}{2}$, 1



g $-\frac{4}{3}$, 0



h $\frac{5}{3}$, $-\frac{1}{3}$



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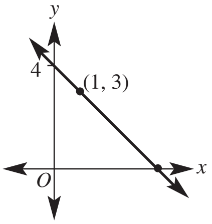
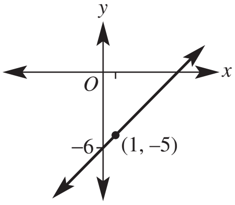
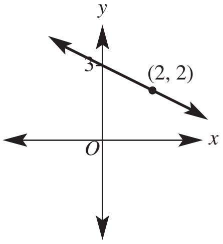
5 **a** –1, 4



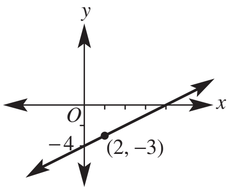
b 1, –6



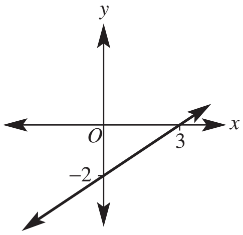
c $-\frac{1}{2}$, 3



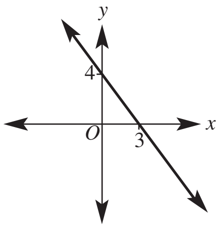
d $\frac{1}{2}$, –4



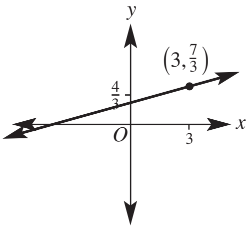
e $\frac{2}{3}$, –2



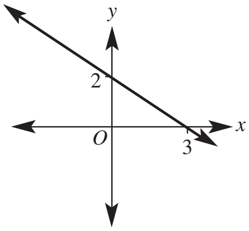
f $-\frac{4}{3}$, 4



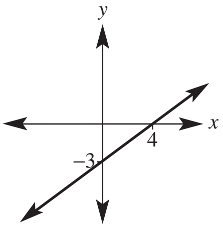
g $\frac{1}{3}$, $\frac{4}{3}$



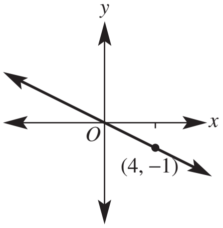
h $-\frac{2}{3}$, 2



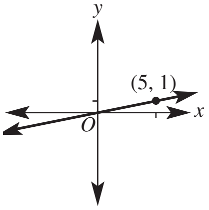
i $\frac{3}{4}$, –3



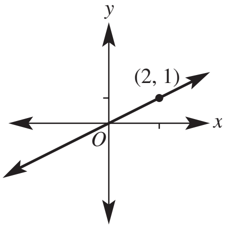
j $-\frac{1}{4}$, 0



k $\frac{1}{5}$, 0

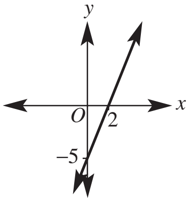


l $\frac{1}{2}$, 0

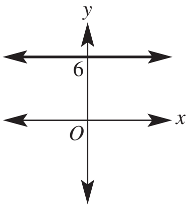


6 **a** D **b** F **c** C **d** B **e** E **f** A

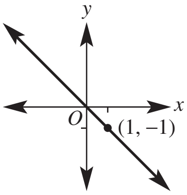
7 **a**



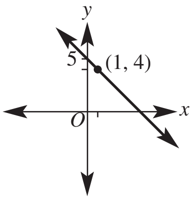
b



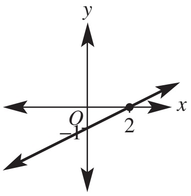
c



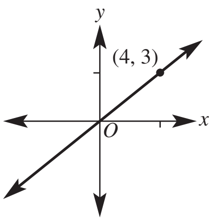
d



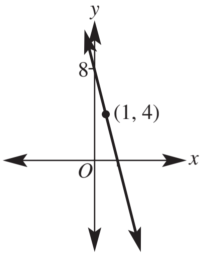
e



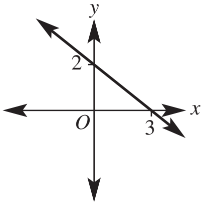
f



g



h



8 C, D, F, H

9 **a** $y = 2x + 2$, y-intercept is 2
b expand brackets and simplify

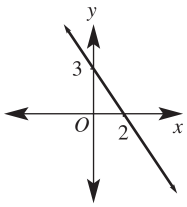
10 $y = k$

11 $y = -\frac{a}{b}x + \frac{d}{b}, m = -\frac{a}{b}, c = \frac{d}{b}$

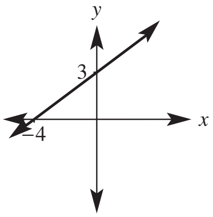
12 a $y = 2x + 3$ **b** $y = -x + 2$
c $y = \frac{5}{3}x + \frac{2}{3}$ **d** $y = -\frac{2}{5}x - \frac{1}{5}$

Progress quiz

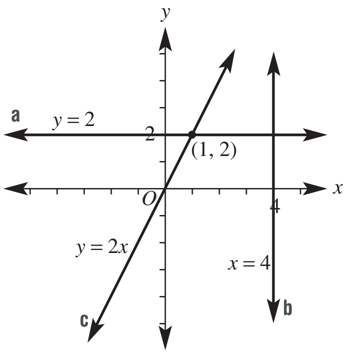
1 a 2, −4 **b** 3, 6
2 a not on line **b** on line
3 a



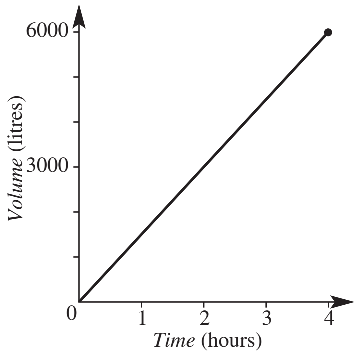
b



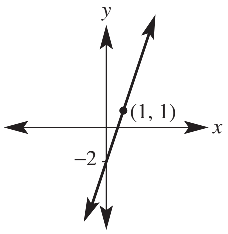
4

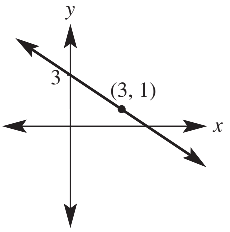


5 a positive, 2 **b** undefined
c zero **d** negative, $-\frac{2}{3}$
6 a $m = 2$ **b** $m = -3$
7 a 1500 L/h
b



c i 1500 **ii** $V = 1500t$
d i 4500 L **ii** 3.5 hours
8 a $y = -5x + 8$ **b** $-3x + y = -5$
9 a 3, −2 **b** $-\frac{2}{3}$, 3





4G

Building understanding

1 a $y = 2x + 5$ **b** $y = 4x - 1$ **c** $y = -2x + 5$
2 a 1 **b** 9 **c** 5

Now you try

Example 13
$y = -2x + 4$
Example 14
$y = \frac{2}{3}x + 6$

Exercise 4G

1 $y = -2x - 6$
2 a $y = x + 3$ **b** $y = 2x - 2$
c $y = 8x + 8$ **d** $y = -x + 4$
3 a $y = \frac{3}{4}x + 3$ **b** $y = -\frac{3}{4}x + 3$ **c** $y = -\frac{5}{4}x + 3$
d $y = \frac{3}{2}x + 4$ **e** $y = \frac{3}{5}x$ **f** $y = -\frac{1}{3}x - 1$
4 a $y = 3x + 5$ **b** $y = -2x - 1$ **c** $y = -3x + 8$
d $y = x - 3$ **e** $y = -3x + 3$ **f** $y = 5x - 1$
g $y = -x + 8$ **h** $y = -3x + 6$ **i** $y = -2x + 2$
j $y = -4x - 9$
5 a i 2 **ii** $y = 2x + 2$
b i −1 **ii** $y = -x + 3$
c i −4 **ii** $y = -4x + 11$
d i 1 **ii** $y = x - 4$
6 $x = 2\frac{1}{2}$
7 $\frac{3}{5}, -\frac{3}{4}$
8 $V = -20t + 120$, $V = 120$ L initially
9 a $y = 5x$ **b** $y = 6.5x + 2$
10 a 1 **b** 8 **c** −6 **d** $\frac{5}{2}$
11 a $c = 3$ **b** $c = 3$
c no, y-intercept is fixed for any given line
12 $y = \frac{b}{a}x$
13 a $y = 2x$ **b** $y = -2x + 4$ **c** $y = 2x + 5$
d $y = -\frac{3}{2}x + 2$ **e** $y = \frac{5}{7}x + \frac{1}{7}$ **f** $y = -\frac{13}{3}x - \frac{11}{3}$

4H

Building understanding

1 a 4 **b** 8 **c** 1 **d** −3
2 a 5.5 **b** 2.5 **c** −1.5 **d** −2.5
3 a 2.24 **b** 8.60 **c** 10.20

Now you try

Example 15

a (2, 3.5) **b** (–1, –1.5)

Example 16

8.06

Exercise 4H

1 **a** (1, 5) **b** (2.5, 2)
2 **a** (3, 3) **b** (2, 2) **c** (4, 1) **d** (–1, 3) **e** (–1, –1) **f** (1.5, 1.5) **g** (0.5, 3) **h** (–1.5, –2.5) **i** (–3, –8.5) **j** (0.5, –2.5)
3 **a** 5.10 **b** 2.83 **c** 5.39 **d** 4.47 **e** 3.61 **f** 2.83 **g** 8.94 **h** 7.21 **i** 6.71
4 $B(8, 0)$; $A(-6, 5)$, $A(-6, 9)$
5 **a** (–3, 1) **b** (1, –4) **c** (8, 2.5)
6 **a** 12.8 **b** 24.2
7 (0, 0), (0, 4), (2, 0), (2, 4)
8 **a** $x = \frac{x_1 + x_2}{2}$ **b** $y = \frac{y_1 + y_2}{2}$
c $M(1, -0.5)$
9 **a** **i** $x_2 - x_1$ **ii** $y_2 - y_1$
iii $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
b Answers may vary.
10 **a** $\frac{1}{3}$ **b** $\frac{1}{3}$
c **i** (1, 1) **ii** (–2, –0.5) **iii** (2, 1.5)
d **i** (–1, 1) **ii** (–2, 5) **iii** (0.4, –1.8) **iv** (–2.4, 2.6)

4I

Building understanding

1 **a** yes **b** no **c** yes **d** no
2 **a** $-\frac{1}{5}$ **b** $\frac{1}{3}$
3 **a** yes **b** no **c** no **d** yes

Now you try

Example 17

$y = 4x - 2$

Example 18

$y = \frac{1}{3}x + 4$

Exercise 4I

1 **a** $y = 2x + 1$ **b** $y = -x + 5$
2 **a** $y = 4x + 8$ **b** $y = -2x - 7$ **c** $y = \frac{2}{3}x - 5$ **d** $y = -\frac{4}{5}x + \frac{1}{2}$
3 **a** $y = -\frac{1}{3}x + 3$ **b** $y = -\frac{1}{5}x + 7$ **c** $y = \frac{1}{2}x - 4$ **d** $y = x + 4$ **e** $y = \frac{1}{7}x - \frac{1}{2}$ **f** $y = -x + \frac{5}{4}$
4 **a** **i** $y = 1$ **ii** $y = -2$
b **i** $x = 3$ **ii** $x = -3$
c **i** $x = 2$ **ii** $x = 0$
d **i** $y = 7$ **ii** $y = -\frac{1}{2}$
5 **a** $y = -3x + 9$ **b** $y = \frac{1}{2}x + \frac{5}{2}$ **c** $y = -\frac{1}{5}x + 6\frac{1}{5}$ **d** $y = x + 5$
6 $y = 0$, $y = x + 3$, $y = -x + 3$
7 $y = 2x - 10$ or $y = 2x + 10$
8 **a** **i** $-\frac{3}{2}$ **ii** –5 **iii** 7 **iv** $\frac{11}{3}$
b $-\frac{b}{a}$
9 **a** **i** $-\frac{1}{2}$ **ii** 3
b **i** 1 **ii** $-\frac{1}{7}$
10 **a** $y = -\frac{1}{2}x + 4$ **b** $y = \frac{2}{3}x + 1\frac{2}{3}$ **c** $y = 2x - 3$ **d** $y = -2x - 5\frac{1}{2}$ **e** $y = \frac{3}{7}x - \frac{5}{7}$ **f** $y = -\frac{5}{6}x + \frac{1}{6}$

4J

Building understanding

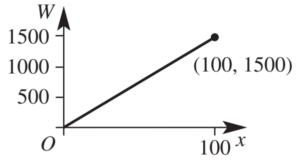
1 B
2 **a** **i** \$1200 **ii** \$1500 **iii** \$2200
b $A = 1000 + 100n$
3 **a** 46 **b** 3.5 **c** 2 **d** 3

Now you try

Example 19

a $W = 500 + 10x$

b

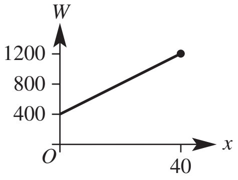


c \$750 **d** 35

Exercise 4J

1 **a** $W = 20x + 400$

b



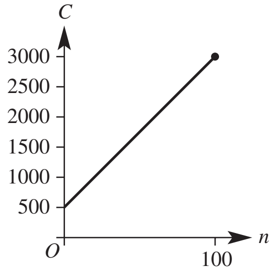
(Note: This graph should really be a series of points but a full line segment is used as a guide.)

c \$640 **d** 30

ISBN 978-1-108-77288-4

2 **a** $C = 25n + 500$

b



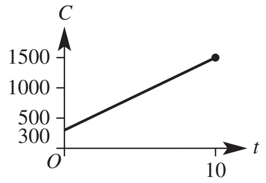
(Note: This graph should really be a series of points but a full line segment is used as a guide.)

c \$1500 **d** 70

3 **a** $C = 50n + 40$ **b** \$240 **c** \$640

4 **a** $C = 120t + 300$

b



c \$1020 **d** 3 hours

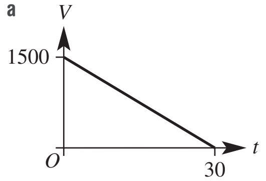
5 **a** $F = 18t + 12$ **b** 3 minutes **c** 2.5 minutes

6 **a** $V = 4000 - 20t$ **b** 2200 L

c 200 minutes or 3 hours 20 minutes

d 2 hours 55 minutes or 175 minutes

7 **a**



b $V = 1500 - 50t$

c the number of litres drained per minute

d 1250 L **e** 15 minutes

8 **a** 80 km/h

b rate changes, i.e. new gradient = 70, $d = 50 + 70t$

9 **a** 20 m/s **b** −20 m/s **c** 350 m **d** 17.5 s

e increasing altitude at rate of 20 m/s

10 **a** **i** $C = 0.4n + 20$ **ii** 160

b **i** $P = 0.8n - 20$ **ii** 25 **iii** 120

11 **a** $C = 10x + 6700$ **b** \$8700

c 630 **d** \$11700

e 670 **f** $P = 10x - 6700$

g $T = \dfrac{10x - 6700}{x}$ **h** 1340

4K

Building understanding

1 **a** (1, 3) **b** (−1, 2) **c** (2, 2)

2 **a** True **b** False **c** False **d** True

3 **a** yes **b** yes **c** yes

Now you try

Example 20

a yes **b** no

Example 21

(2, −1)

Exercise 4K

1 **a** yes **b** no

2 **a** yes **b** no **c** no

d yes **e** no **f** no

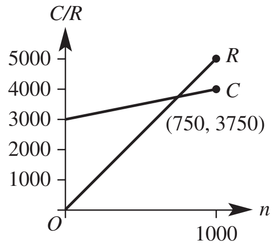
3 **a** (2, 2) **b** (3, 2) **c** (2, −4)

d (3, 2) **e** (2, 1) **f** (3, 7)

g (1, 2) **h** (2, 4) **i** (4, 1)

j (−1, 3) **k** (−1, −5) **l** (1, 3)

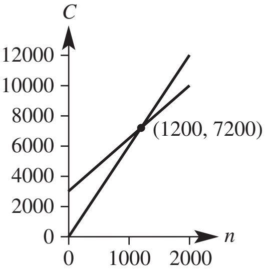
4 **a**



(Note: This graph should really be a series of points but a full line segment is used as a guide.)

b 750

5 1200 DVDs



(Note: This graph should really be a series of points but a full line segment is used as a guide.)

6 10 seconds

7 parallel lines, i.e. same gradient

8 **a** (0, 0) **b** (0, 0)

c no intersection **d** no intersection

e (0, 1) **f** (0, 3)

9 **a** 2 **b** −3

10 **a** (1, 3) **b** $A = 6.75$ units^2

11 **a** 18.75 units^2 **b** 24 units^2 **c** 16 units^2

d 27 units^2 **e** 7 units^2

Problems and challenges

1 20 days

2 2.5 hours

3 (0.5, 5.5), diagonals intersect at their midpoint

4 $k = -2.5$ or 13.5

5 length $AB =$ length AC

6 $A = 13.5$ square units, $P = 20$ units

7 (6, 7)

8 **a** BD: $y = -2x + 1$, AC: $y = \frac{1}{2}x - \frac{3}{2}$

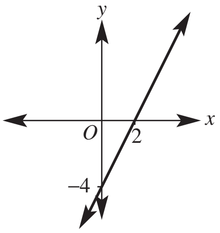
b $b = -4$, $x = 11$ **c** 80 units^2

9 $k = 11.5$ or −10.5

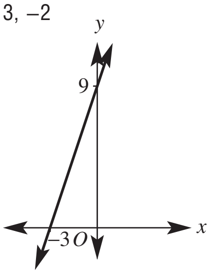
Short-answer questions

1 **a** −2, 4 **b** 3, −2

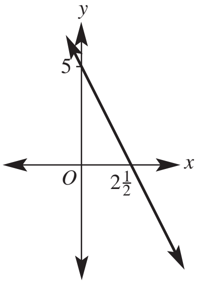
2 **a**



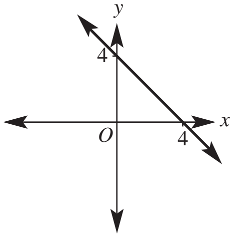
b



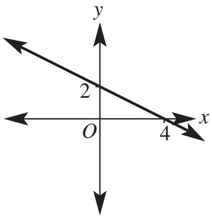
c



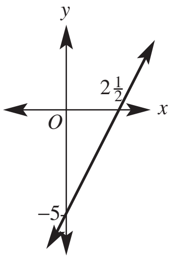
d



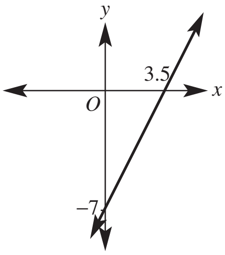
e



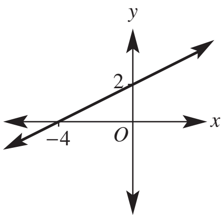
f



g



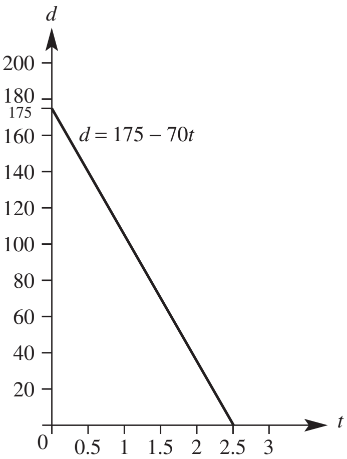
h



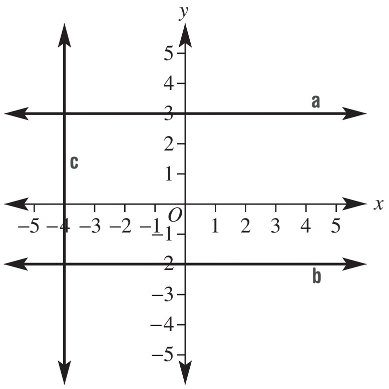
3 **a** 175 km

b 2.5 hours

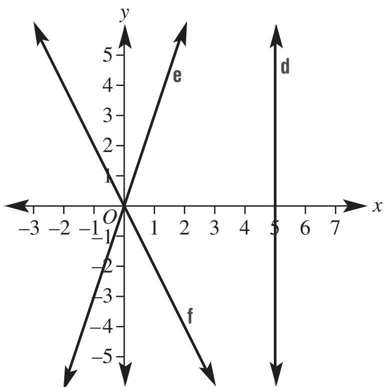
c



4 **a** $y = 3$ **b** $y = -2$ **c** $x = -4$



d $x = 5$ **e** $y = 3x$ **f** $y = -2x$

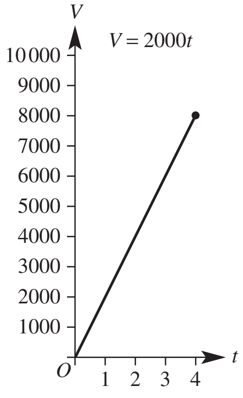


5 **a** 2 **b** −1 **c** $-\frac{5}{2}$

d $\frac{4}{3}$ **e** 2 **f** $-\frac{10}{3}$

6 **a** 2000 L/h

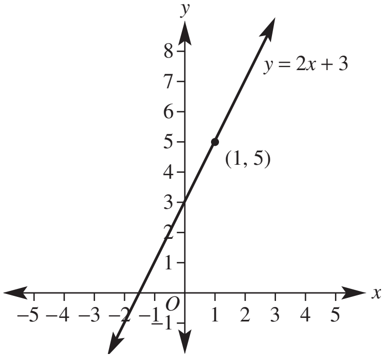
b



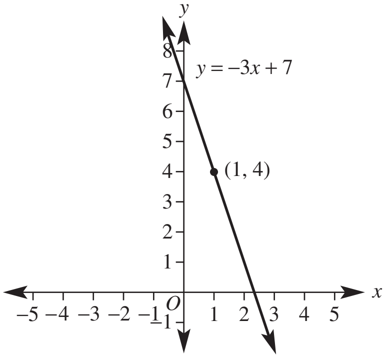
c $V = 2000t$

d 2.5 hours

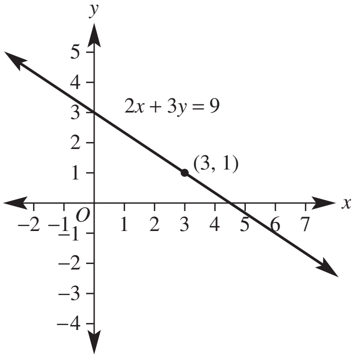
7 **a** gradient = 2 y-intercept = 3



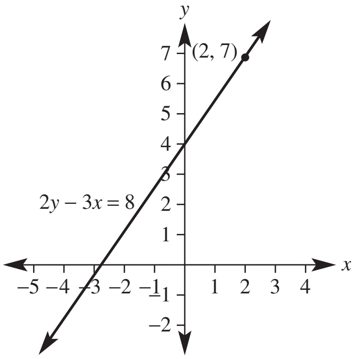
b gradient = −3 y-intercept = 7



c gradient = $-\frac{2}{3}$ y-intercept = 3



d gradient = $\frac{3}{2}$ y-intercept = 4



8 **a** $y = 3x + 2$ **b** $y = -2x + 6$ **c** $y = \frac{4}{3}x - 5$

9 **a** $y = \frac{x}{2} - 1$ **b** $y = -3x + 6$

c $y = 3x - 2$ **d** $y = -2x + 2$

10 **a** **i** $M(4, 6)$ **ii** 5.66

b **i** $M(7.5, 4.5)$ **ii** 7.07

c **i** $M(0, 4)$ **ii** 7.21

d **i** $M(-3, 2.5)$ **ii** 9.85

11 **a** $n = 9$ **b** $n = 10$ **c** $n = 6$

12 **a** $y = 2x + 4$ **b** $y = -x - 3$

c $y = -\frac{1}{2}x - 1$ **d** $y = 3x + 4$

e $y = 3x + 1$ **f** $3x + 2y = 8$ or $y = -\frac{3x}{2} + 4$

13 **a** no **b** yes

14 **a** (2, 0) **b** (−2, 4)

Multiple-choice questions

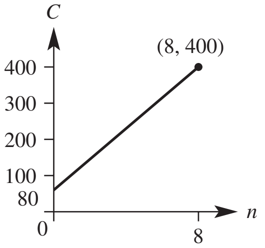
1 C **2** D **3** A **4** C **5** E

6 B **7** B **8** A **9** D **10** D

Extended-response questions

1 **a** **i** \$40/h, \$80

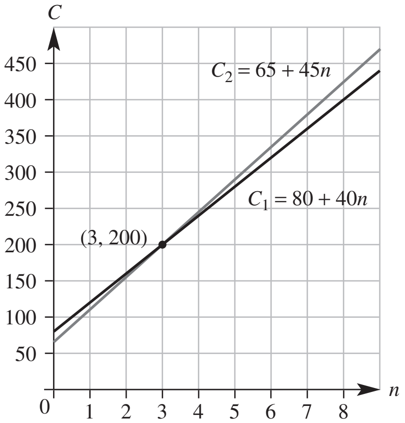
ii



iii \$180 **iv** 5 hours

b **i** $C = 65 + 45n$

ii



c (3, 200) **d** after 3 hours

2 **a** $C = 8v + 90$

b **i** \$90 **ii** \$8 per vase

c 23 vases

Chapter 5

5A

Building understanding

1 a 50 mm b 280 cm c 52.1 cm
d 0.837 m e 4600 m f 2.17 km
2 825 cm, 2.25 cm
3 a $a = 3, b = 6$ b $a = 12, b = 4$
c $a = 6.2, b = 2$ d $a = 9, b = 7$

Now you try

Example 1
a 22 m b 20 cm

Example 2
7 mm

Exercise 5A

1 a 12 m b 24 mm c 32 cm
2 a 27 cm b 18 km c 10 m
3 a 90 cm b 80 cm c 170 cm
d 30.57 m e 25.5 cm f 15.4 km
4 a 9 cm b 4015 m c 102.1 cm
5 a 8000 mm b 110 m c 1 cm
d 20 mm e 0.284 km f 62.743 km
6 a $x = 4$ b $x = 2.2$ c $x = 14$
d $x = 9.5$ e $x = 6$ f $x = 4.2$
7 108 m
8 a 86 cm b 13.6 m
9 a $x = 2$ b $x = 2.1$ c $x = 7$
10 88 cm
11 a $P = 2a + 2b$ b $P = 4x$ c $P = 2a + b$
d $P = 2x + 2y$ e $P = 4(a + b)$ f $P = 2x$
12 All vertical sides add to 13 cm and all horizontal sides add to 10 cm.
13 a 25 cm, 75 cm b 40 cm, 60 cm
c 62.5 cm, 37.5 cm d 10 cm, 20 cm, 30 cm, 40 cm
14 a i 96 cm ii 104 cm iii 120 cm
b $P = 4(20 + 2x), \therefore P = 8x + 80$
c i 109.6 cm ii 136.4 cm
d i $x = 1.25$ ii $x = 2.75$
e no, as with no frame the picture has a perimeter of 80 cm

5B

Building understanding

1 a 2.8 cm b 96 mm
2 a 6π b 12π c $3 + 2\pi$
d $12 + 3\pi$ e $8 + 2\pi$ f $3 + \frac{3\pi}{4}$
3 a $\frac{1}{4}$ b $\frac{1}{2}$ c $\frac{3}{4}$
d $\frac{1}{6}$ e $\frac{5}{12}$ f $\frac{5}{8}$

Now you try

Example 3
a 25.13 m b 6.79 mm

Example 4
a 12π b $16 + 8\pi$

Exercise 5B

1 a 50.27 m b 16.38 m
2 a 87.96 cm b 9.42 mm c 12.57 km
3 a 9.14 cm b 14.94 m c 33.13 cm
d 10.00 cm e 20.05 m f 106.73 km
4 a 12.56 m b 62.8 cm c 22 mm d 44 m
5 a 14π b 4π c 41π
d 10π e 20π f 11π
6 a $8 + 2\pi$ b $4 + 2\pi$ c $10\pi + 20$
d $12 + 2\pi$ e $5\pi + 6$ f $5\pi + 8$
7 28.27 m
8 4.1 m
9 31.42 cm
10 a 188.50 cm
b i 376.99 cm ii 1979.20 cm
c 531
11 a 11.5π m b $2.4 + 0.6\pi$ cm c $21 + 3.5\pi$ m
d $5 + 1.875\pi$ km e $40 + 20\pi$ mm f $23 + 5.75\pi$ m
12 a $r = \frac{C}{2\pi}$
b i 1.6 cm ii 4.0 m
c $d = \frac{C}{\pi}$ d 67 cm
13 a 131.95 m b 791.68 m
c i 3.79 ii 15.16
d 63.66 m

5C

Building understanding

1 a 6 b 16 c 12
d 1 e 12 f 153
2 a rectangle b circle c rhombus/kite
d sector of circle e triangle f trapezium
g parallelogram h square i semicircle
3 a $\frac{1}{4}$ b $\frac{1}{3}$ c $\frac{3}{4}$
d $\frac{1}{6}$ e $\frac{5}{8}$ f $\frac{5}{18}$
4 a 300 cm and 100 cm
b 30 000 cm^2

Now you try

Example 5
a 1.421 m^2 b 316 mm^2

Example 6
a 40 m^2 b 36 mm^2 c 7.2 m^2

Example 7
a 80 km^2 b 34 m^2

Example 8
a 36.32 m^2 b 21.99 cm^2

Exercise 5C

1 a i 2.36 cm^2 ii 4.8 m^2
b i 41 600 cm^2 ii 350 mm^2
2 a 200 mm^2 b 5 cm^2 c 21 000 cm^2
d 21 m^2 e 1000 m^2 f 3.2 km^2
3 a 24 m^2 b 10.5 cm^2 c 20 km^2
d 25.2 m^2 e 15 m^2 f 36.8 m^2
4 a 21 mm^2 b 12 cm^2 c 17 cm^2
d 63 m^2 e 6.205 m^2 f 15.19 km^2
5 a 12.25 cm^2 b 3.04 m^2 c 0.09 cm^2
d 6.5 mm^2 e 18 cm^2 f 2.4613 cm^2
6 a 21.23 m^2 b 216.51 km^2 c 196.07 cm^2
7 a 7.07 m^2 b 157.08 cm^2 c 19.24 cm^2
d 84.82 m^2 e 26.53 m^2 f 62.86 m^2
8 a 1.5×10^{10} (15 000 000 000) cm^2
b 5 mm^2 c 0.075 m^2
9 500 000 m^2
10 0.175 km^2
11 0.51 m^2
12 12.89%
13 31%
14 a $r = \sqrt{\dfrac{A}{\pi}}$
b i 1.3 cm ii 1.5 m iii 2.5 km
15 a i 64° ii 318°
b as angle would be greater than 360°, which is not possible (28.3 m^2 is the largest area possible, i.e. full circle)
16 a i 1.5 m ii 1.5 m
b 78 m^2 c yes
17 46.7%

5D

Building understanding

1 a semicircle and rectangle b triangle and semicircle
c rhombus and parallelogram
2 a $P = 2 \times 5 + 3 + \frac{1}{2} \times 2\pi r$ $A = bh + \frac{1}{2}\pi r^2$

$$\begin{aligned} &= 10 + 3 + 1.5\pi \\ &= 13 + 1.5\pi \\ &= 17.7 \text{ m} \end{aligned} \qquad \begin{aligned} &= 5 \times 2 + \frac{1}{2} \times \pi \times 1.5^2 \\ &= 10 + 1.125\pi \\ &= 13.5 \text{ m}^2 \end{aligned}$$

b $P = 20 + 12 + 12 + 10 + 6$ $A = lw - \frac{1}{2}bh$

$$\begin{aligned} &= 60 \text{ cm} \end{aligned} \qquad \begin{aligned} &= 12 \times 20 - \frac{1}{2} \times 8 \times 6 \\ &= 240 - 24 \\ &= 216 \text{ cm}^2 \end{aligned}$$

Now you try

Example 9
$P = 16.71$ m, $A = 16.07$ m^2

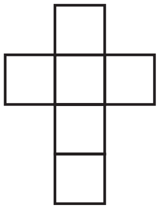
Exercise 5D

1 a 46 m, 97 m^2 b 18.28 m, 22.28 m^2
2 a 34 m, 76 m^2 b 40 m, 90 m^2
c 19.42 m, 26.14 m^2 d 85.42 mm, 326.37 mm^2
3 a 17 cm^2 b 3.5 cm^2 c 21.74 cm^2
d 6.75 m^2 e 189 cm^2 f 115 cm^2
4 a 108 m^2 b 33 cm^2 c 98 m^2
d 300 m^2 e 16 cm^2 f 22.5 m^2
5 a 90 cm^2 b 15 m^2 c 9 m^2
d 7.51 cm^2 e 7.95 m^2 f 180.03 cm^2
g 8.74 mm^2 h 21.99 cm^2 i 23.83 mm^2
6 189.27 m^2
7 68.67 cm^2
8 a 37.70 m, 92.55 m^2 b 20.57 mm, 16 mm^2
c 18.00 cm, 11.61 cm^2 d 12.57 m, 6.28 m^2
e 25.71 cm, 23.14 cm^2 f 33.56 m, 83.90 m^2
9 a 136.3 cm^2 b 42.4 m^2 c 345.6 m^2
10 8 cm
11 a $36 + 18\pi$ b 16 c $12 - \frac{\pi}{8}$
d 2π e $12.96 + 3.24\pi$ f $25 + \frac{75\pi}{4}$
12 7.1 cm
13 a hypotenuse (diameter) would equal 4.24 not 5
b hypotenuse (sloped edge) should be 13 cm not 14 cm
c hypotenuse (diameter) should be 5.83 not 8 m
14 5267.1 cm^2
15 a 34 cm, 18 cm b 226.9 cm^2 c 385.1 cm^2

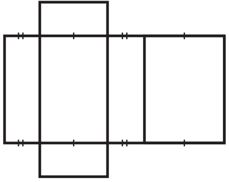
5E

Building understanding

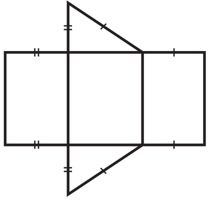
1 a Cube



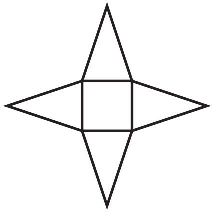
b Rectangular prism



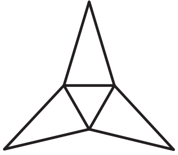
c Triangular prism



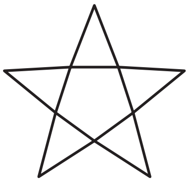
d Square pyramid



e Tetrahedron
(triangular pyramid)



f Pentagonal pyramid



2 **a** $\text{TSA} = 2 \times 8 \times 7 + 2 \times 8 \times 3 + 2 \times 7 \times 3$
$= 112 + 48 + 42$
$= 202\ \text{m}^2$

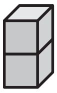
b $\text{TSA} = 2 \times \frac{1}{2} \times 4 \times 3 + 5 \times 7 + 4 \times 7 + 3 \times 7$
$= 12 + 35 + 28 + 21$
$= 96\ \text{cm}^2$

Now you try

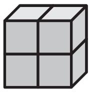
Example 10
a 202 m^2 **b** 85 m^2

Exercise 5E

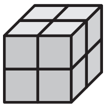
1 **a** 52 m^2 **b** 39 mm^2
2 **a** 242 cm^2 **b** 192 cm^2 **c** 85.76 m^2
3 **a** 224 cm^2 **b** 9.01 m^2
4 **a** 108 cm^2 **b** 199.8 cm^2 **c** 0.96 m^2
d 44.2 m^2 **e** 22 cm^2 **f** 28 cm^2
5 6 m^2
6 14.54 m^2
7 34 000 cm^2
8 **a** 44.4 m^2 **b** 4.44 L
9 **a** 1400 cm^2 **b** 1152 cm^2
10 **a** 10 cm^2



b 16 cm^2



c 24 cm^2



11 **a** [6, 10, 14, 18, 22, 26, 30, 34, 38]
b $S = 4n + 2$
c Answers may vary.
12 **a** 17.7 cm^2 **b** 96 m^2 **c** 204 cm^2
d 97.9 m^2 **e** 137.8 cm^2 **f** 43.3 mm^2

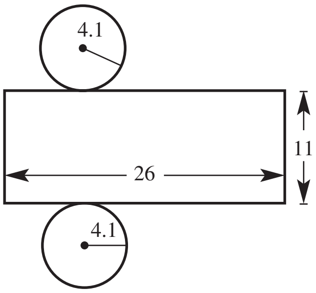
Progress quiz

1 **a** 3.54 m **b** 15.2 cm
2 **a** $C = 25.1$ mm, $A = 50.3$ mm^2
b $C = 37.7$ cm, $A = 113.1$ cm^2
3 **a** 15 cm^2 **b** 72 cm^2 **c** 16π cm^2
4 850 cm^2
5 **a** $P = 29.71$ m, $A = 63.27$ m^2
b $P = 32$ cm, $A = 44$ cm^2
6 103.67 m^2
7 **a** 216 cm^2 **b** 64 cm^2 **c** 198.5 cm^2

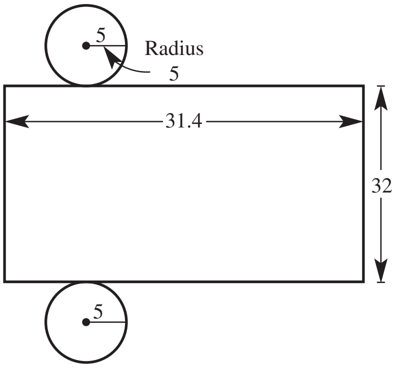
5F

Building understanding

1 **a**



b



2 **a** 22 cm by 10 cm **b** 12.57 cm by 8 cm
c 50.27 m by 9 m

Now you try

Example 11
716.28 cm^2

Example 12
182.52 m^2

Exercise 5F

1 25.13 m^2
2 **a** 471.24 cm^2 **b** 50.27 m^2
3 **a** 44.0 cm^2 **b** 603.2 cm^2 **c** 113.1 m^2
4 395.84 cm^2
5 **a** 251.33 cm^2 **b** 207.35 mm^2 **c** 24.13 m^2
6 **a** 54.56 m^2 **b** 218.23 m^2 **c** 63.98 cm^2
d 71.91 cm^2 **e** 270.80 m^2 **f** 313.65 km^2
g 326.41 m^2 **h** 593.92 m^2 **i** 43.71 mm^2
7 7539.82 cm^2
8 80 424.8 cm^2
9 **a** 18 849.556 cm^2
b **i** 1.88 m^2 **ii** 37.70 m^2
c 239

10 **a** 150π cm^2 **b** 16π m^2

11 Surface area of half cylinder is more than half surface area of a cylinder as it includes new rectangular surface.

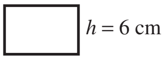
12 **a** $\left(\frac{135\pi}{2}+36\right)$ cm^2 **b** $\left(\frac{70\pi}{3}+12\right)$ cm^2

c $\left(\frac{29\pi}{12}+4\right)$ m^2

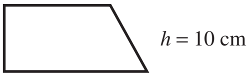
5G

Building understanding

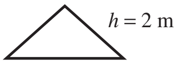
1 **a**



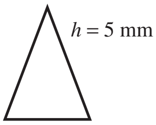
b



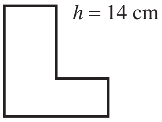
c



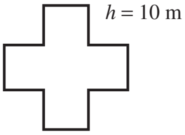
d



e



f



2 **a** 8 cm^3 **b** 84 m^3 **c** 21 mm^3

Now you try

Example 13

a 4800 mm^3 **b** 0.612 m^3

Example 14

a 12 cm^3 **b** 8 cm^3 **c** 15 m^3

Exercise 5G

1 **a** 3 500 000 cm^3 **b** 0.216 cm^3

2 **a** 3000 mm^3 **b** 2 cm^3 **c** 8 700 000 cm^3
d 0.0059 m^3 **e** 10 000 m^3 **f** 0.0000217 km^3
g 3000 mL **h** 200 L **i** 3.5 L
j 21 mL **k** 37 kL **l** 42.9 ML

3 **a** 4 cm^3 **b** 18 m^3 **c** 30 mm^3

4 **a** 4 cm^3 **b** 10.5 m^3 **c** 11.96 cm^3
d 29 cm^3 **e** 14.88 m^3 **f** 8.1351 cm^3
g 108 m^3 **h** 29.82 m^3 **i** 0.382044 cm^3

5 **a** 75 m^3 **b** 30 cm^3 **c** 1.25 cm^3

6 **a** 16 cm^3 **b** 42.875 m^3 **c** 15 cm^3

7 **a** 8 L **b** 125 L **c** 0.36 L **d** 0.48 L

8 8000 cm^3

9 0.19 m^3

10 Yes, the tank only holds 20 L

11 **a** 67.2 cm^3 **b** 28 m^3 **c** 8.9 km^3
d 28 m^3 **e** 0.4 m^3 **f** 29232 mm^3

12 **a** 20 L **b** 75 000 L **c** 8000 L

13 **a** 55 m^2 **b** 825 000 L

14 **a** **i** 1000 **ii** $\frac{1}{1000}$ **iii** 1000
b **i** 1000 000 **ii** 1000 **iii** 1000 000 or 1000^2

15 **a** $V = x^2h$ **b** $V = s^3$ **c** $V = 6t^3$

16 **a** Answers may vary. **b** $\frac{1}{3}$
c Answers may vary.

5H

Building understanding

1 **a** $r = 4$, $h = 10$ **b** $r = 2.6$, $h = 11.1$
c $r = 9$, $h = 23$

2 **a** 2 L **b** 4.3 mL **c** 3700 cm^3
d 1000 L **e** 38 m^3 **f** 200 mL

Now you try

Example 15

a 169.65 m^3 **b** 13.57 cm^3

Example 16

503 L

Exercise 5H

1 **a** 226.19 cm^3 **b** 8.48 m^3

2 **a** 137.44 m^3 **b** 18.85 m^3
c 100.53 cm^3 **d** 68.05 m^3

3 **a** 13 L **b** 503 L **c** 20 L
d 4712 L **e** 589 049 L **f** 754 L

4 **a** 25.133 m^3 **b** 25 133 L

5 37 699 L

6 cylinder by 0.57 m^3

7 **a** 502.65 cm^3 **b** 1.02 m^3 **c** 294.52 m^3
d 35 342.92 m^3 **e** 47.12 cm^3 **f** 1017.88 cm^3

8 **a** 0.707 **b** 2.523

9 **a** 160π m^3 **b** 320π cm^3 **c** 54π km^3
d $\frac{3\pi}{4}$ cm^3 **e** 1500π cm^3 **f** 144π mm^3

10 Answers may vary, but require $h = 2\pi r$.

11 **a** 113.10 cm^3 **b** 10471.98 m^3 **c** 3.73 m^3
d 20.60 cm^3 **e** 858.41 cm^3 **f** 341.29 m^3

Problems and challenges

1 100 L

2 non-shaded is half the shaded area

3 163.4 m^2

4 $\sqrt{200}$ cm = 14.14 cm

5 $\frac{1}{6}$ cm

6 $V = 2\pi^2 r^3$

7 $h = \frac{1 - r^2}{r}$

8 Answers may vary, but $1000 = \pi r^2 h$ needs to hold true for r and h in centimetres. Designers need to consider production costs, material costs and keeping the surface area to a miniumum so that they can maximise profits, as well as the ability to use, stack and market their products. If the container is for cold storage, then fitting into a standard fridge door is also a consideration.

Short-answer questions

1 **a** 380 cm **b** 1270 m **c** 2.73 cm^2
d 52 000 cm^2 **e** 10 000 cm^3 **f** 53.1 cm^3
g 3.1 L **h** 43 mL **i** 2830 L
2 **a** 14 m **b** 51 mm **c** 16.2 cm
3 **a** 4 cm^2 **b** 1122 mm^2 **c** 30.34 mm^2
d 7.5 m^2 **e** 15 cm^2 **f** 3 cm^2
4 **a** 2.5 m^2 **b** 37.4 m^2
5 **a** $A = 28.27$ cm^2, $P = 18.85$ cm
b $A = 5.38$ m^2, $P = 9.51$ m
6 **a** $P = 15.24$ m , $A = 13.09$ m^2
b $P = 14.10$ m, $A = 10.39$ m^2
c $P = 24.76$ km, $A = 33.51$ km^2
7 **a** 8.86 m, 4.63 m^2 **b** 45.56 cm, 128.54 cm^2
8 **a** 46 cm^2 **b** 114 m^2
9 **a** 659.73 mm^2 **b** 30.21 m^2
10 **a** 30 cm^3 **b** 54 m^3 **c** 31.42 mm^3

Multiple-choice questions

1 B **2** C **3** E **4** B **5** A
6 D **7** B **8** E **9** C **10** E

Extended-response questions

1 **a** 517.08 cm **b** $65 **c** 15853.98 cm^2
d 1.58 m^2; claim is correct
2 **a** 1 cm **b** 15.71 cm^2 **c** 125 680 cm^3
d 0.125 68 m^3 **e** 18.85 cm **f** 15 m^2
g $1200

Semester review 1

Reviewing number and financial mathematics

Short-answer questions

1 **a** $\frac{19}{28}$ **b** $\frac{7}{9}$ **c** $\frac{3}{8}$ **d** $1\frac{4}{5}$
2 **a** 60% **b** 31.25% **c** 10% **d** 25%
3 **a** 5 : 3 **b** 55 km/h **c** 2.4 mL/h
4 **a** $67.20 **b** 64.4 cm
5 $892

Multiple-choice questions

1 B **2** E **3** D **4** D **5** A

Extended-response question

a **i** $17 500 **ii** $23 520 **iii** 9 years **iv** 27%
b $23635.69
c **i** Jim by $116 **ii** Jim by $922

Linear and simultaneous equations

Short-answer questions

1 **a** $x = 6$ **b** $x > \frac{9}{2}$ **c** $m = \frac{3}{8}$
d $y = -1$ **e** $a \leqslant \frac{1}{11}$ **f** $x = -\frac{3}{14}$
2 **a** $\frac{m - 3}{2} = 6$
b Noah gets $15 pocket money
3 **a** 155 **b** $I = \frac{2S}{n} - a$ **c** 18
4 **a** $x = 6, y = 3$ **b** $x = -1, y = -5$
c $x = 5, y = -2$ **d** $x = -3, y = 4$

Multiple-choice questions

1 E **2** A **3** C **4** B **5** D

Extended-response question

a **i** $12x + 20 > 74$ **ii** 5 games
b **i** Let \$$x$ be the cost of a raffle ticket and \$$y$ the cost of a badge.
ii $5x + 2y = 11.5$ and $4x + 3y = 12$
iii A raffle ticket costs $1.50 and a badge costs $2.

Pythagoras' theorem and trigonometry

Short-answer questions

1 **a** $x = 15.1$ **b** $x = 5.7$
c $x = 11.2$ **d** $\theta = 29.5°$
2 **a** $x = 13, y = 14.7$ **b** $x = 9.9$
3 **a** 19.21 m **b** 38.7°
4 **a** 16.3 km west **b** 115°

Multiple-choice questions

1 D **2** A **3** C **4** A **5** E

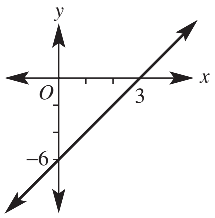
Extended-response question

a 17.75 m **b** 14.3°
c **i** 18.8 m **ii** 6.8 s

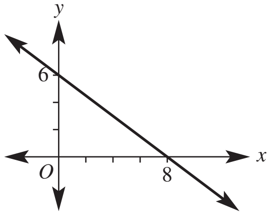
Linear relations

Short-answer questions

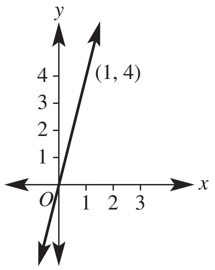
1 a



b



c



2 a $\frac{2}{3}$ b -3 c -2 d $\frac{4}{3}$

3 a $y = -3x + 6$ b $y = 3x - 1$

c $y = 2x$ d $y = -\frac{1}{3}x + 2$

4 (3, 2)

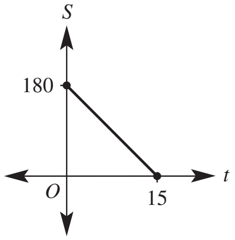
Multiple-choice questions

1 C 2 D 3 B 4 C 5 A

Extended-response question

a 12 kg/h b $S = 180 - 12t$

c



d 15 hours

e i $P = 40 + 25h$ ii $415

Measurement

Short-answer questions

1 a 24.57 m^2 b 36 cm^2

2 4 tins

3 a 216 m^2 b 25.45 m^2

4 a $x = 4$ b $y = 8.5$

Multiple-choice questions

1 C 2 E 3 D 4 A 5 A

Extended-response question

a 45.71 m^2

b i 0.0377 m^2 ii 3.77 m^2

c 1213 d 37.85 m^3 e 19.45 m^3

Chapter 6

6A

Building understanding

1 a 2 b 3 c 3 d 2

2 a 3 b -7 c $\frac{2}{3}$ d y

3 a 3 b 7 c 4 d 13

4 a 2, 3 b 3, 5 c 2, 3, 5 d 7, 11

Now you try

Example 1

a $b \times b \times b \times b$ b $mn \times mn \times mn$ c $5 \times a \times a \times b$

Example 2

a 16 b -27 c $\frac{16}{9}$

Example 3

a $2y^3$ b $\left(\frac{5}{9}\right)^3 \times \left(\frac{3}{7}\right)^2$ c 3^3a^2

Example 4

$2^2 \times 3 \times 5$

Exercise 6A

1 a $a \times a \times a \times a$

b $x \times p \times x \times p \times x \times p \times x \times p \times x \times p \times x \times p$

c $4 \times x \times x \times y \times y \times y \times y$

d $4 \times a \times b \times a \times b \times a \times b$

2 a $b \times b \times b$ b $x \times x \times x$

c $5 \times a \times 5 \times a \times 5 \times a \times 5 \times a$

d $3 \times y \times 3 \times y \times 3 \times y$ e $p \times q \times p \times q$

f $-3 \times s \times s \times s \times t \times t$
g $6 \times x \times x \times x \times y \times y \times y \times y \times y$
h $5 \times y \times z \times y \times z \times y \times z \times y \times z \times y \times z \times y \times z$

3 **a** 36 **b** 16 **c** 243 **d** 12
e -8 **f** -1 **g** 81 **h** 25
i $\frac{8}{27}$ **j** $\frac{9}{16}$ **k** $\frac{1}{216}$ **l** $\frac{25}{4}$
m $-\frac{8}{27}$ **n** $\frac{81}{256}$ **o** $\frac{1}{16}$ **p** $-\frac{3125}{32}$

4 **a** 3^3 **b** 8^6 **c** y^2 **d** $3x^3$
e $4c^5$ **f** 5^3d^2 **g** x^2y^3 **h** 7^3b^2

5 **a** $\left(\frac{2}{3}\right)^4$ **b** $\left(\frac{3}{5}\right)^5$
c $\left(\frac{4}{7}\right)^2\left(\frac{1}{5}\right)^4$ **d** $\left(\frac{7x}{9}\right)^2\left(\frac{y}{4}\right)^3$

6 **a** $3^3x^3y^2$ **b** $(3x)^2(2y)^2$ or $3^22^2x^2y^2$
c $(4d)^2(2e)^2$ or $4^22^2d^2e^2$ **d** $6^3b^2y^3$
e $(3pq)^4$ or $3^4p^4q^4$ **f** $(7mn)^3$ or $7^3m^3n^3$

7 **a** 2×5 **b** 2^3 **c** $2^4 \times 3^2$
d 2^9 **e** $2^3 \times 3^3$ **f** $2^2 \times 5^3$

8 **a** 36 **b** -216 **c** 1 **d** $-\frac{8}{27}$
e -18 **f** 15 **g** -36 **h** 216

9 **a** 4 **b** 8 **c** 5 **d** 2
e -4 **f** -2 **g** $\frac{1}{2}$ **h** 4

10 **a** **i** 10 min **ii** 20 min **iii** 30 min
b $2^{24} = 16\,777\,216$ cells

11 **a** $1000 \times 3^5 = \$243\,000$ **b** 5 years

12 7 months

13 **a** **i** 9 **ii** 9 **iii** -9 **iv** -9
b Same signs give positive when multiplying
c A positive answer is multiplied by the -1 outside the bracket.

14 **a** **i** 8 **ii** -8 **iii** -8 **iv** 8
b A positive cubed is positive.
A negative answer is multiplied by negative.
c A negative number cubed will be negative.
A positive answer is multiplied by negative one.

15 **a** $\frac{1}{8}$ **b** $\frac{1}{16}$ **c** $\frac{1}{125}$
d $\frac{1}{64}$ **e** $\frac{49}{100}$ **f** $\frac{81}{16}$
g $\frac{169}{25}$ **h** $\frac{12769}{100}$ **i** $\frac{289}{25}$

16 **a** LCM = 12, HCF = 2 **b** LCM = 84, HCF = 14
c LCM = 72, HCF = 12 **d** LCM = 30, HCF = 5
e LCM = 360, HCF = 10 **f** LCM = 300, HCF = 10
g LCM = 1764, HCF = 14 **h** LCM = 13068, HCF = 198

6B

Building understanding

1 **a** multiply, base, add **b** divide, base, subtract

2 **a** $3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$
b $\frac{5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5} = 5^2$

3 **a** True **b** True **c** False
d False **e** True **f** False

Now you try

Example 5
a 2^9 **b** 9^4

Example 6
a x^{11} **b** a^3b^7 **c** $28m^5$

Example 7
a a^4 **b** $\frac{2x^3y}{5}$

Example 8
a a^5 **b** $2a^2b^2$

Exercise 6B

1 **a** **i** 2^7 **ii** 5^9
b **i** 3^3 **ii** 6^5

2 **a** 7^6 **b** 8^{10} **c** 3^8 **d** 6^{14}
e 5^3 **f** 10 **g** 9^3 **h** $(-2)^2$

3 **a** x^7 **b** a^9 **c** t^8 **d** y^5
e d^3 **f** y^7 **g** b^8 **h** q^{11}
i x^7y^5 **j** x^9y^4 **k** $5x^4y^9$ **l** $4x^2y^5z$
m $15m^5$ **n** $8e^6f^4$ **o** $20c^7d^2$ **p** $18y^2z^7$

4 **a** a^2 **b** x^3 **c** q^{10} **d** d
e $2b^5$ **f** $\frac{d^5}{3}$ **g** $2a^7$ **h** $2y^8$
i $9m$ **j** $14x^3$ **k** $5y^2$ **l** $6a$
m $\frac{m^5}{4}$ **n** $\frac{w}{5}$ **o** $\frac{a}{5}$ **p** $\frac{x^4}{9}$
q $\frac{4x^6y^3}{3}$ **r** $\frac{3st^2}{7}$ **s** $\frac{4mn}{3}$ **t** $-5x$

5 **a** b^6 **b** y^6 **c** c^7 **d** x
e t **f** p^6 **g** d^6 **h** x^{10}
i $4x^2y^3$ **j** $6b^2g$ **k** $3m^5n^6$ **l** p^5q^4

6 **a** $\frac{m^5}{n^5}$ **b** $\frac{x^4}{y^2}$ **c** a^3b^3
d $\frac{6a^5}{c^7}$ **e** $6f^6$ **f** $12x^4b^2$
g $6k^3m^3$ **h** $\frac{15x^4y}{2}$ **i** $\frac{3m^2n^3}{2}$

7 **a** 12 **b** 8 **c** 3 **d** 3
e 1 **f** 18 **g** 12 **h** 11
i 4 **j** 15 **k** 2 **l** 39

8 **a** $7^2 = 49$ **b** 10 **c** $13^2 = 169$
d $2^3 = 8$ **e** 101 **f** $200^2 = 40\,000$
g $7 \times 31 = 217$ **h** $43 \times 50^2 = 7500$

9 **a** 7 combinations **b** 14 combinations

10 **a** a^5, power of one not added
b x^6, power of one not subtracted
c $\frac{a^2}{2}$, $3 \div 6$ is $\frac{1}{2}$ not 2
d $\frac{x^4}{2}$, numerator power is larger hence x^4 in numerator
e $6x^{11}$, mutiply coefficients not add
f $a^3 \times a = a^4$, order of operations done incorrectly

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11 **a** $4x$ **b** $12x^2$ **c** $10x^3$ **d** $-4x$
e $40x^6$ **f** $\frac{5x}{4}$ **g** $\frac{8}{5}$ **h** $-20x^4$
12 **a** 2^{x+y} **b** 5^{a+b} **c** t^{x+y}
d 3^{x-y} **e** 10^{p-y} **f** 2^{p+q-r}
g 10^{p-q-r} **h** 2^{5a} **i** $a^{3x-2}b^{x+3}$
j $a^{x+y}b^{x+y}$ **k** $a^{x-y}b^{y-x}$ **l** $w^{2-x}b^{x+3}$
m a^{x+y-2} **n** p^aq^{b-5} **o** $4m^{y-3-2x}$
13 Answers may vary.
14 Answers may vary.

6C

Building understanding

1 **a** multiply **b** 1
2 **a** 16, 8, 4, 2, 1 **b** 64, 16, 4, 1
3 **a** $(4 \times 4) \times (4 \times 4) \times (4 \times 4) = 4^6$
b $(a \times a) \times (a \times a) \times (a \times a) \times (a \times a) \times (a \times a) = a^{10}$

Now you try

Example 9
a a^6 **b** $5b^{14}$
Example 10
a 1 **b** -1 **c** 5
Example 11
a y^{18} **b** t^5 **c** m

Exercise 6C

1 **a** **i** y^{12} **ii** m^{18}
b **i** $5m^{16}$ **ii** $2j^{24}$
2 **a** x^{10} **b** b^{12} **c** 3^6 **d** 4^{15}
e 3^{30} **f** 7^{10} **g** $4q^{28}$ **h** $-3c^{10}$
3 **a** 1 **b** 1 **c** 1 **d** 1
e -1 **f** 1 **g** 1 **h** 1
i 5 **j** -3 **k** 4 **l** -6
m 1 **n** 3 **o** 1 **p** 0
4 **a** 4^7 **b** 3^9 **c** x **d** y^{13}
e b^{14} **f** a^{10} **g** d^{24} **h** y^{16}
i z^{25} **j** $a^{11}f^{13}$ **k** $x^{14}y^5$ **l** $5rs^8$
5 **a** 7^2 **b** 4 **c** 3^8 **d** 1 **e** y^3
f h^2 **g** b^6 **h** x^5 **i** y^6
6 **a** $\frac{2}{x^5}$ **b** $\frac{10}{x^3}$ **c** $3x^8$
d $\frac{d^2e}{2}$ **e** $\frac{2m^6n}{5}$ **f** $\frac{a^{12}}{8}$
7 **a** **i** 400 **ii** 6400 **iii** 100
b **i** 800 **ii** 12 800 **iii** 102 400
c 13 years
8 5 ways
9 **a** 4 **b** 1000 **c** 1
d 1 **e** 4 **f** 1
10 **a** 4×5 not $4 + 5$, a^{20}
b power of 2 only applies to x^3, $3x^6$
c power zero applies to whole bracket, 1

11 **a** **i** 2^{24} **ii** $(-2)^{30} = 2^{30}$ **iii** x^{84} **iv** a^{48}
b **i** 2^{abc} **ii** a^{mnp} **iii** x^{6yz}
12 **a** Answers may vary. **b** Answers may vary.
13 **a** 2^{12} **b** 2^{15} **c** 3^6 **d** 3^{20} **e** 5^{10}
f 3^{50} **g** 2^{72} **h** 7^{80} **i** 10^{50}

6D

Building understanding

1 **a** $a^m \times b^m$ **b** $\frac{a^m}{b^m}$
2 **a** $5a \times 5a \times 5a$
$= 5 \times 5 \times 5 \times a \times a \times a$
$= 5^3 \times a^3$
b $\frac{x}{6} \times \frac{x}{6} \times \frac{x}{6}$
$= \frac{x \times x \times x}{6 \times 6 \times 6}$
$= \frac{x^3}{6^3}$

Now you try

Example 12
a $9a^2$ **b** $-27x^6y^9$ **c** $5a^8b^4$
Example 13
a $\frac{32}{x^5}$ **b** $\frac{9m^6}{4n^2p^4}$ **c** $\frac{x^5y^4}{72}$

Exercise 6D

1 **a** $8x^3$ **b** $-27w^9y^3$ **c** $2x^6y^2$
2 **a** $25y^2$ **b** $64a^6$ **c** $9r^2$
d $-81b^4$ **e** $-343r^3$ **f** $(-2)^4h^8 = 16h^8$
g $625c^8d^{12}$ **h** $32x^{15}y^{10}$ **i** $9p^6q^{12}$
j 1 **k** $-4p^8q^2r^2$ **l** $25s^{14}t^2$
3 **a** $\frac{p^3}{q^3}$ **b** $\frac{x^4}{y^4}$ **c** $\frac{64}{y^3}$ **d** $\frac{4}{r^6}$
e $\frac{s^6}{49}$ **f** $\frac{32m^5}{n^5}$ **g** $\frac{27n^9}{8m^{12}}$ **h** $\frac{16r^4}{n^4}$
i $\frac{9f^2}{64g^{10}}$ **j** $\frac{9x^2}{4y^6g^{10}}$ **k** $\frac{27k^3m^9}{64n^{21}}$ **l** $-\frac{25w^8y^2}{4x^6}$
4 **a** $9ab^2$ **b** $27ab^6$ **c** $-12a^8b^8$ **d** $54x^6y^9$
e $-64b^6c^{15}d^3$ **f** $8a^4$ **g** $9a^5$ **h** $-40a^{15}b^3$
i $160m^{15}p^5t^{10}$ **j** $-49d^4f^8g^2$ **k** $1024x^{12}y^3z^9$ **l** $-16a^8b^7$
5 **a** x^{24} **b** $256x^{24}$ **c** $a^{17}b^8$ **d** $a^{10}b^{11}$
e $8m^5n^3$ **f** $12c^8d^7$ **g** $\frac{-27x^6}{125a^{15}b^9}$ **h** $-3a^4b^5$
i $15n$ **j** $a^{11}bc^5$ **k** $x^{11}y^2z$ **l** $\frac{r^9t^{10}}{s}$
6 **a** **i** 8 **ii** 125
b $N = \frac{t^3}{8}$
c **i** 27 **ii** 8
d **i** 8 **ii** 2
7 **a** 2 **b** 4 **c** 2
d 2 **e** 1 **f** 14
8 **a** By simplifying first, there are smaller numbers to raise to powers.
b **i** 8 **ii** 16
iii $\frac{1}{81}$ **iv** $\frac{1}{1000}$

9 **a** False, $(-2)^2 = +(2)^2$ **c** True, $(-3)^3 = -(3)^3$
d True, $(-5)^5 = -(5)^5$ **e** False, $(-4)^4 = +(4)^4$

10 **a** 25 **b** 13 **c** no
d no, $(3-2)^2 \neq 3^2 - 2^2$
e **i** True **ii** False **iii** True **iv** False

6E

Building understanding

1 **a** $\frac{1}{2^2}$ **b** $\frac{1}{3^2}$ **c** $\frac{1}{5^3}$ **d** $\frac{1}{3^3}$

2 **a**

Index form	3^4	3^3	3^2	3^1
Whole number or fraction	81	27	9	3

Index form	3^0	3^{-1}	3^{-2}	3^{-3}
Whole number or fraction	1	$\frac{1}{3}$	$\frac{1}{9} = \frac{1}{3^2}$	$\frac{1}{27} = \frac{1}{3^3}$

b

Index form	10^4	10^3	10^2	10^1
Whole number or fraction	10000	1000	100	10

Index form	10^0	10^{-1}	10^{-2}	10^{-3}
Whole number or fraction	1	$\frac{1}{10}$	$\frac{1}{100} = \frac{1}{10^2}$	$\frac{1}{1000} = \frac{1}{10^3}$

Now you try

Example 14

a $\frac{1}{a^3}$ **b** $\frac{2y^3}{x^4}$

Example 15

a a^3 **b** $\frac{y^4}{x^2}$ **c** $\frac{6b^5}{a^2}$

Example 16

a $\frac{1}{32}$ **b** -32 **c** $\frac{49}{25}$

Exercise 6E

1 **a** **i** $\frac{1}{x^5}$ **ii** $\frac{1}{a^7}$
b **i** $\frac{2b^2}{a^3}$ **ii** $\frac{-7x^4}{y^5}$

2 **a** $\frac{1}{x}$ **b** $\frac{1}{a^4}$ **c** $\frac{1}{b^6}$ **d** $\frac{1}{25}$
e $\frac{1}{64}$ **f** $\frac{1}{9}$ **g** $\frac{5}{x^2}$ **h** $\frac{4}{y^3}$
i $\frac{3}{m^5}$ **j** $\frac{p^7}{q^2}$ **k** $\frac{m}{n^4}$ **l** $\frac{x^4}{y^4}$
m $\frac{2}{a^3b}$ **n** $\frac{7}{r^2s^3}$ **o** $\frac{v^2}{5u^8}$ **p** $\frac{1}{9m^3n^5}$

3 **a** y **b** b^2 **c** m^5 **d** x^4
e $7q$ **f** $3t^2$ **g** $5h^4$ **h** $4p^4$
i ab^2 **j** de **k** $2m^3n^2$ **l** $\frac{x^2y^5}{3}$
m $-\frac{3y^4}{7}$ **n** $-2b^8$ **o** $-\frac{3gh^3}{4}$ **p** $\frac{9u^2t^2}{5}$

4 **a** $\frac{b^3}{a^3}$ **b** $\frac{y^5}{x^2}$ **c** $\frac{h^3}{g^2}$ **d** $\frac{n}{m}$
e $\frac{343}{5}$ **f** $\frac{64}{9}$ **g** $\frac{6}{25}$ **h** 1

5 **a** $\frac{7x^4}{y^3}$ **b** $\frac{u^3}{v^2}$ **c** $\frac{y^3}{5a^3}$ **d** $\frac{2b^5}{a^4c^2}$
e $\frac{5a^2b^2}{6c^4d}$ **f** $\frac{4h^3m^2}{5k^2p}$ **g** $\frac{12w^6}{tu^2v^2}$ **h** $\frac{mn^4x^2}{16y^5}$

6 **a** $\frac{1}{5}$ **b** $\frac{1}{9}$ **c** $\frac{1}{16}$ **d** $-\frac{1}{25}$
e $\frac{1}{25}$ **f** $-\frac{1}{200}$ **g** $-\frac{3}{4}$ **h** $\frac{1}{2}$
i $\frac{1}{36}$ **j** $\frac{1}{8}$ **k** $\frac{4}{25}$ **l** $\frac{7}{81}$
m 8 **n** 100 **o** -250 **p** 16
q -10 **r** 64 **s** $\frac{64}{9}$ **t** $-\frac{27}{64}$
u 100 **v** $\frac{1}{3}$ **w** -2 **x** 49

7 1.95 g

8 **a** -4 **b** -4 **c** -4
d -1 **e** -2 **f** -1

9 **a** negative power only applies to x, $\frac{2}{x^2}$
b $5 = 5^1$ has a positive power, $5a^{-4}$
c $\frac{2}{3^{-2}b^{-2}} = 2 \times 3^2 \times b^2 = 18b^2$

10 **a** $1 \div \frac{2}{3} = 1 \times \frac{3}{2}$
b **i** $\frac{4}{5}$ **ii** $\frac{7}{2}$ **iii** $\frac{3}{x}$ **iv** $\frac{b}{a}$
c $(\text{fraction})^{-1} =$ reciprocal of fraction
d **i** $\frac{9}{4}$ **ii** $\frac{25}{16}$ **iii** 32 **iv** $\frac{27}{343}$

11 **a** 4 **b** 4 **c** 3 **d** 2
e 4 **f** $\frac{3}{2}$ **g** 4 **h** $\frac{7}{3}$

Progress quiz

1 **a** $a \times a \times a \times a$ **b** $5 \times hk \times hk \times hk$
c $2 \times 2 \times 2 \times 2 = 16$ **d** $\frac{-3}{4} \times \frac{-3}{4} \times \frac{-3}{4} = -\frac{27}{64}$

2 **a** $7m^4$ **b** $\left(\frac{2}{3}\right)^2 \times \left(\frac{1}{5}\right)^3$
c $8^3e^4h^3 = (8h)^3e^4$

3 **a** 2^8 **b** a^9 **c** $12k^7m^2$
d 5^8 **e** $2a^6$ **f** $\frac{a^5m^4}{3}$

4 **a** x^6 **b** $3xy$

5 **a** x^{12} **b** $-4q^{42}$

6 **a** 1 **b** 1 **c** 7 **d** -5

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7 **a** a^{14} **b** m^{22} **c** 4 **d** $\frac{4m^8n^4}{3}$

8 **a** $8b^3$ **b** $5h^6j^9k^3$ **c** $-27x^{12}y^6$
d $\frac{64}{c^3}$ **e** $\frac{-8w^3x^9}{125y^6}$ **f** a^8c^2

9 **a** $\frac{1}{m^4}$ **b** $\frac{7y^5}{x^3}$ **c** a^5
d $\frac{c^3}{a^2}$ **e** $-13m^5$ **f** $\frac{-15u^3w^3}{t^2v^2}$

10 **a** $\frac{1}{16}$ **b** 27 **c** $\frac{1}{36}$ **d** $\frac{8}{27}$

6F

Building understanding

1 **a** 10 000 **b** 1000 **c** 100 000
d 1000 **e** 100 000 **f** 10 000
2 **a** 10^5 **b** 10^2 **c** 10^9
3 **a** positive **b** negative **c** positive **d** negative

Now you try

Example 17
a 7.2×10^5 **b** 3×10^{-6}

Example 18
a 6170 **b** 0.00000402

Exercise 6F

1 **a** **i** 4.3×10^4 **ii** 1.49×10^9
b **i** 4×10^{-3} **ii** 6.1×10^{-6}
2 **a** 4×10^4 **b** 2.3×10^{12} **c** 1.6×10^{10}
d -7.2×10^6 **e** -3.5×10^3 **f** -8.8×10^6
g 5.2×10^3 **h** 3×10^6 **i** 2.1×10^4
3 **a** 3×10^{-6} **b** 4×10^{-4} **c** -8.76×10^{-3}
d 7.3×10^{-10} **e** -3×10^{-5} **f** 1.25×10^{-10}
g -8.09×10^{-9} **h** 2.4×10^{-8} **i** 3.45×10^{-5}
4 **a** 6×10^3 **b** 7.2×10^5 **c** 3.245×10^2
d 7.86903×10^3 **e** 8.45912×10^3 **f** 2×10^{-1}
g 3.28×10^{-4} **h** 9.87×10^{-3} **i** -1×10^{-5}
j -4.601×10^8 **k** 1.7467×10^4 **l** -1.28×10^2
5 **a** 57 000 **b** 3 600 000 **c** 430 000 000
d 32 100 000 **e** 423 000 **f** 90 400 000 000
g 197 000 000 **h** 709 **i** 635 700
6 **a** 0.00012 **b** 0.0000046 **c** 0.0000000008
d 0.0000352 **e** 0.3678 **f** 0.000000123
g 0.00009 **h** 0.05 **i** 0.4
7 **a** 6×10^{24} **b** 4×10^7 **c** 1×10^{-10}
d 1.5×10^8 **e** 6.67×10^{-11} **f** 1.5×10^{-4}
g 4.5×10^9
8 **a** 4 600 000 000 **b** 8 000 000 000 000
c 384 000 **d** 0.0038
e 0.00000000000001 **f** 720 000
9 **a** 3.6×10^7 **b** 3.6×10^5 **c** 4.92×10^{-1}
d 3.8×10^{-4} **e** 2.1×10^{-6} **f** 5.2×10^{-8}
g 4×10^{-9} **h** 1.392×10^{-7} **i** 3.95×10^3
j 4.38×10^3 **k** 8.28×10^6 **l** 3×10^{11}
10 **a** $\$1.84 \times 10^9$ **b** $\$2.647 \times 10^9$
11 1.62×10^9 km
12 2.126×10^{-2} g
13 **a** 3.2×10^4 **b** 4.1×10^6 **c** 3.17×10^4
d 5.714×10^5 **e** 1.3×10^4 **f** 9.2×10^1
g 3×10^5 **h** 4.6×10^5 **i** 6.1×10^{-2}
j 4.24 **k** 1.013×10^{-3} **l** 4.9×10^{-4}
m 2×10^{-5} **n** 4×10^{-6} **o** 3.72×10^{-4}
p 4.001×10^{-8}
14 **a** 8×10^6 **b** 9×10^8 **c** 6.25×10^{-4}
d 3.375×10^{-9} **e** 1.25×10^8 **f** 4×10^6
g 9×10^{-4} **h** 2.5×10^4
15 **a** 6×10^6 **b** 8×10^{11} **c** 2×10^4
d 3×10^9 **e** 5.6×10^5 **f** 1.2×10^8
g 1.2×10^3 **h** 9×10^3 **i** 9×10^{-9}
j 7.5×10^{-8} **k** 1.5×10^{-5} **l** 1
16 $5 \times 10^2 = 500$ seconds
17 **a** 3×10^{-4} km $= 30$ cm
b 1×10^{-3} seconds (one thousandth of a second)
$= 0.001$ seconds

6G

Building understanding

1 **a** 57 260, 57 300, 57 000, 60 000
b 0.003661, 0.00366, 0.0037, 0.004
2 **a** yes **b** no **c** no **d** no **e** yes
f yes **g** yes **h** no **i** no

Now you try

Example 19
a 4 **b** 3 **c** 3

Example 20
a 4.9×10^5 **b** 6.5×10^{-2}

Example 21
a 7.71×10^9 **b** 6.94×10^{-2}

Exercise 6G

1 **a** **i** 4 **ii** 2
b **i** 2 **ii** 3
c **i** 2 **ii** 4
2 **a** 3 **b** 4 **c** 5 **d** 2
e 3 **f** 1 **g** 3 **h** 3
i 3 **j** 4 **k** 3 **l** 2
3 **a** 2.42×10^5 **b** 1.71×10^5 **c** 2.83×10^3
d 3.25×10^6 **e** 3.43×10^{-4} **f** 6.86×10^{-3}
g 1.46×10^{-2} **h** 1.03×10^{-3} **i** 2.34×10^1
j 3.26×10^2 **k** 1.96×10^1 **l** 1.72×10^{-1}
4 **a** 4.78×10^4 **b** 2.2×10^4 **c** 4.833×10^6
d 3.7×10^1 **e** 9.95×10^1 **f** 1.443×10^{-2}
g 2×10^{-3} **h** 9×10^{-2} **i** 1×10^{-4}
5 **a** 2.441×10^{-4} **b** 2.107×10^{-6} **c** -4.824×10^{15}
d 4.550×10^{-5} **e** 1.917×10^{12} **f** 1.995×10^8
g 3.843×10^2 **h** 1.710×10^{-11} **i** 1.524×10^8
j 3.325×10^{15} **k** 4.000×10^3 **l** -8.959×10^3

ISBN 978-1-108-77288-4

6 **a** 9.3574×10^1 **b** 2.1893×10^5 **c** 8.6000×10^5
d 8.6288×10^{-2} **e** 2.2985×10^{15} **f** 3.5741×10^{28}
g 6.4000×10^7 **h** 1.2333×10^9 **i** 1.8293
j 5.4459×10^{-1}

7 1.98×10^{30} kg

8 1.39×10^6 km

9 1.09×10^{12} km^3

10 2421×10^3, 24.2×10^5, 2.41×10^6, 0.239×10^7, 0.02×10^8

11 **a** 4.26×10^6 **b** 9.1×10^{-3}
c 5.04×10^{11} **d** 1.931×10^{-1}
e 2.1×10^6 **f** 6.14×10^{-11}

12 should be 8.8×10^{10}

13 **a** **i** 2.30×10^2 **ii** 4.90×10^{-2} **iii** 4.00×10^6
b It is zero
c It clarifies the precision of the number

14 **a** 5.40046×10^{12}
b **i** 4.32×10^{13} **ii** 1.61×10^{19} **iii** 4.01×10^{51}

6H

Building understanding

1 **a** 4, 2 **b** 8, 2 **c** 9, 3
d 27, 3 **e** 16, 4 **f** 64, 4

2 **a** True **b** False **c** True **d** True
e False **f** False **g** False

3 **a** 2.6458 **b** 3.6056 **c** 9.1104

Now you try

Example 22
a $\sqrt{7}$ **b** $\sqrt[6]{3}$

Example 23
a 8 **b** 6

Example 24
a x^3 **b** $a^{\frac{1}{6}}$ **c** b^2

Exercise 6H

1 **a** **i** $\sqrt{8}$ **ii** $\sqrt{15}$
b **i** $\sqrt[4]{3}$ **ii** $\sqrt[7]{7}$

2 **a** $\sqrt{3}$ **b** $\sqrt{7}$ **c** $\sqrt[3]{5}$ **d** $\sqrt[3]{12}$
e $\sqrt[5]{31}$ **f** $\sqrt[7]{18}$ **g** $\sqrt[9]{9}$ **h** $\sqrt[8]{3}$

3 **a** $8^{\frac{1}{2}}$ **b** $19^{\frac{1}{2}}$ **c** $10^{\frac{1}{3}}$ **d** $31^{\frac{1}{3}}$
e $5^{\frac{1}{4}}$ **f** $9^{\frac{1}{5}}$ **g** $11^{\frac{1}{8}}$ **h** $20^{\frac{1}{11}}$

4 **a** 5 **b** 7 **c** 9 **d** 13
e 2 **f** 4 **g** 5 **h** 10
i 2 **j** 3 **k** 5 **l** 2

5 **a** a **b** $a^{\frac{2}{3}}$ **c** a^2 **d** $a^{\frac{5}{2}}$
e $x^{\frac{1}{3}}$ **f** x **g** $x^{\frac{5}{6}}$ **h** x
i y **j** y^2 **k** $y^{\frac{3}{2}}$ **l** $x^{\frac{1}{4}}$
m $x^{\frac{8}{3}}$ **n** $a^{\frac{2}{15}}$ **o** $a^{\frac{3}{8}}$ **p** $n^{\frac{4}{3}}$

6 **a** $a^{\frac{4}{3}}$ **b** $a^{\frac{7}{10}}$ **c** $a^{\frac{23}{21}}$
d $a^{\frac{8}{3}}$ **e** $b^{\frac{1}{6}}$ **f** $x^{\frac{2}{15}}$

7 **a** $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$ **d** $\frac{1}{3}$
e $\frac{1}{5}$ **f** $\frac{1}{3}$ **g** $\frac{1}{10}$ **h** $\frac{1}{4}$

8 **a** $\sqrt{29}$ **b** $\sqrt{13}$ **c** $\sqrt{65}$
d $\sqrt{125}$ **e** $\sqrt{10}$ **f** $\sqrt{1700}$

9 **a** 9 **b** 16 **c** 27 **d** 125
e 32 **f** 32 **g** 729 **h** 3125

10 **a** $a^{\frac{1}{2}+(-\frac{1}{2})} = a^0 = 1$ **b** $a^{\frac{2}{3}+(-\frac{2}{3})} = a^0 = 1$
c $a^{\frac{4}{7}-\frac{4}{7}} = a^0 = 1$ **d** $a^{\frac{5}{6}-\frac{5}{6}} = a^0 = 1$
e $a^{\frac{1}{4}} \times a^{-\frac{1}{4}} = a^{\frac{1}{4}+(-\frac{1}{4})} = a^0 = 1$
f $a^2 \div a^2 = a^{2-2} = a^0 = 1$

11 Brackets needed for fractional power, 9^(1/2) = 3

12 **a** **i** 3 **ii** 5 **iii** 10
b a **c** $(a^2)^{\frac{1}{2}} = a^{2\times\frac{1}{2}} = a$
d **i** 4 **ii** 9 **iii** 36
e a **f** $(a^{\frac{1}{2}})^2 = a^{\frac{1}{2}\times 2} = a$
g **i** a **ii** a **iii** a **iv** a

13 **a** $\frac{4}{5}$ **b** $\frac{3}{7}$ **c** $\frac{2}{9}$ **d** $\frac{2}{3}$
e $\frac{4}{5}$ **f** $\frac{2}{3}$ **g** $\frac{4}{5}$ **h** $\frac{10}{7}$

14 **a** $\frac{2}{3}$ **b** $\frac{12}{7}$ **c** $\frac{5}{2}$ **d** $\frac{5}{6}$

6I

Building understanding

1 **a** like **b** like **c** unlike **d** unlike
e unlike **f** like **g** like **h** unlike

2 **a** both $= 3.162$ **b** both $= 1.732$

Now you try

Example 25
a $8\sqrt{3}$ **b** $-3\sqrt{7}$

Example 26
a $\sqrt{15}$ **b** $\sqrt{6}$

Exercise 6I

1 **a** **i** $5\sqrt{11}$ **ii** $11\sqrt{7}$
b **i** $-3\sqrt{2}$ **ii** $2\sqrt{11}$

2 **a** $8\sqrt{7}$ **b** $8\sqrt{11}$ **c** $9\sqrt{5}$
d $4\sqrt{6}$ **e** $7\sqrt{3} + 2\sqrt{5}$ **f** $9\sqrt{7} + 3\sqrt{5}$
g $-5\sqrt{5}$ **h** $-4\sqrt{7}$ **i** $5\sqrt{7}$
j $-\sqrt{14}$ **k** $7\sqrt{2} - \sqrt{5}$ **l** $3\sqrt{3} + 2\sqrt{7}$

3 **a** $\sqrt{30}$ **b** $\sqrt{21}$ **c** $\sqrt{70}$

d 4 **e** 6 **f** $\sqrt{22}$

g 3 **h** 12 **i** $\sqrt{3}$

j $\sqrt{10}$ **k** $\sqrt{7}$ **l** $\sqrt{3}$

m 3 **n** 4 **o** $\sqrt{7}$

4 **a** $8 - 3\sqrt{3}$ **b** $6\sqrt{2} - \sqrt{3}$ **c** $7\sqrt{5} + 1$

d $\frac{5\sqrt{2}}{6}$ **e** $\frac{7\sqrt{7}}{10}$ **f** $-\frac{3\sqrt{6}}{14}$

g $\frac{2\sqrt{10}}{3}$ **h** $5 + \frac{\sqrt{3}}{3}$ **i** $-\frac{19\sqrt{8}}{56}$

5 **a** $15\sqrt{6}$ **b** $6\sqrt{21}$ **c** $8\sqrt{30}$ **d** $10\sqrt{18}$

e $2\sqrt{3}$ **f** $3\sqrt{6}$ **g** $4\sqrt{14}$ **h** $\frac{\sqrt{2}}{2}$

6 **a** $6\sqrt{15} + 2\sqrt{3}$ **b** $\sqrt{10} + \sqrt{15}$

c $5\sqrt{12} + 15\sqrt{30}$ **d** $14\sqrt{30} - 70$

e $13 - 2\sqrt{39}$ **f** $\sqrt{35} - 10$

7 **a** $2\sqrt{2}$ **b** $2\sqrt{3}$ **c** $3\sqrt{3}$ **d** $3\sqrt{5}$

e $5\sqrt{3}$ **f** $10\sqrt{2}$ **g** $2\sqrt{15}$ **h** $6\sqrt{2}$

8 **a** $5\sqrt{2}$ **b** $\sqrt{2}$ **c** $4\sqrt{2}$ **d** $\sqrt{3}$

e $6\sqrt{2}$ **f** $5\sqrt{3}$ **g** $2\sqrt{5}$ **h** $4\sqrt{3}$

9 **a** $2 + \sqrt{10} + \sqrt{6} + \sqrt{15}$ **b** $3 + \sqrt{6} - \sqrt{15} - \sqrt{10}$

c $6\sqrt{10} + 8\sqrt{5} - 3\sqrt{2} - 4$ **d** $2 + 3\sqrt{2} - 6\sqrt{7} - 9\sqrt{14}$

e 1 **f** 4 **g** 15 **h** 123

i $3 + 2\sqrt{2}$ **j** $15 - 6\sqrt{6}$ **k** $13 - 4\sqrt{3}$ **l** $22 + 4\sqrt{10}$

Problems and challenges

1 **a** 4 **b** 1 **c** 6

2 **a** 6 **b** 30

3 $\frac{9}{4}$

4 **a** $2t^2$ **b** $\frac{2}{t}$

5 100 minutes

6 $-\frac{2}{3}$

7 **a** **i** $2^{\frac{3}{4}}$ **ii** $2^{\frac{7}{8}}$ **iii** $2^{\frac{15}{16}}$

b 2

8 2^7

9 **a** $7\sqrt{2}$ **b** $\frac{3}{\sqrt{2}}$ or $\frac{3\sqrt{2}}{2}$ **c** $12\sqrt{10}$

10 **a–c** Answers may vary.

11 **a** $x = 0$ or 1 **b** $x = 1$ or 2

12 $x = 2^2 \times 3 \times 5^2 = 300$

13 $x = -1, y = \frac{1}{2}$

Short-answer questions

1 **a** 3^4 **b** $2x^3y^2$ **c** $3a^2b^2$ **d** $\left(\frac{3}{5}\right)^3 \times \left(\frac{1}{7}\right)^2$

2 **a** $3^2 \times 5$ **b** $2^2 \times 3 \times 5^2$

3 **a** x^{10} **b** $12a^5b^6c$ **c** $12m^4n^4$

d a^9 **e** x^3y^2 **f** $\frac{b^2}{2a^2}$

4 **a** m^6 **b** $9a^8$ **c** $-32a^{10}b^5$

d $3b$ **e** 2 **f** $\frac{a^6}{27}$

5 **a** $\frac{1}{x^3}$ **b** $\frac{4}{t^3}$ **c** $\frac{1}{9t^2}$

d $\frac{2x^2}{3y^3}$ **e** $\frac{5}{x^6y^3}$ **f** $5m^3$

6 **a** $\frac{x^2}{2y^2}$ **b** $\frac{x^3y^2}{9}$ **c** $8m^7n^3$

7 0.0012, 35.4×10^{-3}, 3.22×10^{-1}, 0.4, 0.007×10^2, 2.35

8 **a** 324 **b** 172 500 **c** 0.2753 **d** 0.00149

9 **a** 7.76×10^9 people **b** 9.63×10^6 km^2

c 3.34×10^{-9} seconds **d** 2.94×10^{-7} m

10 **a** 2.19×10^5 **b** 1.2×10^{-2} **c** 4.32×10^{-7}

11 **a** 1.2×10^{55} **b** 4.3×10^{-5}

12 **a** 2 **b** 5 **c** 7

d 3 **e** $\frac{1}{3}$ **f** $\frac{1}{11}$

13 **a** s^2 **b** $15x^{\frac{5}{2}}$ **c** $9m^{\frac{3}{4}}n^4$ **d** $4a^{\frac{2}{3}}$

14 **a** $7\sqrt{7} + 2$ **b** $\sqrt{3} + 9\sqrt{2}$ **c** 8

d $\sqrt{15}$ **e** $2\sqrt{14}$ **f** $15\sqrt{22}$

g $\sqrt{6}$ **h** 10 **i** $\frac{\sqrt{5}}{2}$

Multiple-choice questions

1 D **2** B **3** E **4** A **5** C **6** B

7 D **8** C **9** E **10** B **11** C **12** A

Extended-response questions

1 **a** $\frac{16x^6}{3}$ **b** $\frac{8b^8}{15a^2}$ **c** $\frac{5m^8}{n^{10}}$ **d** $3x$

2 **a** 5.93×10^{-11} N

b **i** 1.50×10^{11} m **ii** 3.5×10^{22} N

c Earth $a = 9.81$ m/s^2, Mars $a = 3.77$ m/s^2

Acceleration due to gravity on Earth is more than $2\frac{1}{2}$ times that on Mars.

Chapter 7

7A

Building understanding

1 **a** right **b** 180°

c revolution **d** obtuse

e acute **f** 180°

g 90° **h** supplementary

i 180° **j** equal

2 **a** isosceles triangle **b** obtuse angled triangle

c equilateral triangle **d** isosceles triangle

e acute angled triangle **f** scalene triangle

g right-angled triangle

3 **a** acute, 50° **b** right, 90° **c** obtuse, 101°

d reflex, 202° **e** straight, 180° **f** revolution, 360°

4 **a** C **b** S **c** N **d** C

ISBN 978-1-108-77288-4

Now you try

Example 1

a 108° b 18°

Example 2

a right-angled scalene triangle, 65
b acute isosceles triangle, 80
c obtuse scalene triangle, 22

Example 3

a 82 b 100

Exercise 7A

1 a 151° b 61°
2 a i 125° ii 35°
b i 106° ii 16°
c i 91° ii 1°
d i 158° ii 68°
e i 142° ii 52°
f i 133° ii 43°
3 a $a = 63$ b $a = 71$ c $a = 38$
d $a = 147$ e $a = 233$ f $a = 33$
4 a obtuse isosceles, $a = 40$
b acute scalene, $b = 30$
c right-angled scalene, $c = 90$
d equilateral, $d = 60$
e obtuse isosceles, $e = 100$
f right-angled isosceles, $f = 45$
g obtuse scalene, $g = 100$
h equilateral, $h = 60$
i acute scalene, $i = 70$
5 a $s = 120$ b $t = 20$
c $r = 70$ d $a = 60, x = 120$
e $a = 100, b = 140$ f $c = 115, d = 65$
6 a 360° b 90° c 60° d 90°
e 432° f 6° g 720° h 8640°
7 a $x = 56$ b $x = 155$
8 a 90° b 150° c 15°
d 165° e 157.5° f 80°
g 177.5° h 171° i 121.5°
9 $\angle AOB + \angle ABO = 120°$ (exterior angle of triangle)
$\angle AOB = 30°$
(reflex) $x = 330$
10 $AO = BO$ (radii)
ΔAOB is isoceles, 2 sides equal, $\angle AOB = 116°$
$\therefore \angle OAB = 32°$, base angles of isosceles triangle
11 a 160°
b 165°
c $\angle WYZ + a° + b° = 180°$ angle sum of a triangle
$\angle XYZ + \angle WYZ = 180°$ straight line
$\therefore \angle XYZ = a° + b°$
12 Let the interior angles of any triangle be $a°$, $b°$ and $c°$.
Now $a + b + c = 180$
The exterior angles become $180° - a°$, $180° - b°$, $180° - c°$ (straight line)

$$\begin{aligned}\text{Exterior sum} &= (180 - a)° + (180 - b)° + (180 - c)° \\ &= 540° - a° - b° - c° \\ &= 540° - (a + b + c)° \\ &= 540° - 180° \\ &= 360°\end{aligned}$$

13 a $4x = 90, x = 22.5$
b $3x = 180, x = 60$
c $10x = 360, x = 36$
d $2(x + 15) + x = 180, x = 50$
e $2x + 20 = 140, x = 60$
f $6x + 90 = 360, x = 45$

7B

Building understanding

1 a equal b equal c supplementary
2 a 125, alternate angles in || lines
b 110, cointerior angles in || lines
c 80, corresponding angles in || lines
d 96, vertically opposite
e 54, supplementary angles
f 116, cointerior angles on || lines

Now you try

Example 4

a No, cointerior angles are not supplementary.
b Yes, corresponding angles are equal.
c Yes, alternate angles are equal.

Example 5

a $a = 75$ (cointerior angles in parallel lines),
$b = 105$ (vertically opposite angles)
b $a = 38$ (supplementary angles),
$b = 142$ (corresponding angles in parallel lines)

Example 6

115°

Exercise 7B

1 a No, cointerior angles don't add to 180°.
b Yes, corresponding angles are equal.
c No, alternate angles are not equal.
2 a Yes, alternate angles are equal.
b Yes, cointerior angles add to 180°.
c Yes, corresponding angles are equal.
d No, corresponding angles are not equal.
e No, alternate angles are not equal.
f No, cointerior angles do not add to 180°.
3 a $a = 60, b = 120$ b $c = 95, d = 95$
c $e = 100, f = 100, g = 100$ d $a = 110, b = 70$
e $a = 100, b = 80, c = 80$ f $e = 140, f = 140, d = 140$
4 a $x = 70, y = 40$ b $t = 58, z = 122$
c $u = 110, v = 50, w = 50$ d $x = 118$
e $x = 295$ f $x = 79$
5 a 105° b 105° c 56°
d 105° e 90° f 85°
6 a 56 b 120 c 50

7 **a** $180° - a°$ **b** $180° - a°$
c $180° - (a° + b°)$ **d** $180° - (a° + b°)$
e $a° + c°$ **f** $180° - 2c°$

8 $\angle ABC = 100°, \angle BCD = 80°, \angle ABC + \angle BCD = 180°$
$\therefore AB \parallel DC$ as cointerior angles are supplementary.

9 **a** cointerior angles on parallel lines add to 180°
b alternate angles are equal, on parallel lines
c $\Delta ABC \Rightarrow a + b + c = 180$ and these are the three angles of the triangle

10 **a** $\angle BAE = 180° - a°$ (alternate angles and $AB \parallel DE$)
$$\begin{aligned}\angle ABC &= 180° - c° - (180 - a)° \text{ (angle sum of a triangle)}\\ &= 180° - c° - 180° + a°\\ &= -c° + a°\\ &= a° - c°\end{aligned}$$
b $\angle ABD = 180° - (a° + b°)$ (angle sum of triangle ΔABD)
$\angle ABC + \angle ABD = 180°$ (straight line)
$\therefore \angle ABC = a° + b°$
c construct XY through B parallel to AE
$\therefore \angle ABY = a°$ (alternate angles, $AE \parallel XY$)
$\therefore \angle CBY = b°$ (alternate angles, $DC \parallel XY$)
$\therefore \angle ABC = a° + b°$
d construct XY through A parallel to ED
$\angle XAD = 180° - b°$ (cointerior angles, $ED \parallel XY$)
$\angle DAB = 360° - a°$ (revolution)
$$\begin{aligned}\therefore \angle XAB &= 360° - a° - (180° - b°)\\ &= 180° + b° - a°\end{aligned}$$
$\angle ABC = \angle XAB$ (alternate angles and $XY \parallel BC$)
$\therefore \angle ABC = 180° + b° - a°$

7C

Building understanding

1 **a** 5 **b** 7 **c** 4 **d** 11 **e** 9 **f** 12
2 **a** 720° **b** 1080° **c** 1620°
3 **a** parallel **b** right **c** trapezium **d** equal
4 **a** convex quadrilateral **b** non-convex hexagon
c non-convex heptagon

Now you try

Example 7
a 60 **b** 80 **c** 109
Example 8
a $a = 215$ **b** $a = 120, b = 60$

Exercise 7C

1 **a** 105 **b** 100 **c** 105
2 **a** 115 **b** 149 **c** 30
d 121 **e** 140 **f** 220
3 **a** 110 **b** 70 **c** 54 **d** 33 **e** 63 **f** 109
4 **a** 110 **b** 150 **c** 230 **d** 20
e $b = 108, a = 72$ **f** $b = 140, a = 40$
g $b = 120, a = 240$ **h** $b = 128\frac{4}{7}, a = 231\frac{3}{7}$
i 108
5 **a** parallelogram, rectangle, kite
b rectangle, square
c square, rectangle
d square, rhombus, kite
6 **a** 16 **b** 25 **c** 102
7 **a** 255 **b** 86 **c** 115
d 37 **e** 28 **f** 111
8 A parallelogram has opposite sides parallel and equal. Rectangles, squares and rhombi have these properties (and more) and are therefore all parallelograms.
9 **a** $S = 180(n - 2)$ **b** $I = \frac{180(n - 2)}{n}$
c $E = \frac{360}{n}$ **d** 36°
10 **a** **i** one **ii** two **iii** five **b** $(n - 3)$
11 $(180 - a) + (180 - b) + (180 - c) + (180 - d) + (180 - e) = 360$
(sum of exterior angles is 360°)
$$\begin{aligned}180 + 180 + 180 + 180 + 180 - (a + b + c + d + e) &= 360\\ 900 - (a + b + c + d + e) &= 360\\ a + b + c + d + e &= 540\end{aligned}$$
a $a + b + c + d + e + f = 720$
b $a + b + c + d + e + f + g = 900$

7D

Building understanding

1 **a** size **b** $\Delta ABC \equiv \Delta STU$ **c** SAS, RHS, AAS
2 **a** **i** XY **ii** XZ **iii** YZ
b **i** $\angle A$ **ii** $\angle B$ **iii** $\angle C$
3 **a** $\Delta ABC \equiv \Delta FGH$ **b** $\Delta DEF \equiv \Delta STU$

Now you try

Example 9
a SSS **b** AAS **c** RHS **d** SAS
Example 10
a $x = 4, y = 8$ **b** $a = 75, b = 15$

Exercise 7D

1 **a** SAS **b** RHS **c** AAS **d** SSS
2 **a** SAS **b** AAS **c** RHS **d** SSS
3 **a** $x = 3, y = 4$ **b** $x = 2, y = 6$
c $a = 105, b = 40$ **d** $a = 65, b = 85$
e $x = 2.5, b = 29$ **f** $a = 142, x = 9.21, b = 7$
g $y = 4.2, a = 28$ **h** $a = 6.5, b = 60$
4 **a** $\Delta ABC \equiv \Delta GHI$ (RHS) **b** $\Delta DEF \equiv \Delta GHI$ (SSS)
c $\Delta ABC \equiv \Delta DEF$ (SAS) **d** $\Delta ABC \equiv \Delta GHI$ (AAS)
e $\Delta ACB \equiv \Delta DEF$ (SAS)
5 $\Delta DEF \equiv \Delta BRP$
$\Delta LMN = \Delta KIJ$
$\Delta BCD \equiv \Delta FGH$
$\Delta MNO \equiv \Delta RQP$
6 **a** $BC = 13$ **b** $BC = 85$
7 No, they can all be different sizes, one might have all sides 2 cm and another all sides 5 cm.

8 **a** one given, the other pair are vertically opposite
b AAS

9 **a** SSS **b** equal

10 **a** one given ($BA = BC$) and side BD is common
b SAS
c $\Delta ABD \equiv \Delta CBD$
d $\angle ADB = \angle CDB$ (corresponding angles in congruent triangles)
but $\angle ADB + \angle CDB = 180^\circ$ (straight angle)
$\therefore \angle ADB = \angle CDB = 90^\circ$
and AC is perpendicular to DB

11 **a–e** Answers may vary.

7E

Building understanding

1 **a** BD **b** AC **c** AC

2 $OA = OB$ radii of circle centre O

3 **a** $\angle ECD$ **b** $\angle CBA$ **c** $\angle DEC$

4 **a** SSS **b** $\angle BMC$

Now you try

Example 11
In ΔABC and ΔEDC:
$\angle ACB = \angle ECD$ (vertically opposite) (A)
$\angle ABC = \angle EDC$ (alternate angles in parallel lines) (A)
$BC = DC$ (given) (S)
$\therefore \Delta ABC \equiv \Delta EDC$ (AAS)

Example 12
a In ΔABC and ΔADC:
$AB = AD$ (given) (S)
$BC = DC$ (given) (S)
AC is common (S)
$\therefore \Delta ABC \equiv \Delta ADC$ (SSS)
b Since $\Delta ABC \equiv \Delta ADC$ then $\angle ABC = \angle ADC$.

Exercise 7E

1 $AB = CB$ (given)
BD is common
$\angle ADB = \angle CDB = 90^\circ$
$\therefore \Delta ABD \equiv \Delta CBD$ (RHS)

2 **a** $AD = CD$ (given)
$\angle DAB = \angle DCB = 90^\circ$ (given)
DB is common
$\therefore \Delta ABD \equiv \Delta CBD$ (RHS)

b AC is common
$AD = AB$ (given)
$\angle DAC = \angle BAC$ (given)
$\therefore \Delta ADC \equiv \Delta ABC$ (SAS)

c AC is common
$\angle ADC = \angle ABC$ (given)
$\angle DAC = \angle BAC$ (given)
$\therefore \Delta ADC \equiv \Delta ABC$ (AAS)

d AC is common
$AD = AB$ (given)
$DC = BC$ (given)
$\therefore \Delta ADC \equiv \Delta ABC$ (SSS)

e $AC = DC$ (given)
$BC = EC$ (given)
$\angle ACB \equiv \angle DCE$ (vertically opposite)
$\therefore \Delta ABC \equiv \Delta DEC$ (SAS)

f $AC = EC$ (given)
$\angle CAB = \angle CED$ (alternate angles, $AB \parallel DE$)
$\angle ACB = \angle ECD$ (vertically opposite)
(or $\angle CBA = \angle CDE$ (alternate angles, $AB \parallel DE$))
$\therefore \Delta ABC \equiv \Delta EDC$ (AAS)

g $DC = BC$ (given)
$\angle EDC = \angle ABC$ (alternate angles, $DE \parallel AB$)
$\angle DCE = \angle BCA$ (vertically opposite)
(or $\angle DEC = \angle BAC$ (alternate angles, $AB \parallel DE$))
$\therefore \Delta CDE \equiv \Delta CBA$ (AAS)

h BD is common
$AD = CD$ (given)
$\angle ADB = \angle CDB$ (given)
$\therefore \Delta ABD \equiv \Delta CBD$ (SAS)

i AC is common
$AB = CD$ (given)
$BC = DA$ (given)
$\therefore \Delta ABC \equiv \Delta CDA$ (SSS)

j BD is common
$\angle ABD = \angle CDB$ (alternate angles, $AB \parallel CD$)
$\angle ADB = \angle CBD$ (alternate angles, $AD \parallel CB$)
$\therefore \Delta ABD \equiv \Delta CDB$ (AAS)

k $OA = OC$ (radii)
OB is common
$AB = CB$ given
$\therefore \Delta AOB \equiv \Delta COB$ (SSS)

l $OA = OD$ and $OB = OC$ (radii)
$\angle AOB = \angle COD$ (vertically opposite)
$\Delta AOB \equiv \Delta COD$ (SAS)

3 **a** $DC = BC$ (given)
$EC = AC$ (given)
$\angle DCE = \angle BCA$ (vertically opposite)
$\therefore \Delta ABC \equiv \Delta EDC$ (SAS)

b $\angle EDC = \angle ABC$ (corresponding angles in congruent triangles)
$\therefore AB \parallel DE$ (alternate angles are equal)

4 **a** $AE = CD$ (given)
$BE = BD$ (given)
$\angle ABE = \angle CBD$ (vertically opposite with $\angle ABE$ given 90°)
$\therefore \Delta ABE \equiv \Delta CBD$ (RHS)

b $\angle EAB = \angle DCB$ (corresponding angles in congruent triangles)
$\therefore AE \parallel CD$ (alternate angles equal)

5 **a** DB is common
$AB = CD$ (given)
$AD = CB$ (given)
$\therefore \Delta ABD \equiv \Delta CDB$ (SSS)

b $\angle ADB = \angle CBD$ (corresponding angles in congruent triangles)
$\therefore AD \parallel BC$ (alternate angles equal)

6 **a** $OB = OC$ (radii)
$OA = OD$ (radii)
$\angle AOB = \angle DOC$ (vertically opposite)
$\therefore \Delta AOB \equiv \Delta DOC$ (SAS)
b $\angle ABO = \angle DCO$ (corresponding angles in congruent triangles)
$\therefore AB \parallel CD$ (alternate angles equal)

7 **a** BD is common
$AD = CD$ (given)
$\angle ADB = \angle CDB$ (given)
$\therefore \Delta ABD \equiv \Delta CBD$ (SAS)
b $\angle ABD = \angle CBD$ (corresponding angles in congruent triangles)
and $\angle ABD + \angle CBD = 180°$ (straight line)
$\therefore \angle ABD = \angle CBD = 90°$ and AC is perpendicular to BD

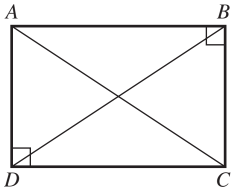
8 **a** DB is common
$\angle ABD = \angle CBD$ (given 90°)
$\angle ADB = \angle CDB$ (given)
$\therefore \Delta ABD \equiv \Delta CBD$ (AAS)
b $AD = CD$ (corresponding side in congruent triangles)
$\therefore \Delta ACD$ is isosceles (2 equal sides)

9 Consider ΔOAD and ΔOBD
OD is common
$OA = OB$ (radii)
$AD = BD$ (given)
$\therefore \Delta OAD \equiv \Delta OBD$ (SSS)
$\angle ODA = \angle ODB = 90°$ (corresponding angles in congruent triangles are equal and supplementary to a straight line)
$\therefore OC \perp AB$

10 Consider ΔADC and ΔCBA
AC is common
$\angle DAC = \angle BCA$ (alternate angles, $AD \parallel BC$)
$\angle DCA = \angle BAC$ (alternate angles, $DC \parallel AB$)
$\therefore\ \Delta ADC \equiv \Delta CBA$ (AAS)
So $AD = BC$, $AB = DC$ are equal corresponding sides in congruent triangles.

11 $AB = DC$ (opposite sides of parallelogram)
$\angle AEB = \angle CED$ (vertically opposite)
$\angle BAE = \angle DCE$ (alternate angles $DC \parallel AB$)
$\therefore\ \Delta ABE \equiv \Delta CDE$ (AAS)
So $AE = CE$ and $BE = DE$, corresponding sides in congruent triangles

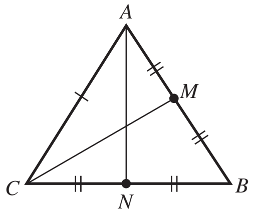
12



Consider ΔADC and ΔBCD
$AD = CB$ and DC is common (opposite sides of a rectangle are equal)
$\angle ADC = \angle BCD = 90°$ (angles of a rectangle)
$\therefore \Delta ADC \equiv \Delta BCD$ (SAS)
So $AC = BD$ (corresponding sides in congruent triangles)
$\therefore$ The diagonals of a rectangle are equal

13 **a** Consider ΔABE and ΔCDE
$AB = CD$ (sides of a rhombus)
$\angle ABE = \angle CDE$ (alternate angles, $AB \parallel CD$)
$\angle BAE = \angle DCE$ (alternate angles, $AB \parallel CD$)
$\therefore \Delta ABE \equiv \Delta CDE$ (AAS)
b Consider ΔDCE and ΔBCE
CE is common
$DC = BC$ (sides of a rhombus)
$DE = BE$ (corresponding sides in congruent triangles)
$\therefore \Delta DCE \equiv \Delta BCE$ (SSS)
$\angle DEC = \angle BEC$ (corresponding angles in congruent triangles)
$\angle DEC + \angle BEC = 180°$ (straight line)
$\therefore \angle DEC = \angle BEC = 90°$
and $AE = CE$ (corresponding sides in congruent triangles)
$\therefore AC$ bisects BD at 90°

14



Let ΔABC be any equilateral triangle $AB = CB = AC$.
Step one
Join C to M, the midpoint of AB.
Prove $\Delta CAM \equiv \Delta CBM$ (SSS).
$\therefore \angle CAM = \angle CBM$ (corresponding angles in congruent triangles)
Step two
Join A to N, the midpoint of CB.
Prove $\Delta ANC \equiv \Delta ANB$.
$\therefore \angle ACN = \angle ABN$ (corresponding angles in congruent triangles)
Now $\angle CAB = \angle ABC = \angle ACB$
and as $\angle CAB + \angle ABC + \angle ACB = 180°$
(angle sum of ΔABC)
$\angle CAB = \angle ABC = \angle ACB = 60°$

Progress quiz

1 **a** 37° **b** 127°
2 **a** 60 **b** 56 **c** 50
d 142 **e** 50 **f** 84
3 **a** yes, as there is a pair of equal alternate angles
b no, as the pair of cointerior angles are not supplementary
4 **a** $a = 68$ (alternate angles equal in parallel lines)
$b = 68$ (corresponding angles equal in parallel lines)
b $w = 98$ (alternate angles equal in parallel lines)
c $x = 92$ (angle sum of a quadrilateral)
d $x = 60$ (exterior angle of a regular hexagon)
5 **a** AAS **b** SAS
6 AC is common
$DC = BC$ given
$\angle ACD = \angle ACB$ (given 90°)
$\therefore \Delta ABC \equiv \Delta ADC$ (SAS)

ISBN 978-1-108-77288-4

$AB = AD$ (corresponding sides in congruent triangles)
$\therefore \Delta ABD$ is isosceles

7 **a** $AE = DC$ given
$\angle ABE = \angle DBC$ (vertically opposite angles)
$\angle EAB = \angle CDB$ (alternate angles, $AE \parallel CD$)
$\therefore \Delta ABE \equiv \Delta DBC$ (AAS)
b 62° **c** 7 cm **d** 21 cm^2

7F

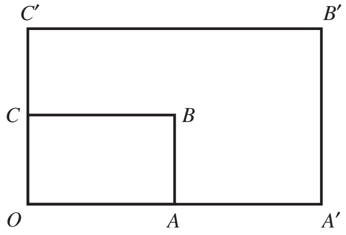
Building understanding

1 **a** $\angle F$ **b** $\angle D$ **c** GH **d** AE **e** 2
2 **a** double **b** double **c** double
d 2 **e** yes
3 **a** OA' is a quarter of OA **b** OD' is a quarter of OD
c $\frac{1}{4}$ **d** yes
4 **a** yes **b** 8 cm **c** 25 m

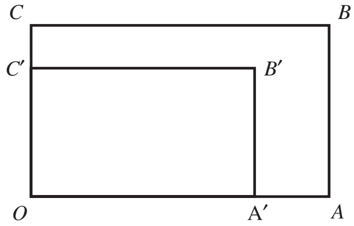
Now you try

Example 13

a



b



Example 14
a 2 **b** 6 **c** 8

Exercise 7F

1 **a** $\Delta A'B'C'$ should have sides $\frac{1}{3}$ that of ΔABC
b $\Delta A'B'C'$ should have sides double that of ΔABC
2 **a** $A'B'C'D'$ should have sides lengths $\frac{1}{2}$ that of $ABCD$
b $A'B'C'D'$ should have side lengths 1.5 times that of $ABCD$
3 **a** **i** 2 **ii** 14 **iii** 10
b **i** 1.5 **ii** 9 **iii** 8
c **i** 1.5 **ii** 45 **iii** 24
d **i** 0.4 **ii** 1 **iii** 1.4
e **i** 2.5 **ii** 0.6 **iii** 2
f **i** 1.75 **ii** 3.5 **iii** 3
4 **a** **i** 2 **ii** (0, 0)
b **i** 0.5 **ii** (0, 0)
c **i** 0.5 **ii** (3, 0)
d **i** 3 **ii** (1, 0)
5 **a** **i** 3.6 m **ii** 9 m **iii** 2.7 m
b **i** 5.4 m **ii** 6.3 m
c **i** 6 m **ii** 3 m
6 **a** 12.7 cm **b** 3 cm **c** 3 m
7 **a** $a > 1$ **b** $a < 1$ **c** $a = 1$
8 **a** all angles of any square equal 90°, with only 1 side length
b all angles in any equilateral triangle equal 60° with only 1 side length
c length and width might be multiplied by different numbers
d two isosceles triangles do not have to have the same size equal angles
9 $\frac{1}{k}$
10 **a** 100 000 cm = 1 km **b** 24 cm
11 **a** N/A
b **i** $\frac{l}{2}$ **ii** $\frac{l}{4}$ **iii** $\frac{l}{128}$
c **i** $\frac{3}{4}$ **ii** $\frac{9}{16}$ **iii** $\frac{243}{1024}$
d zero

7G

Building understanding

1 **a** E **b** C **c** DF
d BC **e** $\angle A$ **f** $\angle E$
2 2.5
3 **a** SAS, AAA, and RHS **b** shape, size

Now you try

Example 15
a SAS **b** SSS **c** RHS **d** AAA

Example 16
a AAA **b** 10.5

Exercise 7G

1 **a** AAA **b** RHS **c** SSS **d** SAS
2 **a** RHS **b** AAA **c** SAS **d** SSS
3 **a** ΔABC ||| ΔGHI **b** ΔABC ||| ΔMNO
c ΔABC ||| ΔADE **d** ΔHFG ||| ΔHJI
e ΔADC ||| ΔAEB **f** ΔABD ||| ΔECD
4 **a** AAA **b** 12
5 **a** RHS **b** 8

6 a i 1.6 **ii** 14.4
b i 3.5 **ii** 5
7 a AAA **b** 15
8 a i AAA **ii** 6.5
b i AAA **ii** 10
c i AAA **ii** 24
9 a 2 **b** 16 **c** 2.8
10 a ΔDEF **b** ΔDEF **c** ΔABC **d** ΔDEF
11 $\angle ACB = 25°$, AAA
12 $\angle WXY = 55°$, not similar as angles not equal
13 2 pairs of equal alternate angles are always formed
14 AAA, in congruency a side length is needed for the triangles to be the same size, in similarity it is not needed.
15 a

Triangle	Original	Image 1	2	3
Length scale factor	1	2	3	4
Area	4	16	36	64
Area scale factor	1	4	9	16
Volume	12	96	324	768
Volume scale factor	1	8	27	64

b Area scale factor = (length scale factor)^2
c n^2
d i 100 **ii** 400 **iii** 10 000
e $\frac{1}{4}$
f volume scale factor = (length scale factor)^3
g n^3
h i 125 **ii** 1000 **iii** $\frac{1}{8}$

7H

Building understanding

1 $\angle C$
2 a $\angle ACB$ and $\angle ECD$
b $\angle BAC = \angle DEC$ and $\angle CBA = \angle CDE$
3 a $\angle C$
b i AC **ii** DB

Now you try

Example 17
a $\angle ACE = \angle BCD$ (common)
$\angle CAE = \angle CBD$ (corresponding angles in parallel lines)
$\angle AEC = \angle BDC$ (corresponding angles in parallel lines)
$\therefore \Delta ACE \mathrel{|||} \Delta BCD$ (AAA)
b $\angle CAD = \angle EAB$ (common)
$\frac{AD}{BA} = \frac{9}{3} = 3$
$\frac{AC}{AE} = \frac{7.5}{2.5} = 3$
$\therefore \Delta ACD \mathrel{|||} \Delta AEB$ (SAS)

Example 18
a All angles are the same (AAA).
b 8 m

Exercise 7H

1 a $\angle AEB = \angle CDB$ (alternate angles, $EA \parallel DC$)
$\angle EAB = \angle DCB$ (alternate angles, $EA \parallel DC$)
$\angle EBA = \angle DBC$ (vertically opposite)
$\therefore \Delta AEB \mathrel{|||} \Delta CDB$ (AAA)
b $\angle C$ is common
$\frac{CA}{CD} = \frac{6}{2} = 3$
$\frac{CE}{CB} = \frac{9}{3} = 3$
$\therefore \Delta CDB \mathrel{|||} \Delta CAE$ (SAS)
2 a $\angle BAC = \angle DEC$ (alternate angles, $AB \parallel DE$)
$\angle ABC = \angle EDC$ (alternate angles, $AB \parallel DE$)
$\angle ACB = \angle ECD$ (vertically opposite)
$\therefore \Delta ACB \mathrel{|||} \Delta ECD$ (AAA)
b $\angle C$ is common
$\angle CDB = \angle CEA$ (corresponding angles, $AE \parallel BD$)
$\angle CBD = \angle CAE$ (corresponding angles, $AE \parallel BD$)
$\therefore \Delta CBD \mathrel{|||} \Delta CAE$ (AAA)
c $\angle A$ is common
$\angle ABE = \angle ADC$ (given)
$\therefore \Delta ABE \mathrel{|||} \Delta ADC$ (AA)
(note 2 angles is enough – $\angle AEB = \angle ACD$ (angle sum of a triangle))
3 a $\angle D$ is common
$\frac{AD}{CD} = \frac{28}{7} = 4$
$\frac{DB}{DE} = \frac{48}{12} = 4$
$\therefore \Delta ABD \mathrel{|||} \Delta CED$ (SAS)
b $\angle DCE = \angle BCA$ (vertically opposite)
$\frac{EC}{AC} = \frac{2}{5}$
$\frac{DC}{BC} = \frac{3}{7.5} = \frac{2}{5}$
$\therefore \Delta DCE \mathrel{|||} \Delta BCA$ (SAS)
4 a AAA **b** 40 m
5 a AAA **b** 7.5 m
6 6 m **7** 20 m **8** 7.2 m **9** $\frac{55}{6}$
10 a $\angle ADC = \angle ACD = 80°$ (base angles of isosceles ΔADC)
$\angle ACB = 100°$ (straight angle)
$\angle CAB = 60°$ (angle sum of ΔACB)
Now $\angle DAB = 80°$
Proof $\angle DAC = \angle DBA$ (given 20°)
$\angle D$ is common
$\angle ACD = \angle BAD$ (both 80°)
$\therefore \Delta ACD \mathrel{|||} \Delta BAD$ (AAA)
b $DC = \frac{20}{3}$ $CB = \frac{25}{3}$

11 **a** **i** $\angle B$ is common

$\angle DAB = \angle ACB$ (given 90°)

$\therefore \Delta ABD \mathrel{|||} \Delta CBA$ (AAA)

ii $\angle D$ is common

$\angle DCA = \angle DAB$ (given 90°)

$\therefore \Delta ABD \mathrel{|||} \Delta CAD$ (AAA)

b **i** $BD = \frac{25}{3}$ **ii** $AC = 4$ **iii** $AB = \frac{20}{3}$

12 **a** $\angle ACB = \angle ECD$ (vertically opposite)

$\angle CAB = \angle CED$ (alternate angles, $DE \mathrel{|||} BA$)

$\therefore \Delta ABC \mathrel{|||} \Delta EDC$ (AAA)

$\therefore \frac{DC}{BC} = \frac{EC}{AC}$ (ratio of corresponding sides in similar triangles)

$\frac{6}{2} = \frac{EC}{AC}$

$\therefore 3AC = CE$

as $AC + CE = AE$

$AE = 4AC$

b $\angle C$ is common

$\angle DBC = \angle AEC$ (given)

$\therefore \Delta CBD \mathrel{|||} \Delta CEA$ (AAA)

$\therefore \frac{DB}{AE} = \frac{2}{4} = \frac{BC}{CE}$ (ratio of corresponding sides in similar triangles)

$\therefore 4BC = 2CE$

$BC = \frac{1}{2}CE$

c $\angle C$ is common

$\angle CBD = \angle CAE$ (corresponding angles, $BD \parallel AE$)

$\therefore \Delta CBD \mathrel{|||} \Delta CAE$ (AAA)

$\therefore \frac{CB}{CA} = \frac{CD}{CE}$ (ratio of corresponding sides in similar triangles)

$\therefore \frac{5}{7} = \frac{CD}{CE}$

$\therefore 5CE = 7CD$

$\therefore CE = \frac{7}{5}CD$

d $\angle C$ is common

$\angle CBD = \angle CAE$ (given 90°)

$\therefore \Delta CBD \mathrel{|||} \Delta CAE$ (AAA)

$\therefore \frac{BD}{AE} = \frac{CE}{CA}$ (ratio of corresponding sides in similar triangles)

$\frac{2}{8} = \frac{CB}{CA}$

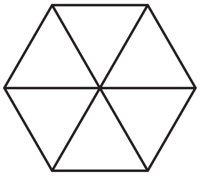
$\frac{1}{4} = \frac{CB}{CB + AB}$

$CB + AB = 4CB$

$\therefore AB = 3CB$

Problems and challenges

1



2 15

3 40°

4 A, B and C should be placed where the three altitudes of the triangle intersect the three sides.

5 45°

6 11

7 $\frac{120}{7}$

Short-answer questions

1 **a** isosceles, $x = 50$, $y = 80$

b right angled, $x = 25$

c obtuse angled, $x = 30$, $y = 110$

2 **a** $a = 30$ (vertically opposite) $b = 150$ (straight angle)

b $x = 60$ (revolution) $y = 120$ (cointerior angles in parallel lines)

c $a = 70$ (alternate angles and parallel lines)

$b = 55$ (angle sum of isosceles triangle)

$c = 55$ (corresponding angles and parallel lines)

3 $\angle ABC = 75°$

4 **a** $a = 70$, $b = 110$ **b** $x = 15$

c $x = 30$ **d** $a = 120$

5 **a** SSS, $x = 60$ **b** not congruent

c RHS, $x = 8$ **d** AAS, $x = 9$

6 **a** AD is common

$\angle ADC = \angle ADB$ (given 90°)

$CD = BD$ (given)

$\therefore \Delta ADC \equiv \Delta ADB$ (SAS)

b **i** $AC = EC$ (given)

$\angle BAC = \angle DEC$ (given)

$\angle ACB = \angle ECD$ (vertically opposite)

$\therefore \Delta ACB \equiv \Delta ECD$ (AAS)

ii as $\angle BAC = \angle DEC$ (alternate angles are equal)

$\therefore AB \parallel DE$

7 For image $\Delta A'B'C'$, $OA' = 3OA$, $OB' = 3OB$, $OC' = 3OC$

8 **a** yes, SAS **b** yes, AAA **c** not similar

9 **a** 3.5 **b** 4 **c** 18

10 **a** $\angle C$ is common

$\angle DBC = \angle EAC = 90°$ (given)

$\therefore \Delta BCD \mathrel{|||} \Delta ACE$ (AAA)

b 5 m

Multiple-choice questions

1 D	**2** A	**3** B	**4** D	**5** C
6 D	**7** E	**8** B	**9** D	**10** A

Extended-response questions

1 **a** $\angle ABC + \angle BCF = 180°$ (cointerior angles, $AB \parallel CF$)

$\therefore \angle BCF = 70°$

$\therefore \angle DCF = 32°$

Now $\angle EDC + \angle DCF = 32° + 148°$
$= 180°$

$\therefore DE \parallel CF$ as cointerior angles add to 180°

b $\angle CDA = 40°$ (revolution)

reflex $\angle BCD = 270°$ (revolution)

$\angle DAB = 30°$ (angle sum in isosceles triangle)
$BADC$ quadrilateral angle sum 360°
$\therefore a = 360 - (30 + 270 + 40)$
$= 20$
or other proof

c $AB = CD$ (given)
$\angle ABC = \angle DCB$ (given)
BC is common
$\therefore \Delta ABC \equiv \Delta DCB$ (SAS)
$\therefore AC = BD$ (corresponding sides in congruent triangles)

2 **a** $\angle ECD$
b $\angle ABC = \angle EDC$ (given 90°)
$\angle ACB = \angle ECD$ (vertically opposite)
$\therefore \Delta ABC \mathbin{|||} \Delta EDC$ (AAA)
c 19.8 m

Chapter 8

8A

Building understanding

1 **a** $x^2, 2x, 3x, 6$
b $x^2 + 3x + 2x + 6 = x^2 + 5x + 6$
2 **a** $2x^2, 2x, 3x, 3$
b $(2x + 3)(x + 1) = 2x^2 + 2x + 3x + 3$
$= 2x^2 + 5x + 3$
3 **a** $x^2 + 5x + x + 5 = x^2 + 6x + 5$
b $x^2 + 2x - 3x - 6 = x^2 - x - 6$
c $21x^2 + 6x - 14x - 4 = 21x^2 - 8x - 4$
d $12x^2 - 16x - 3x + 4 = 12x^2 - 19x + 4$

Now you try

Example 1
a $x^2 + 6x + 8$
b $x^2 + 2x - 15$
c $2x^2 - 11x + 12$
d $12x^2 - 11x - 15$

Exercise 8A

1 **a** **i** $x^2 + 7x + 12$ **ii** $x^2 + 9x + 14$
b **i** $x^2 + 4x - 12$ **ii** $x^2 + x - 12$
c **i** $2x^2 - 9x + 10$ **ii** $3x^2 - 14x + 8$
d **i** $10x^2 + 7x - 12$ **ii** $28x^2 + 23x - 15$
2 **a** $x^2 + 7x + 10$ **b** $b^2 + 7b + 12$ **c** $t^2 + 15t + 56$
d $a^2 + 8a + 7$ **e** $y^2 + 12y + 20$ **f** $m^2 + 16m + 48$
g $x^2 - x - 12$ **h** $x^2 - 4x - 32$ **i** $x^2 - 4x - 12$
j $x^2 + 9x - 10$ **k** $x^2 - 3x + 2$ **l** $x^2 - 9x + 20$
3 **a** $8x^2 + 26x + 15$ **b** $6x^2 + 7x + 2$
c $15x^2 + 17x + 4$ **d** $6x^2 + x - 15$
e $24x^2 + 23x - 12$ **f** $6x^2 - x - 2$
g $10x^2 - 31x - 14$ **h** $6x^2 + 5x - 6$
i $16x^2 - 16x - 5$ **j** $18x^2 - 27x + 10$
k $15x^2 - 11x + 2$ **l** $21x^2 - 37x + 12$
4 **a** $a^2 + ac + ab + bc$ **b** $a^2 + ac - ab - bc$
c $yz - y^2 - xz + xy$ **d** $2x^2 - 3xy - 2y^2$
e $2a^2 - ab - b^2$ **f** $6x^2 + xy - y^2$
g $6a^2 + 4a - 3ab - 2b$ **h** $12x^2 - 25xy + 12y^2$
i $3x^2y - yz^2 - 2xyz$
5 **a** $x^2 + 9x + 20$
b **i** 56 m^2 **ii** 36 m^2
6 **a** $2x^2$ **b** $2x^2 - 30x + 100$
7 **a** $150 - 50x + 4x^2$ **b** 66 m^2
8 **a** 3 **b** 2
c 6, 6 **d** 2, 18
e 2, 6 **f** 3, 15
g $2x, 5x, 3$ **h** $3x, 15x, 4$
i $7x, 3, 17x$ **j** $3x, 4, 11x$
9 **a** $a = 3, b = 2$ or $a = 2, b = 3$
b $a = -3, b = -2$ or $a = -2, b = -3$
c $a = 3, b = -2$ or $a = -2, b = 3$
d $a = 2, b = -3$ or $a = -3, b = 2$
10 **a** $x^3 + 2x^2 + 2x + 1$ **b** $x^3 - 3x^2 + 5x - 6$
c $4x^3 - 4x^2 + 9x - 4$ **d** $x^3 + 2x^2 - 2x + 3$
e $10x^3 - 17x^2 + 7x - 6$ **f** $8x^3 - 18x^2 + 35x - 49$
g $x^3 + ax - a^2x + a^2$ **h** $x^3 - 2ax^2 + a^3$
i $x^3 + a^3$ **j** $x^3 - a^3$
11 $x^3 + 6x^2 + 11x + 6$

8B

Building understanding

1 **a** $+3x + 9 = x^2 + 6x + 9$
b $+5x + 25 = x^2 + 10x + 25$
c $-2x + 4 = x^2 - 4x + 4$
d $-7x + 49 = x^2 - 14x + 49$
2 **a** **i** $x^2 + 6x + 9$
ii $x^2 + 22x + 121$
iii $x^2 + 30x + 225$
b **i** $x^2 - 4x + 4$
ii $x^2 - 18x + 81$
iii $x^2 - 60x + 900$
3 **a** $+4x - 16 = x^2 - 16$
b $-10x - 100 = x^2 - 100$
c $+2x - 2x - 1 = 4x^2 - 1$
d $-12x + 12x - 16 = 9x^2 - 16$

Now you try

Example 2
a $x^2 - 6x + 9$ **b** $9x^2 + 24x + 16$
Example 3
a $x^2 - 16$ **b** $25x^2 - 9y^2$

Exercise 8B

1 **a** **i** $x^2 - 8x + 16$ **ii** $x^2 + 6x + 9$
b **i** $9x^2 + 12x + 4$ **ii** $16x^2 - 40x + 25$
2 **a** $x^2 + 2x + 1$ **b** $x^2 + 6x + 9$ **c** $x^2 + 4x + 4$
d $x^2 + 10x + 25$ **e** $x^2 + 8x + 16$ **f** $x^2 + 18x + 81$
g $x^2 - 4x + 4$ **h** $x^2 - 12x + 36$ **i** $x^2 - 2x + 1$
j $x^2 - 6x + 9$ **k** $x^2 - 18x + 81$ **l** $x^2 - 14x + 49$

3 **a** $4x^2 + 4x + 1$ **b** $4x^2 + 20x + 25$
c $9x^2 + 12x + 4$ **d** $9x^2 + 6x + 1$
e $25x^2 + 20x + 4$ **f** $16x^2 + 24x + 9$
g $49 + 28x + 4x^2$ **h** $25 + 30x + 9x^2$
i $4x^2 - 12x + 9$ **j** $9x^2 - 6x + 1$
k $16x^2 - 40x + 25$ **l** $4x^2 - 36x + 81$
m $9x^2 + 30xy + 25y^2$ **n** $4x^2 + 16xy + 16y^2$
o $49x^2 + 42xy + 9y^2$ **p** $36x^2 + 60xy + 25y^2$
q $16x^2 - 72xy + 81y^2$ **r** $4x^2 - 28xy + 49y^2$
s $9x^2 - 60xy + 100y^2$ **t** $16x^2 - 48xy + 36y^2$
u $81x^2 - 36xy + 4y^2$

4 **a** $9 - 6x + x^2$ **b** $25 - 10x + x^2$
c $1 - 2x + x^2$ **d** $36 - 12x + x^2$
e $121 - 22x + x^2$ **f** $64 - 32x + 4x^2$
g $4 - 12x + 9x^2$ **h** $81 - 36x + 4x^2$
i $100 - 80x + 16x^2$

5 **a** $x^2 - 1$ **b** $x^2 - 9$ **c** $x^2 - 64$
d $x^2 - 16$ **e** $x^2 - 144$ **f** $x^2 - 121$
g $x^2 - 81$ **h** $x^2 - 25$ **i** $x^2 - 36$
j $25 - x^2$ **k** $4 - x^2$ **l** $49 - x^2$

6 **a** $9x^2 - 4$ **b** $25x^2 - 16$ **c** $16x^2 - 9$
d $49x^2 - 9y^2$ **e** $81x^2 - 25y^2$ **f** $121x^2 - y^2$
g $64x^2 - 4y^2$ **h** $100x^2 - 81y^2$ **i** $49x^2 - 25y^2$
j $36x^2 - 121y^2$ **k** $64x^2 - 9y^2$ **l** $81x^2 - 16y^2$

7 **a** **i** x^2 **ii** $x^2 - 4$
b No, they differ by 4.

8 **a** $20 - 2x$
b $(20 - 2x)(20 - 2x) = 400 - 80x + 4x^2$
c 196 cm^2 **d** 588 cm^3

9 **a** $a + b$
b $(a + b)(a + b) = a^2 + 2ab + b^2$
c $a - b$
d $(a - b)(a - b) = a^2 - 2ab + b^2$
e $4ab$
f ab, so yes area of four courts is $4ab$

10 **a** $x^2 - 1$
b No, area of rectangle is 1 square unit less.

11 **a** $a^2 - b^2$
b **i** $(a - b)^2 = a^2 - 2ab + b^2$
ii $b(a - b) = ab - b^2$
iii $b(a - b) = ab - b^2$
c yes, $a^2 - 2ab + b^2 + ab - b^2 + ab - b^2 = a^2 - b^2$

12 **a** $x^2 + 4x$ **b** $-4x + 1$
c $x^2 + 6x - 9$ **d** $-x^2 - 2x$
e 1 **f** $4x$
g $-12x - 8$ **h** $-10x + 2$
i $x^2 + 4xy - y^2$ **j** $8x^2 + 18$
k $-8x$ **l** $2x^2 - 12x + 18$
m $9x^2 - 48x + 48$ **n** $x^2 + 6xy + y^2$

8C

Building understanding

1 **a** 4 **b** 6 **c** 25 **d** 8
2 **a** x **b** $2a$ **c** $-3x$ **d** $-2x$
3 **a** **i** 6 **ii** $3x$ **iii** $6x$
b iii
c terms have no common factor

Now you try

Example 4
a $2x$ **b** $5x$

Example 5
a $7(4 - 3a)$ **b** $-3x(3x + 5)$

Example 6
a $(a + b)(4 + a)$ **b** $(4x + 3)(1 - x)$

Exercise 8C

1 **a** **i** $5a$ **ii** $3b$
b **i** $2x$ **ii** $5x$

2 **a** $2x$ **b** $6a$ **c** 2 **d** 4
e 3 **f** 1 **g** $3x$ **h** $3n$
i $2y$ **j** $2x$ **k** $2xy$ **l** $5ab$

3 **a** $7(x + 1)$ **b** $3(x + 1)$ **c** $4(x - 1)$
d $5(x - 1)$ **e** $4(1 + 2y)$ **f** $5(2 + a)$
g $3(1 - 3b)$ **h** $2(3 - x)$ **i** $3(4a + b)$
j $6(m + n)$ **k** $2(5x - 4y)$ **l** $4(a - 5b)$
m $x(x + 2)$ **n** $a(a - 4)$ **o** $y(y - 7)$
p $x(1 - x)$ **q** $3p(p + 1)$ **r** $8x(1 - x)$
s $4b(b + 3)$ **t** $2y(3 - 5y)$ **u** $3a(4 - 5a)$
v $9m(1 + 2m)$ **w** $16x(y - 3x)$ **x** $7ab(1 - 4b)$

4 **a** $-4(2x + 1)$ **b** $-2(2x + 1)$
c $-5(2x + y)$ **d** $-7(a + 2b)$
e $-3(3x + 4)$ **f** $-2(3y + 4)$
g $-5(2x + 3y)$ **h** $-4(m + 5n)$
i $-3x(x + 6)$ **j** $-4x(2x + 3)$
k $-2y(8y + 3)$ **l** $-5a(a + 2)$
m $-2x(3 + 10x)$ **n** $-3p(2 + 5p)$
o $-8b(2 + b)$ **p** $-9x(1 + 3x)$

5 **a** $(x + 3)(4 + x)$ **b** $(x + 1)(3 + x)$
c $(m - 3)(7 + m)$ **d** $(x - 7)(x + 2)$
e $(a + 4)(8 - a)$ **f** $(x + 1)(5 - x)$
g $(y + 3)(y - 2)$ **h** $(x + 2)(a - x)$
i $(2t + 5)(t + 3)$ **j** $(5m - 2)(m + 4)$
k $(4y - 1)(y - 1)$ **l** $(7 - 3x)(1 + x)$

6 **a** $6(a + 5)$ **b** $5(x - 3)$
c $2(4b + 9)$ **d** $x(x - 4)$
e $y(y + 9)$ **f** $a(a - 3)$
g $xy(x - 4 + y)$ **h** $2ab(3 - 5a + 4b)$
i $(m + 5)(m + 2)$ **j** $(x + 3)(x - 2)$
k $(b - 2)(b + 1)$ **l** $(2x + 1)(x - 1)$
m $(3 - 2y)(y - 5)$ **n** $(x + 4)(x + 9)$
o $(y + 1)(y - 3)$

7 **a** $4(x + 2)$ **b** $2(x + 3)$ **c** $10(x + 2)$
d $2(x + 7)$ **e** $2(2x + 3)$ **f** $2(x + 7)$

8 $4x$

9 **a** $t(5 - t)$
b **i** 0 m **ii** 4 m **iii** 6 m
c 5 seconds

10 a 63 **b** 72 **c** −20
d −70 **e** 69 **f** 189
11 a $3(a^2 + 3a + 4)$ **b** $z(5z - 10 + y)$
c $x(x - 2y + xy)$ **d** $2b(2y - 1 + 3b)$
e $-4y(3x + 2z + 5xz)$ **f** $ab(3 + 4b + 6a)$
12 a $-4(x - 3) = 4(3 - x)$
b $-3(x - 3) = 3(3 - x)$
c $-8(n - 1) = 8(1 - n)$
d $-3(b - 1) = 3(1 - b)$
e $-5m(1 - m) = 5m(m - 1)$
f $-7x(1 - x) = 7x(x - 1)$
g $-5x(1 - x) = 5x(x - 1)$
h $-2y(2 - 11y) = 2y(11y - 2)$
13 a $(x - 4)(x - 3)$ **b** $(x - 5)(x + 2)$
c $(x - 3)(x + 3)$ **d** $(x - 4)(3x - 5)$
e $(2x - 5)(3 - x)$ **f** $(x - 2)(2x - 1)$
g $(x - 3)(4 - x)$ **h** $(x - 5)(x - 2)$
i $(x - 3)(x - 2)$

8D

Building understanding

1 a $x^2 - 4$ **b** $x^2 - 49$ **c** $9x^2 - y^2$
2 a 3 **b** 11 **c** $5b$ **d** $7y$
3 a $(x + 4)(x - 4)$
b $x^2 - (12)^2 = (x + 12)(x - 12)$
c $(4x)^2 - (1)^2 = (4x + 1)(4x - 1)$
d $(3a)^2 - (2b)^2 = (3a + 2b)(3a - 2b)$

Now you try

Example 7
a $(x + 4)(x - 4)$ **b** $(2a + 3)(2a - 3)$ **c** $(5a + b)(5a - b)$
Example 8
a $3(a + 3)(a - 3)$ **b** $(x - 1)(x + 5)$

Exercise 8D

1 a i $(x + 7)(x - 7)$ **ii** $(x + 6)(x - 6)$
b i $(4y + 9)(4y - 9)$ **ii** $(5y + 2)(5y - 2)$
c i $(6a + b)(6a - b)$ **ii** $(x + 3y)(x - 3y)$
2 a $(x + 3)(x - 3)$ **b** $(y + 5)(y - 5)$
c $(y + 1)(y - 1)$ **d** $(x + 8)(x - 8)$
e $(x + 4)(x - 4)$ **f** $(b + 7)(b - 7)$
g $(a + 9)(a - 9)$ **h** $(x + y)(x - y)$
i $(a + b)(a - b)$ **j** $(4 + a)(4 - a)$
k $(5 + x)(5 - x)$ **l** $(1 + b)(1 - b)$
m $(6 + y)(6 - y)$ **n** $(11 + b)(11 - b)$
o $(x + 20)(x - 20)$ **p** $(30 + y)(30 - y)$
3 a $(2x + 5)(2x - 5)$ **b** $(3x + 7)(3x - 7)$
c $(5b + 2)(5b - 2)$ **d** $(2m + 11)(2m - 11)$
e $(10y + 3)(10y - 3)$ **f** $(9a + 2)(9a - 2)$
g $(1 + 2x)(1 - 2x)$ **h** $(5 + 8b)(5 - 8b)$
i $(4 + 3y)(4 - 3y)$ **j** $(6x + y)(6x - y)$
k $(2x + 5y)(2x - 5y)$ **l** $(8a + 7b)(8a - 7b)$
m $(2p + 5q)(2p - 5q)$ **n** $(9m + 2n)(9m - 2n)$
o $(5a + 7b)(5a - 7b)$ **p** $(10a + 3b)(10a - 3b)$
4 a $3(x + 6)(x - 6)$ **b** $10(a + 1)(a - 1)$
c $6(x + 2)(x - 2)$ **d** $4(y + 4)(y - 4)$
e $2(7 + x)(7 - x)$ **f** $8(2 + m)(2 - m)$
g $5(xy + 1)(xy - 1)$ **h** $3(1 + xy)(1 - xy)$
i $7(3 + ab)(3 - ab)$
5 a $(x + 8)(x + 2)$ **b** $(x + 5)(x + 1)$
c $(x + 14)(x + 6)$ **d** $(x + 2)(x - 8)$
e $(x - 6)(x - 8)$ **f** $(x + 3)(x - 9)$
g $(10 + x)(4 - x)$ **h** $-x(x + 4)$
i $(17 + x)(1 - x)$
6 a $4(3 + t)(3 - t)$
b i 36 m **ii** 20 m
c 3 seconds
7 a i x^2 **ii** $(30 + x)(30 - x)$
b i 500 cm^2 **ii** 675 cm^2
8 a $(x + 3)(x - 3)$ **b** $(4x + 11)(4x - 11)$
c $(2 + 5a)(2 - 5a)$ **d** $(x + y)(x - y)$
e $(2b + 5a)(2b - 5a)$ **f** $(c + 6ab)(c - 6ab)$
g $(yz + 4x)(yz - 4x)$ **h** $(b + 30a)(b - 30a)$
9 a Factorise each binomial:
$$\begin{aligned}(4x + 2)(4x - 2) &= 2(2x + 1)\,2(2x - 1)\\ &= 4(2x + 1)(2x - 1)\end{aligned}$$
b Take out common factor of 4.
10 $9 - (x - 1)^2$
$= (3 + x - 1)(3 - (x - 1))$ insert brackets when subtracting a binomial
$= (2 + x)(3 - x + 1)$ remember $-1 \times -1 = +1$
$= (2 + x)(4 - x)$
11 a $\left(x + \frac{1}{2}\right)\left(x - \frac{1}{2}\right)$ **b** $\left(x + \frac{2}{5}\right)\left(x - \frac{2}{5}\right)$
c $\left(5x + \frac{3}{4}\right)\left(5x - \frac{3}{4}\right)$ **d** $\left(\frac{x}{3} + 1\right)\left(\frac{x}{3} - 1\right)$
e $\left(\frac{a}{2} + \frac{b}{3}\right)\left(\frac{a}{2} - \frac{b}{3}\right)$ **f** $5\left(\frac{x}{3} + \frac{1}{2}\right)\left(\frac{x}{3} - \frac{1}{2}\right)$
g $7\left(\frac{a}{5} + \frac{2b}{3}\right)\left(\frac{a}{5} - \frac{2b}{3}\right)$ **h** $\frac{1}{2}\left(\frac{a}{2} + \frac{b}{3}\right)\left(\frac{a}{2} - \frac{b}{3}\right)$
i $(x + y)(x - y)(x^2 + y^2)$ **j** $2(a + b)(a - b)(a^2 + b^2)$
k $21(a + b)(a - b)(a^2 + b^2)$ **l** $\frac{1}{3}(x + y)(x - y)(x^2 + y^2)$

8E

Building understanding

1 a $2x - 2$ **b** $-5 + 5a$
c $a^2 + 5a$ **d** $4y - y^2$
e $ax + x + 2a + 2$ **f** $bx - 2b - 3x + 6$
2 a $2 + x$ **b** $5 - x$
c $a + 1$ **d** $1 - a$
3 a $(x - 3)(x - 2)$ **b** $(x + 4)(x + 3)$
c $(2x + 1)(3 - x)$ **d** $(3x - 2)(4 - x)$
e $(5 - x)(3x + 2)$ **f** $(x - 2)(x + 1)$

Now you try

Example 9

a $(x + 4)(x + 2)$ **b** $(x + 2)(x - 3)$

Example 10

$(x - 5)(2x + 3)$

Exercise 8E

1 a i $(x + 5)(x + 4)$ **ii** $(x + 6)(x + 3)$

b i $(x + 4)(x - 5)$ **ii** $(x + 6)(x - 2)$

2 a $(x + 3)(x + 2)$ **b** $(x + 4)(x + 3)$

c $(x + 7)(x + 2)$ **d** $(x - 6)(x + 4)$

e $(x - 4)(x + 6)$ **f** $(x - 3)(x + 10)$

g $(x + 2)(x - 18)$ **h** $(x + 3)(x - 14)$

i $(x + 4)(x - 18)$ **j** $(x - 2)(x - a)$

k $(x - 3)(x - 3c)$ **l** $(x - 5)(x - 3a)$

3 a $(3a + 5c)(b + d)$ **b** $(4b - 7c)(a + d)$

c $(y - 4z)(2x + 3w)$ **d** $(s - 2)(5r + t)$

e $(x + 3y)(4x - 3)$ **f** $(2b - a)(a - c)$

4 a $(x - b)(x + 1)$ **b** $(x - c)(x + 1)$

c $(x + b)(x + 1)$ **d** $(x + c)(x - 1)$

e $(x + a)(x - 1)$ **f** $(x - b)(x - 1)$

5 a $(x - 7)(2x + 1)$ **b** $(x + 5)(x + 2)$

c $(x + 3)(2x - 1)$ **d** $(1 - 2x)(3x + 4)$

e $(x - 5)(11 + a)$ **f** $(3 - 2x)(4y - 1)$

g $(n + 2)(3m - 1)$ **h** $(3 - r)(5p + 8)$

i $(2 - y)(8x + 3)$

6 a $x^2 + 4x - ax - 4a$ **b** $x^2 - dx - cx + cd$

c $2x - xz + 2y - yz$ **d** $ax + bx - a - b$

e $3cx - 3bx - bc + b^2$ **f** $2xy + 2xz - y^2 - yz$

g $6ab + 15ac + 2b^2 + 5bc$ **h** $3my + mz - 6xy - 2xz$

7 a $(x + 2)(x + 5)$ **b** $(x + 3)(x + 5)$

c $(x + 4)(x + 6)$ **d** $(x - 3)(x + 2)$

e $(x + 6)(x - 2)$ **f** $(x - 9)(x - 2)$

8 a Method 1: $a(x + 7) - 3(x + 7) = (x + 7)(a - 3)$

Method 2: $x(a - 3) + 7(a - 3) = (a - 3)(x + 7)$

b i $(x - 3)(b + 2)$ **ii** $(x + 2)(y - 4)$

iii $(2m + 3)(2m - 5n)$ **iv** $(2 - n)(m - 3)$

v $(1 - 2b)(4a + 3b)$ **vi** $(3a - 1)(b + 4c)$

9 Answers may vary.

10 a $(a - 3)(2 - x - c)$ **b** $(2a + 1)(b + 5 - a)$

c $(a + 1)(x - 4 - b)$ **d** $(a - b)(3 - b - 2a)$

e $(1 - a)(c - x + 2)$ **f** $(x - 2)(a + 2b - 1)$

g $(a - 3c)(a - 2b + 3bc)$ **h** $(1 - 2y)(3x - 5z + y)$

i $(x - 4)(3x + y - 2z)$ **j** $(ab - 2c)(2x + 3y - 1)$

8F

Building understanding

1 a $x^2 + 9x + 14$ **b** $x^2 + 14x + 33$

c $x^2 + 7x - 60$ **d** $x^2 - 8x + 12$

2 a 3, 2 **b** 12, 1 **c** 4, 5

d 5, –3 **e** –6, 5 **f** –9, –2

Now you try

Example 11

a $(x + 2)(x + 7)$ **b** $(x + 6)(x - 2)$ **c** $(x - 3)(x - 4)$

Example 12

$3(x + 3)(x - 5)$

Exercise 8F

1 a i $(x + 2)(x + 4)$ **ii** $(x + 1)(x + 2)$

b i $(x + 5)(x - 2)$ **ii** $(x - 5)(x + 3)$

c i $(x - 3)(x - 6)$ **ii** $(x - 1)(x - 3)$

2 a $(x + 6)(x + 2)$ **b** $(x + 9)(x + 1)$

c $(x + 7)(x + 1)$ **d** $(x + 14)(x + 1)$

e $(x + 3)(x + 4)$ **f** $(x + 8)(x + 2)$

g $(x + 5)(x + 3)$ **h** $(x + 4)(x + 5)$

i $(x + 8)(x + 3)$

3 a $(x + 4)(x - 1)$ **b** $(x + 2)(x - 1)$

c $(x + 5)(x - 1)$ **d** $(x + 7)(x - 2)$

e $(x + 5)(x - 3)$ **f** $(x + 10)(x - 2)$

g $(x + 6)(x - 3)$ **h** $(x + 9)(x - 2)$

i $(x + 4)(x - 3)$

4 a $(x - 5)(x - 1)$ **b** $(x - 1)(x - 1)$

c $(x - 1)(x - 4)$ **d** $(x - 8)(x - 1)$

e $(x - 2)(x - 2)$ **f** $(x - 6)(x - 2)$

g $(x - 9)(x - 2)$ **h** $(x - 7)(x - 3)$

i $(x - 3)(x - 2)$

5 a $(x - 8)(x + 1)$ **b** $(x - 4)(x + 1)$

c $(x - 6)(x + 1)$ **d** $(x - 8)(x + 2)$

e $(x - 6)(x + 4)$ **f** $(x - 5)(x + 3)$

g $(x - 4)(x + 3)$ **h** $(x - 12)(x + 1)$

i $(x - 6)(x + 2)$

6 a $2(x + 4)(x + 1)$ **b** $2(x + 10)(x + 1)$

c $3(x + 2)(x + 4)$ **d** $2(x + 10)(x - 3)$

e $2(x - 9)(x + 2)$ **f** $4(x - 1)(x - 1)$

g $2(x - 2)(x + 3)$ **h** $6(x - 6)(x + 1)$

i $5(x - 4)(x - 2)$ **j** $3(x - 5)(x - 6)$

k $2(x - 5)(x + 2)$ **l** $3(x - 4)(x + 3)$

7 a $6x$ **b** $6x$ or $10x$ **c** $x, 4x, 11x$

d $x, 4x, 11x$ **e** $9x, 11x, 19x$ **f** $9x, 11x, 19x$

g $0x, 6x, 15x$ **h** $0x, 24x$

8 a i $x(x + 2)$ **ii** $x^2 + 2x - 15$ **iii** $(x + 5)(x - 3)$

b i 105 m^2 **ii** 48 m^2

9 a $(x + 4)^2$ **b** $(x + 5)^2$ **c** $(x + 15)^2$

d $(x - 1)^2$ **e** $(x - 7)^2$ **f** $(x - 13)^2$

g $2(x + 1)^2$ **h** $5(x - 3)^2$ **i** $-3(x - 6)^2$

10 B, C, E, F

11 a $(x - 1)^2 - 9$ **b** $(x + 2)^2 - 5$

c $(x + 5)^2 - 22$ **d** $(x - 8)^2 - 67$

e $(x + 9)^2 - 74$ **f** $(x - 16)^2 - 267$

Progress quiz

1 a $x^2 + 6x + 8$ **b** $a^2 + 3a - 40$

c $2x^2 + 9x - 18$ **d** $3a^2 - 7ab + 2b^2$

2 a $y^2 + 8y + 16$ **b** $x^2 - 6x + 9$

c $4a^2 - 12a + 9$ **d** $49k^2 + 28km + 4m^2$

3 a $x^2 - 25$ **b** $121x^2 - 81y^2$

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4 **a** $5(5a-3)$ **b** $x(x-7)$
c $-4x(3x+4)$ **d** $(a+3)(2+a)$
e $(8+a)(7-a)$ **f** $(k-4)(k-1)$
5 **a** $(x+9)(x-9)$ **b** $(4a+7)(4a-7)$
c $(5x+y)(5x-y)$ **d** $2(a+5)(a-5)$
e $12(xy+1)(xy-1)$ **f** $(h+11)(h-5)$
6 **a** $(x+7)(x+2)$ **b** $(a+5)(a-4)$ **c** $(x-h)(x+1)$
7 **a** $(x-3)(2x+3)$ **b** $(3a+2)(p-5)$
8 **a** $(x+4)(x+2)$ **b** $(a+5)(a-3)$
c $(m-6)(m-5)$ **d** $2(k-3)(k+4)$

8G

Building understanding

1 **a** 2, 6 **b** 8, –3
c –5, –7 **d** –7, 4
2 **a** $= 2x^2+2x+5x+5$
$= 2x(x+1)+5(x+1)$
$= (x+1)(2x+5)$
b $= 2x^2-3x-4x+6$
$= x(2x-3)-2(2x-3)$
$= (2x-3)(x-2)$
c $= 5x^2+10x-x-2$
$= 5x(x+2)-1(x+2)$
$= (x+2)(5x-1)$
d $= 4x^2+8x+3x+6$
$= 4x(x+2)+3(x+2)$
$= (x+2)(4x+3)$

Now you try

Example 13
$(3x+2)(x+3)$

Example 14
a $(2x+5)(2x-3)$ **b** $(4x-3)(2x-3)$

Exercise 8G

1 **a** $(2x+1)(x+5)$ **b** $(3x+1)(x+3)$
2 **a** $(2x+1)(x+4)$ **b** $(3x+1)(x+2)$
c $(2x+3)(x+2)$ **d** $(3x+2)(x+2)$
e $(5x+2)(x+2)$ **f** $(2x+3)(x+4)$
g $(3x+5)(2x+1)$ **h** $(4x+1)(x+1)$
i $(4x+5)(2x+1)$
3 **a** $(3x+5)(x-1)$ **b** $(5x-4)(x+2)$
c $(2x+3)(4x-1)$ **d** $(2x+1)(3x-8)$
e $(2x+1)(5x-4)$ **f** $(x-3)(5x+4)$
g $(2x-5)(2x-3)$ **h** $(x-6)(2x-3)$
i $(2x-5)(3x-2)$ **j** $(3x-4)(4x+1)$
k $(2x-3)(2x-3)$ **l** $(x+3)(7x-3)$
m $(x+5)(9x-1)$ **n** $(x-2)(3x-8)$
o $(2x-5)(2x+3)$
4 **a** $(2x+1)(5x+11)$ **b** $(3x+4)(5x-2)$
c $(2x-3)(10x-3)$ **d** $(2x+1)(9x-5)$
e $(5x-3)(5x+4)$ **f** $(4x+1)(8x-5)$
g $(3x+2)(9x-4)$ **h** $(3x+1)(11x+10)$
i $(6x-5)(9x+1)$ **j** $(2x-3)(6x-7)$
k $(3x-1)(25x-6)$ **l** $(6x-1)(15x+8)$
5 **a** $2(3x-2)(5x+1)$ **b** $6(x-1)(2x+5)$
c $3(3x-5)(3x-1)$ **d** $7(x-3)(3x-2)$
e $4(3x-2)(3x+5)$ **f** $5(2x-3)(5x+4)$

6 **a** $-(x-2)(2x-3)$ **b** $-(x-1)(5x+8)$
c $-(2x+1)(3x-8)$ **d** $-(x+3)(5x-6)$
e $-(2x-5)(2x-3)$ **f** $-(2x-1)(4x-5)$
7 **a** $(x+4)(2x-5)$
b $(2x-5)(x+4)$
c No, you get the same result.
d **i** $(x+3)(3x-4)$
ii $(x-2)(5x+7)$
iii $(2x-1)(3x+4)$
8 Answers may vary.
9 See answers to Questions **3** and **4**.

8H

Building understanding

1 **a** $\frac{1}{3}$ **b** $\frac{2x}{3}$ **c** $\frac{3}{x}$ **d** $\frac{x+1}{2}$
2 **a** $3(x+2)$ **b** $20(1-2x)$
c $x(x-7)$ **d** $6x(x+4)$
3 **a** $\frac{2(x-2)}{8}=\frac{x-2}{4}$ **b** $\frac{6(2-3x)}{x(2-3x)}$
4 **a** 9 **b** $\frac{1}{12}$

Now you try

Example 15
a $\frac{x-3}{3}$ **b** $\frac{3}{5}$ **c** $x-3$

Example 16
a $\frac{9}{5(x-4)}$ **b** $\frac{2(x-2)}{x}$ **c** $\frac{x-3}{3}$

Exercise 8H

1 **a** **i** $\frac{x-2}{3}$ **ii** $\frac{2}{3(x-1)}$
b **i** $\frac{4}{7}$ **ii** $\frac{2}{3}$
c **i** $x-4$ **ii** $x+7$
2 **a** $\frac{3}{4}$ **b** $\frac{1}{3}$ **c** 4
d $x-5$ **e** $\frac{2(x-1)}{3}$ **f** $\frac{2}{x+4}$
3 **a** $x-1$ **b** $\frac{2(x-3)}{5}$ **c** $\frac{2}{3}$ **d** 2
e $x-3$ **f** $\frac{2(2x+5)}{5}$ **g** $\frac{3}{2}$ **h** $\frac{4}{3}$
4 **a** $x-10$ **b** $x-7$ **c** $x-5$
d $\frac{2}{x+20}$ **e** $\frac{5}{x+6}$ **f** $\frac{3}{x-9}$
5 **a** $\frac{x-4}{2}$ **b** $x-3$ **c** $3(x-3)$ **d** $\frac{x+4}{2(x-5)}$
6 **a** x **b** $\frac{2(x-2)}{x+2}$ **c** $\frac{4}{x+1}$
d $\frac{x-2}{2(x+2)}$ **e** $\frac{2(x+2)^2}{3(x+4)}$ **f** $\frac{5}{x+2}$

7 **a** $x + 2$ **b** $x + 4$ **c** $x - 3$
d $\frac{1}{x + 3}$ **e** $\frac{1}{x - 2}$ **f** $\frac{1}{x - 10}$

8 **a** $(x + 2)(x - 5)$ **b** $\frac{x + 2}{x + 3}$ **c** $\frac{x - 3}{x - 4}$
d $\frac{x - 5}{x + 2}$ **e** $\frac{3x - 1}{2 - 15x}$ **f** $\frac{x - 4}{3 - x}$
g $\frac{x + 5}{2(x + 4)}$ **h** $\frac{3}{2(x + 3)}$

9 **a** -1 **b** -1 **c** -8
d $-\frac{1}{3}$ **e** $-\frac{1}{6}$ **f** $-(x + 3)$

10 **a** $a + 1$ **b** $5(a - 3)$ **c** $\frac{x + 7}{2}$
d $\frac{x + 2}{6}$ **e** $\frac{x + 3}{2}$ **f** $\frac{11}{x - 2}$

11 **a** $-\frac{x + 3}{2}$ **b** $-\frac{x - 7}{3}$ **c** $-\frac{x - 8}{x + 8}$
d $-\frac{2}{x + 2}$ **e** $-\frac{4}{x + 3}$ **f** $\frac{3(x + 1)}{4}$
g $\frac{3(2x + 3)}{2x}$ **h** $-\frac{1}{2(x + 2)}$ **i** $\frac{x}{3}$
j $-\frac{2}{x - 2}$

8I

Building understanding

1 **a** 24 **b** 15 **c** 143 **d** 36
2 **a** $21x$ **b** 2 **c** 8 **d** 90
3 **a** $\frac{3x}{12} + \frac{8x}{12} = \frac{11x}{12}$
b $\frac{25x}{35} - \frac{14x}{35} = \frac{11x}{35}$
c $\frac{2(x + 1)}{4} + \frac{(2x + 3)}{4} = \frac{2x + 2 + 2x + 3}{4} = \frac{4x + 5}{4}$
4 **a** 15 **b** 14 **c** 6

Now you try

Example 17
a $-\frac{5x}{12}$ **b** $\frac{11x}{8}$ **c** $\frac{5x - 8}{6}$

Example 18
a $\frac{5}{3x}$ **b** $\frac{4x + 5}{x^2}$

Exercise 8I

1 **a** **i** $-\frac{2x}{15}$ **ii** $\frac{15x}{28}$
b **i** $\frac{7x}{15}$ **ii** $\frac{11x}{18}$
c **i** $\frac{7x + 5}{12}$ **ii** $\frac{9x - 20}{14}$

2 **a** $\frac{9x}{14}$ **b** $\frac{2x}{5}$ **c** $\frac{x}{8}$ **d** $\frac{14x}{45}$
e $\frac{y}{56}$ **f** $\frac{13a}{22}$ **g** $\frac{2b}{9}$ **h** $\frac{m}{6}$
i $\frac{11m}{12}$ **j** $\frac{15a}{28}$ **k** $\frac{x}{2}$ **l** $-\frac{20p}{63}$
m $-\frac{5b}{18}$ **n** $\frac{61y}{40}$ **o** $\frac{13x}{35}$ **p** $\frac{5x}{12}$

3 **a** $\frac{7x + 11}{10}$ **b** $\frac{7x}{12}$ **c** $\frac{15a - 51}{56}$ **d** $\frac{11y + 9}{30}$
e $\frac{13m + 28}{40}$ **f** $\frac{5x - 13}{24}$ **g** $\frac{11b - 6}{24}$ **h** $\frac{7x}{6}$
i $\frac{7y - 8}{14}$ **j** $\frac{5t - 4}{16}$ **k** $\frac{34 - 10x}{21}$ **l** $\frac{8m - 9}{12}$

4 **a** $\frac{11}{2x}$ **b** $\frac{1}{3x}$ **c** $-\frac{3}{4x}$ **d** $\frac{14}{9x}$
e $\frac{7}{20x}$ **f** $\frac{13}{15x}$ **g** $-\frac{31}{4x}$ **h** $-\frac{29}{12x}$

5 **a** $\frac{3x + 2}{x^2}$ **b** $\frac{5 + 4x}{x^2}$ **c** $\frac{7x + 3}{x^2}$ **d** $\frac{4x - 5}{x^2}$
e $\frac{3 - 8x}{x^2}$ **f** $\frac{x - 4}{x^2}$ **g** $\frac{6x - 7}{2x^2}$ **h** $\frac{9 - 2x}{3x^2}$

6 **a** $\frac{8 + x^2}{4x}$ **b** $\frac{x^2 - 10}{2x}$ **c** $\frac{-6 - 4x^2}{3x}$ **d** $\frac{6 - 5x^2}{4x}$
e $\frac{9x^2 - 10}{12x}$ **f** $\frac{3 - x^2}{9x}$ **g** $\frac{15x^2 - 4}{10x}$ **h** $\frac{-25 - 6x^2}{20x}$

7 **a** $\frac{x}{3}$ **b** $\frac{x}{8}$ **c** $\frac{x}{2}$
d $\frac{x}{5}$ **e** $\frac{8x}{9}$ **f** $\frac{x}{4}$

8 **a** $\frac{4x}{3}$ **b** 200 seconds

9 **a** didn't make a common denominator, $\frac{7x}{15}$
b didn't use brackets: $2(x + 1) = 2x + 2$, $\frac{7x + 2}{10}$
c didn't use brackets: $3(x - 1) = 3x - 3$, $\frac{13x - 3}{6}$
d didn't multiply numerator in $\frac{2}{x}$ by x as well as denominator, $\frac{2x - 3}{x^2}$

10 **a** $\frac{8x + 2}{8} = \frac{2(4x + 1)}{8} = \frac{4x + 1}{4}$
b $\frac{4x + 1}{4}$
c Using denominator 8 does not give answer in simplified form and requires extra steps. Preferable to use actual LCD.

11 **a** $-\frac{43x}{30}$ **b** $\frac{5x}{12}$ **c** $\frac{13x}{24}$
d $\frac{43x - 5}{60}$ **e** $\frac{23x - 35}{42}$ **f** $\frac{29x + 28}{40}$
g $\frac{14}{3x}$ **h** $\frac{1}{6x}$ **i** $-\frac{11}{20x}$
j $\frac{24 - x}{6x^2}$ **k** $\frac{60x - 21}{14x^2}$ **l** $\frac{3 - 4x}{9x^2}$
m $\frac{30 - 2x^2}{15x}$ **n** $\frac{11x^2 - 3}{6x}$ **o** $\frac{18 - 2x^2}{45x}$

8J

Building understanding

1 a $-2x-6$ b $-14-21x$
c $-3x+3$ d $-30+20x$

2 a 9 b $2x$
c x^2 d $(x-2)(x+3)$

3 a $-x-4$ b $3x+16$ c $6-2x$

Now you try

Example 19

a $\frac{x-16}{6}$ b $\frac{10x-5}{12}$

Example 20

a $\frac{5x-5}{(x+2)(x-3)}$ b $\frac{11-3x}{(x-2)^2}$

Exercise 8J

1 a i $\frac{x-22}{20}$ ii $\frac{3x+15}{10}$
b i $\frac{13x-24}{14}$ ii $\frac{2x+23}{12}$

2 a $\frac{1-x}{12}$ b $\frac{2x-14}{15}$ c $\frac{x-9}{6}$
d $\frac{-7x-14}{10}$ e $\frac{9x-4}{8}$ f $\frac{-x-24}{28}$
g $\frac{5x-3}{12}$ h $\frac{1-18x}{15}$ i $\frac{8x-23}{30}$

3 a $\frac{13-x}{6}$ b $\frac{2x+2}{35}$ c $\frac{x-5}{4}$
d $\frac{22x-41}{21}$ e $\frac{17x}{20}$ f $\frac{30-13x}{24}$
g $\frac{14x+4}{9}$ h $\frac{20x-9}{28}$ i $\frac{10-11x}{56}$

4 a $\frac{7x-1}{(x-1)(x+1)}$ b $\frac{7x-7}{(x+4)(x-3)}$
c $\frac{7x+1}{(x-2)(x+3)}$ d $\frac{5x+13}{(x-4)(x+7)}$
e $\frac{4x+15}{(x+2)(x+3)}$ f $\frac{x-26}{(x+4)(x-6)}$
g $\frac{x+9}{(x+5)(x+1)}$ h $\frac{16-6x}{(x-3)(x-2)}$
i $\frac{7-2x}{(x-5)(x-6)}$

5 a $\frac{1-3x}{(x+1)^2}$ b $\frac{-4x-10}{(x+3)^2}$ c $\frac{3x-2}{(x-2)^2}$
d $\frac{18-2x}{(x-5)^2}$ e $\frac{9-x}{(x-6)^2}$ f $\frac{14-3x}{(x-4)^2}$
g $\frac{4x+7}{(2x+1)^2}$ h $\frac{1-12x}{(3x+2)^2}$ i $\frac{20x-1}{(1-4x)^2}$

6 a $\frac{5x-2}{(x-1)^2}$ b $\frac{19x+8}{12x}$
c $\frac{10x^2-11x+4}{20x}$ d $\frac{x^2+7x}{(x-5)(x+1)}$
e $\frac{2x^2-5x-3}{(4-x)(x-1)}$ f $\frac{x^2+2x+1}{(x-3)^2}$
g $\frac{3-2x}{(x-2)^2}$ h $\frac{-x^2-11x}{(2x+1)(x+2)}$
i $\frac{x^2-4x-1}{(x+1)^2}$

7 a $\frac{3x+11}{(x+3)(x+4)(x+5)}$ b $\frac{2-2x}{(x+1)(x+2)(x+4)}$
c $\frac{26-10x}{(x-1)(x-3)(2-x)}$ d $\frac{3x-2}{(x+1)(x-5)}$
e $\frac{2x+9}{(x-4)(3-2x)}$ f $\frac{7x^2+7x}{(x+4)(2x-1)(3x+2)}$

8 a second line $-2 \times (-2) = +4$ not -4
b $\frac{33x+4}{10}$

9 a $\frac{5}{1-x}$ b $\frac{4x-3}{5-x}$ c $\frac{9}{7x-3}$
d $\frac{1-2x}{4-3x}$ e 1 f 0

10 a $\frac{11}{2(x+2)}$ b $\frac{1}{3(x-1)}$
c $\frac{23}{4(2x-1)}$ d $\frac{13-3x}{(x+3)(x-3)}$
e $\frac{5x-6}{2(x+2)(x-2)}$ f $\frac{30x+33}{(3x-4)(3x+4)}$
g $\frac{9x-27}{(x+3)(x+4)(x-5)}$ h $\frac{x+11}{(x-1)(x+3)^2}$
i $\frac{2x+5}{5(x-2)(x-5)}$ j $\frac{-2}{x(x-1)(x+1)}$

8K

Building understanding

1 a 15 b 6 c 28

2 a $4x$ b $2(x+3)$ c 3 d $4(x-1)$

3 a multiply by 5, subtract 3 then divide by 2
b multiply by x then divide by 2

Now you try

Example 21

a $x=12$ b $x=12$

Example 22

a $x=\frac{7}{40}$ b $x=7$

Exercise 8K

1 a i $x=42$ ii $x=-4$
b i $x=-9$ ii $x=14$

ISBN 978-1-108-77288-4

2 **a** 10 **b** 12 **c** 24
d 10 **e** $\frac{6}{7}$ **f** $7\frac{1}{2}$
g −8 **h** 15 **i** 4

3 **a** 13 **b** 1 **c** $1\frac{3}{5}$
d 59 **e** 1 **f** 5
g 8 **h** $3\frac{2}{7}$ **i** $-3\frac{1}{4}$

4 **a** −3 **b** −4 **c** 5
d 1 **e** −1 **f** 6

5 **a** $\frac{1}{16}$ **b** $\frac{1}{12}$ **c** $\frac{8}{15}$
d $\frac{1}{36}$ **e** $\frac{3}{4}$ **f** $\frac{5}{24}$
g $\frac{5}{12}$ **h** $-\frac{1}{6}$ **i** $2\frac{5}{6}$

6 **a** $-\frac{5}{2}$ **b** −5 **c** −19
d −4 **e** −1 **f** −6

7 **a** $\frac{x}{2}+\frac{2x}{3}=4$ **b** $x=3\frac{3}{7}$

8 **a** $\frac{x}{3}+\frac{x}{4}=77$ **b** 132 games

9 **a** $\frac{1}{6}$ **b** 2 **c** 0
d $\frac{9}{13}$ **e** $\frac{3}{7}$ **f** $\frac{2}{11}$
g $1\frac{5}{7}$ **h** 0 **i** −12

10 **a** $\frac{1}{x}+\frac{1}{2x}=\frac{3}{10}$ **b** 5 and 10

11 On the second line, not every term has been multiplied by 12. The $2x$ should be $24x$ to give an answer $x=\frac{3}{29}$.

12 On third line of working, $-2\times(-1)=+2$ not −2, giving answer $x=-4$.

13 **a** $4\frac{1}{2}$ **b** $5\frac{2}{3}$ **c** $1\frac{1}{2}$
d 3 **e** 15 **f** −2

14 **a** $x=2ab$ **b** $x=\frac{2bd}{2a-bc}$
c $x=\frac{ac}{c-b}$ **d** $x=\frac{bd+be-ac}{c}$
e $x=\frac{4c-3b}{3a-4}$ **f** $x=\frac{6b+a}{5}$
g $x=-\frac{a^2}{2a-b-a^2}$ or $\frac{a^2}{a^2-2a+b}$
h $x=\frac{ac}{c-a}$ **i** $x=\frac{ac}{b}$
j $x=\frac{c-a}{b}$ **k** $x=\frac{b-bc}{a-c}$
l $x=\frac{ad-b}{c-d}$ **m** $x=ab-2a$
n $x=\frac{a+b}{1-a}$ **o** $x=\frac{2ab-a^2}{a-b}$

Problems and challenges

1 **a** 48, 49 **b** 33, 35 **c** 12, 15
2 **a** 15 **b** 5
3 $a=2$, $b=1$, $c=7$ and $d=8$
4 **a** $(n+1)^2+1$ or n^2+2n+2 **b** Answers may vary.
5 $(n-1)(n+1)$
  a $n-1$ and $n+1$ are both even, since they are consecutive even numbers one of them is divisible by 4 hence their product is also divisible by 4
  b $n-1, n, n+1$ are 3 consecutive numbers, one of them must be divisible by 3. Since n is prime it must be $n-1$ or $n+1$ so their product is divisible by 3
  c n^2-1 is divisible by 3 and 4 and since they have no common factor it must also be divisible by $3\times4=12$
6 Factorise each expression and cancel.
7 $4x^2-4x+1=(2x-1)^2$, which is always greater than or equal to zero
8 Ryan

Short-answer questions

1 **a** x^2+x-12 **b** $x^2-9x+14$
c $6x^2-5x-6$ **d** $9x^2+3x-12$
2 **a** x^2+6x+9 **b** $x^2-8x+16$
c $9x^2-12x+4$ **d** x^2-25
e $49-x^2$ **f** $121x^2-16$
3 **a** $4(a+3b)$ **b** $3x(2-3x)$
c $-5xy(x+2)$ **d** $(x-7)(x+3)$
e $(2x+1)(x-1)$ **f** $(x-2)(x-6)$
4 **a** $(x+10)(x-10)$ **b** $3(x+4)(x-4)$
c $(5x+y)(5x-y)$ **d** $(7+3x)(7-3x)$
e $(x-12)(x+6)$ **f** $(1-x)(1+x)$
5 **a** $(x-3)(x+6)$ **b** $(2x+5)(2x-1)$
c $(x-4)(3+2b)$
6 **a** $(x+3)(x+5)$ **b** $(x-6)(x+3)$
c $(x-6)(x-1)$ **d** $3(x+7)(x-2)$
e $2(x+4)^2$ **f** $(5x+2)(x+3)$
g $(2x-3)(2x+1)$ **h** $(3x-4)(2x-3)$
7 **a** $x+4$ **b** $\frac{2}{3}$ **c** $\frac{x-3}{5}$
8 **a** $\frac{1}{4}$ **b** $\frac{x-4}{2}$ **c** $\frac{x}{3}$
d $\frac{2}{5}$ **e** $\frac{2x+3}{50x}$ **f** 4
9 **a** $\frac{11x}{12}$ **b** $\frac{5}{4x}$ **c** $\frac{7x-2}{x^2}$
d $\frac{x-13}{24}$ **e** $\frac{8x+11}{(x+1)(x+2)}$ **f** $\frac{15-2x}{(x-4)^2}$
10 **a** $x=20$ **b** $x=\frac{1}{6}$ **c** $x=7$ **d** $x=-1\frac{2}{9}$

Multiple-choice questions

1 A **2** E **3** D **4** C **5** B
6 D **7** A **8** E **9** B **10** A

ISBN 978-1-108-77288-4

Extended-response questions

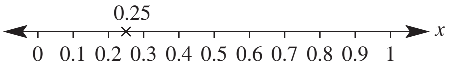
1 a $(x + 3)$ m
  b i no change
    ii $(x^2 - 1)$ m^2, 1 square metre less in area
  c i width $= (x - 3)$ m, decreased by 3 metres
    ii $A = (x + 7)(x - 3) = (x^2 + 4x - 21)$ m^2
    iii $A = 0$ m^2
2 a 400 m^2
  b i $l = w = (20 + 2x)$ m ii $(4x^2 + 80x + 400)$ m^2
  c $\frac{1}{4}$
  d $4x(x + 20)$ m^2
  e $x = 5$

Chapter 9

9A

Building understanding

I a i $\frac{1}{4}$ ii 0.25 iii 25%
  b



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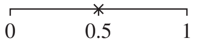
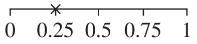
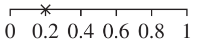
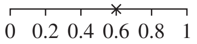
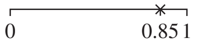
	50%	0.5	$\frac{1}{2}$	
a	25%	0.25	$\frac{1}{4}$	
b	20%	0.2	$\frac{1}{5}$	
c	60%	0.6	$\frac{3}{5}$	
d	85%	0.85	$\frac{17}{20}$	

3 0.15, $\frac{2}{9}$, 1 in 4, 0.28, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{7}$, 2 in 3, 0.7, 0.9

Now you try

Example 1
a $\{1, 2, 3, 4, 5\}$ b $\frac{3}{8}$ c $\frac{5}{8}$
d $\frac{5}{8}$ e $\frac{7}{8}$

Example 2
a $\frac{1}{9}$ b $\frac{8}{9}$ c $\frac{5}{9}$ d $\frac{4}{9}$ e $\frac{2}{3}$ f 1

Exercise 9A

1 a $\{1, 2, 3, 4, 5, 6, 7\}$ b $\frac{1}{7}$ c $\frac{6}{7}$
  d $\frac{2}{7}$ e $\frac{6}{7}$
2 a i $\{2, 6, 7\}$ ii $\frac{1}{2}$ iii $\frac{1}{2}$
    iv $\frac{1}{2}$ v 1
  b i $\{1, 2, 3\}$ ii $\frac{2}{3}$ iii $\frac{1}{3}$
    iv $\frac{5}{6}$ v $\frac{5}{6}$
  c i $\{1, 2, 3\}$ ii $\frac{1}{3}$ iii $\frac{2}{3}$
    iv $\frac{2}{3}$ v $\frac{2}{3}$
  d i $\{1, 2, 3, 4\}$ ii $\frac{1}{7}$ iii $\frac{6}{7}$
    iv $\frac{3}{7}$ v $\frac{5}{7}$
3 a $\frac{1}{2}$ b $\frac{3}{8}$ c $\frac{1}{6}$ d $\frac{1}{4}$ e 1 f 0
4 a $\frac{1}{2}$ b $\frac{5}{8}$ c $\frac{5}{6}$ d $\frac{3}{4}$ e 0 f 1
5 a $\frac{1}{8}$ b $\frac{7}{8}$
6 a $\frac{1}{8}$ b $\frac{1}{4}$ c $\frac{3}{8}$ d $\frac{3}{8}$ e $\frac{5}{8}$
  f 1 g 0 h $\frac{1}{4}$ i $\frac{3}{4}$ j $\frac{3}{4}$
7 a {Hayley, Alisa, Rocco, Stuart}
  b i $\frac{1}{4}$ ii $\frac{1}{2}$ iii $\frac{1}{2}$
8 a $\frac{1}{52}$ b $\frac{1}{13}$ c $\frac{1}{26}$ d $\frac{1}{2}$
  e $\frac{2}{13}$ f $\frac{12}{13}$ g $\frac{23}{26}$ h $\frac{25}{26}$
9 a $\frac{1}{6}$ b $\frac{1}{6}$ c $\frac{5}{6}$ d $\frac{1}{3}$ e $\frac{2}{3}$
  f 1 g $\frac{1}{3}$ h $\frac{1}{2}$ i $\frac{5}{6}$
10 a $\frac{2}{11}$ b $\frac{9}{11}$ c $\frac{4}{11}$ d $\frac{7}{11}$
  e $\frac{7}{11}$ f $\frac{3}{11}$ g $\frac{7}{11}$ h $\frac{4}{11}$
11 Amanda has not taken into account the fact there are two O’s.
12 A, D, F

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13 **a** $\frac{12}{25}$ **b** $\frac{8}{25}$ **c** $\frac{1}{5}$ **d** $\frac{9}{25}$ **e** $\frac{8}{25}$
f $\frac{16}{25}$ **g** $\frac{4}{25}$ **h** $\frac{17}{25}$ **i** 0

14 **a** 31 min **b** $\frac{3}{31}$
c **i** $\frac{4}{31}$ **ii** $\frac{4}{31}$ **iii** $\frac{20}{31}$
iv $\frac{5}{31}$ **v** $\frac{8}{31}$ **vi** $\frac{11}{31}$

9B

Building understanding

1 **a** 26
b **i** 10 **ii** 9 **iii** 4
iv 7 **v** 19
c **i** 12 **ii** 17
2 **a** B **b** D **c** A **d** C
3 **a**

	A	Not A	Total
B	7	8	15
Not B	3	1	4
Total	10	9	19

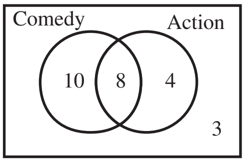
b

	A	Not A	Total
B	2	5	7
Not B	9	4	13
Total	11	9	20

Now you try

Example 3

a



b **i** 22 **ii** 13 **iii** 10
c **i** $\frac{8}{25}$ **ii** $\frac{3}{25}$ **iii** $\frac{4}{25}$

Example 4

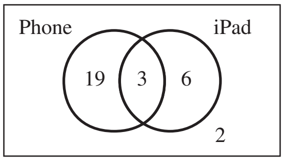
a

	Classes	Not classes	Total
Equipment	8	11	19
Not equipment	10	1	11
Total	18	12	30

b **i** 12 **ii** 1
c **i** $\frac{19}{30}$ **ii** $\frac{1}{3}$ **iii** $\frac{29}{30}$

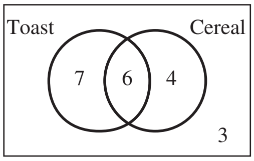
Exercise 9B

1 **a**



b **i** 28 **ii** 21 **iii** 6
c **i** $\frac{1}{10}$ **ii** $\frac{1}{15}$ **iii** $\frac{19}{30}$

2 **a**



b **i** 17 **ii** 7 **iii** 7
c **i** $\frac{3}{10}$ **ii** $\frac{1}{5}$

3 **a** **i** $\frac{2}{5}$ **ii** $\frac{1}{3}$ **iii** $\frac{7}{15}$
iv $\frac{1}{15}$ **v** $\frac{13}{15}$ **vi** $\frac{2}{15}$
b **i** $\frac{3}{7}$ **ii** $\frac{12}{35}$ **iii** $\frac{13}{35}$
iv $\frac{3}{35}$ **v** $\frac{34}{35}$ **vi** $\frac{1}{35}$

4 **a**

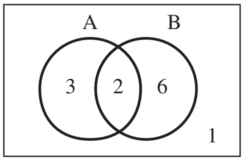
	Cream	Not cream	Total
Ice-cream	5	20	25
Not ice-cream	16	9	25
Total	21	29	50

b **i** 29 **ii** 9
c **i** $\frac{21}{50}$ **ii** $\frac{8}{25}$ **iii** $\frac{41}{50}$

5 **a** **i** $\frac{5}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{3}{8}$
iv $\frac{3}{4}$ **v** $\frac{1}{8}$ **vi** $\frac{1}{4}$
b **i** $\frac{17}{26}$ **ii** $\frac{9}{26}$ **iii** $\frac{11}{26}$
iv $\frac{21}{26}$ **v** $\frac{2}{13}$ **vi** $\frac{5}{26}$

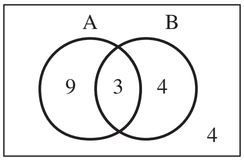
6 **a**

	A	Not A	Total
B	2	6	8
Not B	3	1	4
Total	5	7	12

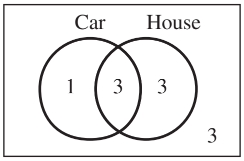


b

	A	Not A	Total
B	3	4	7
Not B	9	4	13
Total	12	8	20



7 a



b 3 **c** $\frac{1}{10}$

8 a

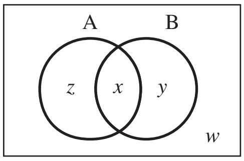
	Rainwater	Not rainwater	Total
Tap water	12	36	48
Not tap water	11	41	52
Total	23	77	100

b 12 **c** $\frac{9}{25}$ **d** $\frac{59}{100}$

9 23

10 $\frac{7}{15}$

11



12 a $T - x - y + z$ **b** $T - x$ **c** $T - y$

d $x - z$ **e** $y - z$ **f** $y - z$ **g** $T - x + z$

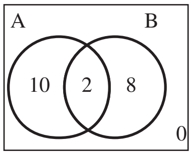
13

	A	Not A	Total
B	⓪	12	12
Not B	11	–4	7
Total	11	⑧	19

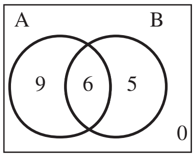
Filling in table so totals add up requires a negative number, which is impossible!

14 4

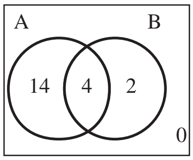
15 a i



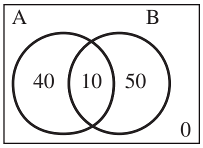
ii



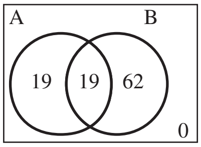
iii



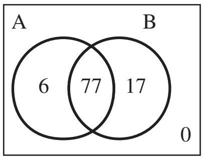
b i



ii



iii



c overlap = (A + B) − total

9C

Building understanding

1 a C **b** D **c** E
d A **e** F **f** B

2 a B **b** D **c** A **d** C

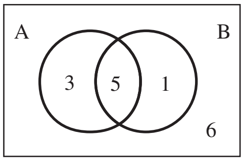
3 a 2 **b** 10 **c** 7 **d** 9

Now you try

Example 5

a i {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}
ii {1, 3, 5, 7, 9, 11, 13, 15}
iii {2, 3, 5, 7, 11, 13}

b



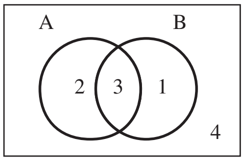
c i {3, 5, 7, 11, 13} **ii** {1, 2, 3, 5, 7, 9, 11, 13}
iii {2, 4, 6, 8, 10, 12, 14} **iv** {2}

d i 8 **ii** $\frac{8}{15}$ **iii** 5 **iv** $\frac{1}{3}$

Exercise 9C

1 a i {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
ii {1, 3, 5, 7, 9} **iii** {2, 3, 5, 7}

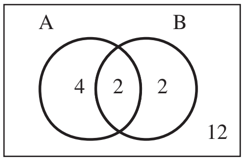
b



c i {3, 5, 7} **ii** {1, 2, 3, 5, 7, 9}
iii {2, 4, 6, 8, 10} **iv** {2}

d i 5 **ii** $\frac{1}{2}$ **iii** 3 **iv** $\frac{3}{10}$

2 a



b i {3, 15} ii {1, 3, 5, 6, 9, 12, 15, 18}
iii {1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20}
iv {1, 5}

c i 4 ii $\frac{1}{5}$ iii 2
iv $\frac{1}{10}$ v 8 vi $\frac{2}{5}$

3 a i True ii True iii False iv False
v True vi False vii True viii False
b i $\frac{1}{2}$ ii $\frac{1}{2}$ iii 0

4 a i $\frac{1}{6}$ ii $\frac{11}{12}$ iii $\frac{5}{12}$
b i $\frac{6}{25}$ ii $\frac{21}{25}$ iii $\frac{8}{25}$
c i $\frac{1}{3}$ ii $\frac{11}{15}$ iii $\frac{2}{5}$
d i $\frac{5}{21}$ ii $\frac{17}{21}$ iii $\frac{4}{7}$

5 a i {Fred, Ron, Rachel} ii {Fred, Rachel, Helen}
iii { } iv {Fred, Rachel}
b i $\frac{3}{4}$ ii $\frac{1}{4}$ iii 0
iv 1 v $\frac{1}{2}$ vi 1

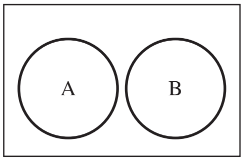
6 a 26 b 5 c 3 d 18
e $\frac{5}{26}$ f $\frac{21}{26}$ g $\frac{3}{26}$ h $\frac{5}{13}$

7 a 11 b 21 c 1 d $\frac{9}{25}$

8 a own both a dog and a cat
b own dogs or cats or both
c does not own a cat
d owns a cat but no dogs

9 B only = $B \cap A'$

10 a yes

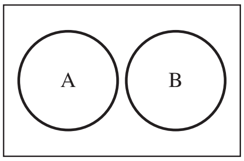


b No, as $A \cup B$ includes only elements from sets A and B.

11

	A	A′	Total
B	$n(A \cap B)$	$n(B \cap A)'$	$n(B)$
B′	$n(A \cap B')$	$n(A' \cap B')$	$n(B')$
Total	$n(A)$	$n(A')$	n(sample space)

12 Mutually exclusive events have no common elements, i.e. $A \cap B = \varnothing$.



13 a i {2, 3, 5, 7, 11, 13, 17, 19}
ii {1, 2, 3, 4, 6, 12} iii {2, 3}
iv {1, 2, 3, 4, 5, 6, 7, 11, 12, 13, 17, 19}
v {1, 4, 6, 12} vi {5, 7, 11, 13, 17, 19}
vii {1, 2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20}
viii {2, 3, 5, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20}
ix {1, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}
b i $\frac{3}{5}$ ii $\frac{1}{10}$ iii $\frac{1}{5}$ iv $\frac{2}{5}$
v $\frac{4}{5}$ vi $\frac{9}{10}$ vii $\frac{9}{10}$ viii $\frac{2}{5}$
c They are equal.

9D

Building understanding

1 a 9 outcomes

	1	**2**	**3**
1	(1, 1)	(2, 1)	(3, 1)
2	(1, 2)	(2, 2)	(3, 2)
3	(1, 3)	(2, 3)	(3, 3)

b 6 outcomes

	A	**B**	**C**
A	X	(B, A)	(C, A)
B	(A, B)	X	(C, B)
C	(A, C)	(B, C)	X

2 a table A b table B
c i $\frac{1}{9}$ ii $\frac{1}{6}$
d i 5 ii 4

Now you try

Example 6

a

		Counter 1	
		R	**B**
Counter 2	**R**	(R, R)	(B, R)
	B	(R, B)	(B, B)

b $\frac{1}{4}$ c $\frac{1}{2}$

Example 7

a

		1st			
		D	**A**	**T**	**A**
2nd	**D**	×	(A, D)	(T, D)	(A, D)
	A	(D, A)	×	(T, A)	(A, A)
	T	(D, T)	(A, T)	×	(A, T)
	A	(D, A)	(A, A)	(T, A)	×

b $\frac{1}{6}$ c $\frac{5}{6}$

ISBN 978-1-108-77288-4

Exercise 9D

1 a

	1	2	3	4
1	(1, 1)	(2, 1)	(3, 1)	(4, 1)
2	(1, 2)	(2, 2)	(3, 2)	(4, 2)
3	(1, 3)	(2, 3)	(3, 3)	(4, 3)
4	(1, 4)	(2, 4)	(3, 4)	(4, 4)

Sample space = { (1, 1), (2, 1), … (4, 4) }, i.e. all pairs from table.

b $\frac{1}{16}$ c $\frac{1}{4}$

2 a

	1	2	3	4	5	6
1	(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)
2	(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
3	(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)
4	(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)	(6, 4)
5	(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)	(6, 5)
6	(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	(6, 6)

b 36

c i $\frac{1}{36}$ ii $\frac{1}{6}$ iii $\frac{1}{12}$ iv $\frac{5}{9}$

3 a

	D	O	G
D	X	(O, D)	(G, D)
O	(D, O)	X	(G, O)
G	(D, G)	(O, G)	X

b $\frac{1}{6}$ c $\frac{2}{3}$

4 a

		A	
		•	∘
B	•	(•, •)	(∘, •)
	∘	(•, ∘)	(∘, ∘)
	∘	(•, ∘)	(∘, ∘)

b 6

c i $\frac{1}{3}$ ii $\frac{1}{2}$ iii $\frac{1}{2}$

5 a

		1st			
		1	2	3	4
2nd	**1**	X	(2, 1)	(3, 1)	(4, 1)
	2	(1, 2)	X	(3, 2)	(4, 2)
	3	(1, 3)	(2, 3)	X	(4, 3)
	4	(1, 4)	(2, 4)	(3, 4)	X

b i $\frac{1}{12}$ ii $\frac{1}{12}$

c i $\frac{1}{6}$ ii $\frac{5}{6}$ iii $\frac{1}{6}$ iv $\frac{1}{2}$

6 a

	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

b i $\frac{1}{16}$ ii $\frac{3}{16}$ iii $\frac{3}{8}$ iv $\frac{3}{16}$ v $\frac{13}{16}$

7 a

	A	B	C	D	E
A	(A, A)	(B, A)	(C, A)	(D, A)	(E, A)
B	(A, B)	(B, B)	(C, B)	(D, B)	(E, B)
C	(A, C)	(B, C)	(C, C)	(D, C)	(E, C)
D	(A, D)	(B, D)	(C, D)	(D, D)	(E, D)
E	(A, E)	(B, E)	(C, E)	(D, E)	(E, E)

b i $\frac{1}{25}$ ii $\frac{1}{5}$ iii $\frac{4}{5}$

c i $\frac{8}{25}$ ii $\frac{1}{25}$

8 a i $\frac{1}{36}$ ii $\frac{1}{12}$ iii $\frac{1}{12}$ iv $\frac{7}{12}$

v $\frac{5}{12}$ vi $\frac{1}{6}$ vii $\frac{1}{6}$ viii 0

b 7, $\frac{1}{6}$

9 a i 169 ii 156

b i 25 ii 12

10 Yes; if one O is removed another remains to be used.

11 a $\frac{1}{25}$ b $\frac{1}{20}$

12 7, 8, 9, 10, 11

13 a $\frac{1}{2500}$ b $\frac{1}{50}$ c $\frac{1}{1250}$

d $\frac{49}{2500}$ e $\frac{23}{1250}$

14 a without replacement b 2652

c $\frac{1}{1326}$

d i $\frac{1}{221}$ ii $\frac{1}{17}$

9E

Building understanding

1 TT, TO, OT, OO

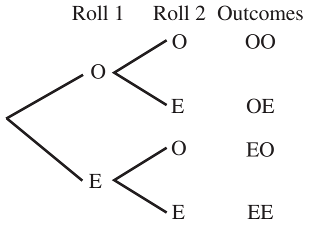
2 AB, AC, BA, BC, CA, CB

ISBN 978-1-108-77288-4

Now you try

Example 8

a

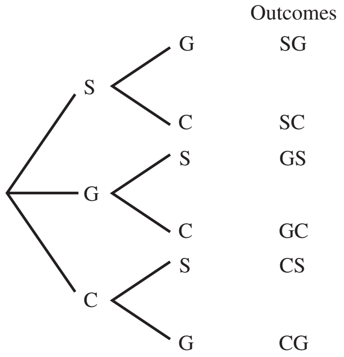


b 4

c **i** $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** $\frac{3}{4}$

Example 9

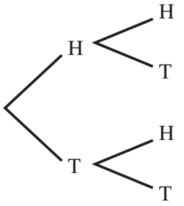
a



b **i** $\frac{1}{3}$ **ii** $\frac{2}{3}$ **iii** 1

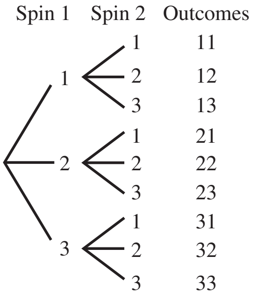
Exercise 9E

1 **a** HH, HT, TH, TT **b** 4



c **i** $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** $\frac{3}{4}$ **iv** $\frac{3}{4}$

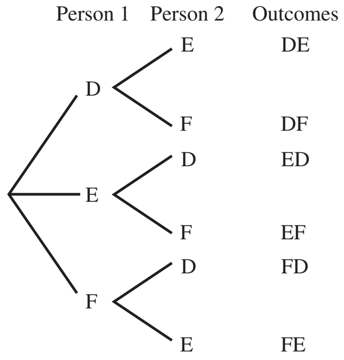
2 **a**



b 9

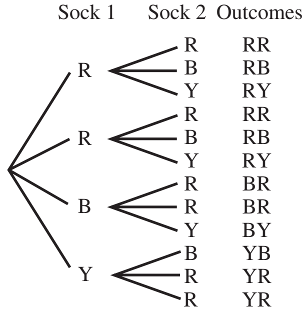
c **i** $\frac{1}{9}$ **ii** $\frac{5}{9}$ **iii** $\frac{8}{9}$ **iv** $\frac{4}{9}$

3 **a**



b **i** $\frac{1}{3}$ **ii** $\frac{2}{3}$ **iii** 1

4 **a**



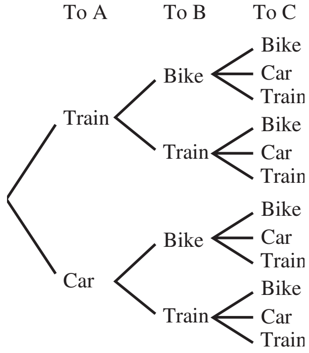
b **i** $\frac{1}{3}$ **ii** $\frac{1}{6}$ **iii** $\frac{1}{6}$ **iv** $\frac{5}{6}$

5 **a** $\frac{4}{9}$ **b** $\frac{2}{9}$ **c** $\frac{1}{27}$ **d** $\frac{8}{27}$

6 **a** $\frac{1}{16}$ **b** $\frac{1}{16}$ **c** $\frac{5}{16}$

d $\frac{11}{16}$ **e** $\frac{15}{16}$

7 **a**



b 12

c **i** $\frac{1}{12}$ **ii** $\frac{1}{3}$ **iii** $\frac{1}{2}$

iv $\frac{1}{4}$ **v** $\frac{2}{3}$

8 **a** $\frac{1}{16}$ **b** $\frac{1}{4}$ **c** $\frac{3}{8}$

d $\frac{1}{4}$ **e** $\frac{1}{16}$

9 **a** 32 **b** 2^n

10 Yes, there is a difference; the probability of obtaining two of the same colour is lower without replacement.

11 **a** $\frac{1}{9}$ **b** $\frac{1}{9}$ **c** $\frac{7}{18}$ **d** $\frac{7}{18}$

e $\frac{1}{2}$ **f** 0 **g** $\frac{1}{6}$ **h** $\frac{2}{3}$

i $\frac{1}{9}$ **j** $\frac{17}{18}$

9F

Building understanding

1 **a** B: $\frac{5}{20} = 0.25$, C: $\frac{30}{100} = 0.3$

b C, as it is a larger sample size

2 **a** 5 **b** 20 **c** 4

Now you try

Example 10

a 0.6 **b** 120

Exercise 9F

1 **a** 0.6

b **i** 60 **ii** 120 **iii** 360

2 **a** $\frac{7}{8}$

b **i** 350 **ii** 4375 **iii** 35

3 **a** $\frac{1}{15}$ **b** $\frac{2}{5}$ **c** $\frac{11}{60}$

4 **a** 20 **b** 40 **c** 60 **d** 40

5 **a** 20

b **i** $\frac{1}{4}$ **ii** $\frac{7}{20}$

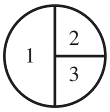
c **i** 25 **ii** 20 **iii** 45

6 **a** 0.64 **b** SEE

7 **a** **i** 0.52 **ii** 0.48 **iii** 0.78

b 78

c $\frac{1}{2}, \frac{1}{4}, \frac{1}{4}$



8 0.41, from the 100 throws as the more times an experiment is carried out the closer the experimental probability becomes to the actual/theoretical probability

9 **a** $\frac{9}{10}$

b No, as the number of throws increases, the experiment should produce results closer to the theoretical probability $\left(\frac{1}{6}\right)$.

10 **a** fair, close to 0.5 chance of tails

b biased, nearly all results are heads

c can't determine on such a small sample

11 **a** $\frac{\text{shaded area}}{\text{total area}} = 0.225 \therefore 100 \text{ shots} \approx 23$

b $\frac{1}{10} \times 100 = 10$

c $\frac{150 - 32}{150} \times 100 \approx 79$

d $\frac{225\pi - 25\pi}{225\pi} \times 100 \approx 89$

12 **a** no **b** true **c** no **d** true

13 3 blue, 2 red, 4 green, 1 yellow

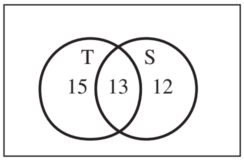
14 2 strawberry, 3 caramel, 2 coconut, 4 nut, 1 mint

Progress quiz

1 **a** {1, 2, 3, 4, 5, 6} **b** {BG, GB, BB, GG}

2 $\frac{4}{11}$

3 **a**



b 13 **c** $\frac{27}{40}$

4 $\frac{3}{10}$ or 0.3

5 **a** **i** {10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}

ii {10, 12, 14, 16, 18, 20}

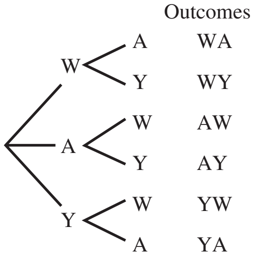
iii {12, 15, 18}

iv {12, 18}

b **i** 2 **ii** $\frac{2}{11}$ **iii** $\frac{5}{11}$

6 **a** $\frac{1}{2}$ **b** $\frac{1}{6}$ **c** $\frac{1}{18}$

7 **a**



b **i** $\frac{2}{3}$ **ii** 1 **iii** $\frac{1}{3}$ **iv** $\frac{1}{3}$

8 **a** 0.4 **b** 0.56

9 140

9G

Building understanding

1 **a** common **b** middle **c** mean

2

	Mean	Median	Mode
a	3	2	2
b	15	16	20

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Now you try

Example 11

a **i** 5 **ii** 4 **iii** 2

b **i** 17 **ii** 16.5 **iii** no mode

Example 12

a \$23 **b** \$37

Exercise 9G

1

	Mean	**Median**	**Mode**
a	6	7	8
b	6	6	2

2

	Mean	**Median**	**Mode**
a	8	6	5, 10
b	11	12	none
c	4	3.5	2.1
d	5	4.5	none
e	3	3.5	–3
f	0	2	3

3 **a** outlier = 33, mean = 12, median = 7.5

b outlier = –1.1, mean = 1.075, median = 1.4

c outlier = –4, mean = –49, median = –59

d outlier = 10, mean = –1.34, median = –2.4

4 **a** yes **b** no **c** no **d** yes

5 **a** 24.67 s **b** 24.8 s

6 **a** 15 **b** 26

7 **a** 90 **b** 60

8 9

9 Answers may vary; examples below.

a 1, 4, 6, 7, 7 is a set **b** 2, 3, 4, 8, 8 is a set

c 4, 4, 4, 4, 4 is a set **d** 2.5, 2.5, 3, 7, 7.5 is a set

e –3, –2, 0, 5, 5 is a set **f** $0, \frac{1}{2}, 1\frac{1}{4}, 1\frac{1}{4}, 2$ is a set

10 **a** \$354 500 **b** \$1 700 000

c (\$354 500 and \$324 000) drops \$30 500

d \$570 667

e (\$570 667 and \$344 800) drops \$225 867

11 An outlier has a large impact on the addition of all the scores and therefore significantly affects the mean. An outlier does not move the middle of the group of scores significantly.

12 **a** 15 **b** 1.2 **c** 1 **d** 1

e No, as it will still lie in the 1 column.

13 **a** 2 **b** 3 **c** 7

14 **a** 76.75

b **i** 71.4, B$^+$ **ii** 75, B$^+$ **iii** 80.2, A

c 81.4; he cannot get an A$^+$

d **i** 43 **ii** 93

e $M = \frac{307 + m}{5}$

f $m = 5M - 307$

9H

Building understanding

1 **a** 9

b **i** 8 min **ii** 35 min

c 21 min **d** 21 min

2 **a** 26

b **i** 9 mm **ii** 15 mm

c skewed **d** symmetrical

Now you try

Example 13

a

Stem	**Leaf**
0	2 3
1	4 4
2	0 3 5
3	1 4 5 8 9
4	3 4 7 8
5	1 6 7
6	4

2 | 3 means 2.3

b 3.65 **c** 1.4

d Data is approximately symmetrical.

Example 14

a

Rockets		**Warriors**
8 7 4 2	6	
8 5 3 2 1	7	5 6 6
8	8	1 2 7 8 9
	9	4 7

7 | 2 means 72

b The Warriors have the highest scores, with scores between 75 and 97. The scores for the Rockets are generally between 62 and 78, with one high score of 88.

Exercise 9H

1 **a** **i**

Stem	**Leaf**
1	8
2	3 3 4
3	2 6 7 8
4	1 5 8
5	5 9
6	2

2 | 3 means 2.3

ii 3.75 **iii** 2.3 **iv** almost symmetrical

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b **i**

Stem	Leaf
1	9 9
2	3 4 6 6 8 8 8
3	2 2 3 5
4	1
5	4

2 | 3 means 23

ii 28 **iii** 28 **iv** skewed

c **i**

Stem	Leaf
1	5 5
2	1 3 3 3 6
3	0 1 1 3 4 5 5 7 9
4	2 2 5 5 5 8
5	0 0 1

3 | 1 means 31

ii 35 **iii** 23, 45 **iv** almost symmetrical

d **i**

Stem	Leaf
32	0
33	3 7 8
34	3 4 5 5 7 8 9
35	2 2 4 5 8
36	1 3 5
37	0

33 | 7 means 33.7

ii 34.85 **iii** 35.2, 34.5 **iv** almost symmetrical

e **i**

Stem	Leaf
15	7 8 9 9 9 9
16	1 4 7 7 7
17	3 5 7 7
18	5 5 7 9
19	3 8
20	0 2

16 | 4 means 164

ii 173 **iii** 159 **iv** skewed

2 **a**

Stem	Leaf
0	5 6 6 8 8 9
1	0 1 1 2 2 2 4 4 5 6 7
2	0 1 2

1 | 2 means 12

b 9

c 12

3 **a** **i**

Set A		Set B
3 2	3	1
9	3	
3 3 0 0	4	0 1 3 4 4
8 7 6 5	4	6 7 8 8 9 9
4 4 3	5	1 3

4 | 6 means 46

ii Set A has values spread between 32 and 54 while set B has most of its values between 40 and 53 with an outlier at 31.

b **i**

Set A		Set B
8 7 4 3	0	1 1 1 2 3 6 6 9 9
9 9 7 6 4	1	2 3 5 8 9
6 5 3 1	2	1 5 6
9 9 6 4 3 2	3	3 4 9
7 3 1	4	3 7 8
7 3 2	5	2 3 7
2 1	6	1 2
8 3	8	3 8
1	9	

1 | 4 means 1.4

ii Sets A and B are similar. The frequency decreases as the numbers increase.

4 **a**

Collingwood		St Kilda
8 3	6	6 8 8
8 7 2 1 0	7	8
9 9 8 2 0	8	0 0 2 2 3 4 7 8
8 7 5 1	9	0 4
4 3 3 3	10	6 9
9 8 5	11	1 3 7 8
7	12	2 5 6
	13	8

8 | 2 means 82

b Collingwood $33\frac{1}{3}\%$, St Kilda $41\frac{2}{3}\%$

c Data for Collingwood is almost symmetrical and based on these results seems to be consistent. St Kilda has groups of similar scores and although less consistent, it has higher scores.

5 **a** 9 days **b** 18°C **c** 7°C

6 **a** 16.1 s **b** 2.3 s **c** yes 0.05 s lower

7 **a**

Battery lifetime

Brand A		Brand B
3	7	2 3 4
	7	5 6 8 9 9
4 2	8	0 1 3
9 8 7 5	8	
4 4 3 2 1 0	9	0 1 2 3 4
9 9 8 7 6 5 5	9	5 6 8 8

8 | 3 represents 8.3 hours

b Brand A, 12; Brand B, 8

c Brand A consistently performs better than Brand B.

8 **a** $c = 2$ **b** 0.02

c **i** 5, 6, 7 or 8 **ii** 0, 1, 2, 3, 4, 5 or 6

9 **a** 48% **b** 15%

c In general, birth weights of babies are lower for mothers who smoke.

10 In symmetrical data the mean is close to the median, as the data is spread evenly from the centre with an even number of data values with a similar difference from the mean above and below it.

11 **a** 52 **b** 17.8

12 **a** mean = 41.06, median = 43

b The median is higher, as more of the scores exist in the higher stems but a few low scores lower the mean.

c The mean is higher for positively skewed data, as the majority of the scores are in the lower stems; however, a few high scores increase the mean.

d symmetrical data

9I

Building understanding

1 $a = 15, b = 2, c = 30, d = 40, e = 20, f = 100$

2 **a** 2 **b** 20

c **i** 30% **ii** 65%

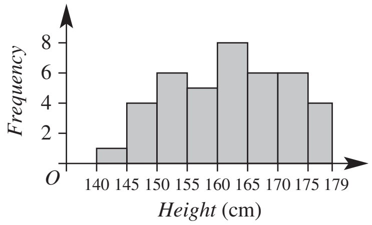
Now you try

Example 15

a

Class interval	Frequency	Percentage frequency
140–	1	2.5
145–	4	10
150–	6	15
155–	5	12.5
160–	8	20
165–	6	15
170–	6	15
175–179	4	10
Total	40	100

b



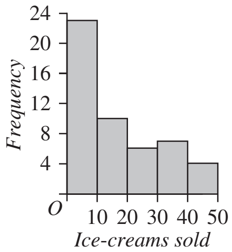
c **i** 5 **ii** 10

Exercise 9I

1 **a**

Class	Frequency	Percentage frequency
0–9	23	46%
10–19	10	20%
20–29	6	12%
30–39	7	14%
40–49	4	8%
	50	100%

b

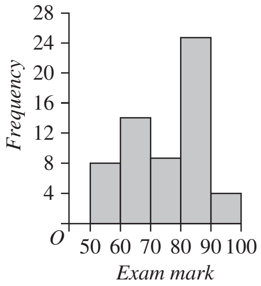


c **i** 33 **ii** 11

2 **a**

Class	Frequency	Percentage frequency
50–59	8	13.33%
60–69	14	23.33%
70–79	9	15%
80–89	25	41.67%
90–99	4	6.67%
	60	100%

b

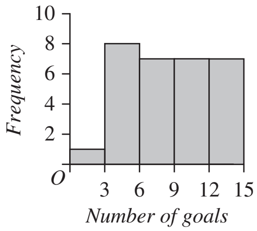


c **i** 22 **ii** 48.3%

3 **a**

Number of goals	Frequency
0–2	1
3–5	8
6–8	7
9–11	7
12–14	7
	30

b



c 9 **d** 7

4 **a** symmetrical data **b** skewed data

5 **a** $a = 6, b = 27.5, c = 17.5, d = 4, e = 12, f = 30, g = 100$

b $a = 1, b = 18, c = 8, d = 24, e = 20, f = 40, g = 100$

6 **a** **i** 20% **ii** 55% **iii** 80% **iv** 75%

v 50%

b **i** 30 **ii** 45

c **i** 2 **ii** 22

7 85.5%

8 because you only have the number of scores in the class interval not the individual scores

9 the number of data items within each class

10 **a** student A

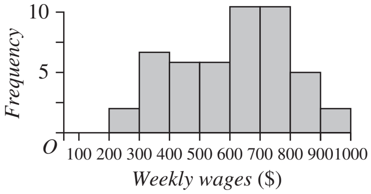
b Make the intervals for their groups of data smaller so that the graph conveys more information.

11 **a** minimum wage: $204; maximum wage: $940

b **i**

Weekly wages ($)	Frequency
200–	2
300–	7
400–	6
500–	6
600–	11
700–	11
800–	5
900–	2

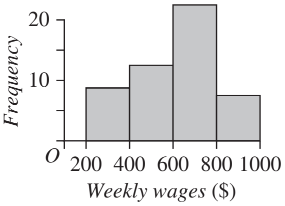
ii



c **i**

Weekly wages	Frequency
200–	9
400–	12
600–	22
800–	7

ii



d The graph with more intervals shows greater detail. As the first graph has each pair of intervals quite similar, these two graphs are quite similar.

9J

Building understanding

1 **a** minimum, maximum

b lower quartile, Q_1, upper quartile, Q_3

2 **a** 6 **b** 7

c **i** 5 **ii** 8

d 3

3 **a** 12 **b** 5

c **i** 3 **ii** 8

d 5

Now you try

Example 16

a 72 76 78 81 83 84 86 90 92

b 20

c **i** 83 **ii** 77 **iii** 88 **iv** 11

Example 17

a 0.4 0.8 1.1 1.2 1.6 1.7 1.8 2.0 2.3 2.3 2.6 2.8 3.1 3.5

b 3.1

c **i** 1.9 **ii** 1.2 **iii** 2.6 **iv** 1.4

d The middle 50% of packages differed by less than 1.4 kg.

Exercise 9J

1

	Range	Q_2	Q_1	Q_3	IQR
a	11	6	2.5	8.5	6
b	23	30	23.5	36.5	13
c	170	219	181.5	284.5	103
d	851	76	28	367	339
e	1.3	1.3	1.05	1.85	0.8
f	34.98	10	0.1	23	22.9

2 a 1446 million
  b i 1595.5 million ii 1405 million
    iii 2068 million iv 663 million
  c The middle 50% of films varied in takings by 663 million.
3 a 110 min
  b i 119.5 min ii 106 min
    iii 130 min iv 24 min
  c The middle 50% of videos rented varied in length by 24 min.
4 a i \$12 000 ii \$547 000 iii \$71 500
    iv \$46 000 v \$78 000 vi \$32 000
  b The middle 50% of prices differs by no more than \$32 000.
  c No effect on Q_1, Q_2 or Q_3 but the mean would increase.
5 a 17.5 b 2.1
6 a 2 b 2
  c No, only the maximum value has changed, so no impact on IQR.
7 No, as the range is the difference between the highest and lowest scores and different sets of two numbers can have the same difference ($10 - 8 = 2$, $22 - 20 = 2$).
8 a Yes, as lowering the highest price reduces the range.
  b No, the middle price will not change.
  c No, as only one value; the highest has changed, yet it still remains the highest, Q_1 and Q_3 remain unchanged.
9 a Yes (3, 3, 3, 4, 4, 4; $\text{IQR} = 4 - 3 = 1$; range $= 1$)
  b Yes (4, 4, 4, 4, 4 has $\text{IQR} = 0$)
10 a i $Q_1 = 25; Q_2 = 26; Q_3 = 27$
    ii $Q_1 = 22; Q_2 = 24.5; Q_3 = 27$
  b 27 jelly beans c 22 jelly beans
  d i $\text{IQR} = 2$ ii $\text{IQR} = 5$
  e Shop B is less consistent than shop A and its data is more spread out.
  f shop A

9K

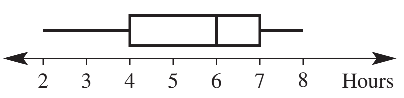
Building understanding

1 a minimum value b lower quartile, Q_1
  c median, Q_2 d upper quartile, Q_3
  e maximum value f scale
  g whisker h box
2 D

Now you try

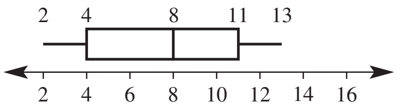
Example 18
a min = 0, max = 30
b 30 c 15 d 15
e 15 f 25 g 20

Example 19
a i 2 ii 8 iii 6
  iv 4 v 7
b

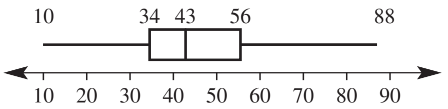


Exercise 9K

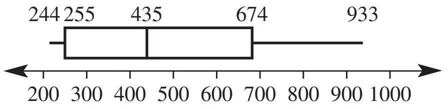
1 a min = 35 cm, max = 60 cm
  b 25 cm c 50 cm d 10 cm
  e 50 cm f 55 cm g 20 babies
2 a min = 0 rabbits, max = 60 rabbits
  b 60 c 25 d 25
  e 35 f 25 g 25
3 a i 2 ii 13 iii 8 iv 4 v 11
    vi



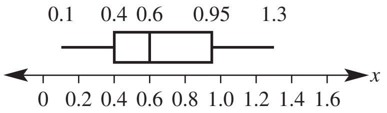
  b i 10 ii 88 iii 43 iv 34 v 56
    vi



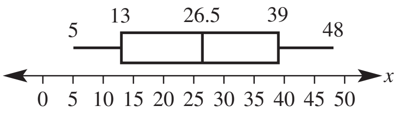
  c i 244 ii 933 iii 435 iv 255 v 674
    vi



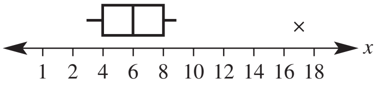
  d i 0.1 ii 1.3 iii 0.6 iv 0.4 v 0.95
    vi



4 a



  b i 100% ii 50% iii 75% iv 25%
5 a Africa b 10 kg c yes, 25 kg
  d i 50% ii 75%
  e African
6 a Mac b 75% c 50%
  d They are the same. Mac has 100% of times within same range as the middle 50% of PC start up times. Mac is more consistent.
7 a Waldren b Yeng c Yeng
  d Yeng, smaller range and IQR
  e Yeng, higher median
8 a 1, 3, 5, 6, 6 b 1, 4, 4, 4, 5, 6
9 No, the median is anywhere within the box, including at times at its edges.
10 Yes, one reason is if an outlier is the highest score, then the mean $> Q_3$; e.g. [1, 1, 3, 5, 6, 7, 256].
11 a $Q_1 = 4$, $Q_2 = 6$, $Q_3 = 8$
  b yes, 17 c 9
  d



  e This calculator may have been used less often than the others, so the battery lasted longer.

Problems and challenges

1 a $2^5 = 32$ b $\frac{3}{16}$ c $\frac{31}{32}$

2 a $\frac{1}{6}$ b $\frac{1}{3}$

3 a i $\frac{1}{4}$ ii $\frac{1}{2}$

b $\frac{1}{6}$ c $\frac{1}{12}$ d $\frac{1}{3}$

4 a mean and median increase by 5, range unchanged

b mean, median and range all double

c mean and median double and decrease 1, range doubles

5 $5m$ grams

6 a 3, 5, 7, 8, 10 or 3, 5, 7, 9, 10 b 2, 2, 8, 12

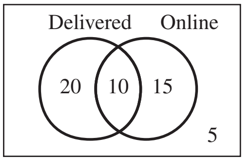
7 1, 4, 6, 7, 7; 2, 3, 6, 7, 7 and 1, 2, 5, 6, 6

8 $\frac{3}{8}$

Short-answer questions

1 a $\frac{2}{3}$ b $\frac{5}{9}$ c $\frac{3}{5}$

2 a



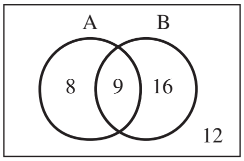
b 15

c i $\frac{1}{5}$ ii $\frac{2}{5}$ iii $\frac{3}{10}$

3 a

	A	A′	Total
B	9	16	25
B′	8	12	20
Total	17	28	45

b



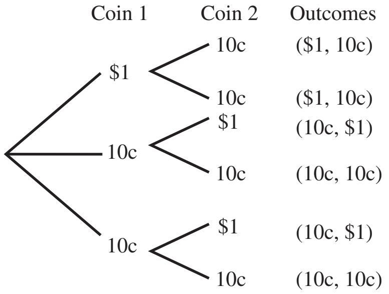
c i $\frac{4}{9}$ ii $\frac{1}{5}$ iii 8 iv 33

4 a 12 outcomes

	1	2	3	4
Red	(red, 1)	(red, 2)	(red, 3)	(red, 4)
Green	(green, 1)	(green, 2)	(green, 3)	(green, 4)
Blue	(blue, 1)	(blue, 2)	(blue, 3)	(blue, 4)

b i $\frac{1}{6}$ ii $\frac{1}{6}$ iii $\frac{2}{3}$

5 a



b $\frac{2}{3}$

6 a $\frac{48}{120} = \frac{2}{5}$ b 14

7 a 26 b 26.5 c 23, 31

8 a

Employee 1		**Employee 2**
9	1	7 7
5 3 1	2	0 4 8 9
9 8 6 5 5 4	3	2 7 7 8
6 5 0	4	0 1 2 5 8
3 1	5	

2 | 4 means 24

b i Employee 1: 36, Employee 2: 37

ii Employee 1: 36, Employee 2: 33

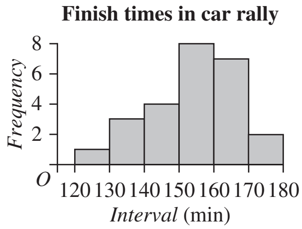
c Employee 1, they have a higher mean and more sales at the high end.

d Employee 1 symmetrical, employee 2 skewed

9 a

Class interval	Frequency	Percentage frequency
120–	1	4
130–	3	12
140–	4	16
150–	8	32
160–	7	28
170–180	2	8
Total	25	100%

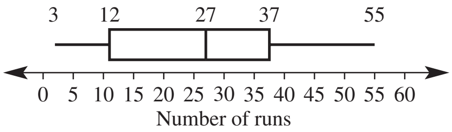
b



c i 4 ii 60%

10 **a** 52 runs

b



c 37 runs

Multiple-choice questions

1 B **2** A **3** D **4** B **5** D
6 B **7** E **8** C **9** B **10** E

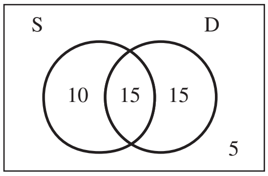
Extended-response questions

1 **a** **i**

		1st spin				
		1	**2**	**3**	**4**	**5**
2nd spin	**1**	(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)
	2	(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)
	3	(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)
	4	(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)
	5	(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)

ii $\frac{12}{25}$ **iii** $\frac{4}{25}$ **iv** 24 **v** \$68

b **i**



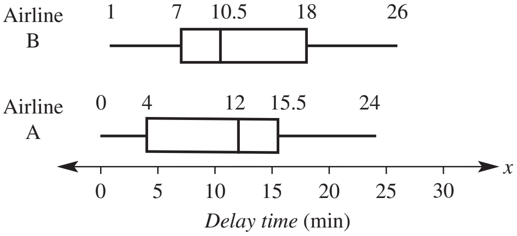
ii 5 **iii** 10 **iv** $\frac{1}{3}$ **v** $\frac{4}{9}$

vi $\frac{7}{9}$, the probability a person did not buy only a sausage

2 **a** Yes, 52 min

b **i** 12 **ii** 4 **iii** 15.5

c and e



d No, the median time is 12 min, so half the flights have less than a 12 minute delay.

f Airline A: range = 24, IQR = 11.5
Airline B: range = 25, IQR = 11
Both airlines have a very similar spread of their data when the outlier is removed.

g There is not much difference when airline A's outlier is removed. It has a marginally better performance with 75% of flights delayed less than 15.5 min compared with 18 min for airline B.

Chapter 10

10A

Building understanding

1 **a** -4 **b** -1 **c** 72
2 **a** no **b** yes **c** yes
d no **e** no **f** yes
3 **a** 1 **b** -3 **c** -2
4 **a** $x - 5 = 0, x = 5$
b $2x + 1 = 0, 2x = -1$ or $x = 3, x = -\frac{1}{2}$ or $x = 3$

Now you try

Example 1
a $x^2 - 3x - 5 = 0$ **b** $2x^2 + 8x - 7 = 0$
c $2x^2 + 3x - 4 = 0$

Example 2
a not a solution **b** is a solution

Example 3
a $x = 0, -5$ **b** $x = -2, 3$ **c** $x = \frac{2}{3}, -\frac{5}{2}$

Exercise 10A

1 **a** $x^2 - 4x - 3 = 0$ **b** $2x^2 + 10x - 3 = 0$
c $4x^2 + x - 2 = 0$
2 **a** $x^2 - 7x - 2 = 0$ **b** $x^2 - 5x + 2 = 0$
c $x^2 - 4x + 1 = 0$ **d** $3x^2 - 6x - 4 = 0$
e $2x^2 + 2x + 1 = 0$ **f** $3x^2 - 3x + 4 = 0$
g $3x^2 + 4 = 0$ **h** $x^2 - 3x - 1 = 0$
i $2x^2 + 5x - 10 = 0$
3 **a** yes **b** yes **c** no **d** no **e** yes
f no **g** yes **h** no **i** no
4 both are solutions
5 both are solutions
6 **a** 0, -1 **b** 0, -5 **c** 0, 2 **d** 0, 7
e -1, 3 **f** 4, -2 **g** -7, 3 **h** $-\frac{1}{2}, \frac{1}{2}$
i 0, -5 **j** $0, \frac{2}{3}$ **k** $0, -\frac{2}{3}$ **l** 0, -2
7 **a** $\frac{1}{2}, -2$ **b** $-2, \frac{1}{3}$ **c** $-\frac{2}{5}, -4$
d $1, \frac{1}{3}$ **e** $-5, -\frac{2}{7}$ **f** $\frac{2}{3}, -\frac{1}{5}$
g $\frac{7}{11}, \frac{13}{2}$ **h** $-\frac{9}{4}, \frac{7}{2}$ **i** $\frac{4}{3}, -\frac{1}{7}$

8 a $0, -3$ **b** $0, 7$ **c** $1, -4$
d $\frac{1}{2}, -6$ **e** $-\frac{3}{2}, \frac{1}{2}$

9 a $4x^2 + x + 1 = 0$ **b** $3x^2 - 3x = 0$
c $3x^2 - x - 4 = 0$ **d** $5x^2 + x - 2 = 0$
e $2x^2 - x - 3 = 0$ **f** $3x^2 - 10x + 6 = 0$
g $x^2 - x - 5 = 0$ **h** $4x^2 - 5x - 6 = 0$

10 a $-1, 2$ **b** $1, 3$ **c** $0, 4$
d $0, -3$ **e** $-4, 1$ **f** $-4, 4$

11 a $(x + 2)(x + 2) = 0$
b both solutions are the same, $x = -2$
c i -3 **ii** 5 **iii** $\frac{1}{2}$ **iv** $\frac{7}{5}$

12 a $(x - 1)(x + 2) = 0$
b $x = 1, x = -2$
c Multiplying by a constant doesn't change a zero value.
d i $-2, 3$ **ii** $0, -2$ **iii** $-1, 3$ **iv** $-2, 5$

13 a i no **ii** no **iii** no **iv** no
b It has no solutions as $(x - 3)^2$ is always $\geqslant 0$, so $(x - 3)^2 + 1 \geqslant 1$

14 a linear **b** quadratic **c** quintic
d cubic **e** quartic **f** quintic

15 a $-2, -1, 3$ **b** $-11, 2, 5$
c $-\frac{2}{3}, \frac{1}{5}, \frac{1}{2}$ **d** $-\frac{10}{7}, -\frac{4}{5}, -\frac{2}{3}, \frac{13}{2}$

10B

Building understanding

1 a 8 **b** x **c** $3x$
2 a $x(x - 3)$ **b** $2x(3x + 2)$
c $4x(1 - 4x)$ **d** $2(x - 2)(x + 2)$
3 a $x = 0, x = 3$ **b** $x = 0, x = -1$
4 $3^2 = 3 \times 3 = 9$ and $(-3)^2 = -3 \times (-3) = 9$

Now you try

Example 4
a $x = 0, -6$ **b** $x = 0, 5$

Example 5
a $x = -7, 7$ **b** $x = -\sqrt{7}, \sqrt{7}$

Exercise 10B

1 a i $x = 0, x = -5$ **ii** $x = 0, x = 3$
b i $x = 0, x = 3$ **ii** $x = 0, x = 4$

2 a $x = 0, x = -3$ **b** $x = 0, x = -7$ **c** $x = 0, x = -4$
d $x = 0, x = 5$ **e** $x = 0, x = 8$ **f** $x = 0, x = 2$
g $x = 0, x = -\frac{1}{3}$ **h** $x = 0, x = \frac{1}{2}$

3 a $x = 0, x = 3$ **b** $x = 0, x = 4$ **c** $x = 0, x = -5$
d $x = 0, x = 3$ **e** $x = 0, x = 3$ **f** $x = 0, x = -4$

4 a $x = 0, x = 3$ **b** $x = 0, x = 2$ **c** $x = 0, x = 4$
d $x = 0, x = -3$ **e** $x = 0, x = -4$ **f** $x = 0, x = -3$

5 a $x = 3, x = -3$ **b** $x = 6, x = -6$ **c** $x = 5, x = -5$
d $x = 12, x = -12$ **e** $x = 9, x = -9$ **f** $x = 20, x = -20$

6 a $x = 2, x = -2$ **b** $x = 3, x = -3$
c $x = 5, x = -5$ **d** $x = 2, x = -2$
e $x = \sqrt{6}, x = -\sqrt{6}$ **f** $x = \sqrt{5}, x = -\sqrt{5}$
g $x = \sqrt{7}, x = -\sqrt{7}$ **h** $x = \sqrt{3}, x = -\sqrt{3}$

7 a $x = 2, x = -2$ **b** $x = 5, x = -5$
c $x = 10, x = -10$ **d** $x = 0, x = -7$
e $x = 0, x = 7$ **f** $x = -1, x = 1$
g $x = 0, x = \frac{2}{3}$ **h** $x = 0, x = \frac{3}{5}$
i $x = -\sqrt{3}, x = \sqrt{3}$

8 a $x = -2, x = 2$ **b** $x = -6, x = 6$ **c** $x = -1, x = 1$
d $x = -1, x = 1$ **e** $x = 0, x = -7$ **f** $x = 0, x = 4$

9 a 0 or 7 **b** 8 or -8 **c** 0 or -4

10 a $(x + 2)(x - 2) = x^2 - 4$ not $x^2 + 4$
b no, $x^2 \geqslant 0$, so $x^2 + 4 \geqslant 4$

11 a $ax^2 + bx = x(ax + b) = 0, x = 0$ is always one solution
b $x = -\frac{b}{a}$

12 a $x = -\frac{4}{3}, x = \frac{4}{3}$ **b** $x = -\frac{6}{5}, x = \frac{6}{5}$
c $x = -\frac{1}{5}, x = \frac{1}{5}$ **d** $x = -\frac{9}{5}, x = \frac{9}{5}$
e $x = -\frac{8}{11}, x = \frac{8}{11}$ **f** $x = -\frac{12}{7}, x = \frac{12}{7}$

13 a $x = -1, x = 5$ **b** $x = -1, x = -9$
c $x = -1, x = 0$ **d** $x = -\frac{2}{5}, x = \frac{8}{5}$
e $x = 1, x = 7$ **f** $x = -1, x = \frac{13}{7}$

10C

Building understanding

1 a $3, 2$ **b** $-5, -2$ **c** $5, -1$ **d** $-4, 3$
2 a $(x + 5)(x + 7)$ **b** $(x + 9)(x - 5)$
c $(x - 2)(x - 8)$
3 a $(x + 5)(x + 4) = 0, x + 5 = 0$ or $x + 4 = 0$, $x = -5$ or $x = -4$
b $(x - 6)(x + 4) = 0, x - 6 = 0$ or $x + 4 = 0$, $x = 6$ or $x = -4$

Now you try

Example 6
a $x = -2, -4$ **b** $x = 5, -2$ **c** $x = 6, 2$

Example 7
a $x = 5$
b $x = 8, -5$

Exercise 10C

1 **a** −4, −6 **b** 11, −3 **c** 4, 9

2 **a** −6, −2 **b** −8, −3 **c** −5, −2 **d** −7, 2
e −6, 2 **f** −10, 3 **g** 4, 8 **h** 3, 6
i 3, 7 **j** −3, 5 **k** −2, 8 **l** −5, 9
m 4, 6 **n** −6, 7 **o** −12, 7 **p** −3, −1
q −3, 9 **r** 2, 10

3 **a** −3 **b** −2 **c** −7 **d** −12 **e** 5
f 8 **g** 6 **h** 9 **i** 10

4 **a** −2, 5 **b** 2, 5 **c** 3 **d** −4, 1
e −7, 2 **f** 4 **g** −6, 2 **h** −6, 1
i 3, 5 **j** −8, 2 **k** −4, −2 **l** −9, 2

5 **a** −2, 3 **b** −5, −3 **c** −2, 8 **d** 2, 3 **e** 2
f −1 **g** 5 **h** −3 **i** −7, 1

6 **a** 2, 3 **b** 3 **c** −10, 2
d −5, 7 **e** −1 **f** 2

7 **a** $x^2 - 3x + 2 = 0$ **b** $x^2 - x - 6 = 0$
c $x^2 + 3x - 4 = 0$ **d** $x^2 - 7x - 30 = 0$
e $x^2 - 10x + 25 = 0$ **f** $x^2 + 22x + 121 = 0$

8 11 am, 6 pm

9 **a** Equation is not written in standard form $x^2 + bx + c = 0$ so cannot apply Null Factor Law in this form.
b $x = -2, x = 3$

10 It is a perfect square: $(x - 1)(x - 1)$, $(x - 1)^2 = 0$, $x = 1$

11 **a** $(x - a)(x - a) = 0$ or $(x - a)^2 = 0$
b $(x - a)(x - b) = 0$

12 **a** $x = -3$ or $x = -\frac{1}{5}$ **b** $x = \frac{2}{3}$ or $x = -2$
c $x = \frac{1}{3}$ or $x = -\frac{1}{2}$ **d** $x = \frac{1}{2}$ or $x = -1$
e $x = \frac{3}{2}$ or $x = -5$ **f** $x = \frac{3}{2}$
g $x = \frac{7}{3}$ or $x = -2$ **h** $x = \frac{3}{5}$ or $x = -4$
i $x = -\frac{5}{3}$

10D

Building understanding

1 **a** −6, 3 **b** 5, −4 **c** 2, −5

2 **a** $x(x + 2) = 8$ **b** 2, −4
c 2, width > 0
d $l = 4$ cm, $w = 2$ cm

Now you try

Example 8
width = 20 cm and length = 32 cm

Exercise 10D

1 $l = 20$ cm, $w = 12$ cm
2 $l = 9$ cm, $w = 4$ cm
3 −8, 6
4 −5, 12
5 −2, 15
6 $L = 3$ m, $W = 23$ m
7 **a** $A = 100 - x^2$ **b** $x = 6$
8 $h = 5$
9 **a** $A = x^2 + 5x + 15$ **b** $x = 7$
10 **a** $x = 3$ (−4 not valid)
b $x = 12$ **c** $x = 25$
11 −8 not valid because dimensions must be >0
12 $x = -2$ or $x = 3$; both valid as both are integers
13 **a** 10, 15, 21
b **i** 28 **ii** 210
c **i** 9 **ii** 15
14 **a** 9, 14
b **i** 8 **ii** 12
15 **a** $A = (20 + 2x)^2 = 4x^2 + 80x + 400$
b 10 cm
16 4 cm

Progress quiz

1 **a** $x^2 - 3x - 8 = 0$ **b** $3x^2 - 4x - 6 = 0$
c $2x^2 - 2x - 3 = 0$ **d** $2x^2 - 3x - 12 = 0$

2 **a** $x = 2$ is not a solution **b** $x = -2$ is a solution
c $x = 3$ is a solution **d** $x = -3$ is a solution

3 **a** $x = 0$ or $x = 7$ **b** $x = 5$ or $x = -2$
c $x = 0$ or $x = -\frac{2}{5}$ **d** $x = \frac{1}{3}$ or $x = \frac{3}{4}$

4 **a** $x = 0$ or $x = -11$ **b** $x = 0$ or $x = 6$
c $x = 0$ or $x = -5$ **d** $x = 0$ or $x = 4$

5 **a** $x = 7$ or $x = -7$ **b** $x = 1$ or $x = -1$
c $x = 2$ or $x = -2$ **d** $x = 3$ or $x = -3$

6 **a** $x = -3$ or $x = -8$ **b** $x = 3$ or $x = 12$
c $x = -5$ or $x = 7$ **d** $x = -9$ or $x = 4$

7 **a** $x = -4$ **b** $x = 7$
c $x = 10$ **d** $x = -3$ or $x = 8$

8 length 9 m; width 6 m

9 number is −12 or 7

10 **a** $x^2 - 5x + 6 = 0$ **b** $x^2 - 5x - 6 = 0$
c $x^2 - 16 = 0$ **d** $x^2 + 6x + 9 = 0$

10E

Building understanding

1 **a** highest **b** parabola **c** intercepts
d vertex **e** lowest **f** zero

2 **a** $x = 3$ **b** $x = 1$ **c** $x = -2$

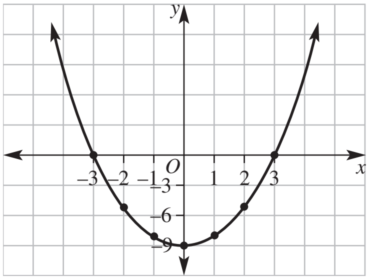
Now you try

Example 9
a $x = 4$ **b** maximum **c** (4, 6)
d 2 and 6 **e** −18

Example 10
a

x	−3	−2	−1	0	1	2	3
y	0	−5	−8	−9	−8	−5	0

b



c **i** minimum **ii** $x = 0$ **iii** (0, −9)
iv −9 **v** −3 and 3

Exercise 10E

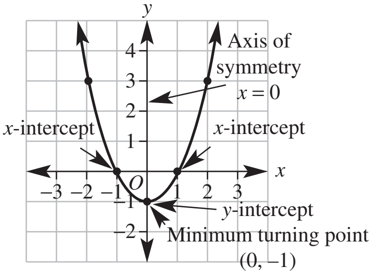
1

	i	ii	iii	iv	v
a	$x = 2$	minimum	(2, −1)	1, 3	3
b	$x = 0$	minimum	(0, −4)	−2, 2	−4
c	$x = 0$	maximum	(0, 3)	−1, 1	3
d	$x = 0$	maximum	(0, 4)	−2, 2	4
e	$x = 2$	minimum	(2, 1)	none	4
f	$x = -1$	maximum	(−1, 7)	−4, 2	6
g	$x = 0$	minimum	(0, 0)	0	0
h	$x = 0$	minimum	(0, −4)	−2, 2	−4
i	$x = 3$	maximum	(3, 4)	1, 5	−5
j	$x = 3$	minimum	$\left(3, -\frac{1}{2}\right)$	2, 4	4
k	$x = 0$	maximum	(0, 2)	−1, 1	2
l	$x = -2$	maximum	(−2, −1)	none	−4

2 **a** $y = x^2 - 1$

x	−2	−1	0	1	2
y	3	0	−1	0	3

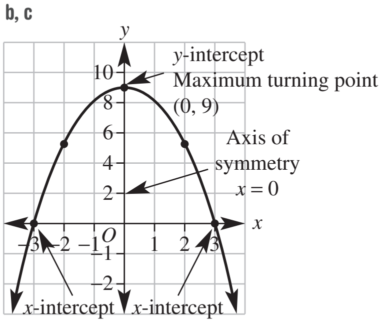
b, c



3 **a** $y = 9 - x^2$

x	−3	−2	−1	0	1	2	3
y	0	5	8	9	8	5	0

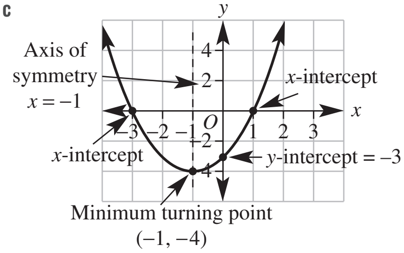
b, c



4 **a** $y = x^2 + 2x - 3$

x	−4	−3	−2	−1	0	1	2
y	5	0	−3	−4	−3	0	5

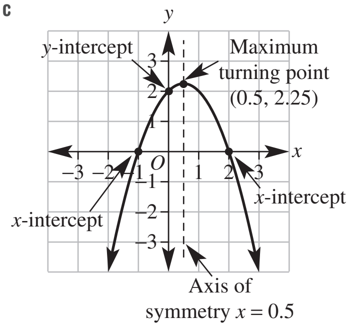
b, c



5 **a** $y = -x^2 + x + 2$

x	−2	−1	0	1	2	3
y	−4	0	2	2	0	−4

b, c



6 **a** **i** 1 s, 3 s
ii one time is on the way up, the other is on the way down
b **i** 2 s **ii** 12 m **iii** 4 s

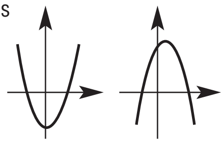
7 **a** 100 m **b** 155 m **c** 5 s
d journey takes 1s longer to go down to the ground

8 **a** $x = 1$ **b** (1, −3)

9 **a** $x = 1$ **b** 2

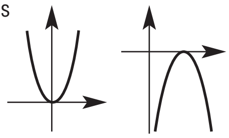
10 **a** $y = x^2$ **b** $y = x^2 - 4$ **c** $y = 1 - x^2$
d $y = x^2 - 2x + 1$ **e** $y = -x^2 - 2x$ **f** $y = x^2 - 3x - 4$

11 a Yes



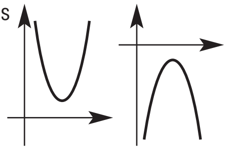
Two x-intercepts

b Yes



One x-intercept

c Yes



Zero x-intercepts

d No

12 a $-2^2 = -1 \times 2^2 = -4$. It is not $(-2)^2 = 4$, so correct solution is $y = -2^2 + 2 \times 2 = -4 + 4 = 0$

b $-(-3)^2 = -1 \times 9 = -9$. It is not $-3^2 = +9$, so correct solution is $y = -3 - (-3)^2 = -3 - 9 = -12$

13 a $x = -2, x = 2$ **b** $x = 0$

c i infinite **ii** one **iii** none

14 a i $x = 0, x = 2$ **ii** $x = -1, x = 3$

b two; a parabola is symmetrical

c one; this is the minimum turning point

d none; -1 is the minimum y-value so there are no values of y less than -1

15 Answers may vary.

10F

Building understanding

1 a $y = 3x^2, y = x^2, y = \frac{1}{2}x^2$

b $y = -3x^2, y = -x^2, y = -\frac{1}{2}x^2$

c $(0, 0)$ **d** $x = 0$

e i $y = 3x^2$ **ii** $y = \frac{1}{2}x^2$

iii $y = -3x^2$ **iv** $y = -\frac{1}{2}x^2$

f i $y = -x^2$ **ii** $y = -3x^2$ **iii** $y = \frac{1}{2}x^2$

2 a positive

b negative

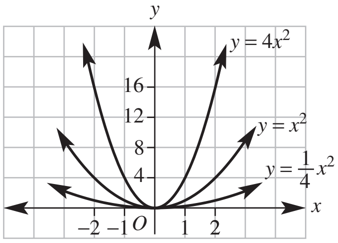
Now you try

Example 11

a

x	-2	-1	0	1	2
$y = x^2$	4	1	0	1	4
$y = 4x^2$	16	4	0	4	16
$y = \frac{1}{4}x^2$	1	$\frac{1}{4}$	0	$\frac{1}{4}$	1

b



c axis of symmetry: $x = 0$, turning point $(0, 0)$

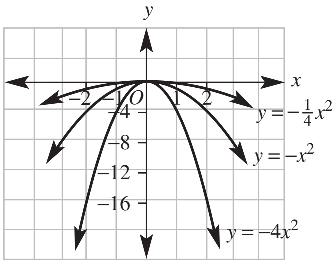
d i narrower **ii** wider

Example 12

a

x	-2	-1	0	1	2
$y = -x^2$	-4	-1	0	-1	-4
$y = -4x^2$	-16	-4	0	-4	-16
$y = -\frac{1}{4}x^2$	-1	$-\frac{1}{4}$	0	$-\frac{1}{4}$	-1

b



c axis of symmetry: $x = 0$, turning point $(0, 0)$

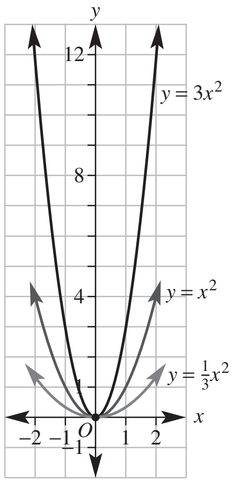
d i narrower **ii** wider

Exercise 10F

1 a

x	-2	-1	0	1	2
$y = x^2$	4	1	0	1	4
$y = 3x^2$	12	3	0	3	12
$y = \frac{1}{3}x^2$	$\frac{4}{3}$	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{4}{3}$

b



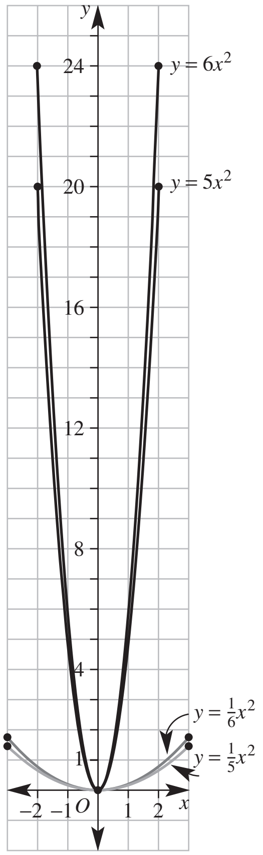
c For all three graphs, the turning point is a minimum at (0, 0) and the axis of symmetry is $x = 0$.

d **i** narrower **ii** wider

2 **a** **i**

x	−2	−1	0	1	2
y	24	6	0	6	24

ii



iii axis of symmetry: $x = 0$; turning point: (0, 0)

iv narrower

b **i**

x	−2	−1	0	1	2
y	20	5	0	5	20

ii See part **a**.

iii axis of symmetry: $x = 0$; turning point: (0, 0); x- and y-intercept: 0

iv narrower

c **i**

x	−2	−1	0	1	2
y	$\frac{2}{3}$	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{2}{3}$

ii See part **a**.

iii axis of symmetry: $x = 0$; turning point: (0, 0)

iv wider

d **i**

x	−2	−1	0	1	2
y	$\frac{4}{5}$	$\frac{1}{5}$	0	$\frac{1}{5}$	$\frac{4}{5}$

ii See part **a**.

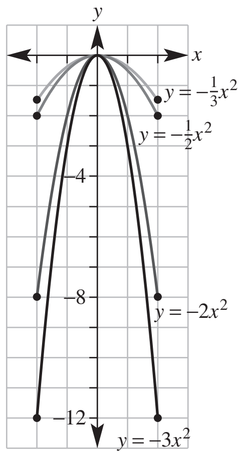
iii axis of symmetry: $x = 0$; turning point: (0, 0)

iv wider

3 **a** **i**

x	−2	−1	0	1	2
y	−8	−2	0	−2	−8

ii



iii axis of symmetry: $x = 0$; turning point: (0, 0); x- and y-intercept: 0

iv narrower

b **i**

x	−2	−1	0	1	2
y	−12	−3	0	−3	−12

ii See part **a**.

iii axis of symmetry: $x = 0$; turning point: (0, 0); x- and y-intercept: 0

iv narrower

c **i**

x	−2	−1	0	1	2
y	−2	$-\frac{1}{2}$	0	$-\frac{1}{2}$	−2

ii See part **a**.

iii axis of symmetry: $x = 0$; turning point: (0, 0); x- and y-intercept: 0

iv wider

d **i**

x	−2	−1	0	1	2
y	$-\frac{4}{3}$	$-\frac{1}{3}$	0	$-\frac{1}{3}$	$-\frac{4}{3}$

ii See part **a**.
iii axis of symmetry: $x = 0$; turning point: (0, 0); x- and y-intercept: 0
iv wider

4 **a** A **b** H

5 **a** D **b** C **c** A
d E **e** F **f** B

6 **a** reflection in the x-axis, dilaton by a factor of 3 from the x-axis
b reflection in the x-axis, dilaton by a factor of 6 from the x-axis
c reflection in the x-axis, dilaton by a factor of $\frac{1}{2}$ from the x-axis
d reflection in the x-axis, dilaton by a factor of 2 from the x-axis
e reflection in the x-axis, dilaton by a factor of 3 from the x-axis
f reflection in the x-axis, dilaton by a factor of $\frac{1}{3}$ from the x-axis

7 **a** $y = -4x^2$ **b** $y = \frac{1}{3}x^2$ **c** $y = -4x^2$ **d** $y = -\frac{4}{3}x^2$

8 No, because both transformations are multiplying to 'a' and multiplication is commutative: $bc = cb$

9 $y = ax^2$, has y-axis as axis of symmetry so it is symmetrical about the y-axis

10 **a** $y = 5x^2$ **b** $y = 7x^2$ **c** $y = x^2$ **d** $y = \frac{7}{4}x^2$
e $y = \frac{4}{25}x^2$ **f** $y = \frac{26}{9}x^2$ **g** $y = 5x^2$ **h** $y = -52x^2$

11 $y = \frac{67}{242\,064}x^2$

10G

Building understanding

1 **a** **i** (−2, 0) **ii** (3, 0)
b left **c** right

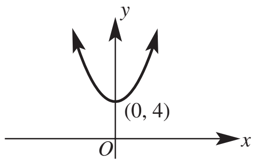
2 **a** **i** (0, −3) **ii** (0, 2)
b down **c** up

3 **a** 3 **b** −4 **c** −4 **d** 25

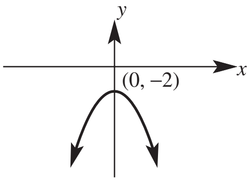
Now you try

Example 13

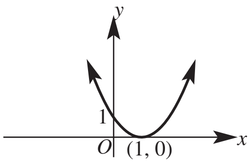
a



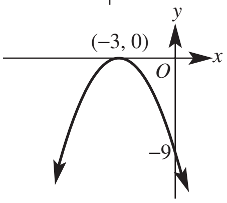
b



c

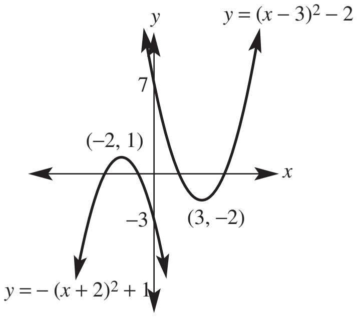


d



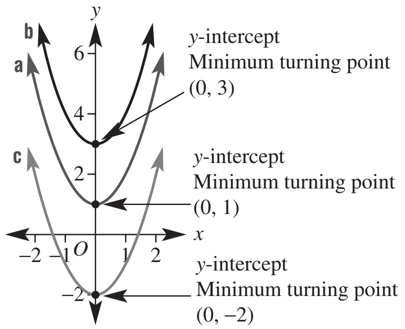
Example 14

a and b

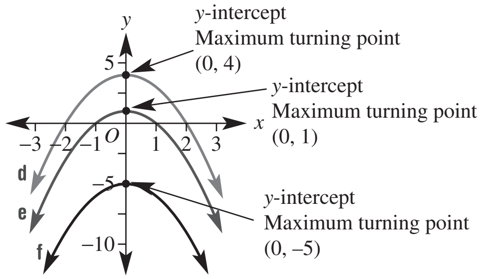


Exercise 10G

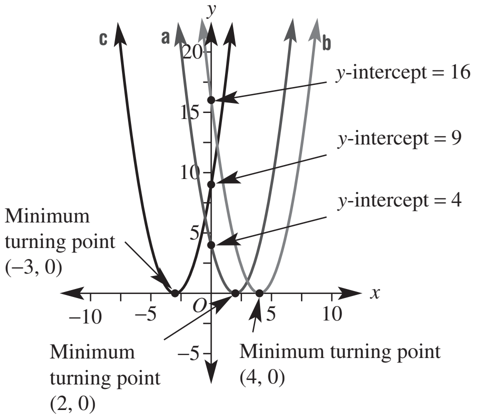
1 **a** $y = x^2 + 1$ **b** $y = x^2 + 3$ **c** $y = x^2 - 2$



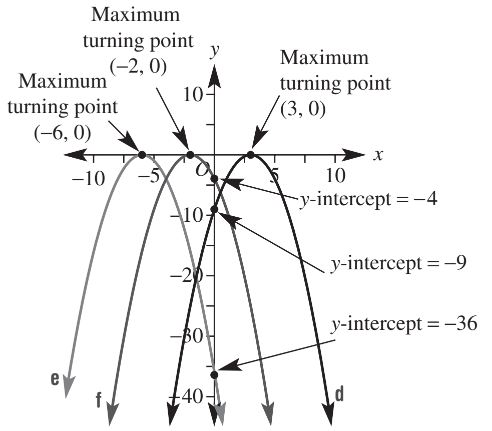
d $y = -x^2 + 4$ **e** $y = -x^2 + 1$ **f** $y = -x^2 - 5$



2 **a** $y = (x - 2)^2$ **b** $y = (x - 4)^2$ **c** $y = (x + 3)^2$

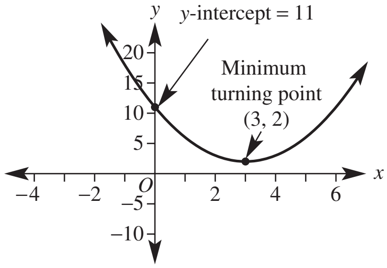


d $y = -(x-3)^2$ **e** $y = -(x+6)^2$ **f** $y = -(x+2)^2$

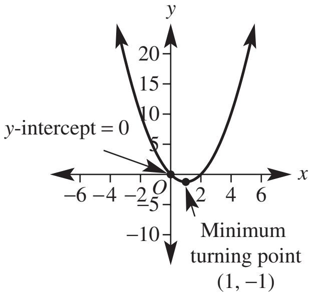


3 **a** **i** (2, 3) **ii** (−1, −1)
b **i** one, left, one, down **ii** two, right, three, up

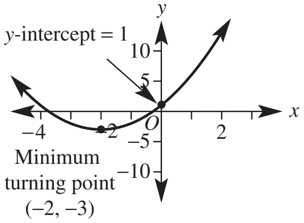
4 **a** $y = (x-3)^2 + 2$



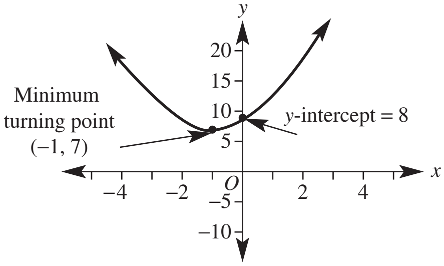
b $y = (x-1)^2 - 1$



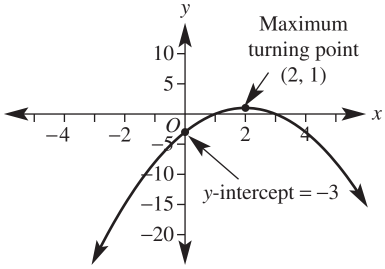
c $y = (x+2)^2 - 3$



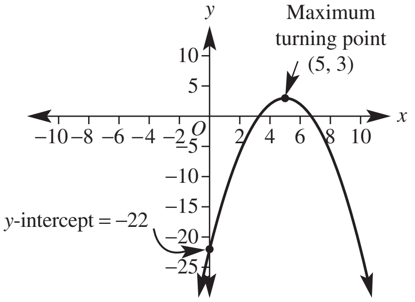
d $y = (x+1)^2 + 7$



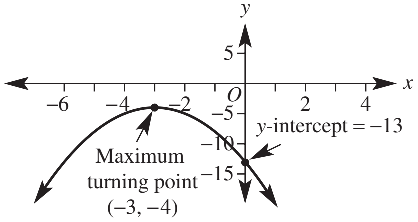
e $y = -(x-2)^2 + 1$



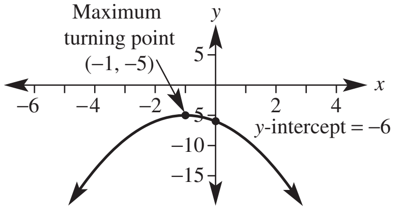
f $y = -(x-5)^2 + 3$



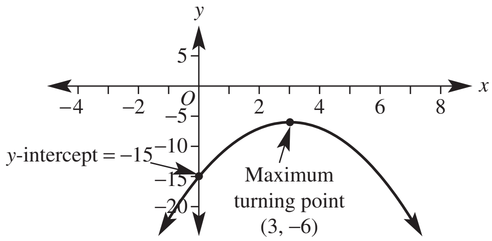
g $y = -(x+3)^2 - 4$



h $y = -(x+1)^2 - 5$



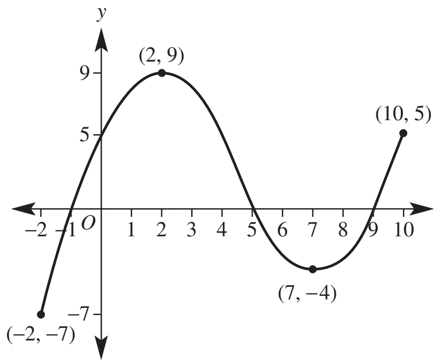
i $y = -(x - 3)^2 - 6$



5 a J b D c B d H e K f G
g A h E i L j I k F l C

6 a $y = (x - 1)^2 + 1$ b $y = (x + 2)^2 + 2$
c $y = (x + 1)^2 - 3$ d $y = -(x - 1)^2 + 4$
e $y = -(x - 2)^2 - 2$ f $y = -(x + 2)^2 + 4$

7 a $y = -(x - 2)^2 + 9$, turning point (2, 9)
$y = (x - 7)^2 - 4$, turning point (7, −4)
b



8 a i $y = -(x + 1)^2 + 3$ ii $y = (x + 3)^2 + 4$
iii $y = (x - 1)^2 - 3$ iv $y = -(x - 5)^2 - 7$
v $y = -x^2 - 2$ vi $y = x^2 - 6$
b i (−1, 3) ii (−3, 4) iii (1, −3)
iv (5, −7) v (0, −2) vi (0, −6)

9 a (h, k) b $y = ah^2 + k$

10 a $(2 - x)^2 = (-1(x - 2))^2 = (-1)^2(x - 2)^2 = (x - 2)^2$, so graphs are the same
b graphs are the same

11 a $y = x^2 + 3$ b $y = x^2 - 4$
c $y = x^2 - 3$ d $y = x^2 - 5$

12 a $y = -x^2 + 4$ b $y = -x^2 + 4$
c $y = -x^2 + 24$ d $y = -x^2 + 10$

13 a $y = (x + 3)^2$ or $y = (x - 5)^2$
b $y = (x - 2)^2$ or $y = (x - 4)^2$
c $y = (x + 4)^2$ or $y = (x - 2)^2$
d $y = x^2$ or $y = (x - 6)^2$

14 a $y = -(x - 1)^2 + 1$ b $y = (x + 2)^2$
c $y = -(x - 3)^2$ d $y = -(x + 3)^2 + 2$
e $y = (x + 1)^2 + 4$ f $y = (x - 3)^2 - 9$

10H

Building understanding

1 a $x(x + 2)$ b $x(x - 3)$
c $(x + 3)(x - 3)$ d $(x + 7)(x - 7)$
e $(x - 4)(x + 3)$ f $(x - 2)(x - 2) = (x - 2)^2$

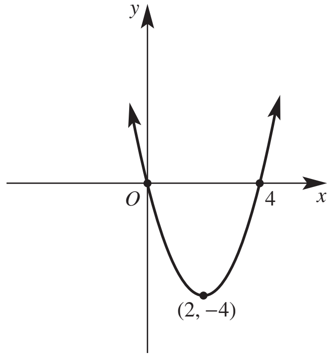
2 a 0 b −3
c −1 d −0.5

3 a −4 b −9 c −1

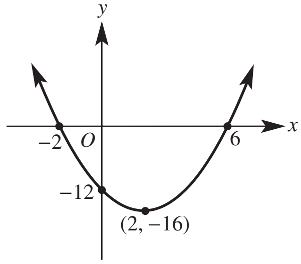
Now you try

Example 15
a i 0 or −5 ii 0
b i −1 or 4 ii −12

Example 16
a $y = x(x - 4)$ b 0 c 0 or 4
d $x = 2$ e (2, −4)
f



Example 17
a $y = (x - 6)(x + 2)$ b −12 c 6 or −2
d $x = 2$ e (2, −16)
f

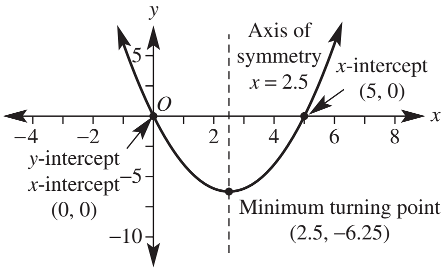


Exercise 10H

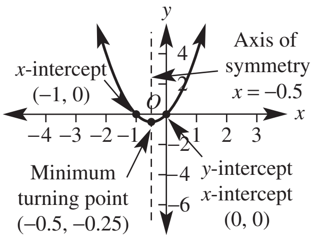
1 a i 0, −2 ii 0
b i 1, −2 ii −8

2 a i 0, −7 ii 0
b i 0, −3 ii 0
c i 0, −4 ii 0
d i 4, −2 ii −8
e i −2, 5 ii −10
f i −3, 7 ii −21
g i −3, 1 ii −6
h i −4, −1 ii 12
i i 2, 3 ii 6

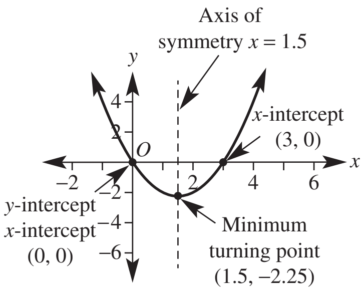
3 a $y = x(x - 5)$



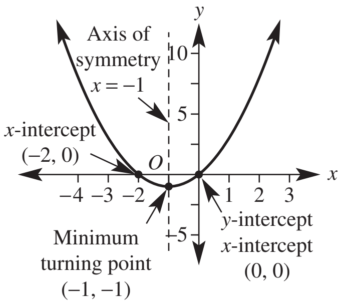
b $y = x(x + 1)$



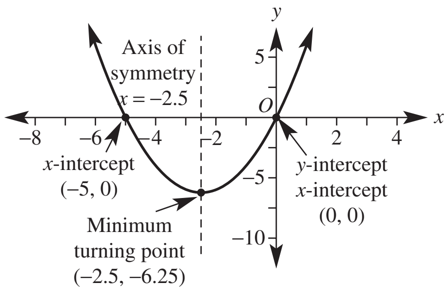
c $y = x(x - 3)$



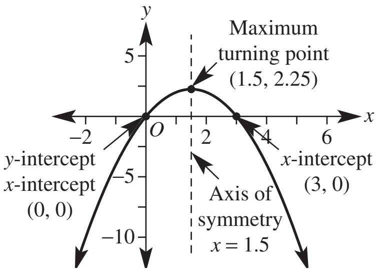
d $y = x(2 + x)$



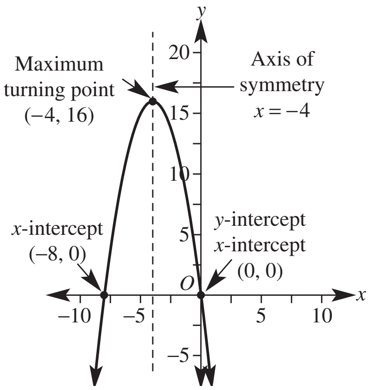
e $y = x(5 + x)$



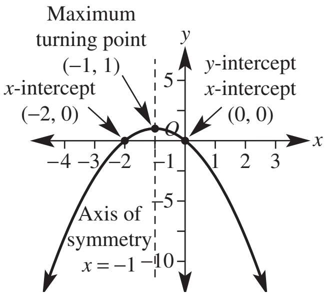
f $y = x(3 - x)$



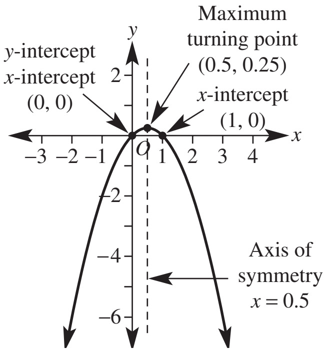
g $y = -x(x + 8)$



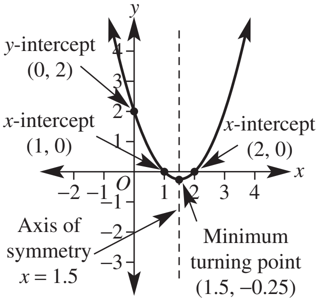
h $y = -x(2 + x)$



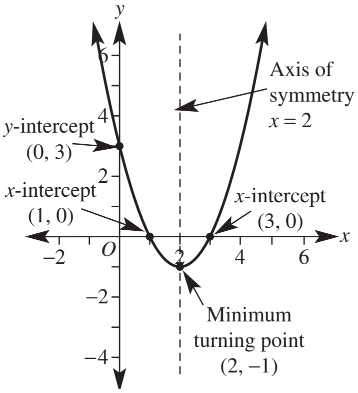
i $y = -x(x - 1)$



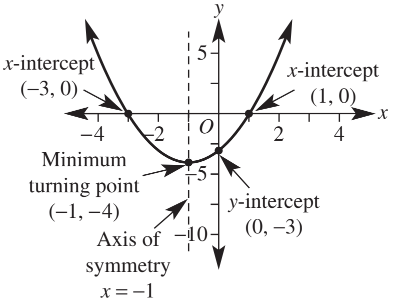
4 **a** $y = (x - 2)(x - 1)$



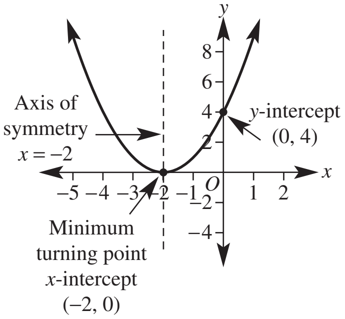
b $y = (x - 3)(x - 1)$



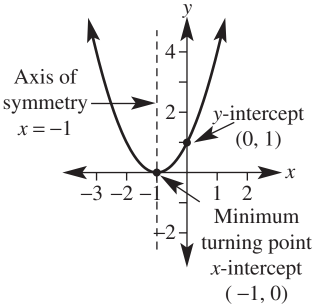
c $y = (x + 3)(x - 1)$



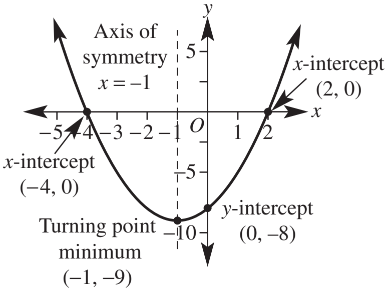
d $y = (x + 2)^2$



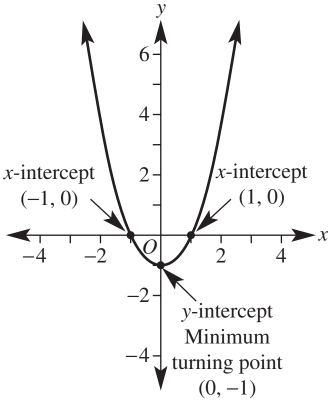
e $y = (x + 1)^2$



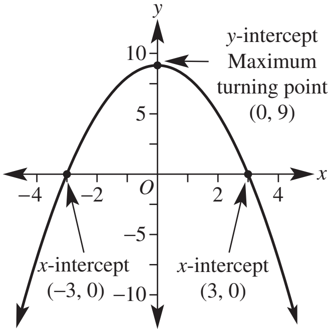
f $y = (x + 4)(x - 2)$



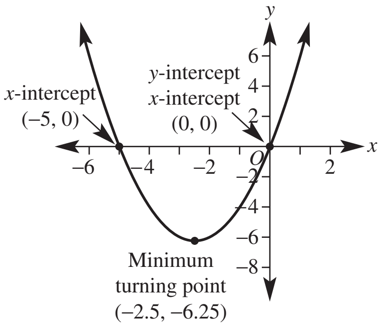
5 **a** $y = x^2 - 1$



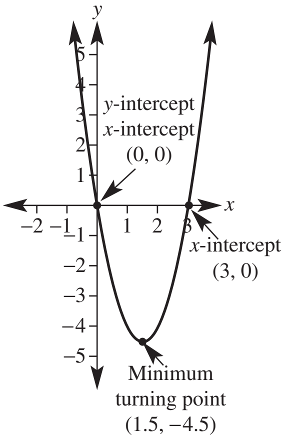
b $y = 9 - x^2$



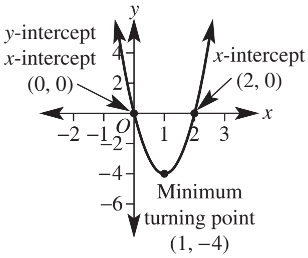
c $y = x^2 + 5x$



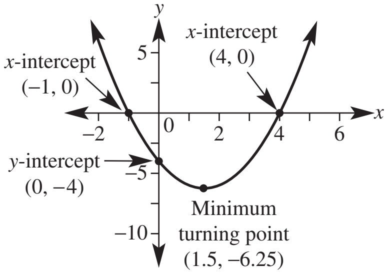
d $y = 2x^2 - 6x$



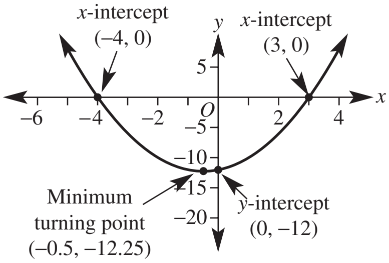
e $y = 4x^2 - 8x$



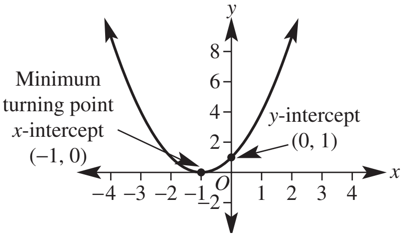
f $y = x^2 - 3x - 4$



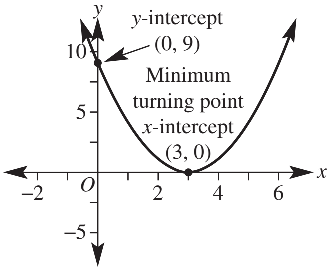
g $y = x^2 + x - 12$



h $y = x^2 + 2x + 1$



i $y = x^2 - 6x + 9$



6	**a** −3	**b** 2	**c** 3
	d −4	**e** −5	**f** 6
7	**a** 30 m	**b** 225 m	
8	**a** 200 m	**b** 1000 m	

9 There is only one x-intercept so this must be the turning point.

10 **a** $y = c$ **b** $y = ab$

11 **a** (1, −16)

b (5, 0), (−3, 0), turning point x-coordinate: $\frac{5 + (-3)}{2} = 1$, turning point(1, −16)

c $y = (x - 2)^2 - 49$, $h = 2$, $k = -49$

12 **b** $y = (x + 3)(x - 1)$ **c** $y = 2(x + 5)(x - 1)$

d $y = -(x + 1)(x - 5)$ **e** $y = -(x + 8)(x + 2)$

f $y = \frac{1}{2}(x + 8)(x - 2)$ **g** $y = -\frac{1}{3}(x + 3)(x - 7)$

Problems and challenges

1 **a** They are square numbers.

b $100^2 = 10\,000$

2 **a** $\frac{5}{3}, -\frac{7}{2}$ **b** 2, − 1 **c** $2^{\frac{2}{3}} + 1$

3 6 seconds

4 $\frac{n(n + 1)}{2}$; $n = 11$

5 $x = 2$

6 **a** 20 units, 21 units, 29 units **b** 17 units

7 **a** $k < 0$ **b** $k = 0$ **c** $k > 0$

8 **a** 10, −10, 11, −11, 14, −14, 25, −25

b 2, −2, 5, −5, 10, −10, 23, −23

9 $x = -3, 1, 2, 6$

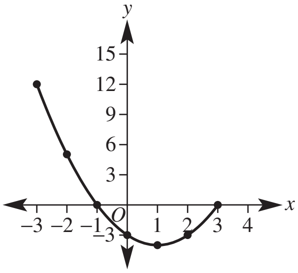
10 $x = -3, -1, 1, 3$

Short-answer questions

1 **a** $y = x^2 - 2x - 3$

x	−3	−2	−1	0	1	2	3
y	12	5	0	−3	−4	−3	0

b



2 **a** 0, −2 **b** 0, 4 **c** −3, 7

d 2, −2 **e** $-1, \frac{2}{5}$ **f** $\frac{1}{2}, \frac{4}{3}$

3 **a** 0, −3 **b** 0, 4 **c** −5, 5

d −9, 9 **e** −2, 2 **f** $-\sqrt{10}, \sqrt{10}$

g −3, −7 **h** −5, 8 **i** 4

4 **a** $x^2 - 5x = 0; x = 0, 5$

b $3x^2 - 18x = 0; x = 0, 6$

c $x^2 + 8x + 12 = 0; x = -6, -2$

d $x^2 - 2x - 15 = 0; x = -3, 5$

e $x^2 - 8x + 15 = 0; x = 3, 5$

f $x^2 + 3x - 4 = 0; x = -4, 1$

5 **a** $x^2 + 2x - 80 = 0; x = 8$ units

b $x^2 + 5x - 24 = 0; x = 3$ units

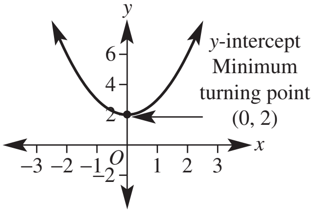
6 **a** **i** $x = 3$ **ii** minimum (3, −2)

b **i** $x = -1$ **ii** maximum (−1, 3)

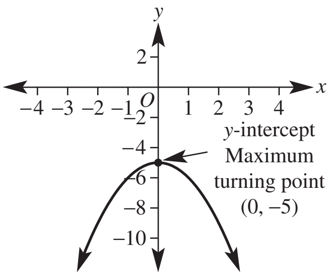
7 **a** $y = 2x^2$ **b** $y = -x^2 + 2$ **c** $y = (x + 1)^2$

d $y = (x - 3)^2$ **e** $y = x^2 - 4$

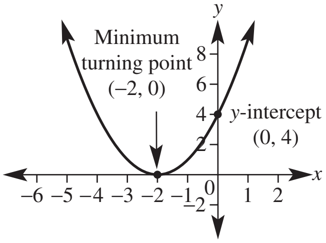
8 **a** $y = x^2 + 2$



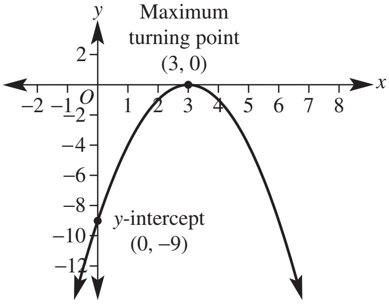
b $y = -x^2 - 5$



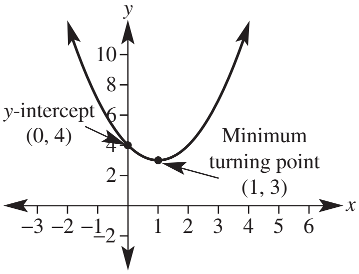
c $y = (x + 2)^2$



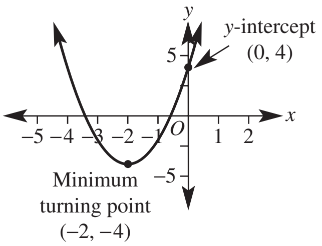
d $y = -(x - 3)^2$



e $y = (x - 1)^2 + 3$

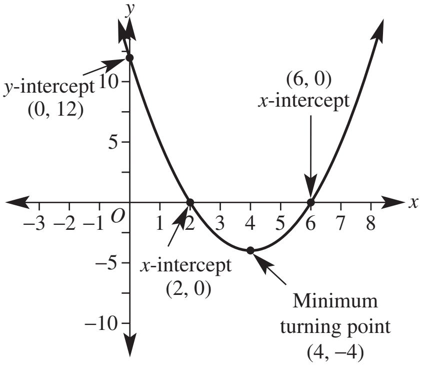


f $y = 2(x + 2)^2 - 4$

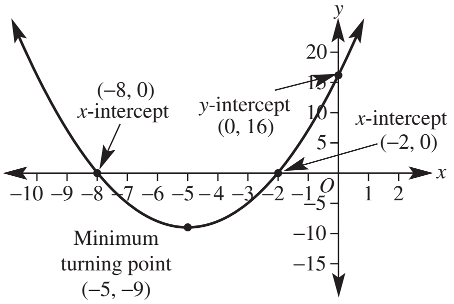


9 **a** $y = x^2$ translated up 2 units
  b $y = x^2$ reflected in the x-axis and translated down 5 units
  c $y = x^2$ translated 2 units left
  d $y = x^2$ reflected in the x-axis then translated 3 units right
  e $y = x^2$ translated 1 unit right then 3 units up
  f $y = x^2$ dilated by a factor of 2 from the x-axis, translated 2 units left and then down 4 units

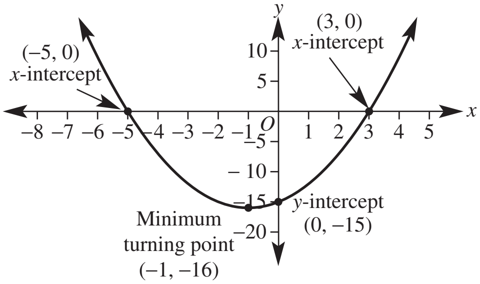
10 **a** $y = x^2 - 8x + 12$



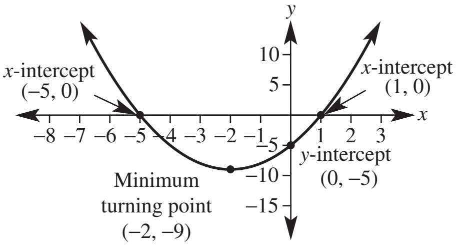
b $y = x^2 + 10x + 16$



c $y = x^2 + 2x - 15$



d $y = x^2 + 4x - 5$



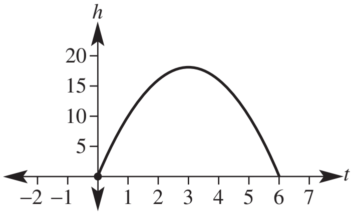
Multiple-choice questions

1 D **2** B **3** C **4** A **5** D
6 B **7** E **8** C **9** A **10** B

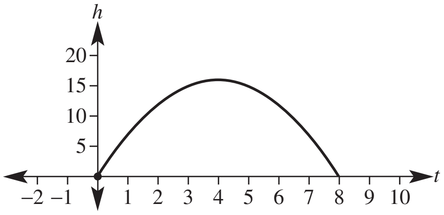
Extended-response questions

1 **a** $x + 5$ **b** $A = x^2 + 5x$
  c $x = 2$ ($x = -7$ not valid as $x > 0$)
  d base = 4 m, height = 7 m

2 **a** **i** (0, 0), (6, 0) **ii** maximum at (3, 18)
  iii



  b **i** (0, 0) **ii** (4, 16) **iii** (0, 0), (8, 0)
  iv



  c **i** Connor's rocket, 8 s
  ii Sam's rocket, 2 m higher
  iii 16 m, the same height

Semester review 2

Indices and surds

Short-answer questions

1 **a** $\frac{a^2b}{2}$ **b** $2x^3y^5$ **c** $8x^6 - 2$
2 **a** $5m^2$ **b** $\frac{2a^8}{3b^5}$ **c** $\frac{3x^2}{y}$
3 **a** 3.07×10^{-2} kg **b** 4.24×10^6 kg
  c 1.22×10^4 seconds **d** 2.35×10^{-7} seconds
4 **a** 12 **b** 3 **c** $2\sqrt{3} + 7\sqrt{5}$ **d** $\sqrt{7}$

Multiple-choice questions

1 C **2** B **3** D **4** E **5** C

Extended-response question

a **i** 74 000 000 000 **ii** 7.4×10^{10}
b 1.87×10^{17} **c** 8.72×10^{-7}

Geometry

Short-answer questions

1 a $a = 100, b = 140$ b $a = 70, b = 55$
c $x = 67, y = 98$ d $x = 35$

2 85°

3 $CB = CD$ (given equal sides)
$\angle ACB = \angle ACD$ (given equal angles)
AC is common
$\therefore \Delta ABC \equiv \Delta ADC$ (SAS)

4 a AAA b 2.4

Multiple-choice questions

1 C 2 B 3 A 4 E 5 B

Extended-response question

a $\angle ABD = \angle ECD$ (given right angles)
$\angle ADB = \angle EDC$ (common angle)
$\angle DAB = \angle DEC$ (corresponding angles in parallel lines are equal)
$\therefore \Delta ABD \equiv \Delta ECD$ (AAA)

b 7.5 m c 3.75 m d 4.3 m

Algebraic techniques

Short-answer questions

1 a $x^2 - 9$ b $x^2 + 4x + 4$ c $6x^2 - 17x + 12$

2 a $2ab(4 + a)$ b $(3m - 5)(3m + 5)$
c $3(b - 4)(b + 4)$ d $(a + 4)(a + 10)$
e $(x + 3)^2$ f $(x - 2)(x + 10)$
g $2(x - 3)(x - 5)$ h $(2x - 3)(x - 4)$
i $(2x - 1)(3x + 4)$

3 a $(x - 3)(x - 1)$ b $(x + 2)(2x - 5)$

4 a i $\frac{3}{2}$ ii $\frac{4}{x + 3}$
iii $\frac{13x + 21}{14}$ iv $\frac{1}{3x}$
v $\frac{11x - 10}{(x + 1)(x - 2)}$ vi $\frac{2x + 25}{(x - 5)(x + 2)}$
b i $x = \frac{17}{12}$ ii $x = -11$

Multiple-choice questions

1 E 2 D 3 B 4 A 5 D

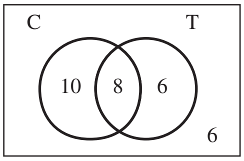
Extended-response question

a $10 - 2x$ and $8 - 2x$
b $(10 - 2x)(8 - 2x) = 80 - 36x + 4x^2$
c 48 m^2
d $4(x - 4)(x - 5)$
e Area of rug is 0 as it has no width.

Probability and statistics

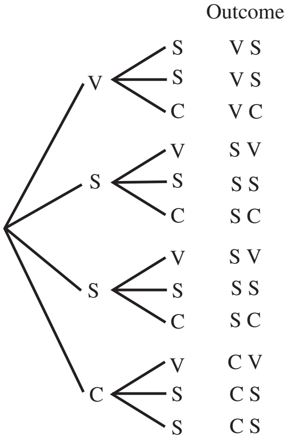
Short-answer questions

1 a



b i 24 ii 16
c i $\frac{1}{5}$ ii $\frac{1}{3}$

2 a



b i $\frac{1}{3}$ ii $\frac{1}{6}$ iii $\frac{1}{2}$

3 a

Stem	Leaf
1	0 1 1 2 3 5 7 8
2	2 5 5 5 6
3	1 2 2

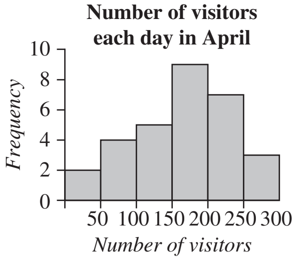
1 | 3 means 13 aces

b mode = 25, median = 20 c skewed

4 a

Class interval	Frequency	Percentage frequency
0–	2	6.7
50–	4	13.3
100–	5	16.7
150–	9	30
200–	7	23.3
250–	3	10
Total	30	100

b



c i 6 ii 60%

ISBN 978-1-108-77288-4

Multiple-choice questions

1 D **2** A **3** C **4** B **5** C

Extended-response question

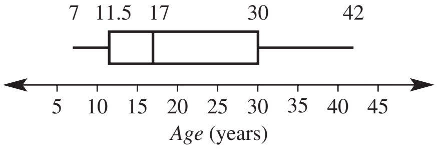
a

		Red		
		1	**2**	**3**
Green	**1**	(1, 1)	(1, 2)	(1, 3)
	2	(2, 1)	(2, 2)	(2, 3)
	3	(3, 1)	(3, 2)	(3, 3)

b $\frac{4}{9}$

c **i** $\frac{1}{3}$ **ii** 18

d **i**

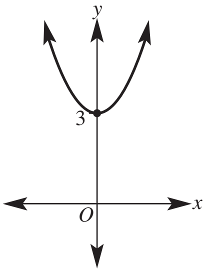


ii 11.5 years **iii** 90

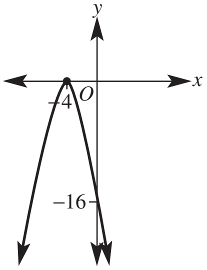
Introduction to quadratic equations and graphs

Short-answer questions

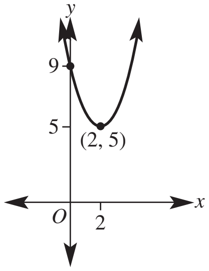
1 **a** $x = -5$ or $x = 3$ **b** $x = \frac{1}{2}$ or $x - \frac{5}{3}$
c $x = 0$ or $x = -2$ **d** $x = -3$ or $x = 3$
e $x = 2$ or $x = 7$ **f** $x = -4$
2 15 m by 8 m
3 **a** translated 3 units up



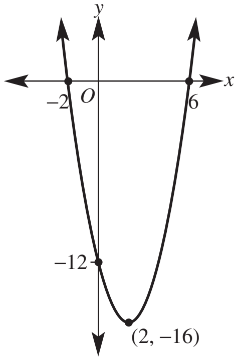
b reflected in the x-axis and translated 4 units left



c translated 2 units right and 5 units up



4 **a** $y = -12$ **b** $x = -2, 6$ **c** (2, −16)
d



Multiple-choice questions

1 B **2** B **3** E **4** C **5** E

Extended-response question

a Lands after 6 seconds.
b **i** 180 cm **ii** 3 seconds
c after 2 seconds and 4 seconds
d